Asynchronous Distributed Charging Protocol for Plug-in Electric Vehicles

***Abstract*—The proliferation of plug-in electric vehicles (PEVs) advocates a distributed paradigm for the coordination of PEV charging. Distinct from existing primal-dual decomposition or consensus methods, this paper proposes a cutting-plane based distributed algorithm considering a peer-to-peer (P2P) communication network, which enables asynchronous coordination among PEVs. First, an equivalent surrogate model is established by exploiting the duality of the original optimization problem, which masks the private information of individual users by equivalent transformation. Then, a cutting-plane based algorithm is derived to solve the surrogate problem in a distributed manner with intrinsic superiority to cope with various asynchrony. Critical implementation issues, including the distributed initialization, distributed cutting-plane generation, and localized stopping criteria, are discussed in detail. Numerical tests on IEEE 37- and 123-node feeders with real data show that the proposed method is resilient to a variety of asynchrony and admits the plug-and-play operation mode. It is expected the proposed methodology provides an alternative path toward the robust protocol for PEV charging.**

***Index Terms*—Asynchronous algorithm, charging protocol, distributed optimization, plug-in electrical vehicles.**

I. Introduction

The past years witnessed the proliferation of plug-in electric vehicles (PEVs). However, their rapid growth inevitably creates new challenges to power system operation. Particularly, as traditional distribution systems were not designed to support simultaneous charging of many PEVs [1], transformer capacity expansion or even reconstruction of the distribution system are needed to meet the growing demand for PEV charging. However, such costly countermeasures could be alleviated or even avoided if the charging behaviors of PEVs are well managed, in either a centralized or distributed manner.

Traditional centralized management of PEV charging needs to collect all PEVs' information, such as positions, available charging time and state of charge (SOC), etc. Hence it may raise severe privacy concerns in individual PEV owners. Moreover, the decision center may suffer from a heavy communication burden and high computational complexity. In this regard, distributed management was developed, where the charging patterns of PEVs are decided locally following a certain coordination scheme. It is expected to better protect the privacy of PEV owners and enable a faster response to environmental changes [2], which is crucially important when numerous PEVs disperse across the distribution network. However, there always exist various kinds of asynchrony in practice due to non-ideal communication such as time delay and packet drop. These regards motivate us to address the protocol of PEV distributed charging in this paper, considering asynchrony resilience and privacy-preserving.

Generally, prior works on distributed PEV charging management can be cast into two branches: non-cooperative strategies and cooperative ones, which are briefly reviewed as follows.

1) Non-cooperative Strategies: Non-cooperative charging strategies oftentimes are partially distributed, where a coordinator is needed to broadcast coordination signals (usually electricity price) and then each PEV reacts to its received signals. One typical distributed non-cooperative strategy relies on a one-way communication broadcast, but the open-loop approach appears to be less effective due to the absence of feedback adjustments. To address this problem, iterative strategies emerge to search for the optimal or quasi-optimal charging profiles, where bi-directional communication between the coordinator and individual PEVs is required and each PEV solves a restricted sub-problem in every round. Under this framework, several distributed charging algorithms are derived, based on Lagrangian dual decomposition [3], projected gradient [4], [5], alternating direction method of multipliers (ADMM) [6]–[9] and theory of non-cooperative games [10], [11], to name a few.

2) Cooperative Strategies: Cooperative strategies are usually investigated under a fully distributed framework, where PEVs collaborate (usually with their immediate neighbors) to achieve a certain optimal target [12]. Such peer-to-peer (P2P) based schemes [13] serve as more flexible, scalable, and robust alternatives since individual PEVs can autonomously achieve coordination with the absence of a coordinator. In this regard, distributed solution algorithms based on Karush-Kuhn-Tucker (KKT) conditions and consensus techniques are designed, see, for example, [13]–[15].

The potential of P2P based cooperative schemes, however, has not been well addressed yet when it comes to the following two critical implementation issues: i) User-state-information (USI) privacy. PEV users are reluctant to disclose their USI (such as SOC, positions, and demand profiles, etc.) neither to a center nor to other users. In regard to the information exchange which is necessary for coordination in a P2P network, privacy issue also remains a concern. ii) Imperfect communication. Considering time delays, packet drops, topology changes, and non-identical computation capabilities of individual users, the participants have to wait for the slowest one to finish before executing their local actions in the next iteration. Though prior works [16]–[18] have achieved many successes on distributed algorithms under asynchronous communication, their convergence results and solution quality rely on the assumptions of maximum time delay. Moreover, it is not trivial to select appropriate parameters and adjust the step size in the updating process.

The cutting-plane theory [19] has recently drawn increasing attention in the community of distributed decision-making.

However, there are few works considering its application to power systems, except [20] and [21] that discuss dynamic economic dispatch and microgrid control, respectively. Different from traditional consensus algorithms, a cutting-plane based consensus algorithm can achieve an agreement on a common query point through iterative constraint exchanges. This salient feature can better decompose the computation into individual agents with the minimal requirements of information synchrony, which inspires a suitable framework for the distributed PEV charging coordination against asynchrony. Existing cutting-plane algorithms, however, only provide results on the limit of the sequence and the optimizer has no idea when to stop the iteration since consensus does not necessarily imply optimality.

In this paper, a novel P2P protocol is proposed for PEVs charging coordination aiming at optimizing the aggregated charging power with both local and global constraints, in consideration of communication asynchrony and privacy preserving. The main contributions of this paper are threefold.

