The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Euclid Contest

Wednesday, April 2, 2025
(in North America and South America)

Thursday, April 3, 2025
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: $2\frac{1}{2}$ hours

Do not open this booklet until instructed to do so.

Number of questions: 10 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.

A Note about Bubbling

Please make sure that you have correctly coded your name, date of birth and grade on the Student Information Form, and that you have answered the question about residency.

1. (a) If $4(x - 2) = 2(x - 4)$, what is the value of x?

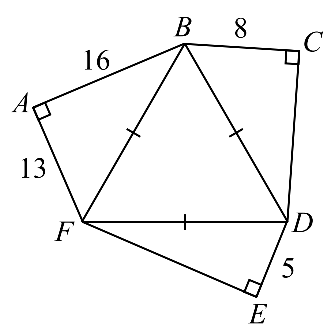
   (b) If $2x = 9$, what is the value of 2^{6x-23}?

   (c) Determine the coordinates of the point of intersection of the lines with equations $y = 3x + 7$ and $y = 7x + 3$.

2. (a) There is one positive integer k for which $3 < \sqrt{k^2 + 4} < 4$. What is this positive integer k?

   (b) What is the sum of the 20 smallest odd positive integers?

   (c) In the diagram, $\triangle BDF$ is equilateral and $\angle FAB = \angle BCD = \angle DEF = 90^\circ$. Also, $AB = 16$, $BC = 8$, $DE = 5$, and $FA = 13$. Determine the perimeter of hexagon $ABCDEF$.

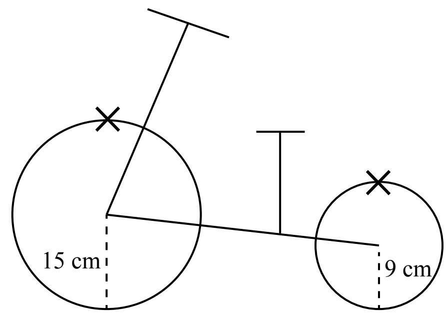


3. (a) Suppose that $p+q+r=18$ and $p+q=5$ and $q+r=9$. What is the value of q?

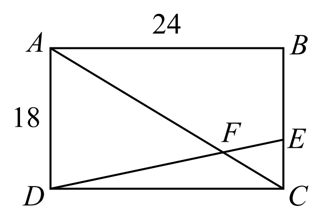
(b) The line with equation $6x+y=24$ has its x-intercept at point P and its y-intercept at point Q. What is an equation of the parabola whose y-intercept is at Q and whose only x-intercept is at P?

(c) Suppose that $\dfrac{1}{2w}=\dfrac{1}{3y}=\dfrac{1}{4z}$ and $\dfrac{1}{2w}+\dfrac{1}{3y}+\dfrac{1}{4z}=\dfrac{1}{24}$. Determine the value of $w+y+z$.

4. (a) Terry's bicycle has a larger front wheel with radius 15 cm and a smaller rear wheel with radius 9 cm, as shown. Terry ties a ribbon to the top of each wheel, and then starts to ride forward. Terry travels d cm forward and stops. Both ribbons are again at the top of the wheels. What is the integer closest to the smallest possible value of d with $d>0$?



(b) In the diagram, $ABCD$ is a rectangle with $AB=24$ and $AD=18$. Also, E is on BC with $EC=6$. If segments DE and AC intersect at F, determine the length of CF.



5. (a) Alice has a lock whose combination consists of three integers a, b, c which need to be entered in that order. The three integers satisfy the following:

- each of a, b and c is between 1 and 40, inclusive;
- a, b and c are all different;
- b is less than a, and b is less than c; and
- one of the integers is 20 and another of the integers is 30.

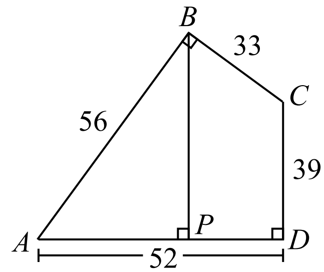
How many possible combinations satisfy these conditions?

(b) For some angles θ, the three numbers $2-2\cos\theta$, $1+\sin\theta$, $2+2\cos\theta$ form a geometric sequence in that order. Determine all possible exact values of $\cos\theta$.

(A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a non-zero constant. For example, 3, 6, 12 is a geometric sequence with three terms.)

6. (a) Twelve points are equally spaced around the entire circumference of a circle. In how many ways can three of these points be chosen so that the triangle that they form has at least two sides of equal length?

