

Lecture #11

- Sorting Algorithms, part II:
 - Quicksort
 - Mergesort
- Trees
 - Introduction
 - Implementation & Basic Properties
 - Traversals: The Pre-order Traversal
- On-your-own Study
 - Full binary trees

But first... STL Challenge

Give me a data structure that I can use to maintain a bunch of people's names and for each person, allows me to easily get all of the streets they lived on.

Assuming I have P total people and each person has lived on an average of E former streets...

What is the Big-Oh cost of:

- A. Finding the names of all people who have lived on "Levering street"?
- B. Determining if "Bill" ever lived on "Westwood blvd"?
- C. Printing out every name along with each person's street addresses, in alphabetical order.
- D. Printing out all of the streets that "Tala" has lived on.

GOOD CODERS...



... KNOW WHAT THEY'RE DOING

Advanced Sorting Algos.

	 Insertion	 Selection	 Bubble	 Shell	 Merge	 Heap	 Quick	 Quick3
 Random								
 Nearly Sorted								
 Reversed								
 Few Unique								

Advanced Sorting Algorithms

Why should you care?

Because this is basically how folks sort stuff in **real life**.

You can sort **billions of values** in **seconds**. SECONDS BABY!

And because you'll be asked about them in **job interviews** and on **exams**.

So pay attention!



Divide and Conquer Sorting

The last two sorts we'll learn (for now) are
Quicksort and Mergesort.

These sorts generally work as follows:

1. **Divide** the elements to be sorted into two groups of roughly equal size.
2. **Sort** each of these smaller groups of elements (conquer).
3. **Combine** the two sorted groups into one large sorted list.

Any time you see "divide and conquer," you should think
recursion... EEK!

The Quicksort Algorithm

1. If the array contains only 0 or 1 element, **return**.
2. Select an arbitrary element **P** from the array (typically the **first element** in the array).
3. Move all elements that are **less than or equal** to **P** to the **left of the array** and all elements **greater than P** to the **right** (this is called **partitioning**).
4. Recursively repeat this process on the left sub-array and then the right sub-array.

Divide

Conquer

13	1	21	30	69	40	77
----	---	----	----	----	----	----

Select an arbitrary item **P** from the array.

Move items **smaller than or equal to P** to the **left** and **larger items** to the **right**; **P** goes in-between.

Recursively repeat this process on the **left items**.

Recursively repeat this process on the **right items**.

QuickSort



EE
Major

MBA

History
Major

Bio
Major

Drop-out

CS
M

USC
rad

Everything
on this side
is **larger**
than item **P**!

Everything on
this side is
smaller than
item **P**!

And item **P** is exactly
in the **right spot** in
between!



History
Major

Bio
Major

USC
Grad

EE
Major

MBA

Drop-out

CS Major

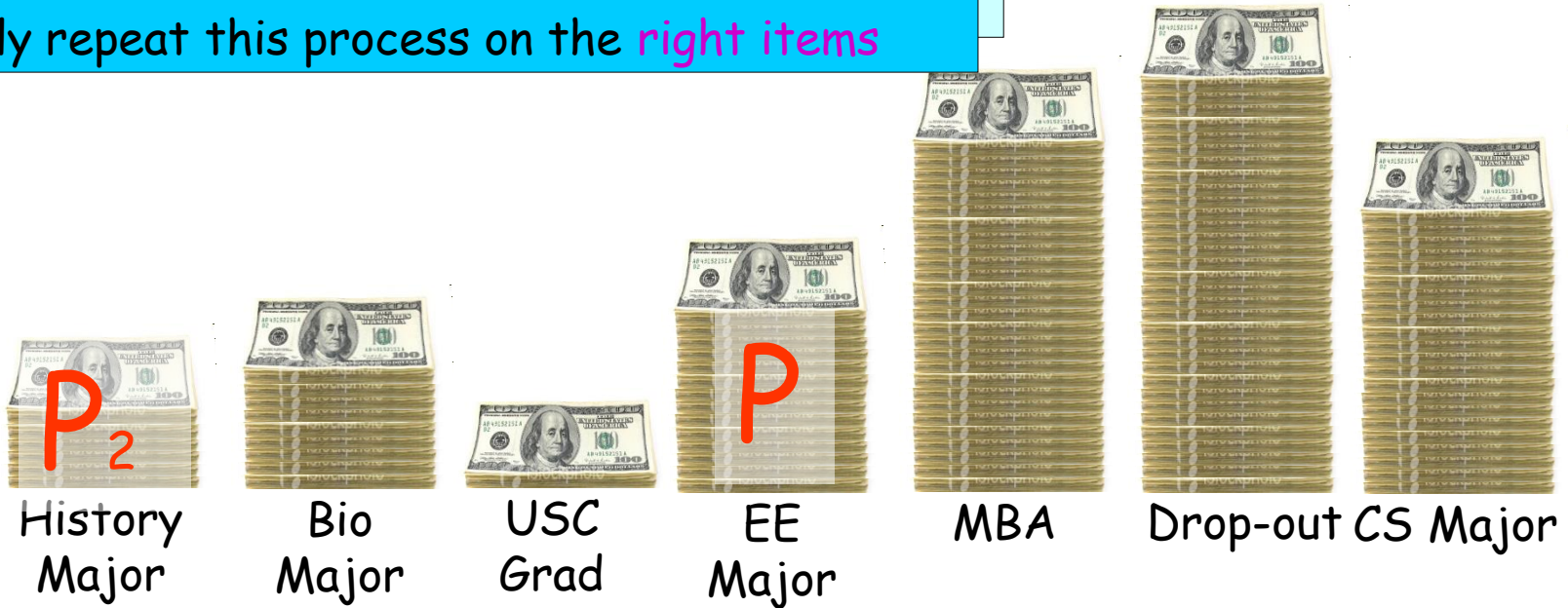
QuickSort

Select an arbitrary item **P** from the array.

Move items **smaller than or equal to P** to the **left** and **larger items** to the **right**; **P** goes in-between.

Recursively repeat this process on the **left items**

Recursively repeat this process on the **right items**



Everything left of EE Major
(our first P) is now **sorted**!

10

Select an arbitrary item **P** from the array.

Move items **smaller than or equal to P** to the **left** and **larger items** to the **right**; **P** goes in-between.

Recursively repeat this process on the **left items**

Recursively repeat this process on the **right items**

QuickSort



USC
Grad



History
Major



Bio
Major



EE
Major



MBA



Drop-out CS Major



USC
Grad



History
Major



Bio
Major



EE
Major



CS Major



MBA



Drop-out

Finally, all items are sorted!

Everything right of EE
Major (our first P) is now
sorted!

D

Quickso

First specifies the starting element of the array to sort.

Last specifies the last element of the array to sort.

Only bother sorting arrays of **at least two** elements!

And here's an actual Quicksort C++ function:

0

7

```
void QuickSort(int Array[], int First, int Last)
{
    if (Last - First >= 1 )
    {
        int PivotIndex;

        PivotIndex = Partition(Array, First, Last);
        QuickSort(Array, First, PivotIndex-1); // left
        QuickSort(Array, PivotIndex+1, Last);  // right
    }
}
```

CONQUER

Apply our QS algorithm to the left half of the array.

DIVIDE

Pick an element.
Move \leq items left
Move $>$ items right

3

CONQUER

Apply our QS algorithm to the right half of the array.

13	1	21	30	69	40	77	46
0	1	2	3	4	5	6	7

The QS Partition Function

The **Partition** function uses the first item as the pivot value and moves **less-than-or-equal items** to the **left** and **larger ones** to the **right**.

```
int Partition(int a[], int low, int high)
{
    int pi = low;
    int pivot = a[low];
    do
    {
        while ( low <= high && a[low] <= pivot )
            low++;
        while ( a[high] > pivot )
            high--;
        if ( low < high )
            swap(a[low], a[high]);
    }
    while ( low < high );
    swap(a[pi], a[high]);
    pi = high;
    return(pi);
}
```

And finally, return the pivot's index in the array (**4**) to the QuickSort function.