1) *Coordinator-free.* A fully-distributed scheduling method based on cutting-plane consensus is proposed for the coordination of PEV charging. The global optimum is achieved without a central coordinator or any security constraints violated. Moreover, unlike previous works [3]–[9] where charging profiles or multipliers of the corresponding optimization problem are collected or exchanged, the proposed method only involves the aggregated information of a surrogate model, which better protects the private user-state-information of PEV owners.

2) *Asynchrony-resilience.* Distinct from primal-dual decomposition methods [16]–[18] and ADMM algorithms [6]–[9], [22] where primal/dual variables are exchanged, the proposed method is designed to exchange cutting planes among neighboring individuals. The resulting algorithm intrinsically admits asynchronous implementations, favoring a strong resilience to a broad variety of asynchrony in practice such as time delays, packet drops, communication topology changes, and non-identical local clocks in local processors.

3) *Convergence-guarantee.* The existing cutting-plane based works heuristically take any locally converged consensus result as the optimal solution [20], [21], which lacks a theoretic convergence guarantee. This paper first unfolds the solution quality of the algorithm and derives a provable stopping criterion, which allows a *completely localized* implementation.

The rest of the paper is organized as follows. The problem description with necessary notations is stated in Section II and Section III derives an equivalent surrogate model. Section IV-V present the distributed solution algorithm based on cutting-plane consensus. Case studies are introduced in Section VI. Finally, Section VII concludes the paper.

II. Notations and Problem Formulation

A. Notations

In this paper, $\mathbb{R}^n$ ($\mathbb{R}^n_+$) depicts the n-dimensional (non-negative) Euclidean space. Use $\mathbb{Z}^+$ to denote the set of positive integers. For a column vector $z \in \mathbb{R}^n$ (matrix $A \in \mathbb{R}^{m\times n}$), $z^\mathsf{T}(A^\mathsf{T})$ denotes its transpose. $\|\cdot\|$ denotes the Euclidean norm. We define the distance from a point x_0 to set $\mathcal{X}$ as $\mathrm{dist}(x_0,\mathcal{X}) := \min_{x\in\mathcal{X}} \|x - x_0\|^2$. For a set $\mathcal{X}$, $|\mathcal{X}|$ denotes the number of elements $\mathcal{X}$. For $x \in \mathbb{R}^1$, $|x|$ denotes the absolute value of x. Given a collection of y_i for i in a certain set $\mathcal{N}$, define $\mathrm{col}(y_j) := (y_1, y_2, \cdots, y_n)^\mathsf{T}$ and denote its vector form by $y := \mathrm{col}(y_i)$. We use $\mathbf{1}$ (resp. $\mathbf{0}$) to denote vector of ones (resp. zeros).

Notations for cutting-plane are given as follows. Given a convex set $\mathcal{S} \subset \mathbb{R}^n$ and a query point $z_q \notin \mathcal{S}$, a half-space

$$h_{z_q} := \left\{ z | a_{z_q}^\mathsf{T} z \le b_{z_q} \right\}, a_{z_q} \in \mathbb{R}^n, b_{z_q} \in \mathbb{R}^1 \qquad (1)$$

is referred to as the cutting-plane of $\mathcal{S}$ and z_q if it satisfies the following properties: (i) $a_{z_q} \neq \mathbf{0}$, (ii) $a_{z_q}^\mathsf{T} z_q > b_{z_q}$ and (iii) $a_{z_q}^\mathsf{T} z \le b_{z_q}, \forall z \in \mathcal{S}$.

Cutting-plane set $H := \cup_{k=1}^m h_k$ is the collection of m single cutting-planes where $h_k := \left\{ z | a_k^\mathsf{T} z \le b_k \right\}$. The induced polyhedron of H is denoted by $\mathcal{H} := \left\{ z | A_H^\mathsf{T} z \le b_H \right\}$, with $A_H := [a_1, \ldots, a_m]$ and $b_H := [b_1, \ldots, b_m]^\mathsf{T}$.

Note that in this paper the union symbol (same for intersection symbol) plays an opposite role on cutting-plane set and the induced polyhedron. For example, given two cutting-plane sets H_1 and H_2, we have $H = H_1 \cup H_2 \Leftrightarrow \mathcal{H} = \mathcal{H}_1 \cap \mathcal{H}_2$, where $\mathcal{H}_1$, $\mathcal{H}_2$ and $\mathcal{H}$ are the polyhedrons directly induced by H_1, H_2 and H, respectively. For convenience of phrasing, let $h^\emptyset := \left\{ z | \mathbf{0}^\mathsf{T} z \le 0 \right\}$ denote an empty cutting-plane.

B. Charging of PEVs

To coordinate PEV charging in a fully distributed manner, we adopt the configuration shown in Fig.1. Each PEV charger is equipped with a local processor with a certain capability of communication and computation. The processor collects data from the user (such as the designated SOC and charging deadline) and data from the PEV (such as the initial SOC and capacity of batteries). The local processor also receives some broadcast information such as the electricity price and enables bi-directional information exchange with its neighbors. Once the optimal charging strategy is derived, the control command is generated and sent to the PEV charger.

C. Communication Network

We consider a generalized asynchronous communication protocol presented in [17], where each processor has its own concept of time defined by a *local clock* $k_i \in \mathbb{Z}^+$. k_i triggers when processor i awakes, independently of other processors, to conduct local computations and update information to its neighbors. When i is idle, it listens for messages from neighbors and stores them to its receiving cache. Then let $k \in \mathbb{Z}^+$ denote a virtual *global clock* that does not exist in reality and is used only for analysis. The relationship between local clocks and the global clock is depicted in Fig.2.

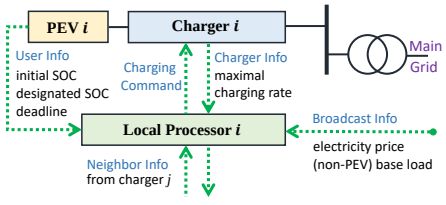

Fig. 1. The schematic of the proposed local processor.

