alme9138project

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1 Understanding Diamond Prices

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1.1 Problem

This will be a regression project seeking to project the price of a diamond based on its other attributes. This is an interesting topic to me because the diamond market is often discussed as one based on artificial scarcity. I am interested to see how closely overall price correlates to each component measurement. Which attributes contribute most and least to the overall price? Is it a linear relationship or does a dollar go further at one end of the scale or the other? This could be useful information for those looking to dazzle on a budget or who are jewel thieves that wish to better target and select their ill-gotten gains.

I will initially take both training and test data from a dataset comprised of 2017 diamond prices. If I can find a price list from another year, I would like to use the candidate models developed in my project to predict current diamond prices. If successful, there should be a predictable difference corresponding to changes in the market and inflation during the intervening time interval.

1.2 Dataset

I will be exploring a dataset comprised of diamond prices. This was obtained from Kaggle and is based on the 2017 Tiffany's pricelist. The dataset is in tabulated form and consists of 53,940 instances each with 10 attributes:

1. **price** price in US dollars (326 to 18,823) - Continuous
2. **carat** weight of the diamond (0.2 to5.01) - Continuous
3. **cut** quality of the cut (Fair, Good, Very Good, Premium, Ideal) - Ordinal Categorical
4. **color** diamond colour, from J (worst) to D (best) - Ordinal Categorical
5. **clarity** a measurement of how clear the diamond is (I1 (worst), SI2, SI1, VS2, VS1, VVS2, VVS1, IF (best)) - Ordinal Categorical
6. **x** length in mm (0 to 10.74) - Continuous
7. **y** width in mm (0 to 58.9) - Continuous
8. **z** depth in mm (0 to 31.8) - Continuous
9. **depth** total depth percentage = z / mean(x, y) = 2 * z / (x + y) (43–79) - Continuous

10. **table** width of top of diamond relative to widest point (43 to 95) - Continuous

Little cleaning was required of the dataset. There appear to be no missing values. I did need to drop the index column and convert the ordinal categorical data to the appropriate type in pandas (cut, color, and clarity). I performed a basic EDA by plotting a histogram (continuous values) or value count bar plot (categorical values) to get a sense of the shape of the data and to note any outliers. Based on a visual analysis, the dataset skews towards the lighter, less expensive diamonds, and higher quality cut diamonds. No obvious outliers were observed.

[3]:
```
import copy
import numpy as np
import pandas as pd
%matplotlib inline
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.formula.api as smf
import statsmodels.api as sm
import itertools
import patsy
from sklearn import datasets
from sklearn.model_selection import cross_val_predict, cross_val_score,␣
 ↪train_test_split
from sklearn import linear_model,metrics
# Set color map to have light blue background
sns.set()
```

[4]:
```
df = pd.read_csv('diamonds.csv')
df = df.drop('Unnamed: 0', 1)
df.head(10)
```

```
C:\Users\melni\AppData\Local\Temp\ipykernel_20752\636088786.py:2: FutureWarning:
In a future version of pandas all arguments of DataFrame.drop except for the
argument 'labels' will be keyword-only.
  df = df.drop('Unnamed: 0', 1)
```

[4]:

	carat	cut	color	clarity	depth	table	price	x	y	z
0	0.23	Ideal	E	SI2	61.5	55.0	326	3.95	3.98	2.43
1	0.21	Premium	E	SI1	59.8	61.0	326	3.89	3.84	2.31
2	0.23	Good	E	VS1	56.9	65.0	327	4.05	4.07	2.31
3	0.29	Premium	I	VS2	62.4	58.0	334	4.20	4.23	2.63
4	0.31	Good	J	SI2	63.3	58.0	335	4.34	4.35	2.75
5	0.24	Very Good	J	VVS2	62.8	57.0	336	3.94	3.96	2.48
6	0.24	Very Good	I	VVS1	62.3	57.0	336	3.95	3.98	2.47
7	0.26	Very Good	H	SI1	61.9	55.0	337	4.07	4.11	2.53
8	0.22	Fair	E	VS2	65.1	61.0	337	3.87	3.78	2.49
9	0.23	Very Good	H	VS1	59.4	61.0	338	4.00	4.05	2.39

[5]:
```
df.describe()
```

[5]:
```
              carat         depth         table         price             x  \
count  53940.000000  53940.000000  53940.000000  53940.000000  53940.000000
mean       0.797940     61.749405     57.457184   3932.799722      5.731157
std        0.474011      1.432621      2.234491   3989.439738      1.121761
min        0.200000     43.000000     43.000000    326.000000      0.000000
25%        0.400000     61.000000     56.000000    950.000000      4.710000
50%        0.700000     61.800000     57.000000   2401.000000      5.700000
75%        1.040000     62.500000     59.000000   5324.250000      6.540000
max        5.010000     79.000000     95.000000  18823.000000     10.740000

                  y             z
count  53940.000000  53940.000000
mean       5.734526      3.538734
std        1.142135      0.705699
min        0.000000      0.000000
25%        4.720000      2.910000
50%        5.710000      3.530000
75%        6.540000      4.040000
max       58.900000     31.800000
```

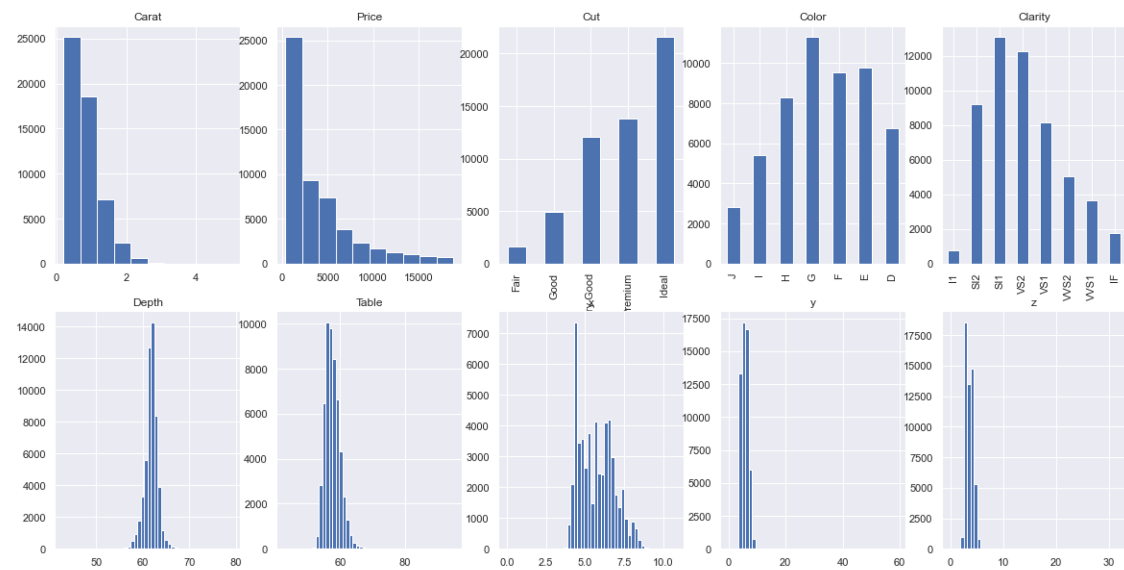
[6]:
```
#Test for missing values
df.isnull().values.any()
```