4	1	12	13	30	52	40	99	77	35	47	56
0	1	2	3	4	5	6	7	8	9	10	11

Big-oh of Quicksort

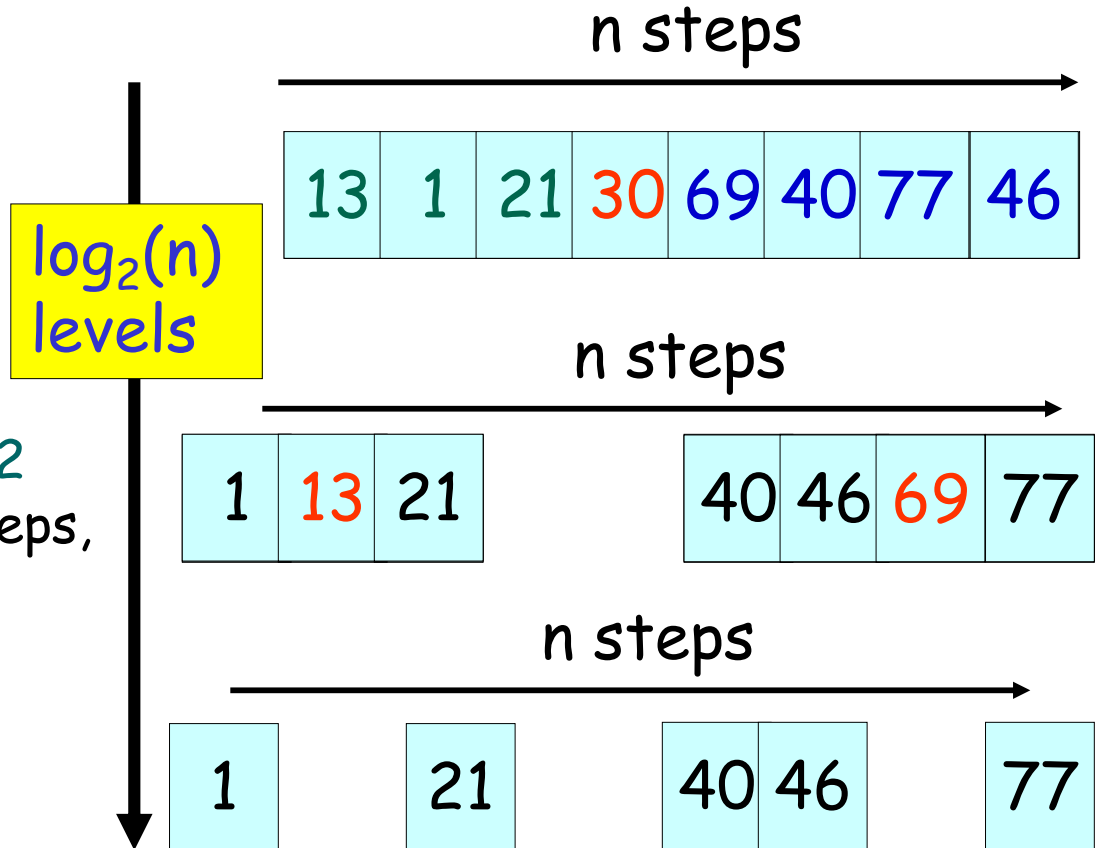
We first **partition** the array, at a cost of n steps.

Then we repeat the process for each half...

We **partition** each of the 2 halves, each taking $n/2$ steps, at a total cost of n steps.

Then we repeat the process for each half...

We **partition** each of the 4 halves, each taking $n/4$ steps, at a total cost of n steps.



So at each level, we do n operations, and we have $\log_2(n)$ levels, so we get: $n \log_2(n)$.

Quicksort - Is It Always *Fast*?

Are there any kinds of input data where Quicksort is either **more** or **less efficient**?

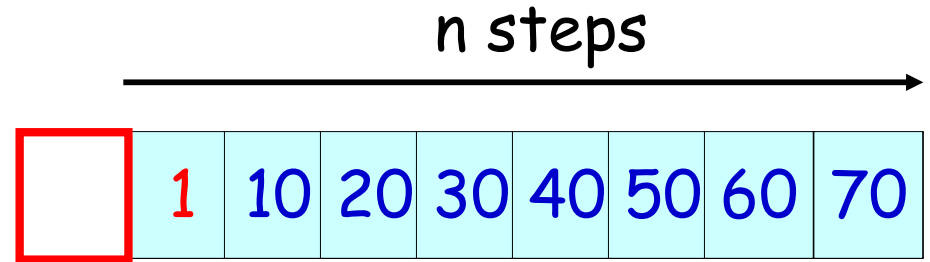
Yes! If our array is **already sorted** or **mostly sorted**, then quicksort becomes **very slow**!

1	10	20	30	40	50	60	70
---	----	----	----	----	----	----	----

Let's see why.

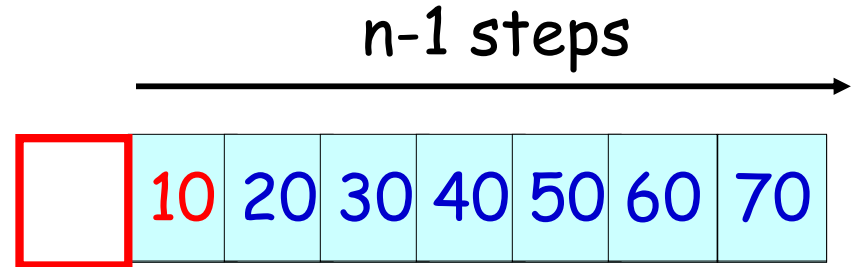
Worst-case Big-oh of Quicksort

We first **partition** the array, at a cost of n steps.

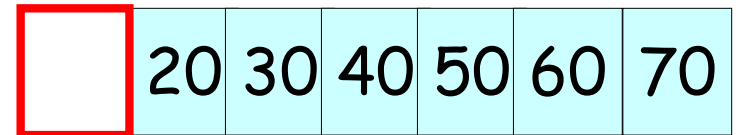


Then we repeat the process for the ~~left~~ & right groups...

Ok, let's **partition** our **right group** then.

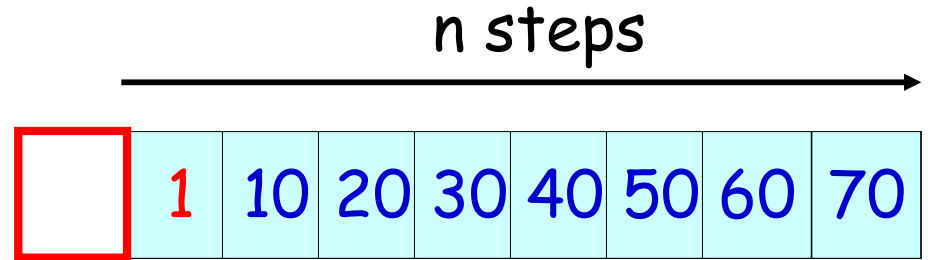


Then we repeat the process for the ~~left~~ & right groups...



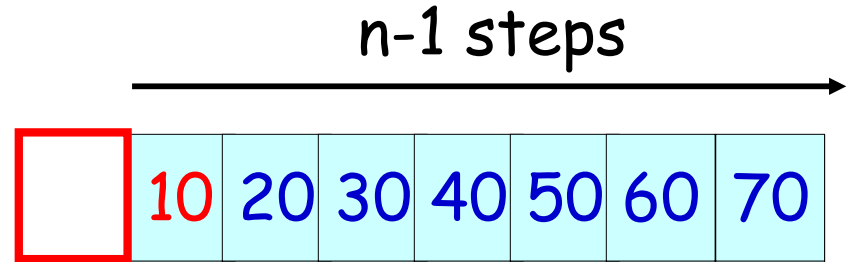
Worst-case Big-oh of Quicksort

We first **partition** the array, at a cost of n steps.



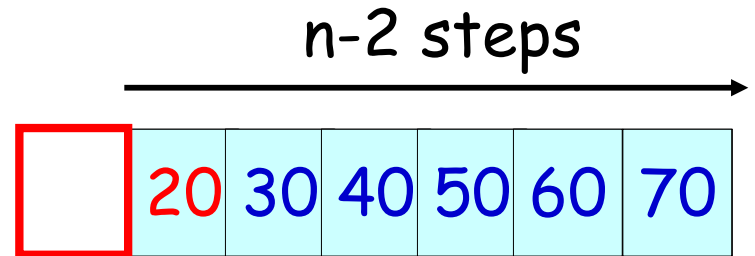
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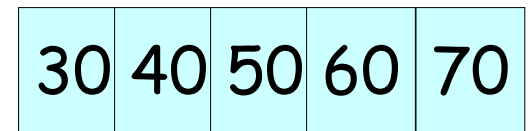


Then we repeat the process for the ~~left~~ & right groups...

Ok, let's **partition** our **right group** then.



Then we repeat the process for the ~~left~~ & right groups...



Worst-case Big-oh of Quicksort

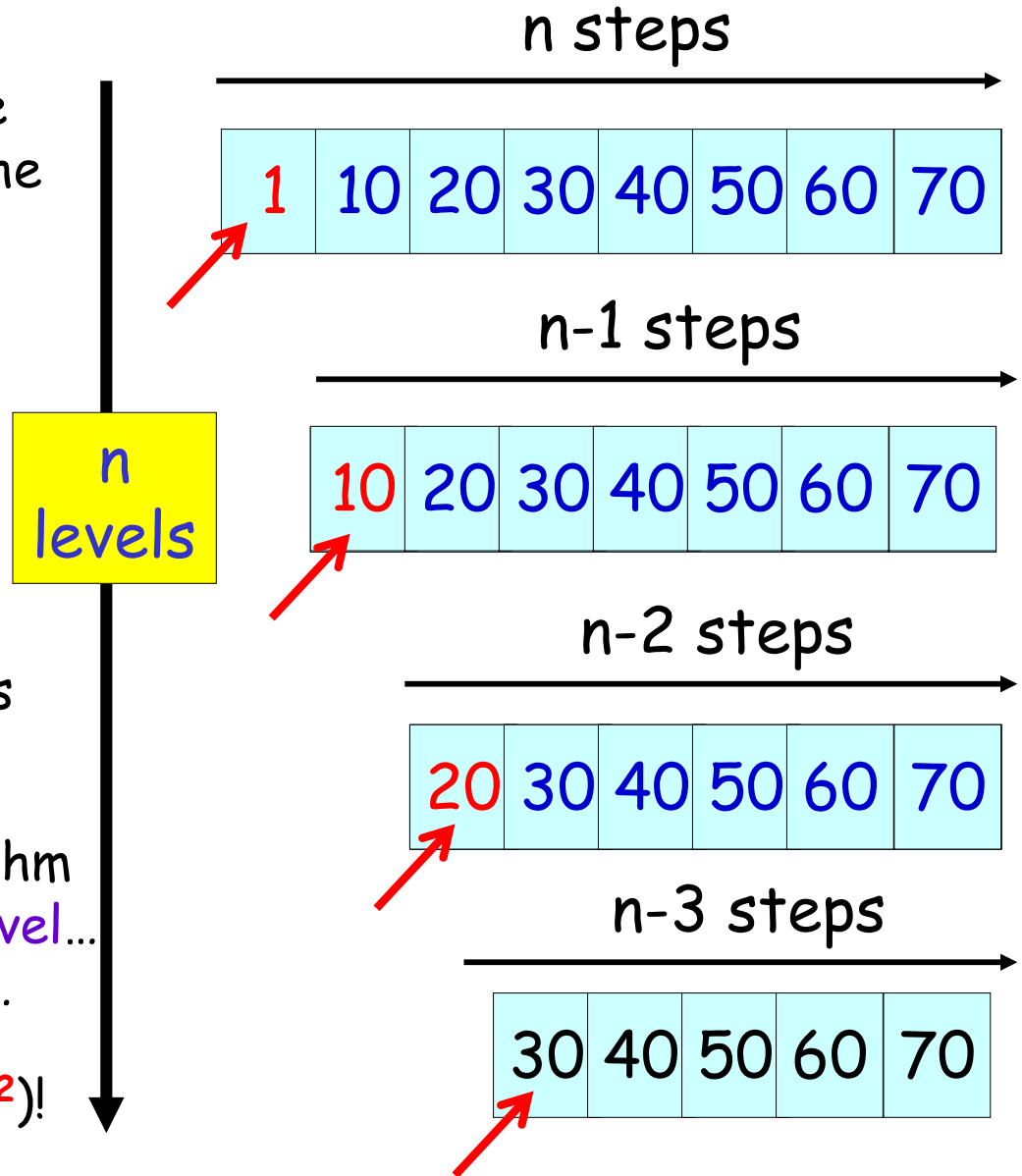
What you'll notice is that each time we partition, we remove **only one item** off the left side!