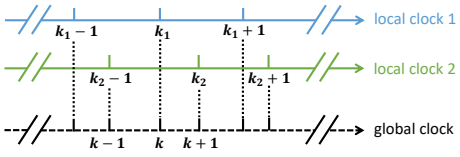

Fig. 2. An instance of two local clocks and the virtual global clock.

To consider the change of communicating network, we model the topology at global clock k as a k-dependent directed graph $\{\mathcal{G}_k = (\mathcal{N}, \mathcal{E}_k)\}_{k\in\mathbb{Z}^+}$, where $\mathcal{N} := \{1, 2, \ldots, n\}$ is the set of local charger processors and $\mathcal{E}_k \subseteq \mathcal{N} \times \mathcal{N}$ is the edge set at k. If processor i transmits messages to processor j at k, there is an edge from node i to j at k, denoted by $(i, j) \in \mathcal{E}_k$. For an edge set $\mathcal{E}_k$, we denote by $\mathcal{N}^-_{i,\mathcal{E}_k} := \{j|(i, j) \in \mathcal{E}_k\}$ the out-neighbors of processor i and by $\mathcal{N}^+_{i,\mathcal{E}_k} := \{j|(j, i) \in \mathcal{E}_k\}$ its in-neighbors.

In this paper, we assume a minimal assumption on the connectivity of the time-varying network $\{\mathcal{G}_k\}_{k\in\mathbb{Z}^+}$ as follows.

***Assumption* 1** ($\overline{\mathrm{T}}$-strongly connected [23])**.** There exists $\overline{\mathrm{T}} \in \mathbb{Z}^+$ such that the graph with edge set $\bigcup_{\tau=s\overline{\mathrm{T}}}^{(s+1)\overline{\mathrm{T}}-1} \mathcal{E}_\tau$ is strongly connected for every $s \in \mathbb{Z}^+$.

Assumption 1 says, the information propagation between those processors that communicate directly is bounded by $\overline{\mathrm{T}}$ consecutive global time clocks. It is a typical setting in multi-agent optimization and control under a changing topology.

D. Battery Model of PEVs

Consider there are n PEVs to be charged over time horizon $\mathcal{T}$. Denote by $\mathcal{N}$ the set of PEV processors. For processor i, let $p_{it} \in \mathbb{R}^1$ denote the charging power of PEV i at time t and we use $p_i \in \mathbb{R}^{|\mathcal{T}|}$ to denote the column vector of p_{it} over the entire time horizon of $\mathcal{T}$ for simplicity, i.e., $p_i := \mathrm{col}(p_{it})$.

Each PEV is available for load dispatch once it is plugged in and before the charging is completed. For $i \in \mathcal{N}$, limits on the total charging amount should be satisfied, leading to

$$(\mathrm{SOC}_i^{\mathrm{d}} - \mathrm{SOC}_i^0) \cdot \mathrm{Cap}_i \le \eta_i \cdot \sum\nolimits_{t\in\mathcal{T}} p_{it} \tag{2a}$$

$$(\mathrm{SOC}_i^{\max} - \mathrm{SOC}_i^0) \cdot \mathrm{Cap}_i \ge \eta_i \cdot \sum\nolimits_{t\in\mathcal{T}} p_{it} \tag{2b}$$

SOC_i^0 and $\mathrm{SOC}_i^{\mathrm{d}}$ stand for the initial and designated state of charge of PEV i respectively. Cap_i is its battery capacity and η_i scales the charging efficiency. Constraint (2b) implies that the charging process stops once the battery reaches its maximum state of charge $\mathrm{SOC}_i^{\max}$.

Each PEV can charge only after it plugs in at a certain time slot $t_i^0 \in \mathcal{T}$ and before it leaves at $t_i^{\mathrm{d}} \in \mathcal{T}$, where $t_i^0 < t_i^{\mathrm{d}}$. Hence we have constraints (3) for PEV i, enforcing p_{it} be zero when t is out of the range between t_i^0 and t_i^{d}.

$$p_{it} \cdot (t - t_i^0) \ge 0, \ \forall t \in \mathcal{T} \tag{3a}$$

$$p_{it} \cdot (t - t_i^{\mathrm{d}}) \le 0, \ \forall t \in \mathcal{T} \tag{3b}$$

At each time slot, the charging power of a PEV is assumed constant but can vary from 0 to its maximum charging power $p_i^{\max}$ at different time slots. Thus, we have

$$0 \le p_{it} \le p_i^{\max}, \ \forall t \in \mathcal{T}. \tag{4}$$

Combining 2-4, the individual feasible charging region over the time horizon of $\mathcal{T}$ is depicted by (5). Note that $\mathcal{P}_i$ should be kept to PEV processor i itself, as the information about involved arrival time, departure time and battery state are regarded as private.

$$\mathcal{P}_i := \{\ p_i \in \mathbb{R}^{|\mathcal{T}|} \mid p_i \text{ satisfies (2)-(4) }\},\ i \in \mathcal{N} \tag{5}$$

E. Coordination of PEV Charging

The coordination of PEV charging (CoC) aims to minimize the total cost while satisfying system operation constraints and individual charging demands. It can be formulated as follows.

$$\text{CoC}: \quad \min \ \sum\nolimits_{i\in\mathcal{N}} f_i(p_i) \tag{6a}$$

$$\text{s.t.} \quad \sum\nolimits_{i\in\mathcal{N}} p_i \le \mathrm{F} \quad (\text{dual multiplier: } \pi) \tag{6b}$$

$$p_i \in \mathcal{P}_i, \forall i \in \mathcal{N}. \tag{6c}$$

where, p_i is a $|\mathcal{T}|$-dimensional decision vector representing the charging power of PEV i during time horizon $|\mathcal{T}|$. The charging cost of PEV i is denoted by a convex function $f_i(p_i) : \mathbb{R}^{|\mathcal{T}|} \to \mathbb{R}$ with respect to p_i.