[6]: False

[7]:
```
#Change type of caterical data to category and order
df.cut.astype(pd.api.types.CategoricalDtype(categories = ["Fair", "Good", "Very␣
↪Good", "Premium", "Ideal"], ordered = True))
df.color.astype(pd.api.types.CategoricalDtype(categories = ["J", "I", "H", "G",␣
↪"F", "E", "D"], ordered = True))
df.clarity.astype(pd.api.types.CategoricalDtype(categories = ["I1", "SI2",␣
↪"SI1", "VS2", "VS1", "VVS2", "VVS1", "IF"], ordered = True));
```

[8]:
```
#Plot histograms or value counts plots for each attribute
fig, axs = plt.subplots(2, 5, figsize=(20,10))
axs[0,0].set_title('Carat')
axs[0,0].hist(df.carat);
axs[0,1].set_title('Price')
axs[0,1].hist(df.price);
axs[0,2].set_title('Cut')
df['cut'].value_counts()[['Fair', 'Good', 'Very Good', 'Premium', 'Ideal']].
↪plot(kind='bar', ax=axs[0,2]);
axs[0,3].set_title('Color')
df['color'].value_counts()[["J", "I", "H", "G", "F", "E", "D"]].
↪plot(kind='bar', ax=axs[0,3]);
axs[0,4].set_title('Clarity')
```

```
df['clarity'].value_counts()[["I1", "SI2", "SI1", "VS2", "VS1", "VVS2", "VVS1",␣
 ↪"IF"]].plot(kind='bar', ax=axs[0,4]);
axs[1,0].set_title('Depth')
axs[1,0].hist(df.depth, bins=50);
axs[1,1].set_title('Table')
axs[1,1].hist(df.table, bins=50);
axs[1,2].set_title('x')
axs[1,2].hist(df.x, bins=50);
axs[1,3].set_title('y')
axs[1,3].hist(df.y, bins=50);
axs[1,4].set_title('z')
axs[1,4].hist(df.z, bins=50);
```



1.3 Model Development

I initially performed an ordinary least squares regression using all predictors, assuming no interactions to identify predictors that did not contribute to price. This identified both y and z based on the $p > 0.05$ and a confidence interval that includes 0. Overall, this also provides a basic expectation for candidate models, with an R^2 value of 0.92. Initially I used the entire data set. Later, when I formally select a model, I will split the data into training and test sets.

[9]:
```
#Perform OLS regression using all predictors
model_tr = smf.ols(formula='price~carat+cut+color+clarity+depth+table+x+y+z',␣
 ↪data=df).fit()
model_tr.summary()
```

[9]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.920
Model:                            OLS   Adj. R-squared:                  0.920
Method:                 Least Squares   F-statistic:                 2.688e+04
Date:                Mon, 02 May 2022   Prob (F-statistic):               0.00
Time:                        17:40:32   Log-Likelihood:            -4.5573e+05
No. Observations:               53940   AIC:                         9.115e+05
Df Residuals:                   53916   BIC:                         9.117e+05
Df Model:                          23
Covariance Type:            nonrobust
================================================================================
====
                       coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
----
Intercept         2184.4774    408.197      5.352      0.000    1384.409
2984.546
cut[T.Good]        579.7514     33.592     17.259      0.000     513.911
645.592
cut[T.Ideal]       832.9118     33.407     24.932      0.000     767.433
898.391
cut[T.Premium]     762.1440     32.228     23.649      0.000     698.978
825.310
cut[T.Very Good]   726.7826     32.241     22.542      0.000     663.591
789.975
color[T.E]        -209.1181     17.893    -11.687      0.000    -244.189
-174.047
color[T.F]        -272.8538     18.093    -15.081      0.000    -308.316
-237.392
color[T.G]        -482.0389     17.716    -27.209      0.000    -516.763
-447.315
color[T.H]        -980.2667     18.836    -52.043      0.000   -1017.185
-943.348
color[T.I]       -1466.2445     21.162    -69.286      0.000   -1507.723
-1424.766
color[T.J]       -2369.3981     26.131    -90.674      0.000   -2420.615
-2318.181
clarity[T.IF]     5345.1022     51.024    104.757      0.000    5245.095
5445.110
clarity[T.SI1]    3665.4721     43.634     84.005      0.000    3579.949
3750.995
clarity[T.SI2]    2702.5863     43.818     61.677      0.000    2616.702
2788.471
clarity[T.VS1]    4578.3979     44.546    102.779      0.000    4491.087
4665.708
```