And if we only remove one item off the left side each time...

We're going to have to go through this **partitioning** process **n times** to process the entire array!

And if the partition algorithm requires **$\sim n$ steps** at **each level**...
And we go **n levels deep**...

Then our algorithm is $O(n^2)$!



Other Quicksort Worst Cases?

So, as you can see, an array that's **mostly in order** will require an average of **N^2 steps!**

As you can probably guess, **Quicksort** also has the same problem with arrays that are in **reverse order!**

So if you happen to know your data will be **mostly sorted** (or in **reverse**) **order**, avoid Quicksort!

It's a **DOG!**



QuickSort Questions

Can QuickSort be applied easily to sort items within a linked list?

Is QuickSort a "stable" sort?

Does QuickSort use a fixed amount of RAM, or can it vary?

Can QuickSort be parallelized across multiple cores?

When might you use QuickSort?

Mergesort

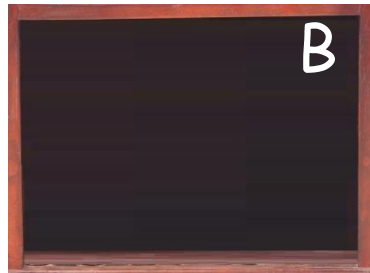
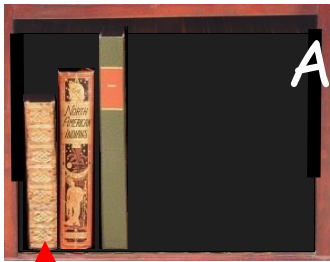
The Mergesort is another extremely efficient sort - yet it's pretty easy to understand.



But before we learn the **Mergesort**, we need to learn another algorithm called "**merge**".

Mergesort

The basic **merge** algorithm takes **two-presorted arrays** as inputs and **outputs a combined, third sorted array**.



Merge Algorithm

Consider the left-most book in both shelves
 Take the smallest of the two books
 Add it to the new shelf
 Repeat the whole process until all books are moved



1. Initialize counter variables $i1$, $i2$ to zero
2. While there are more items to copy...
 - If $A1[i1]$ is less than $A2[i2]$
 - Copy $A1[i1]$ to output array B and $i1++$
 - Else
 - Copy $A2[i2]$ to output array B and $i2++$
3. If either array runs out, copy the entire contents of the other array over

By always selecting and moving the **smallest book** from either shelf we guarantee all of our books will end up sorted!

Merge Algorithm in C++

```
void merge(int data[],int n1, int n2)
{
    int i=0, j=0, k=0;
    int *temp = new int[n1+n2];
    int *sechalf = data + n1;

    while (i < n1 || j < n2)
    {
        if (i == n1)
            temp[k++] = sechalf[j++];
        else if (j == n2)
            temp[k++] = data[i++];
        else if (data[i] <= sechalf[j])
            temp[k++] = data[i++];
        else
            temp[k++] = sechalf[j++];
    }
    for (i=0;i<n1+n2;i++)
        data[i] = temp[i];
    delete [] temp;
}
```

Here's the C++ version of our **merge** function!

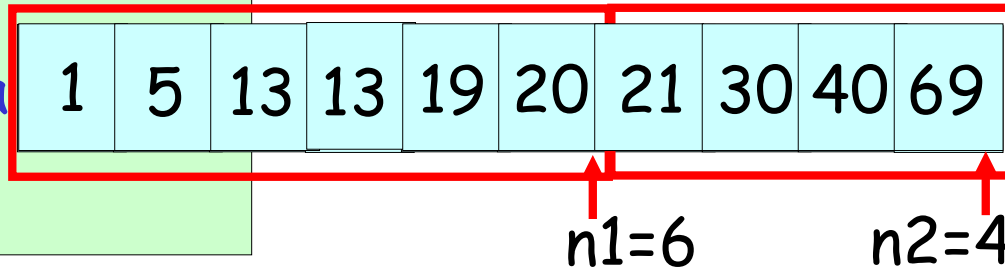
Instead of passing in **A1**, **A2** and **B...**

you pass in an array called **data** and two sizes: **n1** and **n2**

Notice how this function uses **new/delete** to allocate a temporary array for merging.

data holds the merged contents at the end.

data



Mergesort

OK - so what's the full mergesort algorithm:

Mergesort function :

1. If array has one element, then return (it's sorted).
2. Split up the array into two equal sections
3. Recursively call Mergesort function on the left half
4. Recursively call Mergesort function on the right half
5. Merge the two halves using our **merge** function

Ok, let's see how to mergesort a shelf full of books!




1. If array has 1 item, then return
2. Split array in two equal sections
3. Call Mergesort on the left half
4. Call Mergesort on the right half
5. **Merge** the halves back together



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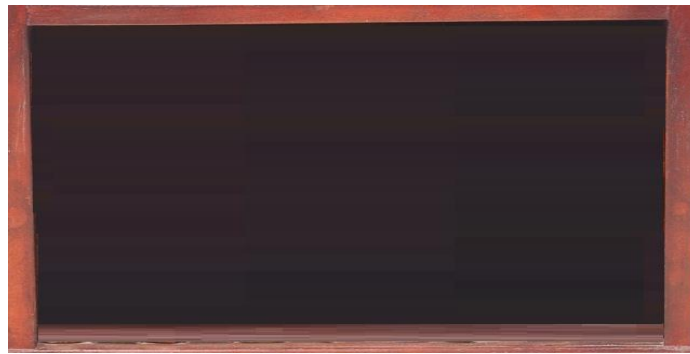
- 
1. If array has 1 item, then return
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- A red arrow points from the bottom of the third list box to the top of this final list box.



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↑
i1



↑
i2

And our array is
sorted!!!!

Mergesort - One Final Detail

While I showed the Mergesort moving books into a bunch of small piles...



i1 i2



The real algorithm sorts the data in-place in the array...

and only uses a separate array for merging.

Let's see how it really works!

Big-oh of Mergesort



$\log_2 n$ levels deep
 Why? Because we
 keep dividing our
 piles in half...
 until our piles are
 just 1 book!



Big-oh of Mergesort

n items merged



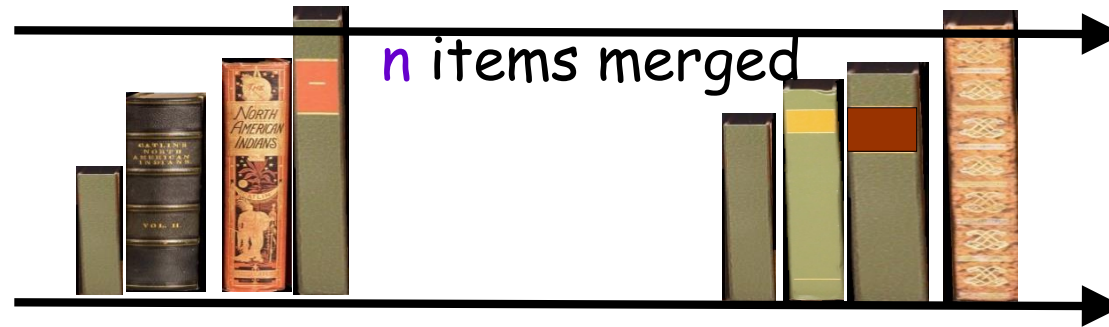
n items merged

$\log_2 n$ levels deep

Why? Because we keep dividing our piles in half...

until our piles are just 1 book!

Big-oh of Mergesort



n items merged



$\log_2 n$ levels deep

Why? Because we keep dividing our piles in half...

until our piles are just 1 book!

Overall, this gives us $n \cdot \log_2(n)$ steps to sort n items of data. Not bad! 😊

Mergesort - Any Problem Cases

So, are there any cases where mergesort is less efficient?

No! Mergesort works equally well regardless of the ordering of the data...



However, because the merge function needs secondary arrays to merge, this can slow things down a bit...

In contrast, quicksort doesn't need to allocate any new arrays to work.

MergeSort Questions

Can MergeSort be applied easily to sort items within a linked list?

Is MergeSort a "stable" sort?

Are there any special uses for MergeSort that other sorts can't handle?

Can MergeSort be parallelized across multiple cores?