(b) In the diagram, quadrilateral $ABCD$ has $\angle ABC = \angle CDA = 90^\circ$, $AB = 56$, $BC = 33$, $CD = 39$, and $DA = 52$. Point P is on AD so that BP is perpendicular to AD. Determine the exact length of BP.



7. (a) The functions f and g are defined by the tables of values below:

x	$f(x)$
1	5
2	3
3	4
4	1
5	2

x	$g(x)$
1	3
2	1
3	4
4	5
5	2

The functions f^{-1} and g^{-1} are the inverse functions of f and g, respectively. If $f^{-1}(g^{-1}(a)) = 3$, what is the value of a?

(b) Determine all pairs (x, y) of real numbers that satisfy the following system of equations:

$$x^2 - 8xy + 16y^2 = 0$$
$$(\log_{10} x)^2 + 2(\log_{10} x)(\log_{10} y) + (\log_{10} y)^2 = 4$$

8. (a) Leilei, Jerome and Farzad write a test independently. The probability that Leilei passes the test and Jerome fails the test is $\frac{3}{20}$. The probability that Jerome passes and Farzad fails is $\frac{1}{4}$. The probability that Leilei and Farzad both pass is $\frac{2}{5}$. Determine the probability that at least one of Leilei, Jerome and Farzad fails the test.

(b) The integer 7447 is a palindrome because it reads the same forwards and backwards. Suppose that positive integers m and n are palindromes between 1001 and 9999, inclusive, with $m > n$. Determine the number of pairs (m, n) for which the difference $m - n$ is a multiple of 35.

9. Suppose that $p(x) = qx^3 - rx^2 - sx + t$ for some positive integers $q < r < s < t$ which form an arithmetic sequence.

(a) Show that $x = 1$ is a root of $p(x)$.

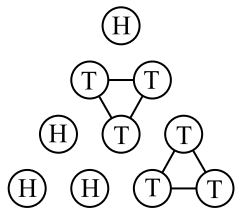
(b) Suppose that the average of q, r, s, t is 19 and that $p(x)$ has three rational roots. Determine the roots of $p(x)$.

(c) Prove that, for every positive integer $n > 3$, there are at least two arithmetic sequences of positive integers $q < r < s < t$ with common difference $2n$ for which $p(x)$ has three rational roots.

(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant, called the common difference. For example, $3, 5, 7, 9$ are the first four terms of an arithmetic sequence.)

10. An equilateral triangle is formed using n rows of coins. There is 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on, up to n coins in the nth row. Initially, all of the coins show heads (H). Carley plays a game in which, on each turn, she chooses three mutually adjacent coins and flips these three coins over. To win the game, all of the coins must be showing tails (T) after a sequence of turns. An example game with 4 rows of coins after a sequence of two turns is shown.

Below (a), (b) and (c), you will find instructions about how to refer to these turns in your solutions.

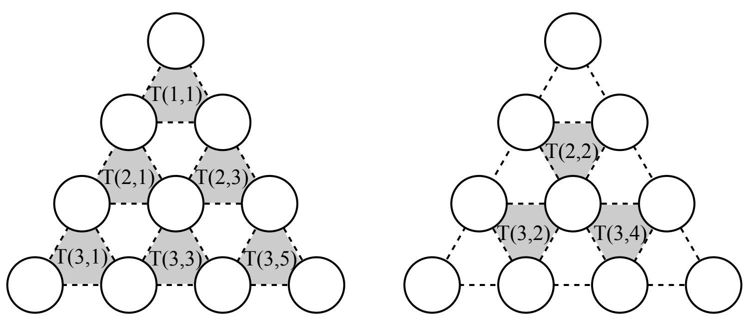


(a) If there are 3 rows of coins, give a sequence of 4 turns that results in a win.

(b) Suppose that there are 4 rows of coins. Determine whether or not there is a sequence of turns that results in a win.

(c) Determine all values of n for which it is possible to win the game starting with n rows of coins.

Note: For a triangle with 4 rows of coins, there are 9 possibilities for the set of three coins that Carley can flip on a given turn. These 9 possibilities are shown as shaded triangles below:



Participants should use the names for these moves shown inside the 9 shaded triangles when answering (b). Participants should adapt this naming convention in a suitable way when answering parts (a) and (c).

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For students...

Thank you for writing the 2025 Euclid Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

If you are graduating from secondary school, good luck in your future endeavours! If you will be returning to secondary school next year, encourage your teacher to register you for the 2025 Canadian Senior Mathematics Contest, which will be written in November 2025.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2025/2026 contests
- Look at our free online courseware
- Use our free Problem Set Generator to create problem sets for curriculum support and enrichment
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results