Congestion due to feeder head capacity limit is considered in (6b). The right-hand-side parameter F is a $|\mathcal{T}|$-dimension vector that stands for the maximum available total charging power to avoid overload on the feeder head at each time slot. F is determined by the distribution system operator (DSO) and delivered to at least one processor. Note that F can also be properly designed to achieve a valley-filling purpose or other demand response aims. Besides the coupling constraints, individual charging demand is captured by (6c) where $\mathcal{P}_i$ is the feasible region of processor i over the time horizon of $\mathcal{T}$ and its specific form is given by (5). In practice, usually $f_i(\cdot)$, $\mathcal{P}_i$ and p_i are private information of PEV i, which is accessed by processor i only and should not be disclosed to others.

For the convex problem CoC (6), we make the following regular assumptions throughout the paper.

***Assumption* 2.** (i) The functions f_i are σ_i-strongly convex with respect to Euclidean norm $\|\cdot\|$ [24, Chapter 9.1.2].

(ii) The Slater's condition [24, Chapter 5.2.3] holds for CoC, i.e., there exist p_i in the interior of $\mathcal{P}_i$ such that (6b) holds.

Note that if Assumption 2 (i) does not hold, we can apply smoothing techniques by adding sufficiently small quadratic items to f_i without revising the optimal solution [25]. Assumption 2 (ii) is made to guarantee strong duality holds.

Definition 1 (Primal Optimal)**.** $(p_i^*, f_i^*)_{i\in\mathcal{N}}$ is primal optimal to the CoC problem if p_i^* is optimal for (6). Also, we take the notations that $f_i^* := f_i(p_i^*)$ and the vector $p^* \in \mathbb{R}^{n|\mathcal{T}|}$ being the collections of $\{p_i^*\}_{i\in\mathcal{N}}$.

III. A Surrogate Model of CoC

In this section, first we decompose (6) by constructing an equivalent transformation, then derive a surrogate problem to mask the private information of individual PEVs.

A. Dual Decomposition

First of all, the Lagrangian function [24, Chapter 5.1.1] of (6) is given by

$$L(\pi, p) = \sum\nolimits_{i\in\mathcal{N}} f_i(p_i) + \pi^{\mathsf{T}}\left(-\mathrm{F} + \sum\nolimits_{i\in\mathcal{N}} p_i\right) \quad (7)$$

where $\pi \in \mathbb{R}^{|\mathcal{T}|}$ is the dual variable vector corresponding to the global constraint (6b). We can observe that the Lagrangian function can be written in the following separable form

$$L(\pi, p) = \sum\nolimits_{i\in\mathcal{N}} L_i(\pi, p_i) \quad (8a)$$

$$L_i(\pi, p_i) = \begin{cases} f_i(p_i) + \pi^{\mathsf{T}}(p_i - \mathrm{F}), i = i^* \\ f_i(p_i) + \pi^{\mathsf{T}} p_i, \forall i \in \mathcal{N}\backslash\{i^*\} \end{cases} \quad (8b)$$

where i^*, the one who is informed of the value of F, is (randomly) pre-determined. Then define a set of local dual functions as

$$D_i(\pi) := \min_{p_i\in\mathcal{P}_i} L_i(\pi, p_i), i \in \mathcal{N}. \quad (9)$$

Note that $-D_i(\pi)$ are convex functions with respect to π [24, Chapter 3.2.3]. Then the dual problem of (6) is formulated into a convex maximization problem as follows.

$$\max_{\pi\geq 0} \sum\nolimits_{i\in\mathcal{N}} D_i(\pi) \quad (10)$$

The transformation from (6) to (10) is built on the Lagrangian decomposition [26, Chapter 4.3.1], by dualizing the coupling constraint (6b) to obtain a separate structure as in (10). The equivalence is ensured by noting the strong duality holds under Assumption 2. We denote by $p_i(\pi)$ the unique optimal solution to the inner problem with local dual functions.

$$p_i(\pi) := \arg\min_{p_i\in\mathcal{P}_i} L_i(\pi, p_i), \forall i \in \mathcal{N} \quad (11)$$

Once the optimal solution of (10) is obtained, the optimal solution to the primal CoC problem (6) is uniquely determined by (11). Note that, without knowing the $f_i(\cdot)$ and $\mathcal{P}_i$ of other PEV processors, one cannot infer other PEV's optimal charging profile even if the optimal π is disclosed, since the dual decomposition process protects the private information.