Sorting Overview

Sort Name	Stable/ Non-stable	Notes
Selection Sort	Unstable	Always $O(n^2)$, but simple to implement. Can be used with linked lists. Minimizes the number of item-swaps (important if swaps are slow)
Insertion Sort	Stable	$O(n)$ for already or nearly-ordered arrays. $O(n^2)$ otherwise. Can be used with linked lists. Easy to implement.
Bubble Sort	Stable	$O(n)$ for already or nearly-ordered arrays (with a good implementation). $O(n^2)$ otherwise. Can be used with linked lists. Easy to implement. Rarely a good answer on an interview!
Shell Sort	Unstable	$O(n^{1.25})$ approx. OK for linked lists. Used in some embedded systems (eg, in a car) instead of quicksort due to fixed RAM usage.
Quick Sort	Unstable	$O(n \log_2 n)$ average, $O(n^2)$ for already/mostly/reverse ordered arrays or arrays with the same value repeated many times. Can be used with linked lists. Can be parallelized across multiple cores. Can require up to $O(n)$ slots of extra RAM (for recursion) in the worst case, $O(\log_2 n)$ avg.
Merge Sort	Stable	$O(n \log_2 n)$ always. Used for sorting large amounts of data on disk (aka "external sorting"). Can be used to sort linked lists. Can be parallelized across multiple cores. Downside: Requires n slots of extra memory/disk for merging - other sorts don't need extra RAM.
Heap Sort	Unstable	$O(n \log_2 n)$ always. Sometimes used in low-RAM embedded systems because of its performance/low memory req'ts.

Challenge Problems

1. Give an algorithm to efficiently determine which element occurs the largest number of times in the array.
2. What's the best algorithm to sort 1,000,000 random numbers that are all between 1 and 5?

when your code is meant to be
 $O(N \log N)$ but it's been 30
minutes and it still hasn't finished
 $N=3$

breath in



boi

Trees



Trees

Why should you care?

Trees are used to organize data in many software applications, including:

Databases (B-TREES)

Data Compression (Huffman Trees)

Bitcoin (Merkle Trees)

Medical Diagnosis (Decision Trees)

And because you'll be asked about them in **job interviews** and on **exams**.

So pay attention!



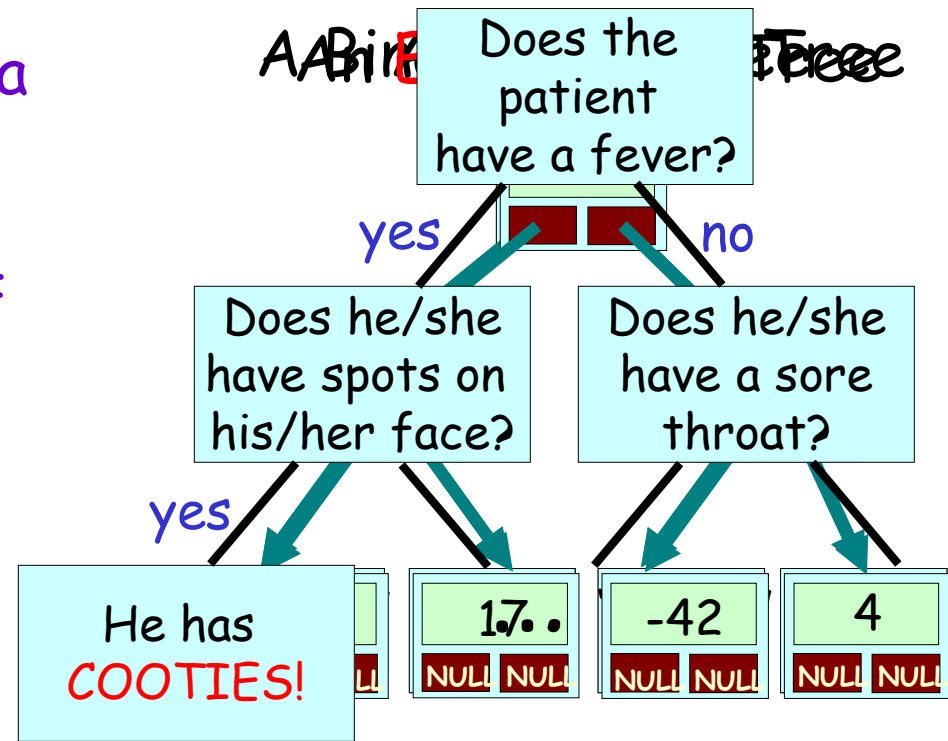
Trees

"I think that I shall never see a data structure as lovely as a tree." - Carey Nachenberg

A **Tree** is a special **linked list-based data structure** that has many uses in Computer Science:

- To organize hierarchical data
- To make information easily searchable
- To simplify the evaluation of mathematical expressions
- To make decisions

A **Decision** Tree



Basic Tree Facts

1. Trees are made of **nodes** (just like linked list nodes).
2. Every tree has a "root" pointer.
3. The top node of a tree is called its "root" node.
4. Every node may have zero or more "children" nodes.
5. A node with 0 children is called a "leaf" node.
6. A tree with no nodes is called an "empty tree."

```

struct node
{
    int value;    // some value
    node *left, *right;
};

node *rootPtr;
  
```

Leaf node

0 children

2 children

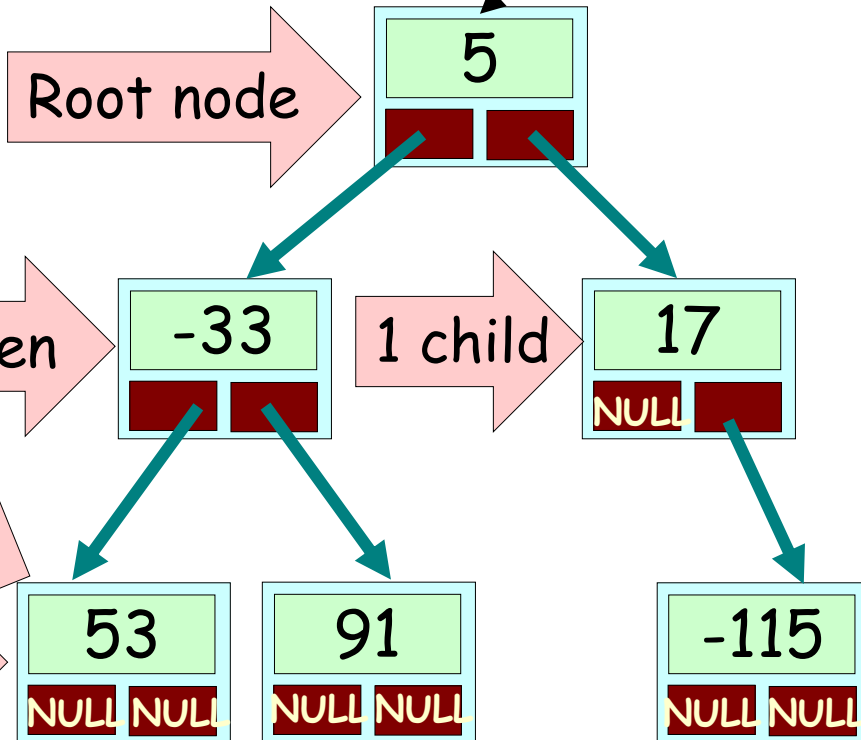
1 child

Empty tree

root ptr
NULL

Root node

root ptr



But instead of just one next pointer, a tree node can have **two or more next pointers!**

The tree's **root pointer** is like a linked list's **head pointer!**

Tree Nodes Can Have Many Children

A tree node can have more than just two children:

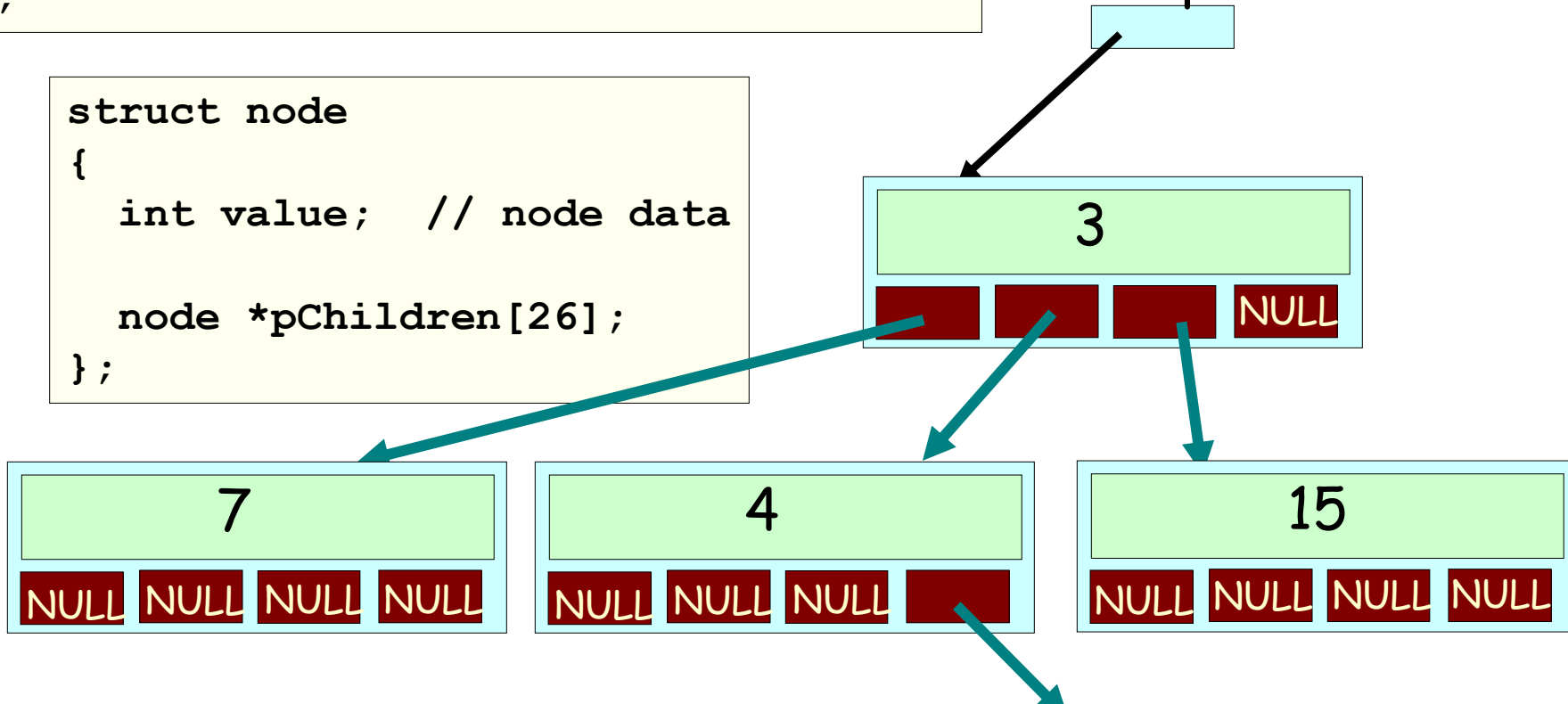
```
struct node
{
    int value;    // node data

    node *pChild1, *pChild2, *pChild3, ...;
};
```

```
struct node
{
    int value;    // node data

    node *pChildren[26];
};
```