```
clarity[T.VS2]     4267.2236     43.853     97.306      0.000     4181.270
4353.177
clarity[T.VVS1]    5007.7590     47.160    106.187      0.000     4915.326
5100.192
clarity[T.VVS2]    4950.8141     45.855    107.967      0.000     4860.938
5040.690
carat              1.126e+04     48.628    231.494      0.000     1.12e+04
1.14e+04
depth               -63.8061      4.535    -14.071      0.000      -72.694
-54.918
table               -26.4741      2.912     -9.092      0.000      -32.181
-20.767
x                 -1008.2611     32.898    -30.648      0.000    -1072.741
-943.781
y                     9.6089     19.333      0.497      0.619      -28.284
47.502
z                   -50.1189     33.486     -1.497      0.134     -115.752
15.515
==============================================================================
Omnibus:                    14433.356   Durbin-Watson:                   1.183
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           565680.446
Skew:                           0.577   Prob(JB):                         0.00
Kurtosis:                      18.823   Cond. No.                     7.14e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 7.14e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
"""
```

1.3.1 Correlation Matrix

Next I used a correlation matrix to identify predictors that are strongly correlated, indicating that they can be combined. Unsurprisingly, carat was strongly correlated with x, y, and z values. I will plan to use carat as an overall measure for the effect of these "size" predictors and drop the x, y, and z predictors from the model.

[10]:
```
df.corr()
```

[10]:

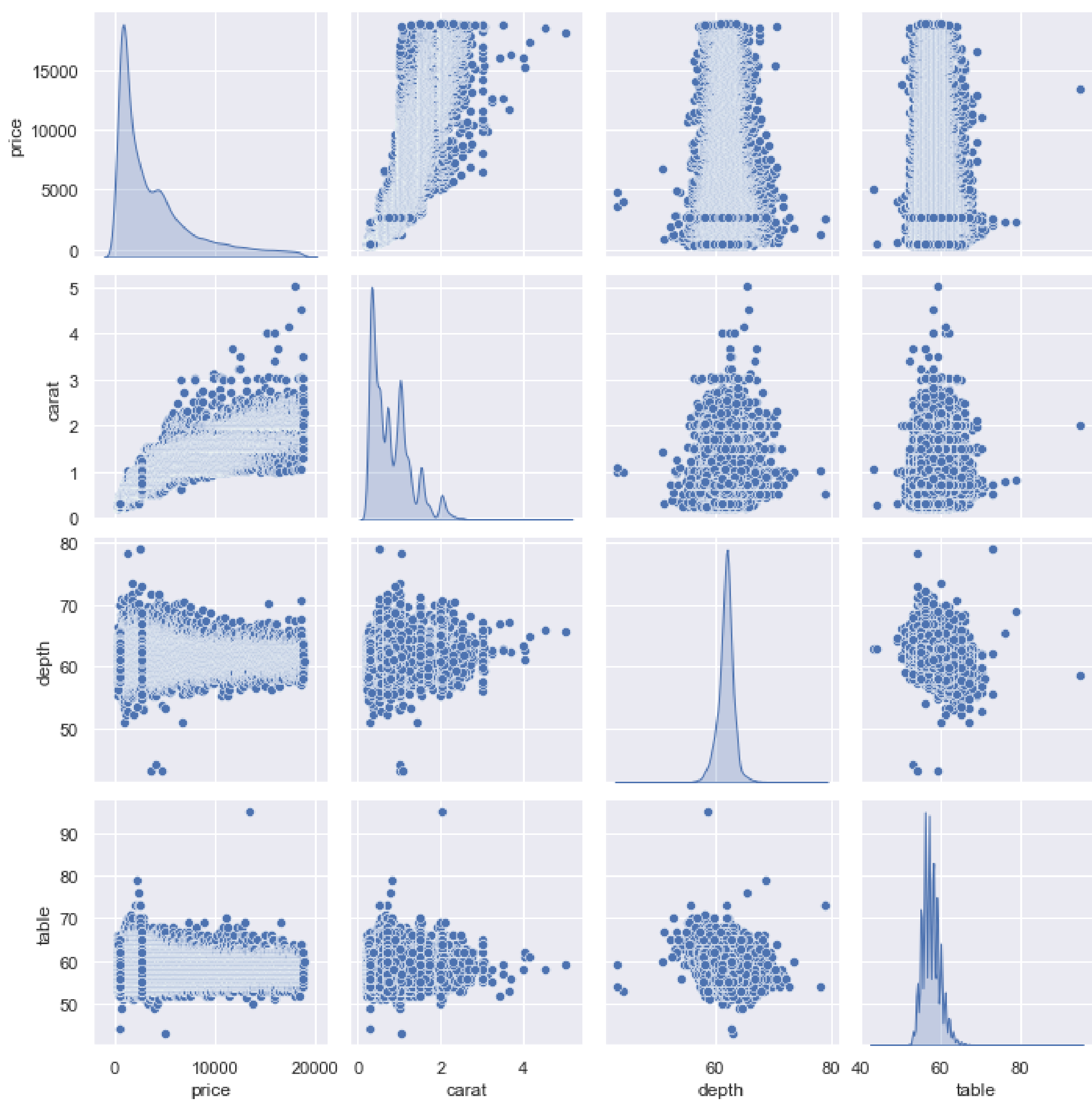
	carat	depth	table	price	x	y	z
carat	1.000000	0.028224	0.181618	0.921591	0.975094	0.951722	0.953387
depth	0.028224	1.000000	-0.295779	-0.010647	-0.025289	-0.029341	0.094924
table	0.181618	-0.295779	1.000000	0.127134	0.195344	0.183760	0.150929
price	0.921591	-0.010647	0.127134	1.000000	0.884435	0.865421	0.861249
x	0.975094	-0.025289	0.195344	0.884435	1.000000	0.974701	0.970772