B. Further Transformation

Inspired by Dantzig-Wolfe decomposition [27], we construct a transformation on (10). New decision variables of the reformulated optimization problem, $z \in \mathbb{R}^{|\mathcal{T}|+n}$, consists of two parts: $\pi \in \mathbb{R}^{|\mathcal{T}|}$ and $d \in \mathbb{R}^n$, as shown in (12). $d := \mathrm{col}(d_i)$ where d_i is introduced to replace $D_i(\pi)$ in (10).

$$z := [\underbrace{\pi_1, \ldots, \pi_{|\mathcal{T}|}}_{\pi\in\mathbb{R}^{|\mathcal{T}|}}, \underbrace{d_1, \ldots, d_n}_{d\in\mathbb{R}^n}]^{\mathsf{T}} \quad (12)$$

The relationship between π and d_i, which is originally described by $d_i = D_i(\pi)$, is now captured by the feasible region of z which is denoted by $\mathcal{Z}_i$ and identified as

$$\mathcal{Z}_i := \{z | \pi \geq 0; d_i \leq D_i(\pi)\}, \forall i \in \mathcal{N}. \quad (13)$$

Note that $\mathcal{Z}_i$ is convex set since $-D_i(\pi)$ is a convex function with respect to π. Then, the dual problem (10) is equivalently converted into the convex maximization problem (14)

$$\max_z e^{\mathsf{T}} z, \text{ s.t. } z \in \mathcal{S}^{\mathrm{D}} \quad (14)$$

where, $e := (\mathbf{0}^{\mathsf{T}}, \mathbf{1}^{\mathsf{T}})^{\mathsf{T}}$ and $\mathcal{S}^{\mathrm{D}} := \cap_{i\in\mathcal{N}} \mathcal{Z}_i$.

IV. Asynchronous Distributed Algorithm

In this section, we develop a cutting-plane based distributed algorithm to solve the surrogate optimization problem (14) over a completely asynchronous P2P communication network while preserving private information of PEV processors.

A. Cutting-Plane based Asynchronous Distributed Algorithm

Considering each PEV processor runs at its local clock k_i, the asynchronous algorithm is derived as in Algorithm 1.

Algorithm 1 Asynchronous PEV Charging for processor i

Input: The local feasible region $\mathcal{P}_i$ of processor i

Iteration at k_i: Suppose processor i's clock ticks at k_i. Then it is activated to update its cutting-plane set as follows:

Step 1: Reading Phase

Get cutting-plane set from its in-neighbors' output cache. Generate temporary cutting-plane set according to (15).

$$H^{[i]}_{\mathrm{tmp}}(k_i) = \left(\cup_{j\in\mathcal{N}_i^+} H^{[j]}(k_j)\right) \cup H^{[i]}(k_i) \quad (15)$$

Step 2: Computation Phase

Solve linear programming

$$z^{[i]}(k_i) := \arg\max_z e^{\mathsf{T}} z - \rho\|z\|^2 \text{ s.t. } z \in \mathcal{H}^{[i]}_{\mathrm{tmp}}(k_i) \quad (16)$$

where a regularization item $-\rho\|z\|^2$ is added to the objective $e^{\mathsf{T}} z$ to derive the unique optimal solution that has minimal Euclidean norm and ρ is a sufficiently small positive number. $\mathcal{H}^{[i]}_{\mathrm{tmp}}(k_i)$ is the polyhedron induced by $H^{[i]}_{\mathrm{tmp}}(k_i)$. Then, shrink $H^{[i]}_{\mathrm{tmp}}(k_i)$ by remaining the set of active constraints.

Based on $z^{[i]}(k_i)$, generate a new cutting-plane $h_i(z^{[i]}(k_i))$ the specific form of which is given in (19). Then, update local cutting-plane set according to (17).

$$H^{[i]}(k_i + 1) = H^{[i]}_{\mathrm{tmp}}(k_i) \cup h_i(z^{[i]}(k_i)) \quad (17)$$

Step 3: Writing Phase

Write $H^{[i]}(k_i + 1)$ to its output cache. Update local clock by $k_i = k_i + 1$.

The basic idea of the Algorithm 1 is as follows. A set of cutting planes are generated by each processor and is individually updated in every round of iterations. After reading the cutting-plane set from neighbors' output cache, the local processor collects all received cutting-planes and its own cutting-planes to form a polyhedron, denoted by $\mathcal{H}^{[i]}_{\mathrm{tmp}}(k_i)$ for processor i in the k_i round of local iteration. The polyhedron can be regarded as an approximation of the feasible region $\mathcal{S}^{\mathrm{D}}$. Moreover, in each round of iteration, additional cutting planes are added to constantly shrink the polyhedron, leading to a more and more accurate estimation. Mathematically, this procedure is similar to the outer approximation method. Since $\mathcal{H}^{[i]}(k_i) \subseteq \mathcal{H}^{[i]}(k_i - 1)$ holds for each iteration, $\mathcal{H}^{[i]}(k_i)$ will eventually approach $\mathcal{S}^{\mathrm{D}}$. In this way, each local processor only needs to know its own $\mathcal{Z}_i$, and iteratively approaches the feasible region $\mathcal{S}^{\mathrm{D}}$. Once the consensus on $\mathcal{S}^{\mathrm{D}}$ is achieved, the consensus value of z is obtained. Then each PEV processor can extract its optimal charging profile via (11).

For Algorithm 1, we have the following useful remark.

Remark 1 (Asynchrony-resilience)**.** Note that no global clock is required in Algorithm 1, implying intrinsic permission for asynchronous computation and updates. As demonstrated in

Section VI, the proposed distributed algorithm is also resilient to other imperfect communication such as packet drops.