root ptr



Binary Trees

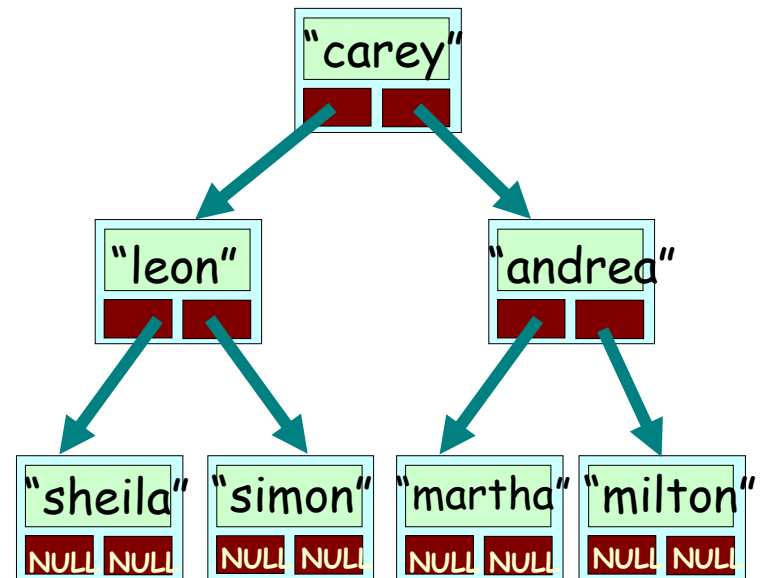
A **binary tree** is a special form of tree. In a binary tree, every node has at most **two children nodes**:

A left child and a right child.

```
struct BTNODE // binary tree node
{
    string value; // node data

    BTNODE *pLeft, *pRight;
};
```

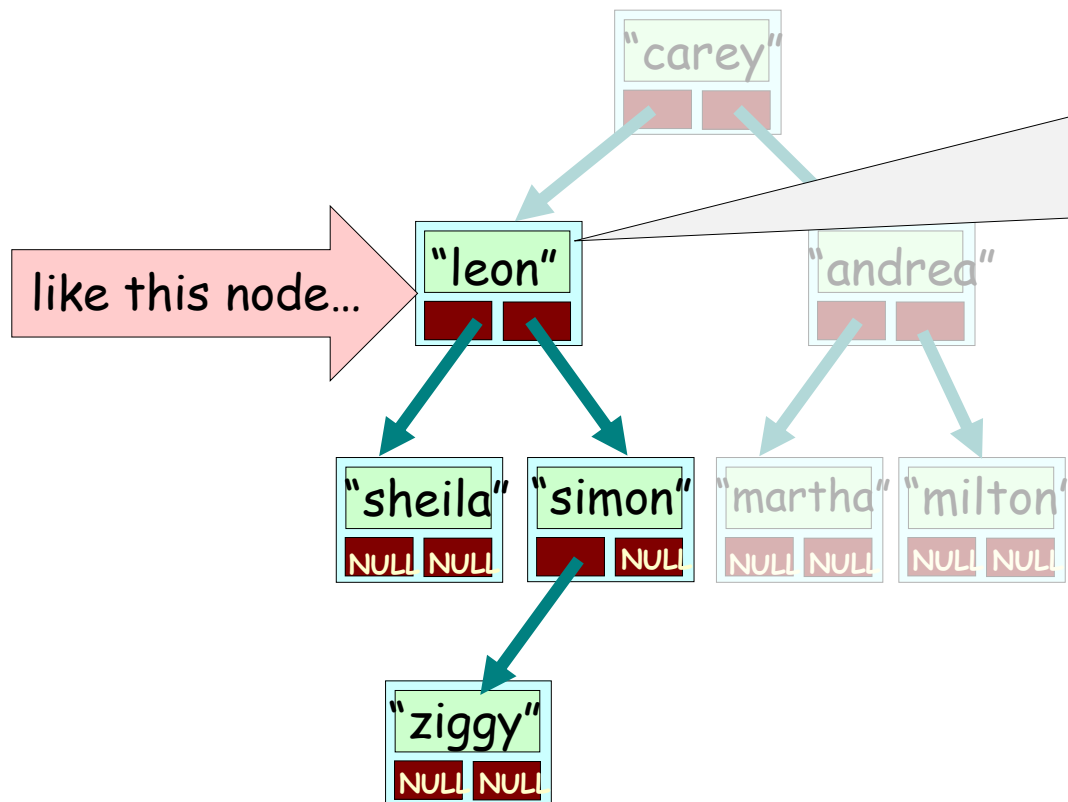
A **Binary** Tree



Binary Tree Subtrees

We can pick any node in the tree...

And then focus on its "**subtree**" - which includes it and all of nodes below it.

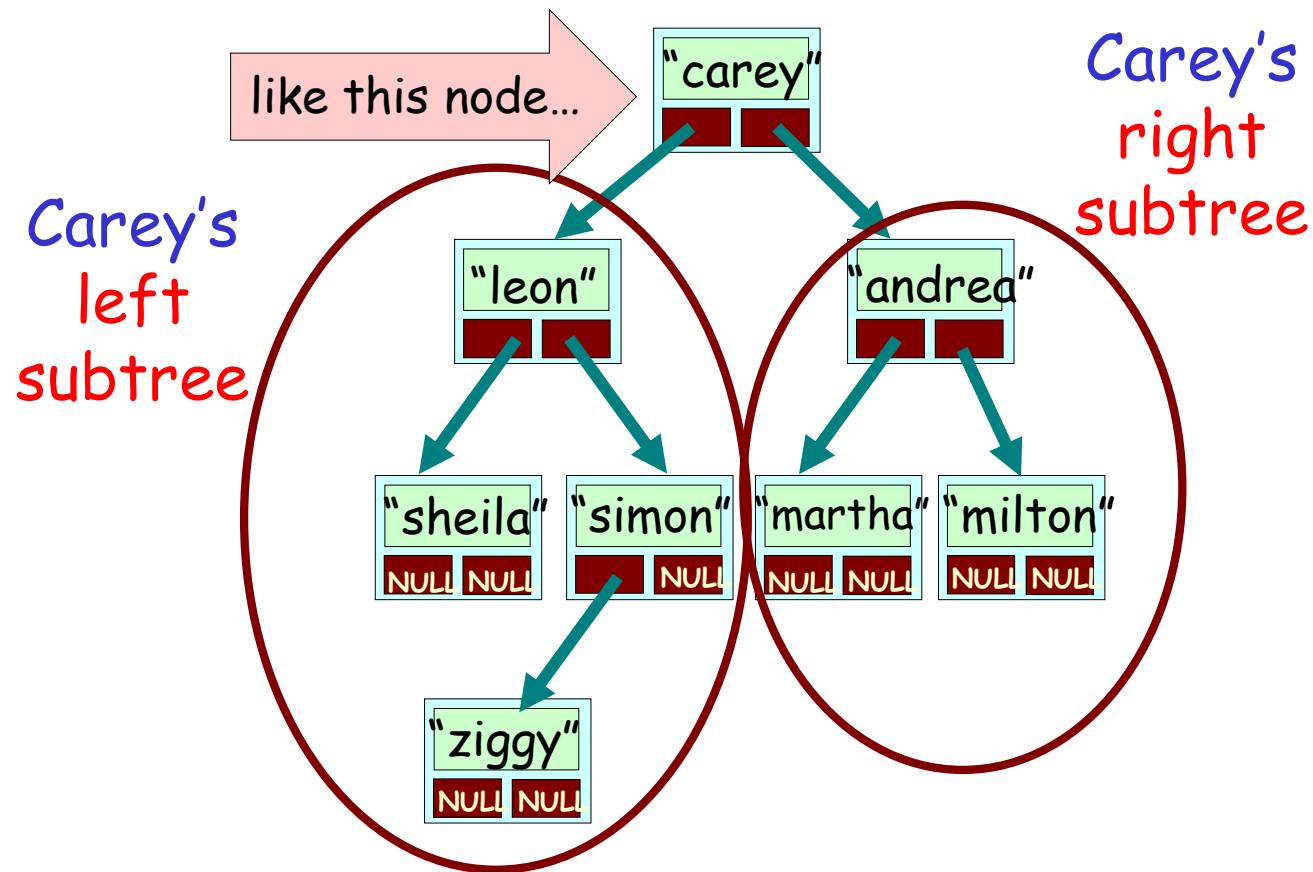


This subtree includes four different nodes...