```
y       0.951722 -0.029341  0.183760  0.865421  0.974701  1.000000  0.952006
z       0.953387  0.094924  0.150929  0.861249  0.970772  0.952006  1.000000
```

1.3.2 Scatterplot Matrix

I plotted the remaining quantitative variables in a scatterplot matrix to get a general sense of any correlation. Price and carat appear to have a positive correlation that is linear (though potentially polynomial). Depth and table do not appear to have a strong correlation with price. I expect the best model will make use of the categorical variables.

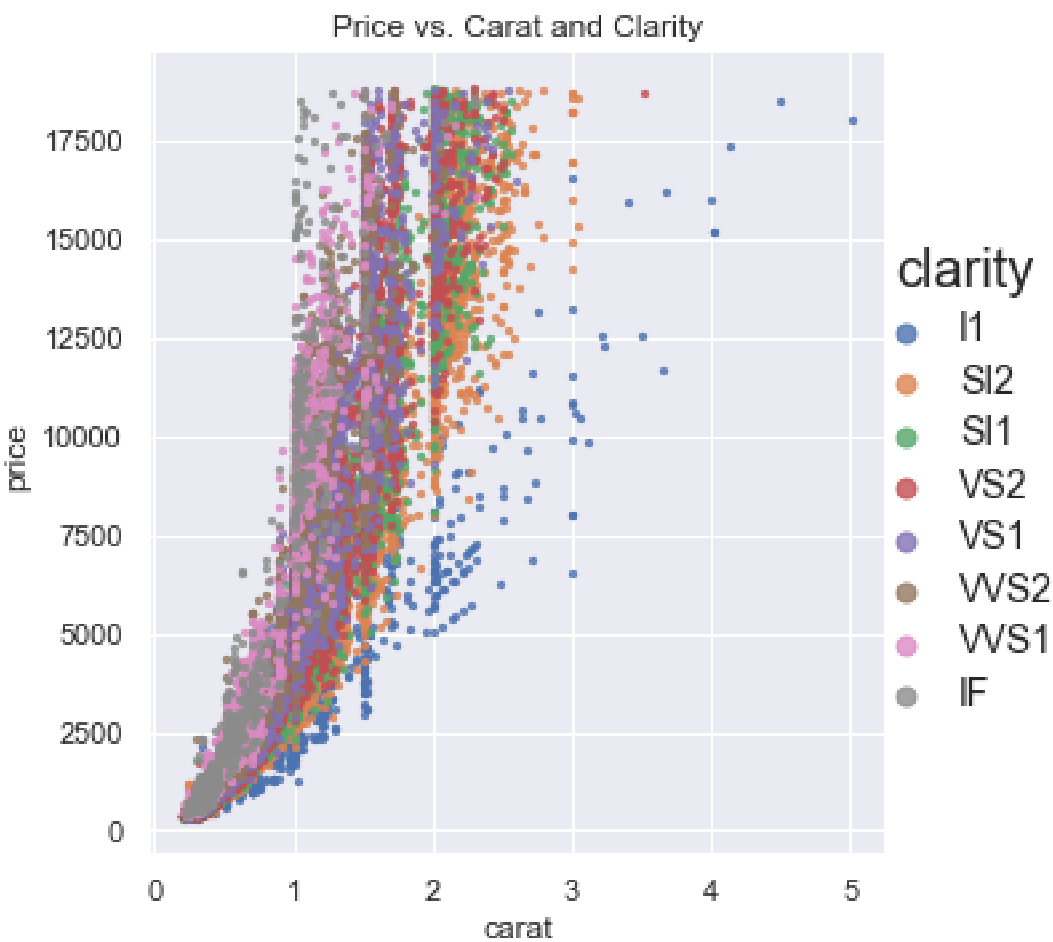
[11]:
```
sns.pairplot(df[['price', 'carat', 'depth', 'table']], diag_kind='kde');
```



1.3.3 Categorical Scatterplots

Using the base of a scatterplot comparing carat to price, I plotted each categorical value using hue. If a certain category routinely lies along a relatively higher or lower slope, this indicates that category has importance as a predictor. Higher quality ratings are at the bottom of the legend, lower quality at the top. From this, I can see that clarity is important to the price in a fairly organized manner. Higher quality color also leads to a higher price, but there is more mixing. Cut appears to be less conclusive.

[12]:
```
#plot carat vs. price with clarity hued
clarplot = sns.lmplot( x="carat", y="price", data=df, fit_reg=False, hue_order␣
↪= ["I1", "SI2", "SI1", "VS2", "VS1", "VVS2", "VVS1", "IF"], hue='clarity',␣
↪legend=True, scatter_kws={"s": 5}).set(title='Price vs. Carat and Clarity');
plt.setp(clarplot._legend.get_title(), fontsize=20);
plt.setp(clarplot._legend.get_texts(), fontsize=14);
for lh in clarplot._legend.legendHandles:
    lh._sizes = [50]
```



[13]:
```
#plot carat vs. price with cut hued
cutplot = sns.lmplot( x="carat", y="price", data=df, fit_reg=False, hue_order =␣
↪["Fair", "Good", "Very Good", "Premium", "Ideal"], hue='cut', legend=True,␣
↪scatter_kws={"s": 5}).set(title='Price vs. Carat and Cut');
plt.setp(cutplot._legend.get_title(), fontsize=20);
plt.setp(cutplot._legend.get_texts(), fontsize=14);
for lh in cutplot._legend.legendHandles:
    lh._sizes = [50]
```

[14]:
```
#plot carat vs. price with color hued
colplot = sns.lmplot( x="carat", y="price", data=df, fit_reg=False, hue_order =␣
↪["J", "I", "H", "G", "F", "E", "D"], hue='color', legend=True,␣
↪scatter_kws={"s": 5}).set(title='Price vs. Carat and Color');
plt.setp(colplot._legend.get_title(), fontsize=20);
plt.setp(colplot._legend.get_texts(), fontsize=14);
for lh in colplot._legend.legendHandles:
    lh._sizes = [50]
```

1.3.4 Model Selection

Based on my findings from the exploratory data analysis, I believe I will use a subset selection method to determine a good polynomial, linear model. Subset selection will be a good method because I have already decided to drop 3 of the predictors, leaving me with a p of only 6. This is a low enough p that a subset selection will remain efficient and complete. I will test the quality of the top candidate models with cross validation. With a relatively large data set, any overfitting should be evident.

[15]:
```
param_list = ['carat', 'table', 'depth', 'cut', 'color', 'clarity']
r2_list_ind = []
best_models_ind = []
for k in range(len(param_list)):
    bestr2 = 0
    best_params = ''
    model_combos = itertools.combinations(param_list, k+1)
    test_params = ['+'.join(i) for i in model_combos]
    for test_param in test_params:
```

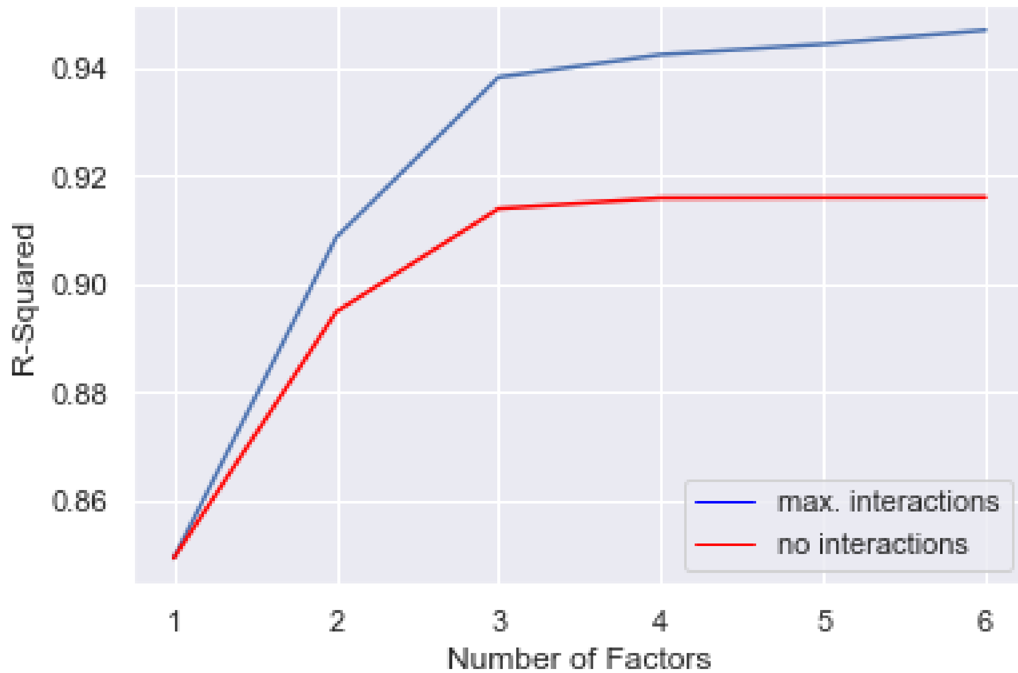
```
        curr_model = smf.ols(formula = 'price~' + test_param, data=df).fit()
        if curr_model.rsquared > bestr2:
            bestr2 = curr_model.rsquared
            best_params = test_param
    r2_list_ind.append(bestr2)
    best_models_ind.append(best_params)
```

I also suspect some predictors may be related, so I will also run the same subset selection with the * operator, replacing any models with an improved r^2

[16]:
```
best_models = copy.copy(best_models_ind)
r2_list = copy.copy(r2_list_ind)
for k in range(len(param_list)):
    model_combos = itertools.combinations(param_list, k+1)
    test_params = ['*'.join(i) for i in model_combos]
    for test_param in test_params:
        curr_model = smf.ols(formula = 'price~' + test_param, data=df).fit()
        if curr_model.rsquared > r2_list[k]:
            r2_list[k] = curr_model.rsquared
            best_models[k] = test_param
```

[31]:
```
print(best_models_ind)
print(r2_list_ind)
print(best_models)
print(r2_list)

xticks = np.arange(1,7, 1)

plt.plot(xticks, r2_list);
plt.plot(xticks, r2_list_ind, color ='red');

plt.xlabel('Number of Factors')
plt.ylabel('R-Squared');

custom_lines = [plt.Line2D([0], [0], color='blue', lw=1),
                plt.Line2D([0], [0], color='red', lw=1)]
plt.legend(custom_lines,['max. interactions', 'no interactions']);
```

```
['carat', 'carat+clarity', 'carat+color+clarity', 'carat+cut+color+clarity',
'carat+table+cut+color+clarity', 'carat+table+depth+cut+color+clarity']
[0.8493305264354858, 0.8948552484076189, 0.9139612406281767, 0.915940554017946,
0.9160129678812533, 0.916054322423958]
['carat', 'carat*clarity', 'carat*color*clarity', 'carat*cut*color*clarity',
'carat*table*cut*color*clarity', 'carat*table*depth*cut*color*clarity']
[0.8493305264354858, 0.908660479361379, 0.9382805875502322, 0.9424372697580661,
0.944376205819862, 0.9469584256217356]
```



In fact, it appears that including interactions improved the performance of each on the entire data set, with all parameters included having the maximum R^2. However, these last few models will be immensely complex and uninterpretable for potentially little benefit in practice. I will conduct a cross validation test to compare the independent models to the dependent models. I will use a 50-50 split for train and test sets for the formal investigation, as well as a 3-way split using the sklearn cross_validation_score function for quick tests.

[67]:
```
#Split dataset into test and train sets, each 50% of the dataset
X = df[param_list]
X = pd.get_dummies(data=X, drop_first=True)
y = df['price']
lr = linear_model.LinearRegression()
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.50)
```

[100]:
```
#function that will take an sklearn model and return a Dataframe listing the␣
↪features and coefficients
def summarize(model):
    coefs = np.array(model.coef_)
    coefs = np.insert(coefs,0,model.intercept_)
    features = np.array(model.feature_names_in_)
    features = np.insert(features,0,'Intercept')
    return pd.DataFrame({'Features': features, 'Coefficients' : coefs})