B. Distributed Initialization of Cutting-planes

To develop the cutting-plane based distributed algorithm, each local processor has to generate an initial cutting-plane set $H^{[i]}(0)$ without knowing the whole picture of the feasible region $\mathcal{S}^{\mathrm{D}}$. To guarantee convergence, it is required that $\mathcal{S}^{\mathrm{D}} \subseteq \mathcal{H}^{[i]}(0)$ and $\max_{z\in\mathcal{H}^{[i]}(0)} e^{\mathsf{T}} z < \infty$. To this end, we utilize the observation that the objective of (6), which represents a total costs of PEV charging, must have an upper bound in practice. Since strong duality holds for (6) and (14), there also exists an upper bound of the equivalent maximization problem (14). Hence, each processor can individually choose a properly large number $M_i > 0$ according to historical data, and construct an initial cutting-plane set as

$$\mathcal{H}^{[i]}(0) = \left\{ z | \pi \geq 0; e^{\mathsf{T}} z \leq M_i \right\}. \tag{18}$$

C. Distributed Generation of Cutting-planes

The consensus on feasible region depends upon the generation of new cutting-planes. In this subsection, we omit k_i for succinctness. Given the query point $z^{[i]}$ and a target set $\mathcal{Z}_i$, $h_i(z^{[i]})$ is generated as the cutting plane separating $z^{[i]}$ and $\mathcal{Z}_i$ if $z^{[i]}$ is not inside $\mathcal{Z}_i$. In order to identify if $z^{[i]}$ is within $\mathcal{Z}_i$, processor i needs to compare the values of $D_i(\pi^{[i]})$ and $d_i^{[i]}$ according to the definition of $\mathcal{Z}_i$ in (13). Then processor i generates the new cutting plane according to

$$h_i(z^{[i]}) = \begin{cases} h^{\emptyset}, & \text{if } d_i^{[i]} \leq D_i(\pi^{[i]}) \\ \left\{ z | d_i \leq L_i(\pi, p_i(\pi^{[i]})) \right\}, & \text{otherwise} \end{cases} \tag{19}$$

where $p_i(\cdot)$ is defined in (11).

D. Convergence and Optimality of Algorithm 1

Definition 2 (Dual Optimal)**.** (z^*, D^*) is dual optimal to the CoC problem if $z^* = (\pi^{*\mathsf{T}}, d^{*\mathsf{T}})^{\mathsf{T}}$ is optimal for problem (14) and $D^* := \sum_{i\in\mathcal{N}} d_i^*$ (or $D^* := e^{\mathsf{T}} z^*$).

Since additional constraints are added to the maximization problem while inactive constraints are pruned in every round of communication, the objective value sequence $\{e^{\mathsf{T}} z^{[i]}(k_i)\}_{k_i\in\mathbb{Z}^+}$ is monotonically non-increasing with respect to k_i. This is an inherent characteristic of the constraint-exchanging method. Thus convergence of the Algorithm 1 is warranted, since $\{e^{\mathsf{T}} z^{[i]}(k_i)\}_{k_i\in\mathbb{Z}^+}$ is monotonically non-increasing and lower bounded by D^*. The optimality of Algorithm 1 is guaranteed by classic cutting-plane theory [19], [21]. Specifically, as the feasible region $\mathcal{S}^{\mathrm{D}}$ is closed and compact, when Assumptions 1 and 2 hold, the limit point of sequence $\{z^{[i]}(k_i)\}_{k_i\in\mathbb{Z}^+}$ lies in $\mathcal{S}^{\mathrm{D}}$, implying D^* is greater than or equal to the limit of $\{e^{\mathsf{T}} z^{[i]}(k_i)\}_{k_i\in\mathbb{Z}^+}$.Thus, the convergence and optimality of Algorithm 1 is ensured as $\lim_{k_i\to\infty} e^{\mathsf{T}} z^{[i]}(k_i) = D^*, \forall i \in \mathcal{N}$.

V. Derivation of Local Stopping Criterion

In practice, we are more concerned with the quality of the solutions obtained within *finite* rounds of iterations. We define the *truncated local solution* of individual processor as

Definition 3 (Truncated Local Solution)**.** Recall the structure of $z^{[i]}(k_i)$:

$$z^{[i]}(k_i) = [\underbrace{\pi_1^{[i]}(k_i), \ldots, \pi_{|\mathcal{T}|}^{[i]}(k_i)}_{\pi^{[i]}(k_i)}, \underbrace{d_1^{[i]}(k_i), \ldots, d_n^{[i]}(k_i)}_{d^{[i]}(k_i)}]. \tag{20}$$

We denote by $J^{[i]}(k_i) := e^{\mathsf{T}} z^{[i]}(k_i) - \rho \|z^{[i]}(k_i)\|^2$ for succinctness. $(z^{[i]}(k_i), J^{[i]}(k_i))$ is called the truncated local solution of processor i at its local clock k_i.

By implementing the proposed distributed Algorithm 1, each local processor will derive a sequence of local truncated solutions as defined in Definition 3, during the iterations. It is crucial to find an appropriate stopping criterion for measuring consensus and optimality of the distributed Algorithm 1. First we will introduce the popular centralized criterion, then extend it to a *completely localized* one. We will prove that the *local criterion* serves as a sufficient condition of the *global criterion*,

A. Global Criterion

The global objective error at iteration k is given as follows.

Global Criterion: The global criterion is set as

$$\max_{i,j\in\mathcal{N}} |J^{[i]}(k_i) - J^{[j]}(k_j)| < \epsilon^2, \tag{21}$$

where k_i, k_j are the latest local clocks of processor i and j respectively and $\epsilon > 0$ is a pre-set tolerance.

If the global criterion is satisfied with a pre-specified convergence tolerance ϵ, consensus on the objective value is regarded as been encountered. Empirically, the algorithm terminates when the global criterion is met. The underlying rationale is that the temporary objective value $J^{[i]}(k_i)$ is the maximum of $e^{\mathsf{T}} z - \rho\|z\|^2$ in the polyhedron $H_{\mathrm{tmp}}^{[i]}(k_i)$ which is an outer-approximation to the feasible region $\mathcal{S}^{\mathrm{D}}$. Thus, the consensus on objective value can be approximated to the consensus on the feasible region $\mathcal{S}^{\mathrm{D}}$, and then consequently consensus on solution z can be achieved. The global criterion, however, entails temporary objective values from all local processors. It implies that individuals are not able to implement this criterion locally. Moreover, a consensus on objective value may not necessarily imply that the optimal D^* is achieved as D^* is not known a priori for individual processors. We will showcase in section VI that the global criterion may fail. To circumvent these issues, a *local criterion* is proposed below.