It has the "leon" node as its root.

Binary Tree Subtrees

If we pick a node from our tree...
we can also identify its **left** and **right sub-trees**.



Operations on Binary Trees

The following are common operations that we might perform on a Binary Tree:

- enumerating all the items
- searching for an item
- adding a new item at a certain position on the tree
- deleting an item
- deleting the entire tree (destruction)
- removing a whole section of a tree (called **pruning**)
- adding a whole section to a tree (called **grafting**)

We'll learn about many of these operations over the next two classes.

```

struct BTNODE // node
{
    int value; // data
    BTNODE *left, *right;
};

```

```

main()
{

```

```

    BTNODE *temp, *pRoot

```

```

    pRoot = new BTNODE;
    pRoot->value = 5;

```

```

    temp = new BTNODE;
    temp->value = 7;
    temp->left = NULL;
    temp->right = NULL;
    pRoot->left = temp;

```

```

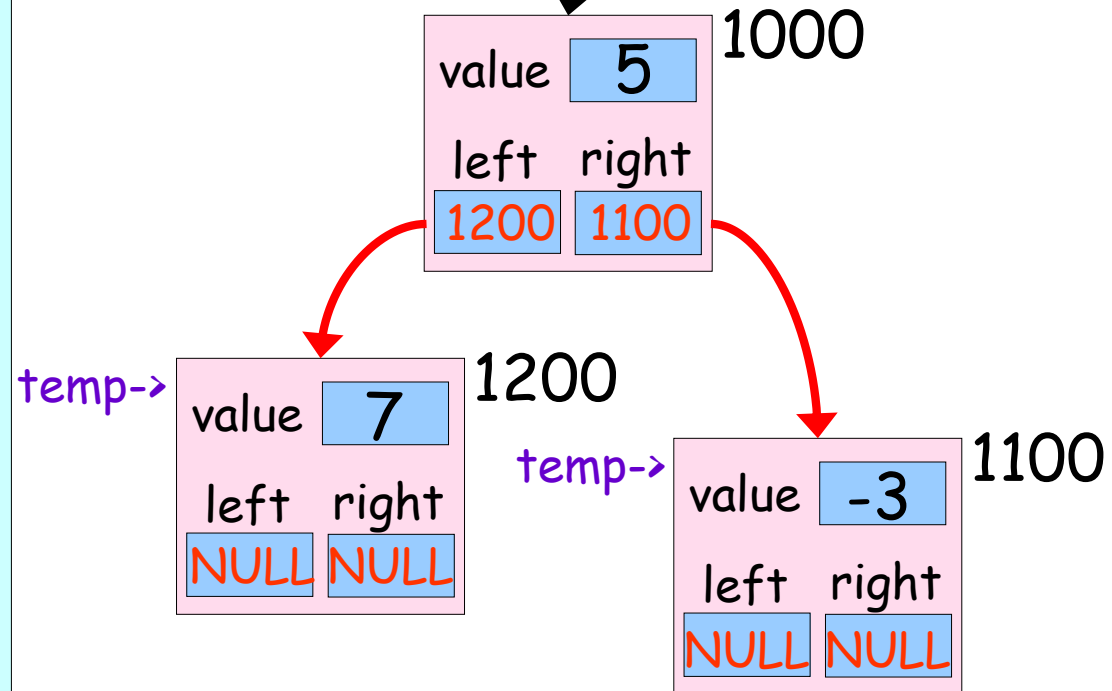
    temp = new BTNODE;
    temp->value = -3;
    temp->left = NULL;
    temp->right = NULL;
    pRoot->right = temp;
    // etc...

```

As with **linked lists**, we use **dynamic memory** to allocate our **nodes**.

A Simple Tree Example

temp ████████
 pRoot 1000



And of course, later we'd have to delete our tree's nodes.

We've created a binary tree... now what?

Now that we've created a binary tree, what can we do with it?

Well, next class we'll learn how to use the binary tree to speed up searching for data.

But for now, let's learn how to iterate through each item in a tree, one at a time.

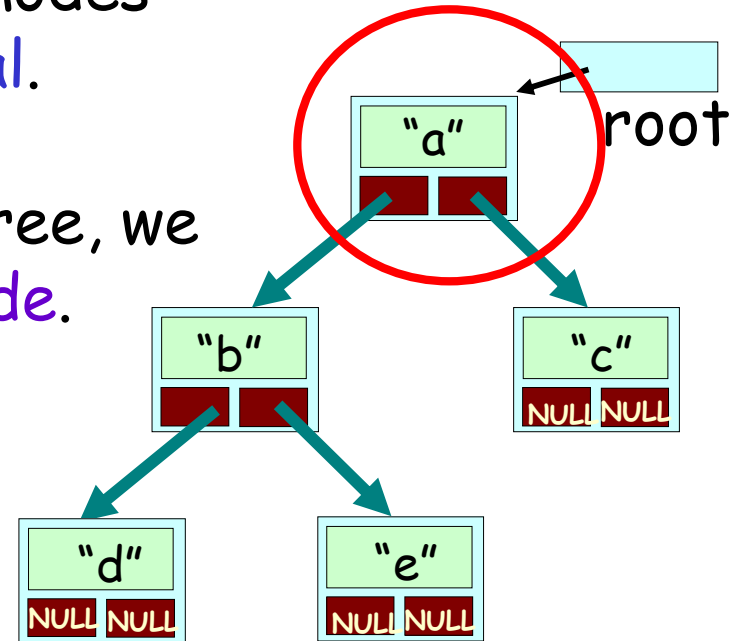
This is called "traversing" the tree, and there are several ways to do it.

Binary Tree Traversals

When we iterate through all the nodes in a tree, it's called a **traversal**.

Any time we traverse through a tree, we always start with the **root node**.

There are four common ways to traverse a tree.



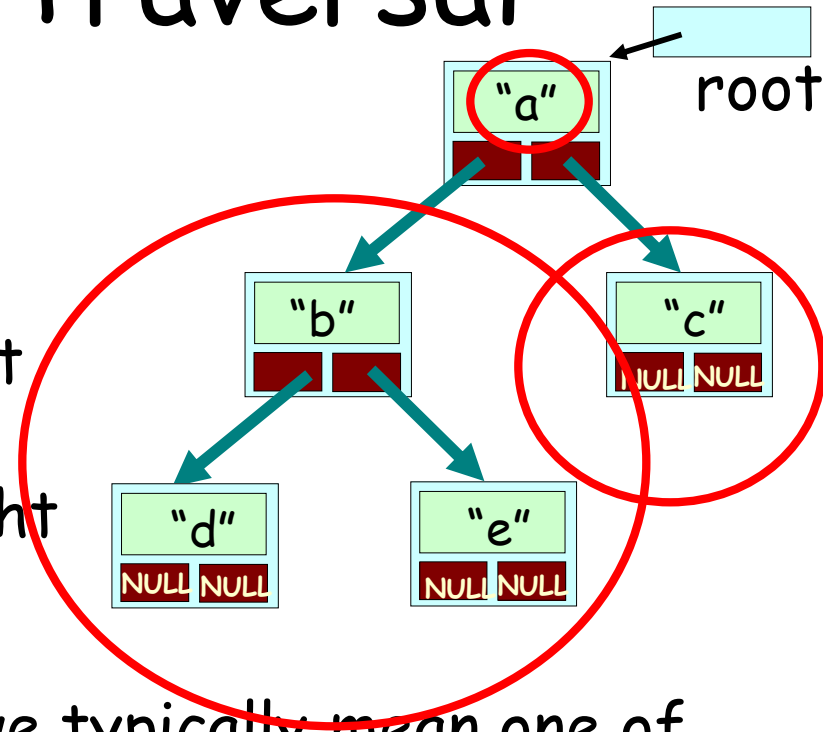
Each technique differs in the **order** that each node is visited during the traversal:

1. Pre-order traversal
2. In-order traversal
3. Post-order traversal
4. Level-order traversal

The Preorder Traversal

Preorder:

1. Process the current node.
2. Process the nodes in the left sub-tree.
3. Process the nodes in the right sub-tree.



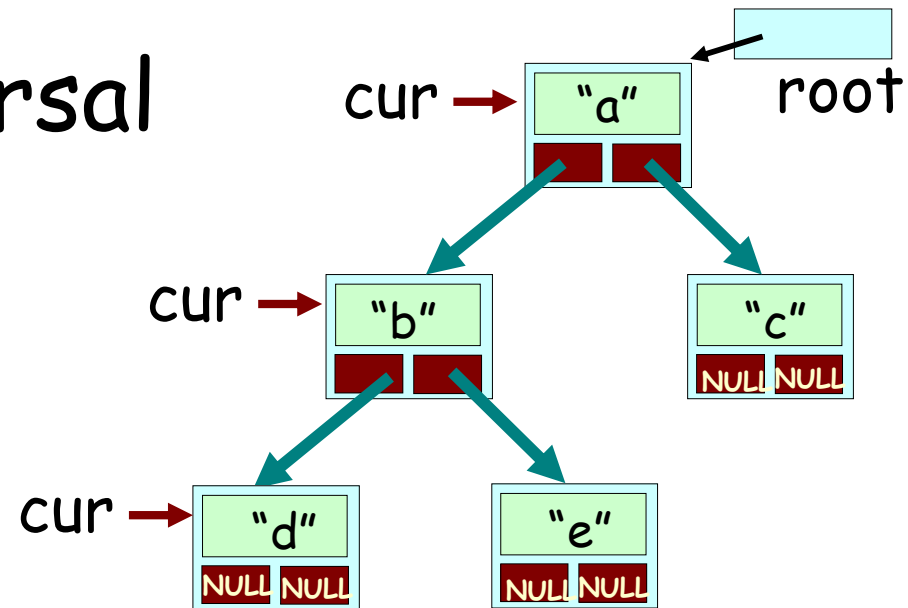
By “**process the current node**” we typically mean one of the following:

1. Print the current node's value out.
2. Search the current node to see if its value matches the one you're searching for.
3. Add the current node's value to a total for the tree
4. Etc...

