SEEM 5840 Term Project

Topic 1 — China Government Bond

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1 Introduction

There is an asset portfolio including 6 Chinese government bonds (CGBs) [1], all of which are fixed-rate, interest-paying bonds (coupon bonds), and simple interest with ACT/ACT as the benchmark. Among them, 3 bonds are short (-50, -20, -30 respectively), another 3 are long. 4 bonds pay the coupon every year, and 2 semi-annually. The start and end time, as well as other details are as follows:

A	B	C	D	E	F	G	H	I	J	K	L
证券代码	证券简称	起息日期	债券期限(年) [单位] 年	利率类型	票面利率(发行时) [单位] %↓	计息方式	计息基准	息票品种	到期日期	每年付息次数	本金金额（百万）
180001.IB	18附息国债01	2018-01-18	5.0000	固定利率	3.8100	单利	ACT/ACT	附息	2023-01-18	1	-50
180005.IB	18附息国债05	2018-03-08	7.0000	固定利率	3.7700	单利	ACT/ACT	附息	2025-03-08	1	-20
180019.IB	18附息国债19	2018-08-16	10.0000	固定利率	3.5400	单利	ACT/ACT	附息	2028-08-16	2	5
190003.IB	19附息国债03	2019-03-07	3.0000	固定利率	2.6900	单利	ACT/ACT	附息	2022-03-07	1	10
160023.IB	16附息国债23	2016-11-03	10.0000	固定利率	2.7000	单利	ACT/ACT	附息	2026-11-03	2	5
190002.IB	19附息国债02	2019-02-21	2.0000	固定利率	2.4400	单利	ACT/ACT	附息	2021-02-21	1	-30

Figure 1: Details of 6 CGBs

To manage this portfolio, we try to calculate the portfolio value on each business day based on the daily data released by the Chinese government in 2020. Then a method is designed to calculate the 10-day 99% value at risk (VaR). Finally, back-testing for 2020 is conducted to test whether the method designed to calculate the VaR is reasonable or not, and exceptions are analyzed if any.

2 Calculation of the portfolio value for the year of 2020

According to official data released by the Chinese government, there are 249 working days in 2020, and 17 yields (interest rates) at differing maturity dates every day. A total of 17*249=4233 pieces of data.

For each working day, calculate the value of every bond on that day, then the cumulative sum of 6 bonds is the portfolio value. For a bond, there are several steps to calculate its present value:

a) calculate how long from now to end-date / maturity date, 1 year = 360 days (convert to year unit).
b) calculate how many cash flows remaining,
c) find the interest rates of each cash flow, annualize those interest rates.
d) use formular to calculate the present value of the bond.

Here we refer to the basic principle of China bond valuation[2]:

The valuation of China bonds is based on the yield curve of China bonds and

other market information, and the discount model of cash flow is mainly adopted. Below is the formula:

$$PV = \frac{C/f}{(1+y/f)^{w}} + \frac{C/f}{(1+y/f)^{w+1}} + \cdots + \frac{C/f}{(1+y/f)^{w+n-1}} + \frac{M}{(1+y/f)^{w+n-1}}$$

PV -- the price of the bond;
Y -- valuation yield;(For each individual bond, it is the yield to maturity of the corresponding year.)
C -- coupon rate;
F -- the frequency of annual interest payments;
N -- the number of remaining interest payment cycles. N-1 is the number of remaining interest payment cycles;
W -- D/ the actual number of days in the current interest payment cycle;
M -- face value;
D -- the number of days between the valuation date and the latest coupon date.

In our project, according to the yield curve on every working day, coupons and the principle that have different maturity dates correspond to different interest rates on that day. Therefore, the formular should be adjusted. And the whole procedure is as follows:

