ECE M146 Spring 2020 Optional Diagnostic Quiz
Machine Learning
Instructor: Lara Dolecek Maximum score: 50 points

This quiz is optional and will not be graded. Please time and grade yourself for this diagnostic quiz. Solution will be uploaded on Friday. You should be able to do this quiz in 30 mins.

Good luck!

Instructions

- You may find the following useful.
  - $\int \exp(ax)dx = \frac{\exp(ax)}{a} + C$ where C is an arbitrary constant
  - $||\boldsymbol{x}||_1 = \sum_{i=1}^{n} |x_i|$, $||\boldsymbol{x}||_\infty = \max_i |x_i|$
  - $\frac{d(\ln(x))}{dx} = \frac{1}{x}$

1. Calculus **(10 pts)**

   (a) Let $f(\boldsymbol{x}) = \ln(ax_1x_2 + bx_1 + cx_1^2)$. What is the partial derivative of f with respect to x_1?

   (b) Evaluate $\int_b^\infty a \exp(-a(x-b))dx$ where $a > 0$?

2. Probability **(14 pts)**
Suppose A and B are two events. Which of these statements is **true/false**? Explain.

(a) A and B are mutually exclusive events then $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

(b) $P(A \cap B \cap C) = P(A)P(B|A)P(C|B)$

(c) A is a Gaussian random variable with $\mu = 0$ and $\sigma^2 = 1$ then $P(A = 0) = 0.5$

(d) If A and A^c are independent, where A^c denotes the complement of event A then $0 < P(A) < 1$

(e) Assume X is a random variable. The variance of X is defined as $Var(X) = E[(X - E[X])^2]$. Prove that $Var(aX + b) = a^2 Var(X)$.

3. Linear Algebra **(16 pts)**
   Consider the vector $\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

   (a) Compute $\frac{||\boldsymbol{x}||_2}{||\boldsymbol{x}||_1}$.

   (b) Compute $\boldsymbol{x}\boldsymbol{x}^T$.

   (c) Find $c \in \mathbb{R}$ such that $||c\boldsymbol{x}||_2 < 1$

4. MATLAB Question **(10 pts)**
Read the following MATLAB code and answer the questions.

```
A = [1, 2; 3, 4];
B = [1, 0; 0, 2];
C = A * B;
D = A .* B;
E = det(A);
F = inv(B);
```

(a) Write A and B in matrix form.

(b) Write C and D in matrix form.

(c) What is the value of E? Write F in matrix form.