#function that will take a parameter string and the operator used and return a␣
↪list of parameters (including dummy parameters)
```

```python
def param_to_cols(parameterstring, operator):
    params = parameterstring.split(operator)
    return_list = []
    for param in params:
        if param == 'cut':
            return_list = return_list + ['cut_Good', 'cut_Ideal', 'cut_Premium', 'cut_Very Good']
        elif param == 'clarity':
            return_list = return_list + ['clarity_IF', 'clarity_SI1', 'clarity_SI2', 'clarity_VS1', 'clarity_VS2', 'clarity_VVS1', 'clarity_VVS2']
        elif param == 'color':
            return_list = return_list + ['color_E', 'color_F', 'color_G', 'color_H', 'color_I','color_J']
        else:
            return_list = return_list + [param]
    return return_list

#function that will interpret the list of best models and return a fitted model based on the number of features desired and the predictors/targets passed
def create_model(train_preds, train_target, feature_list, k_num):
    train_set = train_preds[param_to_cols(feature_list[k_num-1], '+')]
    return lr.fit(train_set, train_target)

#function that will remove excess predictors based on selected number of features and based on the best subset list passed
def trim_preds(test_preds, feature_list, k_num):
    test_set = test_preds[param_to_cols(feature_list[k_num-1], '+')]
    return test_set

#iterate through models in the best_models_ind list (those with no interactions) and store the performance of each:
#train_r2_list holds R^2 scores for the training set
#test_r2_list holds R^2 scores for the test set
#model_details is a dictionary that holds the parameter string as keys and summaries of the related values (DataFrames) as values
train_r2_list = ['N/A']
test_r2_list = ['N/A']
model_details = {}
for i in range(6):
    curr_model = create_model(X_train, y_train, best_models_ind, i+1)
    curr_train_x = trim_preds(X_train, best_models_ind, i+1)
    curr_test_x = trim_preds(X_test, best_models_ind, i+1)
    train_r2_list.append((best_models_ind[i], metrics.r2_score(y_train, curr_model.predict(curr_train_x))))
    test_r2_list.append((best_models_ind[i], metrics.r2_score(y_test, curr_model.predict(curr_test_x))))
```

```
    model_details[best_models_ind[i]] = summarize(curr_model)
```

Having tested and stored the results of all the non-interactive models, I want to do a quick cross-validation test of the interactive models since those will require more resources to test. Using sklearn cross_validation_score, I determined that "carat * clarity" and "carat * color * clarity" hold the most promise. I will also more fully test "carat * clarity + carat * cut" as I suspec the triple interaction will be practically impossible to interpret without much improving the predictve power of the model.

[21]:
```
#Cross validation to see which of these is worth investigating based on test R^2
for model in best_models:
    y, X = patsy.dmatrices("price ~" + model, data=df)
    print(model)
    print(cross_val_score(lr, X, y, cv=3))
```

```
carat
[-0.13319395  0.69010218 -0.6115836 ]
carat*clarity
[ 0.34115369  0.7765628  -0.3444758 ]
carat*color*clarity
[ 0.40300134  0.80661172 -0.26872628]
carat*cut*color*clarity
[-2.76111296e+19 -1.68302502e+18 -1.59736997e+23]
carat*table*cut*color*clarity
[-2.01168611e+18  5.22146190e-01 -9.30359379e-01]
carat*table*depth*cut*color*clarity
[-2.40409945e+16 -5.75875878e+12 -1.20110568e+18]
```

To test the models, I expand the training and test predictor sets by hand to include the interactions. I will be investigating the following models: |Model|Coded Shorthand| |-|-| |carat * clarity|carclar| |carat * clarity + carat * color|carclarpluscarcol| |carat * clarity * color|carclarcol| |carat * clarity + carat * color + cut + table + depth|carclarpluscarcolplus| |carat * color * clarity + cut + table + depth|carclarcolplus|

[101]:
```
#Based on cross validation, I will investigate (carat*clarity), (carat*clarity␣
↪+ carat*color), and (carat*color*clarity)
#Copy the base DataFrames and trim to match the desired features
X_train_carclar = copy.copy(X_train)
X_test_carclar = copy.copy(X_test)
X_train_carclarpluscarcol = copy.copy(X_train)
X_test_carclarpluscarcol = copy.copy(X_test)
X_train_carclarcol = copy.copy(X_train)
X_test_carclarcol = copy.copy(X_test)

X_train_carclar = X_train_carclar[param_to_cols('carat*clarity', '*')]
X_test_carclar = X_test_carclar[param_to_cols('carat*clarity', '*')]
X_train_carclarpluscarcol =␣
↪X_train_carclarpluscarcol[param_to_cols('carat*clarity*color', '*')]
```

```
X_test_carclarpluscarcol =␣
↪X_test_carclarpluscarcol[param_to_cols('carat*clarity*color', '*')]
X_train_carclarcol =␣
↪X_train_carclarpluscarcol[param_to_cols('carat*clarity*color', '*')]
X_test_carclarcol =␣
↪X_test_carclarpluscarcol[param_to_cols('carat*clarity*color', '*')]
```

[102]:
```
clar_list = ['clarity_IF', 'clarity_SI1', 'clarity_SI2', 'clarity_VS1',␣
↪'clarity_VS2', 'clarity_VVS1', 'clarity_VVS2']
color_list = ['color_E', 'color_F', 'color_G', 'color_H', 'color_I','color_J']

for clar in clar_list:
    #mash up clarity and color
    X_train_carclar['carat:'+clar] =␣
↪X_train_carclar['carat']*X_train_carclar[clar]
    X_test_carclar['carat:'+clar] = X_test_carclar['carat']*X_test_carclar[clar]
    #clarity and color are done, let's prepare the second set now
    X_train_carclarpluscarcol['carat:'+clar] =␣
↪X_train_carclarpluscarcol['carat']*X_train_carclarpluscarcol[clar]
    X_test_carclarpluscarcol['carat:'+clar] =␣
↪X_test_carclarpluscarcol['carat']*X_test_carclarpluscarcol[clar]
    #and the third
    X_train_carclarcol['carat:'+clar] =␣
↪X_train_carclarcol['carat']*X_train_carclarcol[clar]
    X_test_carclarcol['carat:'+clar] =␣
↪X_test_carclarcol['carat']*X_test_carclarcol[clar]
for col in color_list:
    X_train_carclarpluscarcol['carat:'+col] =␣
↪X_train_carclarpluscarcol['carat']*X_train_carclarpluscarcol[col]
    X_test_carclarpluscarcol['carat:'+col] =␣
↪X_test_carclarpluscarcol['carat']*X_test_carclarpluscarcol[col]
    #now the second is done, finishing the third
    X_train_carclarcol['carat:'+col] =␣
↪X_train_carclarcol['carat']*X_train_carclarcol[col]
    X_test_carclarcol['carat:'+col] =␣
↪X_test_carclarcol['carat']*X_test_carclarcol[col]
for clar in clar_list:
    for col in color_list:
        X_train_carclarcol[clar+':'+col] =␣
↪X_train_carclarcol[clar]*X_train_carclarcol[col]
        X_test_carclarcol[clar+':'+col] =␣
↪X_test_carclarcol[clar]*X_test_carclarcol[col]
        X_train_carclarcol['carat:'+clar+':'+col] = X_train_carclarcol[clar+':
↪'+col]*X_train_carclarcol['carat']
        X_test_carclarcol['carat:'+clar+':'+col] = X_test_carclarcol[clar+':
↪'+col]*X_test_carclarcol['carat']
```