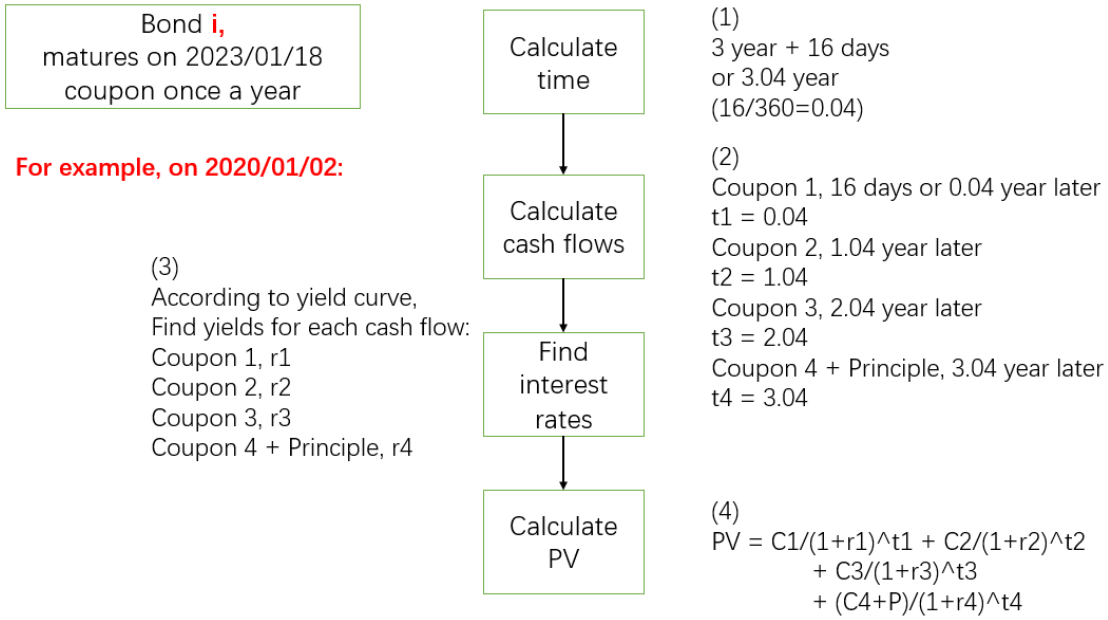


Figure 2: Procedure of how to calculate present value

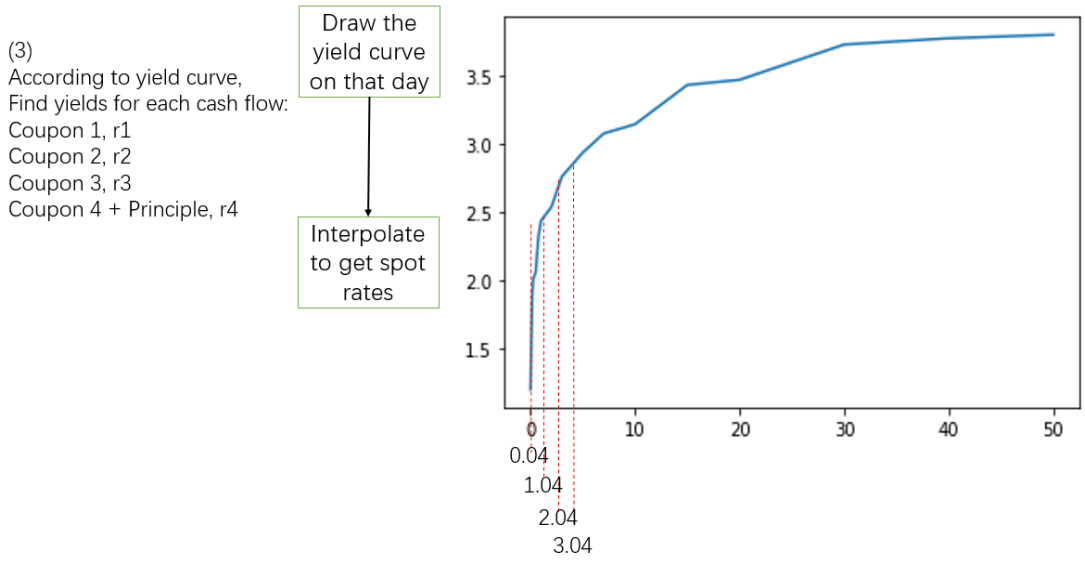
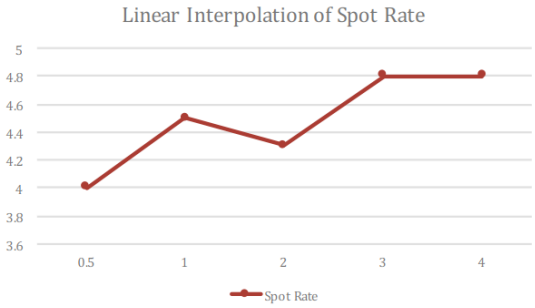


Figure 3:Details about how to find yields in (3)

For the (3), interpolate to find the interest rates, simple linear interpolation is used. And the basic formular is:

Assume that spot rate y_{t_1} *and* y_{t_0} *for time* t_1 *and* t_0*, then for time t* $(t_0 \le t \le t_1)$

$$y_t = y_{t_0} + \frac{y_{t_1} - y_{t_0}}{t_1 - t_0}(t - t_0)$$



Tenor	Spot Rate
0.5	4
1	4.5
2	4.3
3	4.8
4	4.8

Figure 4: Simple linear interpolation) [3]

After those procedures, the value of one bond for certain date can be calculated, apply the same method to all other bonds to get the cumulative sum, namely, the portfolio value on that day. Then, a loop can be utilized to iterate all working days (249 days). Here is the screenshot of the top 6 days:

1		date	180001.IB	180005.IB	180019.IB	190003.IB	160023.IB	190002.IB	value
2	0	2020/1/2	-53.3694	-21.4309	5.247481	10.24588	4.925688	-30.6318	-85.0131
3	1	2020/1/3	-53.3493	-21.413	5.246432	10.24448	4.924152	-30.6335	-84.9807
4	2	2020/1/6	-53.4246	-21.4471	5.251547	10.25451	4.927281	-30.6493	-85.0877
5	3	2020/1/7	-53.4455	-21.4374	5.250578	10.25709	4.928009	-30.6454	-85.0926
6	4	2020/1/8	-53.5096	-21.4541	5.254465	10.25993	4.932484	-30.6817	-85.1985
7	5	2020/1/9	-53.5108	-21.4661	5.260592	10.25814	4.935211	-30.6801	-85.2031

...

