

Exercise 7

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Problem 1.

Solution. (a) $p \wedge q$; (b) $p \wedge \neg q$; (c) $\neg p \wedge \neg q$; (d) $p \vee q$. □

Problem 2.

Proof. (a)

$$\neg(p \wedge \neg q) \equiv \neg p \vee \neg \neg q \equiv \neg p \vee q.$$

(b) From the truth table, we conclude that the truth values of LHS and RHS are the same for all truth assignments.

p	q	r	$p \vee q$	$\neg p \vee r$	$\neg p \wedge q$	$p \wedge r$	$(p \vee q) \wedge (\neg p \vee r)$	$(\neg p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	F	T	T	T
T	T	F	T	F	F	F	F	F
T	F	T	T	T	F	T	T	T
T	F	F	T	F	F	F	F	F
F	T	T	T	T	T	F	T	T
F	T	F	T	T	T	F	T	T
F	F	T	F	T	F	F	F	F
F	F	F	F	T	F	F	F	F

□

Problem 3.

Proof. Let \mathcal{J} be a truth assignment such that $\forall \phi \in \Phi, \llbracket \phi \rrbracket_{\mathcal{J}} = \mathbf{T}$. If $\llbracket p \rrbracket_{\mathcal{J}} = \mathbf{T}$, then $\llbracket \neg p \rrbracket_{\mathcal{J}} = \mathbf{F}$, and then $\llbracket r \rrbracket_{\mathcal{J}} = \mathbf{T}$ follows from $\llbracket \neg p \vee r \rrbracket_{\mathcal{J}} = \mathbf{T}$. If $\llbracket q \rrbracket_{\mathcal{J}} = \mathbf{T}$, then $\llbracket \neg q \rrbracket_{\mathcal{J}} = \mathbf{F}$, and then $\llbracket r \rrbracket_{\mathcal{J}} = \mathbf{T}$ follows from $\llbracket \neg q \vee r \rrbracket_{\mathcal{J}} = \mathbf{T}$. By proof by cases, $\llbracket r \rrbracket_{\mathcal{J}} = \mathbf{T}$. □

Problem 4.

Proof. Let $\mathcal{J} : \{p, q\} \rightarrow \{\mathbf{T}, \mathbf{F}\}$ be such that $\llbracket p \rrbracket_{\mathcal{J}} = \mathbf{F}, \llbracket q \rrbracket_{\mathcal{J}} = \mathbf{T}$. Then by definition $\llbracket p \vee q \rrbracket_{\mathcal{J}} = \mathbf{T}, \llbracket \neg p \vee \neg q \rrbracket_{\mathcal{J}} = \mathbf{T}, \llbracket p \wedge \neg q \rrbracket_{\mathcal{J}} = \mathbf{F}$. Thus $\Phi \not\models p \wedge \neg q$. □

Problem 5.

Proof. (a) For each \mathcal{J} such that $\llbracket \phi \rrbracket_{\mathcal{J}} = \mathbf{T}$ and $\forall \chi \in \Phi, \llbracket \chi \rrbracket_{\mathcal{J}} = \mathbf{T}$, $\llbracket \psi \rrbracket_{\mathcal{J}} = \mathbf{T}$ follows from that $\Phi \models \psi$. Thus $\Phi, \phi \models \psi$ by definition.

(b) Suppose $\Phi, \psi \models \chi$. For any \mathcal{J} such that $\forall \phi \in \Phi, \llbracket \phi \rrbracket_{\mathcal{J}} = \mathbf{T}$, if $\llbracket \psi \rrbracket_{\mathcal{J}} = \mathbf{T}$ then $\llbracket \chi \rrbracket_{\mathcal{J}} = \mathbf{T}$ follows from that $\Phi, \psi \models \chi$, and then $\llbracket \neg \psi \vee \chi \rrbracket_{\mathcal{J}} = \mathbf{T}$ by definition; if $\llbracket \psi \rrbracket_{\mathcal{J}} = \mathbf{F}$ then $\llbracket \neg \psi \rrbracket_{\mathcal{J}} = \mathbf{T}$, and then $\llbracket \neg \psi \vee \chi \rrbracket_{\mathcal{J}} = \mathbf{T}$ by definition. By proof by cases, $\Phi \models \neg \psi \vee \chi$.

Suppose $\Phi \models \neg \psi \vee \chi$. For any \mathcal{J} such that $\llbracket \psi \rrbracket_{\mathcal{J}} = \mathbf{T}$ and $\forall \phi \in \Phi, \llbracket \phi \rrbracket_{\mathcal{J}} = \mathbf{T}$, $\llbracket \neg \psi \vee \chi \rrbracket_{\mathcal{J}} = \mathbf{T}$ follows from that $\Phi \models \neg \psi \vee \chi$. $\llbracket \psi \rrbracket_{\mathcal{J}} = \mathbf{T}$ implies that $\llbracket \neg \psi \rrbracket_{\mathcal{J}} = \mathbf{F}$. Thus $\llbracket \chi \rrbracket_{\mathcal{J}} = \mathbf{T}$ follows from that $\llbracket \neg \psi \vee \chi \rrbracket_{\mathcal{J}} = \mathbf{T}$. That implies that $\Phi, \psi \models \chi$. The proof is completed. □

Problem 6.

Proof. (a) Suppose $\phi \models \psi$. Then we only consider the black rows of the truth table (and discard the gray row). We conclude that $\phi \wedge \psi$ and ϕ have the same truth values under all truth assignments, and so do $\phi \vee \psi$ and ψ , as desired.

ϕ	ψ	$\phi \wedge \psi$	$\phi \vee \psi$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

(b) $\phi \wedge \psi \models \phi$ implies that $\phi \vee (\phi \wedge \psi) \equiv (\phi \wedge \psi) \vee \phi \equiv \phi$. $\phi \models \phi \vee \psi$ implies that $\phi \wedge (\phi \vee \psi) \equiv \phi$. That proves the absorption rules.

□