The Pre-order Traversal

Output:

a b d



```

void PreOrder(Node *cur)
{
    if (cur == NULL)           // if empty, return...
        return;

    cout << cur->value;        // Process the current node.

    PreOrder(cur->left);        // Process nodes in left sub-tree.
    PreOrder(cur->right);       // Process nodes in left sub-tree.
}
    
```

```

main()
{
    Node *root;

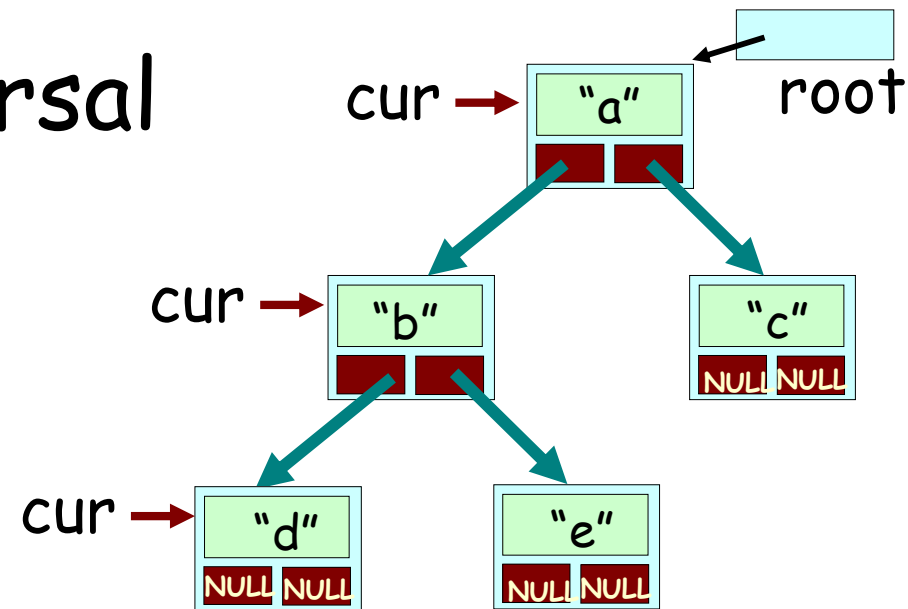
    ...

    PreOrder(root);
}
    
```

The Pre-order Traversal

Output:

a b d



```
void PreOrder(Node *cur)
```

```
{
  if (cur == NULL)      // if empty, return...
    return;
```

```
  cout << cur->value;    // Process the current node.
```

```
  PreOrder(cur->left);    // Process nodes in left sub-tree.
```

```
  PreOrder(cur->right);   // Process nodes in left sub-tree.
```

```
}
```

```
}
```

```
}
```

```
}
```

n()

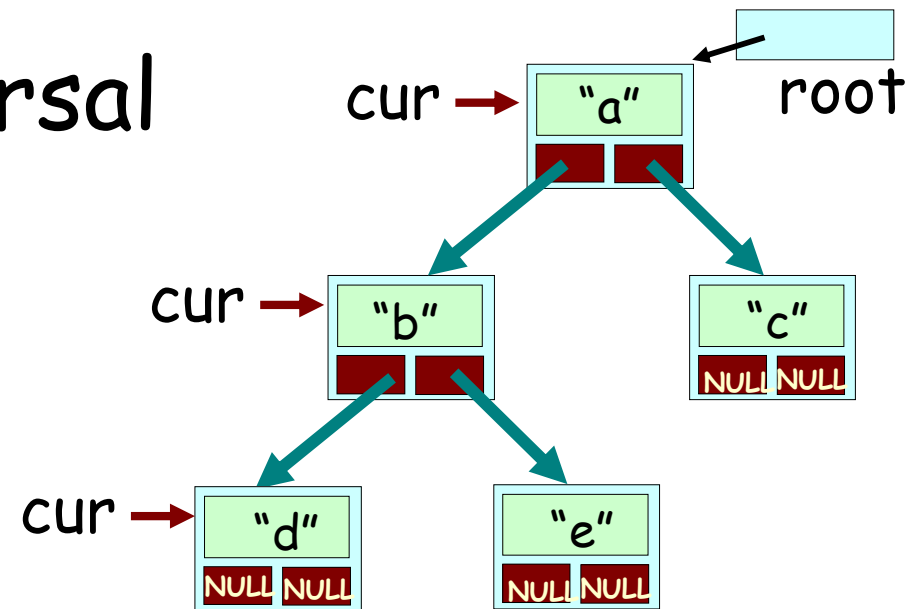
Node *root;

PreOrder(root);

The Pre-order Traversal

Output:

a b d



```
void PreOrder(Node *cur)
```

```
{
{
{   if (cur == NULL)           // if empty, return...
    return;

    cout << cur->value;        // Process the current node.

    PreOrder(cur->left);       // Process nodes in left sub-tree.
    PreOrder(cur->right);      // Process nodes in left sub-tree.
}
}
```

```
main()
```

```
Node *root;
```

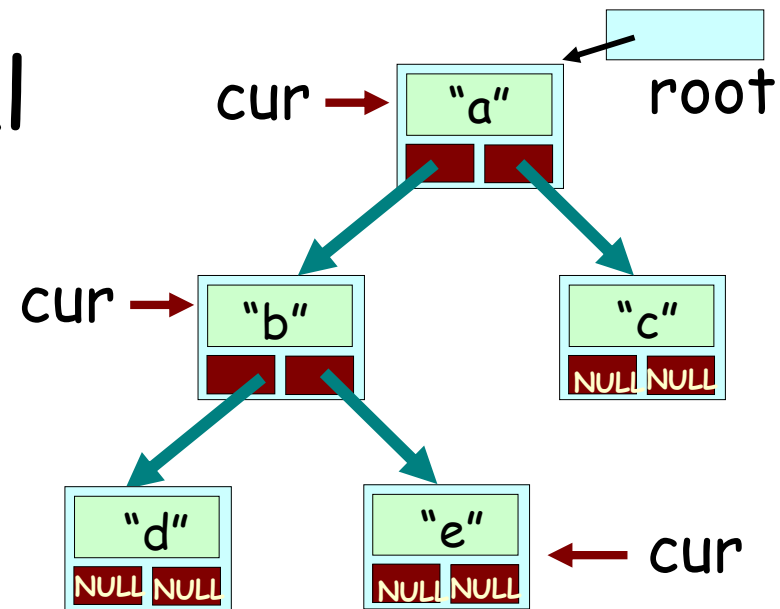
```
...
```

```
PreOrder(root);
```

The Pre-order Traversal

Output:

a b d e



```
void PreOrder(Node *cur)
```

```
{
{
{   if (cur == NULL)           // if empty, return...
    return;

    cout << cur->value;        // Process the current node.
    PreOrder(cur->left);       // Process nodes in left sub-tree.
    PreOrder(cur->right);      // Process nodes in left sub-tree.
}
}
}
```

```
main()
```

```
Node *root;
```

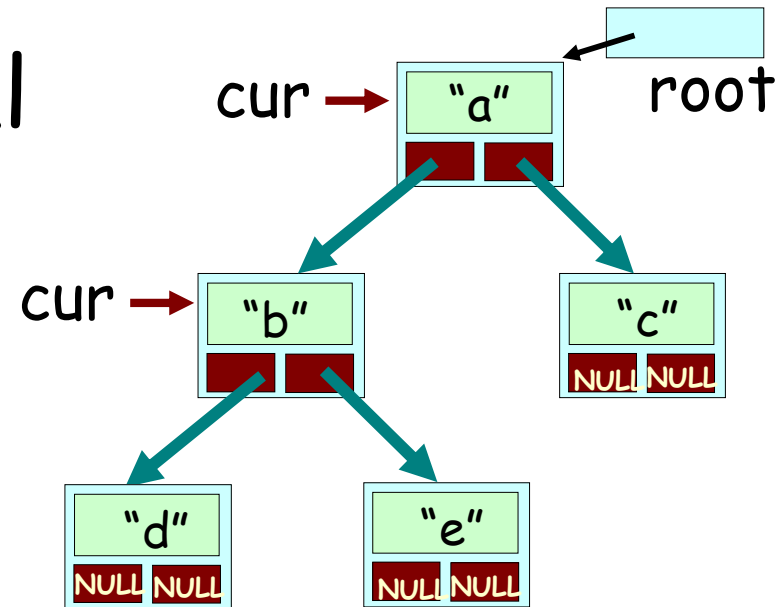
```
...
```

```
PreOrder(root);
```

The Pre-order Traversal

Output:

a b d e



```
void PreOrder(Node *cur)
```

```
{
{   if (cur == NULL)           // if empty, return...
```

```
    return;
```

```
    cout << cur->value;        // Process the current node.
```

```
    PreOrder(cur->left);        // Process nodes in left sub-tree.
```

```
    PreOrder(cur->right);       // Process nodes in left sub-tree.
```

```
}
```

```
}
```

```
main()
```

```
{
```

```
    Node *root;
```

```
    ...
```