Figure 5: Portfolio value of top 5 business days, 1/2-1/9

3 Calculation of the 10-day 99% value at risk (VaR)

Here we calculate the 1-day 99% VaR firstly, based on the historical data, namely, the data in 2019. Then use the time horizon formula to derive the 10-day 99% VaR: [3]

$$T\text{-day VaR} = 1\text{-day VaR} \times \sqrt{T}$$

Figure 6: Time horizon, T = 10

When calculating the 1-day 99% VaR, cumulative loss distribution is applied:

1) Use the same method to calculate the portfolio value of each day in 2019 from 4/1. because there 2 bonds issued in February and March. Here is the screenshot of the top 5 days:

2019/4/1	-52.0945	-20.7986	5.199024	10.00632	4.912949	-30.0158	-82.7906
2019/4/2	-52.0833	-20.7723	5.186434	10.00354	4.900773	-30.0076	-82.7725
2019/4/3	-51.9384	-20.715	5.160798	9.976506	4.88215	-30.0143	-82.6482
2019/4/4	-51.8702	-20.6809	5.14924	9.971794	4.87732	-30.0025	-82.5552
2019/4/8	-51.8712	-20.673	5.156378	9.974423	4.881391	-29.9673	-82.4993
2019/4/9	-51.8105	-20.6293	5.142958	9.965213	4.871324	-29.9467	-82.407

Figure 7:Portfolio value of year 2019, 4/1-4/9

2) Calculate the loss, which is the daily changes of the portfolio value, and sort the results as shown in the figure below:

scenario	Vi - V(i-1)
157	-0.12866761
82	-0.12773168
209	-0.12264952
83	-0.11403402
123	-0.10634101
72	-0.10037683
170	-0.09691173
100	-0.0849432
101	-0.08374457
96	-0.07919275
246	-0.07643265
146	-0.07206792
147	-0.06935943
111	-0.06905467
149	-0.06841351
91	-0.06251343

...

Figure 8: The sorted changes of the portfolio

3) Take the top 1% as the 1-day 99% VaR, and then use the above time horizon to get the 10-day 99% VaR.

the 99% 1-day VaR	-0.12773168
the 99% 10-day VaR	-0.40392304

4 Back-testing by using VaR for the year of 2020:

Calculate every 10-day portfolio value change of year 2020 and sort the results.

10 day portfolio value change	sorted
-0.17485385	-0.95018993
-0.2218842	-0.82784985
-0.14359135	-0.82585948
-0.17702701	-0.82287662
1.73185206	-0.79722144
1.70394826	-0.77832272
1.70380309	-0.76455359
1.67068066	-0.76194031
1.25882771	-0.73997491
1.26813463	-0.70006905

Figure 9: The sorted every 10-day portfolio value change

Since the 99% 10-day VaR is -0.4024, we can find that there are 30 exceptions in the 239 pieces of data, 30/239=0.1255 = 12.55%, which is obviously exceeds the expected 99% 10-day VaR.

After analysis, the followings are the reasons behind:

1) When calculating the 99% 1-day VaR, it is based on cumulative loss distribution and the top 1% is taken as the VaR, the cutoff is obvious, and the approximation may not be accurate enough in comparison with model-building.
2) When calculating the 99% 10-day VaR, we directly multiplied the single-day VaR by the square root of 10 according to the time horizon formula, which "are exactly true when the changes in the value of the portfolio on successive days have independent identical normal distributions with mean zero", but "In other cases, they are approximations". [3]
3) The variance of the daily change in portfolio value in 2020 is much greater than that in 2019, about 13 times, which means the VaR in 2019 is much smaller than that in 2020.

variance of the daily change in 2019	0.001787547
variance of the daily change in 2020	0.0234723
	0.0234723 / 0.001787547 = 13.13

5 Conclusion

Generally, the CGBs are very stable, and the risk is relatively low when compared with other financial instruments, but risk management is necessary. On the one hand, you need to track the status of the portfolio, on the other hand, even for the same bonds, they may have different risks in different years/time.
From this project, several lectures are learned,

1) When estimating the VaR, historical data can be an approximation, but may not be accurate.

2) Changes on successive days are not independent identical normal distribution, therefore, when deriving 10-day VaR just by multiplying 1-day VaR with $\sqrt{T}$, it causes errors.

3) Different years may have different variances/volatilities, more factors should be introduced to help estimate the VaR.

Reference

[1] chinabond.com.cn

[2] 中央国债登记结算有限责任公司企业标准

[3] Risk Management and Financial Institutions