B. Local Criterion

Given a pre-set tolerance $\epsilon > 0$, two conditions constituting the *local criterion* are given: both Condition 1 and 2 are stated in localized form; Condition 1 claims to have stagnation on local objective updating within K_0 iterative steps; Condition 2 guarantees a bounded distance from the truncated local solution $z^{[i]}(k_i)$ in hand to the local feasible region $\mathcal{Z}_i$.

Condition 1. For processor $i \in \mathcal{N}$, $|J^{[i]}(k_i) - J^{[i]}(k_i - K_0)| < \epsilon^2$ where $K_0 := (n-1)\overline{\mathrm{T}}$ is a constant. $\overline{\mathrm{T}}$ is the communicating topology parameter stated in Assumption 1.

Condition 2. For processor $i \in \mathcal{N}$, $d_i^{[i]}(k_i) < D_i(\pi^{[i]}(k_i)) + \epsilon$.

Then a *local criterion* can be designed as follows:

Global Criterion: For any processor $i \in \mathcal{N}$, it stops at its local clock k_i when Condition 1 and 2 are fulfilled with a pre-set tolerance $\epsilon > 0$.

The proposed local criterion enables a measure to estimate the distance between a truncated local solution $J^{[i]}(k_i)$ (or $z^{[i]}(k_i)$) and the dual optimum D^* (or z^*). In the next subsection, we will show by a theoretical proof that though D^* (either z^*) does not appear in the Condition 1 and 2, the two conditions together guarantee bounded error of a truncated local solution, with respect to consensus, optimality, and feasibility.

C. Justification of the Local Criterion

1) Consensus: A consensus result among distributed multiple processors is guaranteed by implementing Algorithm 1 with the local criterion given in subsection V-B. This is justified by the following theorem.

Theorem 1. Let Assumptions 1,2 hold and Condition 1 of the local criterion hold for all processors in $\mathcal{N}$, then the global criterion (21) is satisfied.

Theorem 1 implies that the local criterion is a sufficient condition for the global criterion.

To prove Theorem 1, first we take the following notations.

Let $\tau^{[i]}(k_i) : \mathbb{Z}^+ \to [0, \infty)$ denote the mapping from the local clock of processor i which is denoted by k_i to a continuous time interval $[0, \infty)$. Let $\tau(k) : \mathbb{Z}^+ \to [0, \infty)$ denote the mapping from the global clock k to the continuous time interval $[0, \infty)$.

Truncated local solutions are defined with individual local clocks. To compare them, we project all the truncated local solutions to the common global clock. Note that for any processor $i \in \mathcal{N}$ and any global time slot $k \in \mathbb{Z}^+$, there exists a unique local time slot $\hat{k}_i \in \mathbb{Z}^+$ such that $\tau^{[i]}(\hat{k}_i) \leq \tau(k) < \tau^{[i]}(\hat{k}_i + 1)$. Hence we can define

$$\hat{J}^{[i]}(k) := J^{[i]}(\hat{k}_i). \tag{22}$$

where $J^{[i]}(\hat{k}_i)$ is the truncated local solution of processor i at $\hat{k}_i$, as stated in Definition 3.

By the above notations, the truncated local solutions are mapped to the series $\{\hat{J}^{[i]}(k)\}_{k\in\mathbb{Z}^+}$ over the global clock k. Since $J^{[i]}(k_i)$ is monotonously non-increasing with respect to k_i, $\hat{J}^{[i]}(k)$ is also monotonously non-increasing with respect to k, by recalling that $\tau^{[i]}(k_i), \tau(k)$ is a monotonously increasing map from discrete time index to continuous time horizon.

We start the proof of Theorem 1 with the following lemma.

Lemma 1. Let Assumptions 1 and 2 hold. For $i \in \mathcal{N}$, $k_i, k \in \mathbb{Z}^+$, if $\tau^{[i]}(k_i) \leq \tau(k) < \tau^{[i]}(k_i + 1)$, then $J^{[i]}(k_i - K) \geq \hat{J}^{[i]}(k - K)$ holds for any positive integer $K < \min\{k_i, k\}$.

Proof. For any given integer $K < \min\{k_i, k\}$, according to the definition of $\hat{J}^{[i]}$ in (22), there exists a unique $\hat{k}_i \in \mathbb{Z}^+$ such that $\hat{J}^{[i]}(k - K) = J^{[i]}(\hat{k}_i)$ and

$$\tau^{[i]}(\hat{k}_i) \leq \tau(k - K) < \tau^{[i]}(\hat{k}_i + 1). \tag{23}$$

Note that $\tau^{[i]}(k_i) \leq \tau(k)$ due to the definition of τ. From the relationship of global clock and local clocks, we have

$$\tau^{[i]}(k_i - K) \leq \tau(k - K). \tag{24}$$

Combining (23) with (24), we have

$$\tau^{[i]}(k_i - K) \leq \tau(k - K) < \tau^{[i]}(\hat{k}_i + 1). \tag{25}$$

From $\tau^{[i]}(k_i - K) < \tau^{[i]}(\hat{k}_i + 1)$ we have $k_i - K < \hat{k}_i + 1$ by noting the monotonicity of mapping τ. It is equivalent to

$$k_i - K \leq \hat{k}_i \tag{26}$$

since $k_i - K$ and $\hat{k}_i$ are both positive integers.