```
#Now make carat*clarity + carat*color + (other 3 predictors) and c*c + c*c +␣
↪c*c + (other 2)
X_train_carclarpluscarcolplus = copy.copy(X_train_carclarpluscarcol)
X_test_carclarpluscarcolplus = copy.copy(X_test_carclarpluscarcol)
X_train_carclarcolplus = copy.copy(X_train_carclarcol)
X_test_carclarcolplus = copy.copy(X_test_carclarcol)
X_train_carclarpluscarcolplus['table'] = X_train['table']
X_test_carclarpluscarcolplus['table'] = X_test['table']
X_train_carclarpluscarcolplus['depth'] = X_train['depth']
X_test_carclarpluscarcolplus['depth'] = X_test['depth']
X_train_carclarcolplus['table'] = X_train['table']
X_test_carclarcolplus['table'] = X_test['table']
X_train_carclarcolplus['depth'] = X_train['depth']
X_test_carclarcolplus['depth'] = X_test['depth']
for cut in ['cut_Good', 'cut_Ideal', 'cut_Premium', 'cut_Very Good']:
    X_train_carclarpluscarcolplus[cut] = X_train[cut]
    X_test_carclarpluscarcolplus[cut] = X_test[cut]
    X_train_carclarcolplus[cut] = X_train[cut]
    X_test_carclarcolplus[cut] = X_test[cut]
```

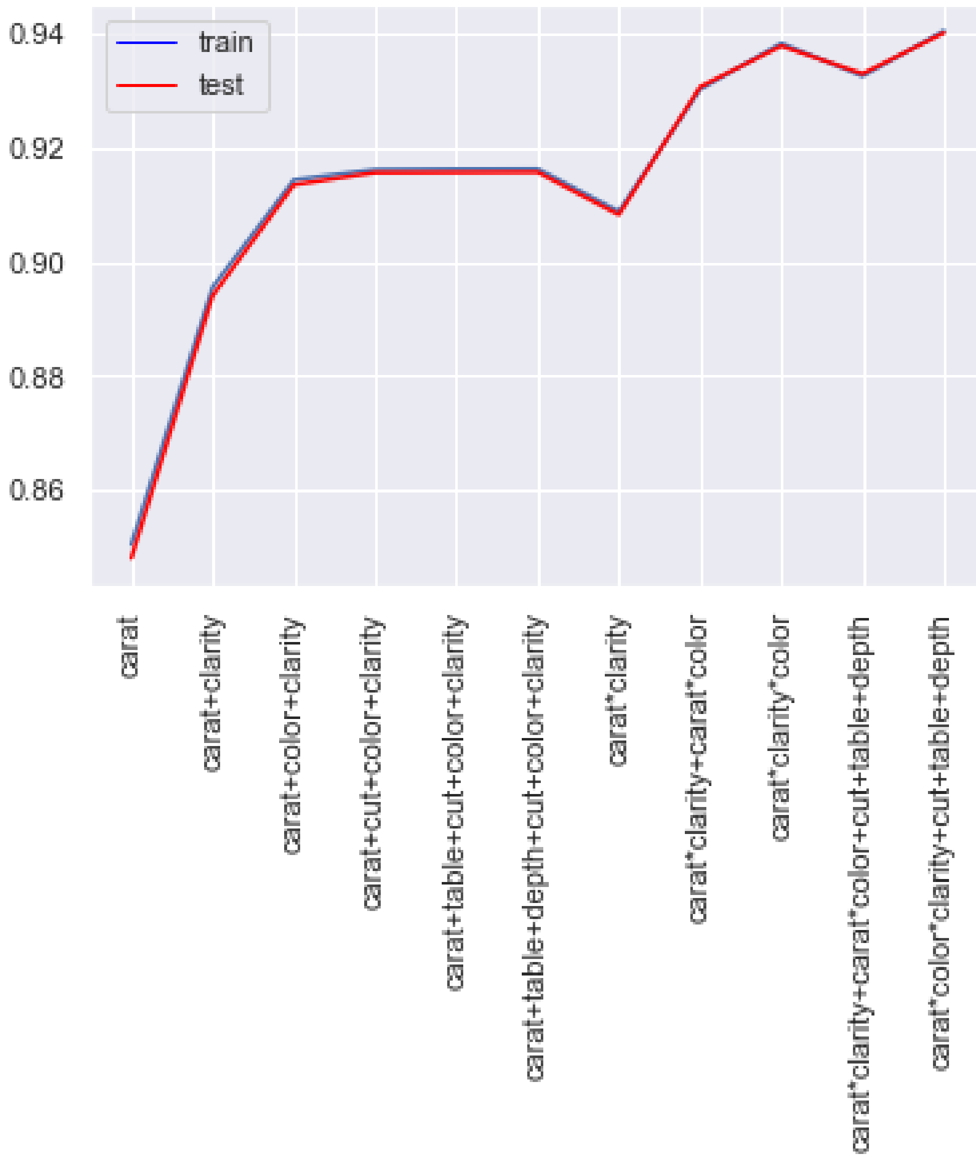
[103]:
```
#carat*clarity
model_carclar = lr.fit(X_train_carclar, y_train)
train_r2_list.append(('carat*clarity', metrics.r2_score(y_train, model_carclar.
↪predict(X_train_carclar))))
test_r2_list.append(('carat*clarity', metrics.r2_score(y_test, model_carclar.
↪predict(X_test_carclar))))
model_details['carat*clarity'] = summarize(model_carclar)
#carat*clarity+carat*color
model_carclarpluscarcol = lr.fit(X_train_carclarpluscarcol, y_train)
train_r2_list.append(('carat*clarity+carat*color', metrics.r2_score(y_train,␣
↪model_carclarpluscarcol.predict(X_train_carclarpluscarcol))))
test_r2_list.append(('carat*clarity+carat*color', metrics.r2_score(y_test,␣
↪model_carclarpluscarcol.predict(X_test_carclarpluscarcol))))
model_details['carat*clarity+carat*color'] = summarize(model_carclarpluscarcol)
#carat*color*clarity
model_carclarcol = lr.fit(X_train_carclarcol, y_train)
train_r2_list.append(('carat*clarity*color', metrics.r2_score(y_train,␣
↪model_carclarcol.predict(X_train_carclarcol))))
test_r2_list.append(('carat*clarity*color', metrics.r2_score(y_test,␣
↪model_carclarcol.predict(X_test_carclarcol))))
model_details['carat*clarity*color'] = summarize(model_carclarcol)
#carat*color+carat*clarity+cut+table+depth
model_carclarpluscarcolplus = lr.fit(X_train_carclarpluscarcolplus, y_train)
train_r2_list.append(('carat*clarity+carat*color+cut+table+depth', metrics.
↪r2_score(y_train, model_carclarpluscarcolplus.
↪predict(X_train_carclarpluscarcolplus))))
```