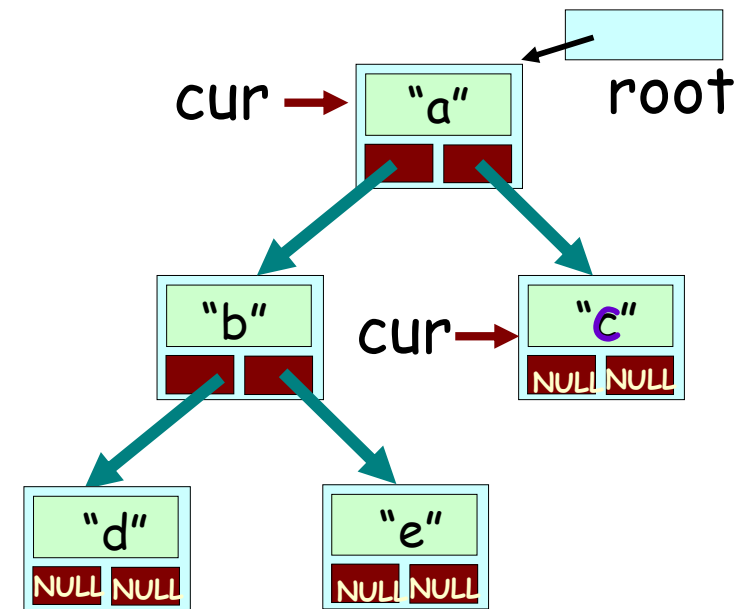
```
    PreOrder(root);
```

```
}
```

The Pre-order Traversal

Output:

a b d e c



```

void PreOrder(Node *cur)
{
    if (cur == NULL)        // if empty, return...
        return;

    cout << cur->value;      // Process the current node.

    PreOrder(cur->left);     // Process nodes in left sub-tree.
    PreOrder(cur->right);    // Process nodes in left sub-tree.
}
  
```

```

main()

Node *root;

...

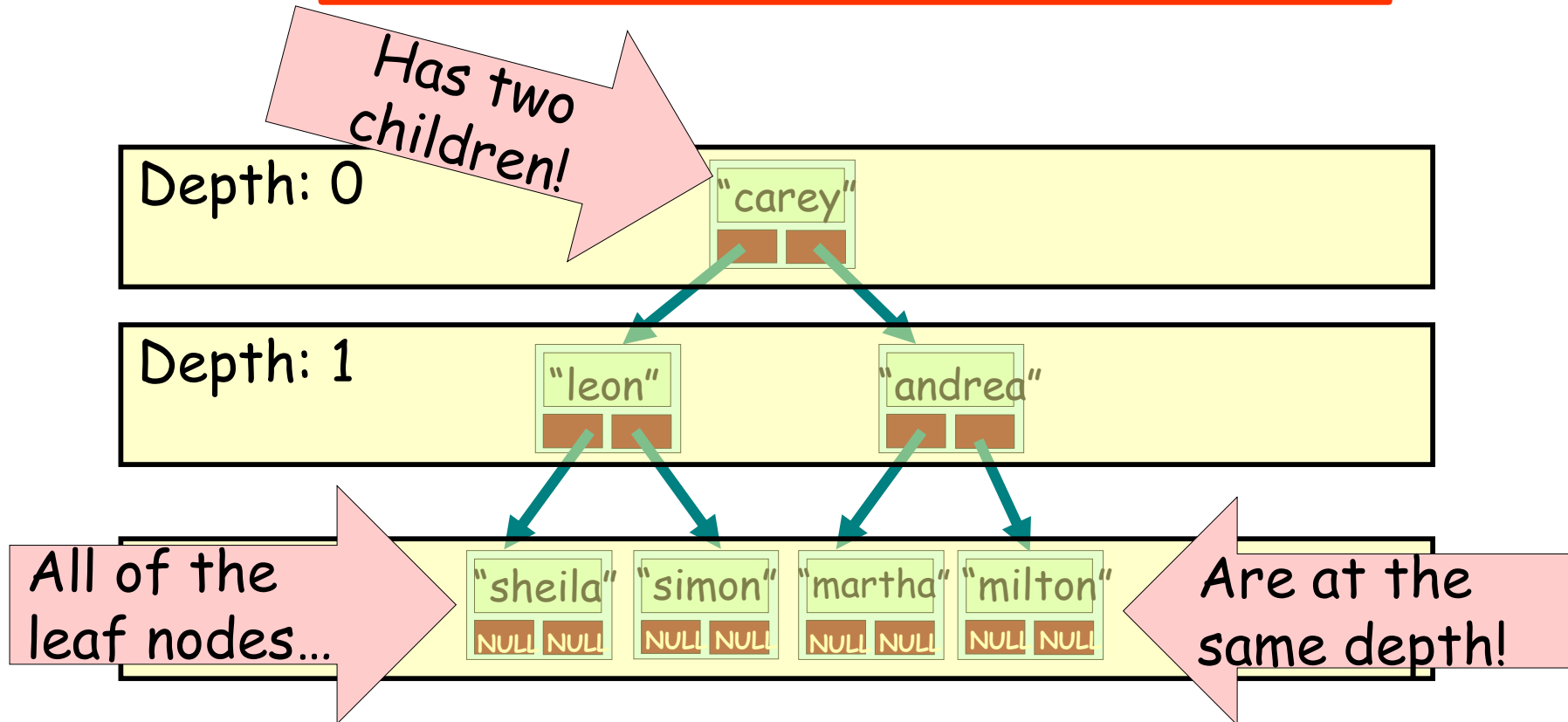
PreOrder(root);
  
```

Appendix - On Your Own Study

- Full Binary Trees

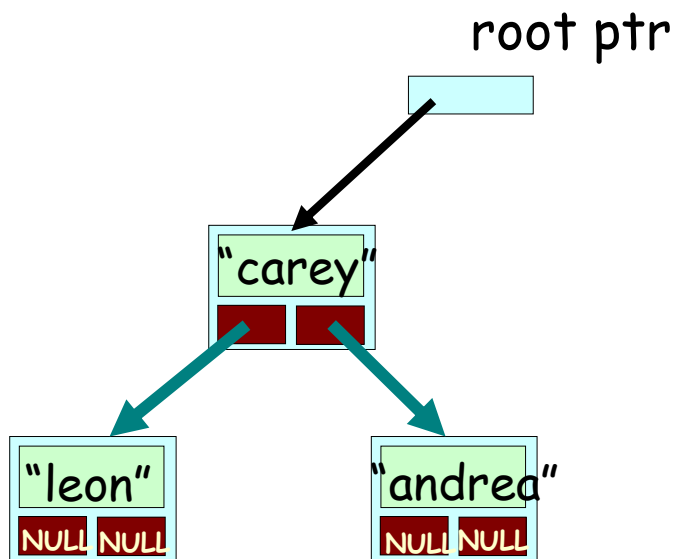
Full Binary Trees

A **full binary tree** is one in which **every leaf node has the same depth,** and **every non-leaf has exactly two children.**

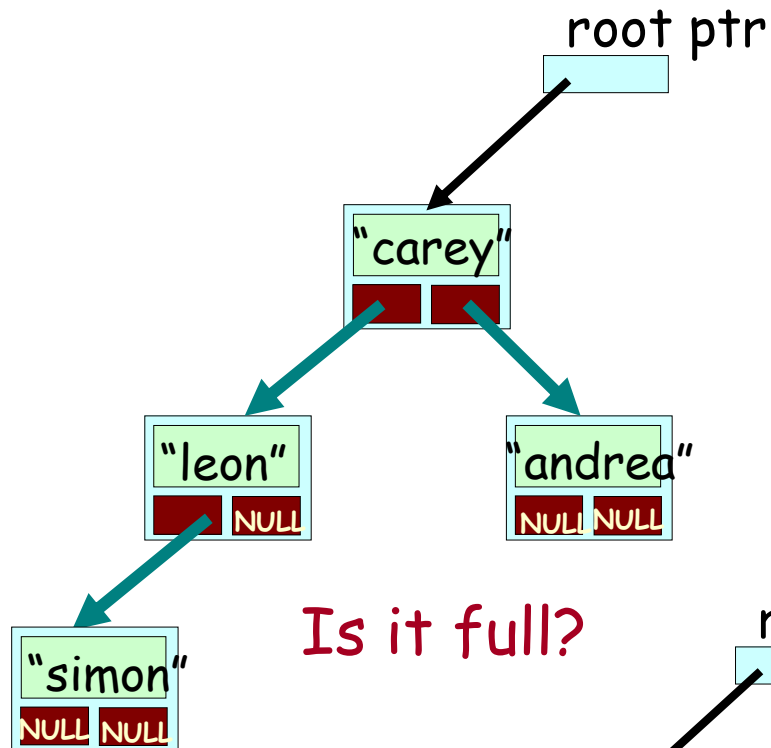


Full Binary Trees

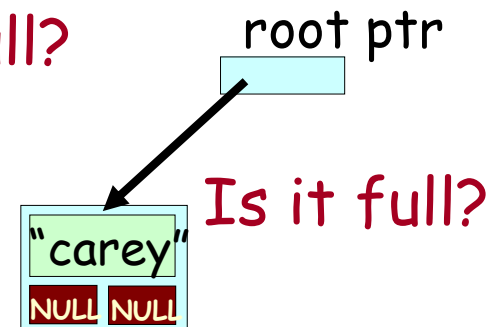
A **full binary tree** is one in which every leaf node has the same depth, and every non-leaf has exactly two children.



Is it full?



Is it full?



Is it full?