```
test_r2_list.append(('carat*clarity+carat*color+cut+table+depth', metrics.
    ↪r2_score(y_test, model_carclarpluscarcolplus.
    ↪predict(X_test_carclarpluscarcolplus))))
model_details['carat*clarity+carat*color+cut+table+depth'] =␣
    ↪summarize(model_carclarpluscarcolplus)
#carat*color*clarity+cut+table+depth
model_carclarcolplus = lr.fit(X_train_carclarcolplus, y_train)
train_r2_list.append(('carat*color*clarity+cut+table+depth', metrics.
    ↪r2_score(y_train, model_carclarcolplus.predict(X_train_carclarcolplus))))
test_r2_list.append(('carat*color*clarity+cut+table+depth', metrics.
    ↪r2_score(y_test, model_carclarcolplus.predict(X_test_carclarcolplus))))
model_details['carat*color*clarity+cut+table+depth'] =␣
    ↪summarize(model_carclarcolplus)
```

[104]:
```
test_r2_list
```

[104]:
```
['N/A',
 ('carat', 0.8481242962072413),
 ('carat+clarity', 0.8940802386678173),
 ('carat+color+clarity', 0.9135087184800096),
 ('carat+cut+color+clarity', 0.915642720775098),
 ('carat+table+cut+color+clarity', 0.9156847830775754),
 ('carat+table+depth+cut+color+clarity', 0.9157336485382679),
 ('carat*clarity', 0.9082985399475849),
 ('carat*clarity+carat*color', 0.9306630262053311),
 ('carat*clarity*color', 0.9378315786539743),
 ('carat*clarity+carat*color+cut+table+depth', 0.9329558382024644),
 ('carat*color*clarity+cut+table+depth', 0.9401280060463929)]
```

[105]:
```
to_train_plot = train_r2_list[1:]
to_test_plot = test_r2_list[1:]
x1, y1 = zip(*to_train_plot)
x2, y2 = zip(*to_test_plot)
plt.plot(x1, y1);
plt.plot(x1, y2, color='red');
plt.xticks(rotation = 90);

custom_lines = [plt.Line2D([0], [0], color='blue', lw=1),
                plt.Line2D([0], [0], color='red', lw=1)]
plt.legend(custom_lines,['train', 'test']);
```



As shown in the graph above, there does not seem to be much evidence of overfitting between the training and test sets. Though the models that include three interactive terms (carat * color * clarity) have the highest R^2 values, I will select (carat * clarity + carat * color) as my primary model going forward since my goal is inference, not prediction. With 3 degrees of interactions, the features list becomes unwieldy and many of the features gain a p-value greater than 0.05.

In a final attempt to boost the model's predictive power, I want to test the influence of polynomial features. Since carat is the most significant predictor and carat/color are categorical, I will focus on various polynomials of carat.

[106]:
```
X_train_half = copy.copy(X_train_carclarpluscarcol)
X_test_half = copy.copy(X_test_carclarpluscarcol)
```

```
X_train_square= copy.copy(X_train_carclarpluscarcol)
X_test_square = copy.copy(X_test_carclarpluscarcol)

X_train_half['sqrt(carat)'] = np.sqrt(X_train_half['carat'])
X_test_half['sqrt(carat)'] = np.sqrt(X_test_half['carat'])
X_train_square['carat^2'] = X_train_square['carat']**2
X_test_square['carat^2'] = X_test_square['carat']**2

#carat*clarity+carat*color+carat^0.5
model_half = lr.fit(X_train_half, y_train)
train_r2_list.append(('carat*clarity+carat*color+carat^0.5', metrics.
 ↪r2_score(y_train, model_half.predict(X_train_half))))
test_r2_list.append(('carat*clarity+carat*color+carat^0.5', metrics.
 ↪r2_score(y_test, model_half.predict(X_test_half))))
model_details['carat*clarity+carat*color+carat^0.5'] = summarize(model_half)
#carat*clarity+carat*color+carat^2
model_square = lr.fit(X_train_square, y_train)
train_r2_list.append(('carat*clarity+carat*color+carat^2', metrics.
 ↪r2_score(y_train, model_square.predict(X_train_square))))
test_r2_list.append(('carat*clarity+carat*color+carat^2', metrics.
 ↪r2_score(y_test, model_square.predict(X_test_square))))
model_details['carat*clarity+carat*color+carat^2'] = summarize(model_square)
```

[107]:

```
to_train_plot = train_r2_list[1:]
to_test_plot = test_r2_list[1:]
x1, y1 = zip(*to_train_plot)
x2, y2 = zip(*to_test_plot)
plt.plot(x1, y1);
plt.plot(x1, y2, color='red');
plt.xticks(rotation = 90);

custom_lines = [plt.Line2D([0], [0], color='blue', lw=1),
                plt.Line2D([0], [0], color='red', lw=1)]
plt.legend(custom_lines,['train', 'test']);
```

This graph shows that adding a $\sqrt{carat}$ element to the model improves the R^2 score by approximately 0.03. This is not insignificant. Higher-order polynomial elements do not improve the model further.

One last test: Since the goal of this regression is inference, a simpler model would be preferable (this is–minimize interactions.) I will also investigate the effect of adding the $\sqrt{carat}$ element to the most promising non-interactive models.

[108]:
```
#first with all terms
X_train_half_ind = copy.copy(X_train)
X_test_half_ind = copy.copy(X_test)
X_train_half_ind['sqrt(carat)'] = np.sqrt(X_train_half_ind['carat'])
```

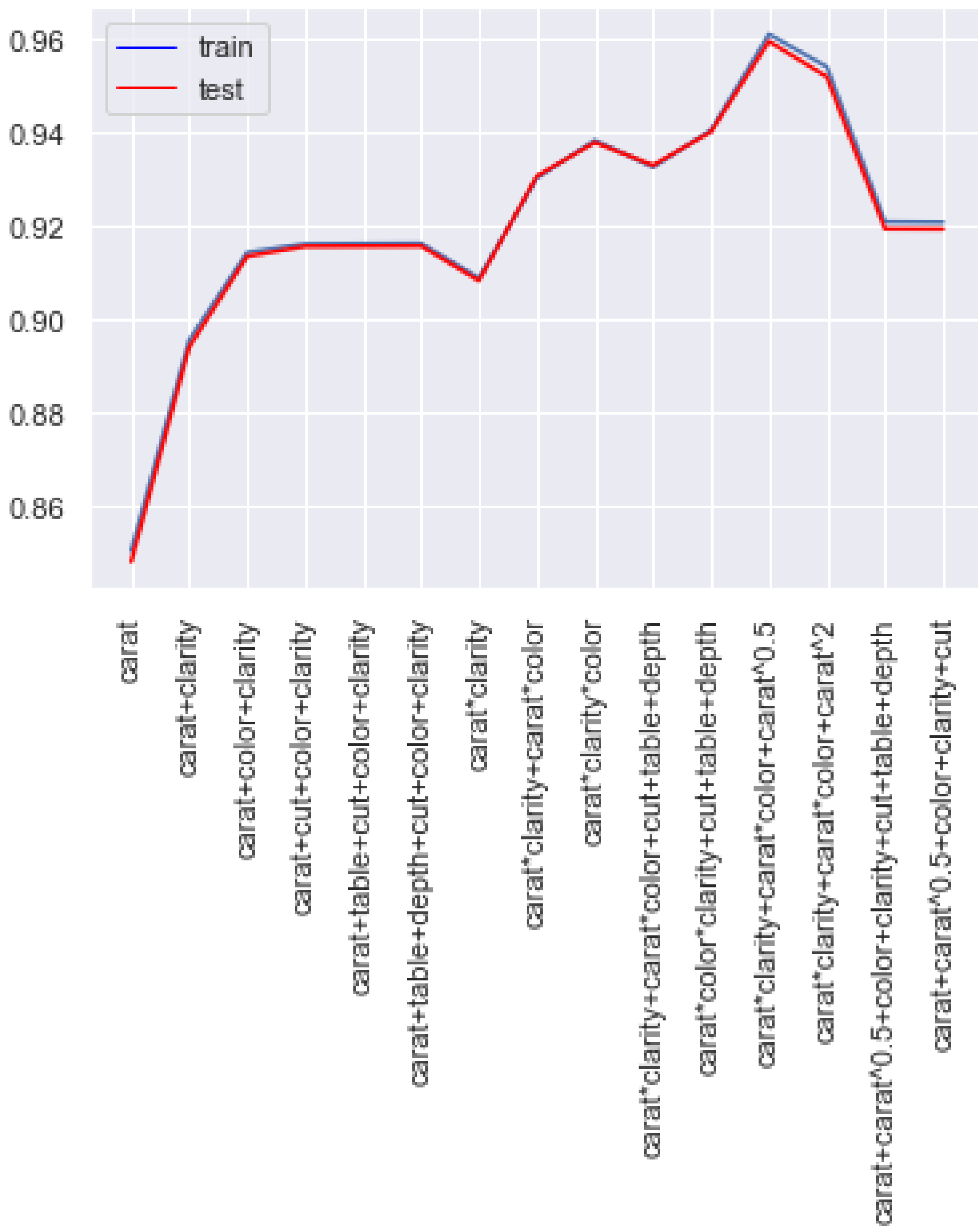
```
X_test_half_ind['sqrt(carat)'] = np.sqrt(X_test_half_ind['carat'])
model_half_ind = lr.fit(X_train_half_ind, y_train)
train_r2_list.append(('carat+carat^0.5+color+clarity+cut+table+depth', metrics.
 ↪r2_score(y_train, model_half_ind.predict(X_train_half_ind))))
test_r2_list.append(('carat+carat^0.5+color+clarity+cut+table+depth', metrics.
 ↪r2_score(y_test, model_half_ind.predict(X_test_half_ind))))
model_details['carat+carat^0.5+color+clarity+cut+table+depth'] =␣
 ↪summarize(model_half_ind)
```

[109]:

```
#and now with the "four c's"
X_train_half_simple = X_train_half_ind.drop(['table', 'depth'], axis=1)
X_test_half_simple = X_test_half_ind.drop(['table', 'depth'], axis=1)
model_half_simple = lr.fit(X_train_half_simple, y_train)
train_r2_list.append(('carat+carat^0.5+color+clarity+cut', metrics.
 ↪r2_score(y_train, model_half_simple.predict(X_train_half_simple))))
test_r2_list.append(('carat+carat^0.5+color+clarity+cut', metrics.
 ↪r2_score(y_test, model_half_simple.predict(X_test_half_simple))))
model_details['carat+carat^0.5+color+clarity+cut'] =␣
 ↪summarize(model_half_simple)
```

[110]:

```
to_train_plot = train_r2_list[1:]
to_test_plot = test_r2_list[1:]
x1, y1 = zip(*to_train_plot)
x2, y2 = zip(*to_test_plot)
plt.plot(x1, y1);
plt.plot(x1, y2, color='red');
plt.xticks(rotation = 90);

custom_lines = [plt.Line2D([0], [0], color='blue', lw=1),
                plt.Line2D([0], [0], color='red', lw=1)]
plt.legend(custom_lines,['train', 'test']);
```



Both of the non-interactive models perform 0.05 worse than the best model in terms of R^2. However, that is an acceptable tradeoff since my goal is inference. I will publish both models.

Best model for inference, with an R^2 of approximately 0.92:

	Feature	Coefficient
0	Intercept	-4133.85
1	carat	12683.2
2	cut_Good	605.443
3	cut_Ideal	891.796

	Feature	Coefficient
4	cut_Premium	749.356
5	cut_Very Good	765.298
6	color_E	-216.682
7	color_F	-268.101
8	color_G	-478.187
9	color_H	-986.394
10	color_I	-1491.48
11	color_J	-2351.42
12	clarity_IF	5354.24
13	clarity_SI1	3645
14	clarity_SI2	2674.89
15	clarity_VS1	4575.06
16	clarity_VS2	4258.89
17	clarity_VVS1	4969
18	clarity_VVS2	4908.48
19	sqrt(carat)	-7223.69

Best model for prediction, with an R^2 of approximately 0.96:

	Feature	Coefficient
0	Intercept	10792.2
1	carat	16138.6
2	clarity_IF	-5167.04
3	clarity_SI1	-4053.27
4	clarity_SI2	-3410.62
5	clarity_VS1	-4494.97
6	clarity_VS2	-4309.55
7	clarity_VVS1	-4925.45
8	clarity_VVS2	-4826.71
9	color_E	86.9598
10	color_F	31.9883
11	color_G	258.312
12	color_H	522.668
13	color_I	681.04
14	color_J	1229.55
15	carat:clarity_IF	10983.8
16	carat:clarity_SI1	6317.42
17	carat:clarity_SI2	4774.94
18	carat:clarity_VS1	8037.26
19	carat:clarity_VS2	7389.61
20	carat:clarity_VVS1	9908.02
21	carat:clarity_VVS2	9351.86
22	carat:color_E	-372.779
23	carat:color_F	-469.281
24	carat:color_G	-1138.81

	Feature	Coefficient
25	carat:color_H	-1996.39
26	carat:color_I	-2711.78
27	carat:color_J	-3945.89
28	sqrt(carat)	-23520.5

Full model R^2 Results

	Model	R^2 Train	R^2 test
0	carat	0.85052	0.848124
1	carat+clarity	0.895576	0.89408
2	carat+color+clarity	0.914368	0.913509
3	carat+cut+color+clarity	0.916185	0.915643
4	carat+table+cut+color+clarity	0.916283	0.915685
5	carat+table+depth+cut+color+clarity	0.916316	0.915734
6	carat*clarity	0.908884	0.908299
7	carat*clarity+carat*color	0.930232	0.930663
8	carat*clarity*color	0.938297	0.937832
9	carat*clarity+carat*color+cut+table+depth	0.932545	0.932956
10	carat*color*clarity+cut+table+depth	0.94048	0.940128
11	carat*clarity+carat*color+carat^0.5	0.96103	0.959492
12	carat*clarity+carat*color+carat^2	0.954045	0.951891
13	carat+carat^0.5+color+clarity+cut+table+depth	0.920903	0.919384
14	carat+carat^0.5+color+clarity+cut	0.920818	0.919335