Since $J^{[i]}(k_i)$ is monotonously non-increasing with respect to k_i, from (26) we have

$$J^{[i]}(k_i - K) \geq J^{[i]}(\hat{k}_i) = \hat{J}^{[i]}(k - K) \tag{27}$$

which completes the proof. □

Now we prove Theorem 1 by contradiction.

Proof. Suppose that for a given global time k, Condition 1 is fulfilled by all processors in $\mathcal{N}$. Then suppose for the sake of contradiction that there exists $i, j \in \mathcal{N}$ and their latest local clock $k_i, k_j \in \mathbb{Z}^+$ such that $|J^{[j]}(k_j) - J^{[i]}(k_i)| > \epsilon^2$, $\tau^{[i]}(k_i) \leq \tau(k) < \tau^{[i]}(k_i+1)$, and $\tau^{[j]}(k_j) \leq \tau(k) < \tau^{[j]}(k_j+1)$. By taking the notation of $\hat{J}$, we have

$$|\hat{J}^{[j]}(k) - \hat{J}^{[i]}(k)| > \epsilon^2 \tag{28}$$

with

$$\hat{J}^{[j]}(k) = J^{[j]}(k_j), \hat{J}^{[i]}(k) = J^{[i]}(k_i). \tag{29}$$

Without loss of generality, we assume that $J^{[j]}(k_j) > J^{[i]}(k_i)$. Thus (28) is equivalent to

$$\hat{J}^{[j]}(k) > \epsilon^2 + \hat{J}^{[i]}(k). \tag{30}$$

Since processor $i \in \mathcal{N}$ satisfies Condition 1, we have

$$|J^{[i]}(k_i) - J^{[i]}(k_i - K_0)| < \epsilon^2 \tag{31}$$

which is equivalent to

$$J^{[i]}(k_i - K_0) < J^{[i]}(k_i) + \epsilon^2 \tag{32}$$

by noting $J^{[i]}(k_i)$ is monotonously non-increasing over k_i.

From Lemma 1 and the relationship that $\tau^{[i]}(k_i) \leq \tau(k) < \tau^{[i]}(k_i + 1)$, we have

$$\hat{J}^{[i]}(k - K_0) \leq J^{[i]}(k_i - K_0). \tag{33}$$

Then combining (33), (32), (29) and (30), we have the following relationship.

$$\hat{J}^{[i]}(k - K_0) \leq J^{[i]}(k_i - K_0) < J^{[i]}(k_i) + \epsilon^2 \tag{34a}$$

$$J^{[i]}(k_i) + \epsilon^2 = \hat{J}^{[i]}(k) + \epsilon^2 < \hat{J}^{[j]}(k) \tag{34b}$$

Next we denote $k_0 := k - K_0$. Construct a series of index sets $\{I_s\}_{s\geq k_0}$ by recurrence to characterize the information propagation from processor i starting at global time k_0 as

$$I_s := \begin{cases} \{i\}, s = k_0 \\ I_{s-1} \cup \{m \in \mathcal{N} : \exists l \in I_{s-1}, \text{s.t.} (l, m) \in \mathcal{E}_s\}, s > k_0 \end{cases} \tag{35}$$

Algorithm 1 guarantees that for any $s \geq k_0$ and any $m \in I_s$,

$$\hat{J}^{[m]}(s) \leq \hat{J}^{[i]}(k_0) \tag{36}$$

as the active constraints of i at time k_0 is available to m till time s.

Since the communication graph is $\overline{\mathrm{T}}$-strongly connected and the length of a path from i to j is at most $n - 1$, $I_{k_0+(n-1)\overline{\mathrm{T}}}$ must contain all the nodes in $\mathcal{N}$. Recall the definition that $k_0 := k - K_0$ and $K_0 := (n - 1)\overline{\mathrm{T}}$, I_k must contains all the nodes in $\mathcal{N}$. Thus $j \in I_k$, and from (36) we have

$$\hat{J}^{[j]}(k) \leq \hat{J}^{[i]}(k_0). \tag{37}$$

which contradicts (34b) by recalling that $k_0 = k - K$. □

2) Optimality: Next we discuss the dual optimality of the proposed local criterion since strong duality holds for the CoC problem (6) under Assumption 2. Specifically, the optimality of truncated local solutions derived from the local criterion to the dual problem (14) can be characterized by the following theorem.

Theorem 2 (Optimality)**.** Assume Assumptions 1 and 2 hold. If Conditions 1 and 2 of the local criterion are satisfied for all processors in $\mathcal{N}$, then $\forall i \in \mathcal{N}$, $|e^\mathsf{T} z^{[i]}(k_i) - D^*| \in \mathcal{O}(\epsilon)$.

We start the proof of Theorem 2 with the following lemma.

Lemma 2. Assume Assumptions 1 and 2 hold. If Conditions 1 and 2 hold for all processors in $\mathcal{N}$, then $\forall i, j \in \mathcal{N}$, there is

$$d_j^{[i]}(k_i) - D_j(\pi^{[i]}(k_i)) \le B\epsilon. \tag{38}$$

where B is a finite positive constant.

Proof. For simplicity, we omit the k_i in $d^{[i]}(k_i), \pi^{[i]}(k_i)$ and the k_j in $d^{[j]}(k_j), \pi^{[j]}(k_j)$ in the following proof.

Since Conditions 1 and 2 hold for all processors in $\mathcal{N}$, then for any $i, j \in \mathcal{N}$ and $i \neq j$, we have

$$d_j^{[i]} - D_j(\pi^{[i]}) = d_j^{[i]} - \min_{p_j \in \mathcal{P}_j} L_j(\pi^{[i]}, p_j) \tag{39a}$$

$$\le d_j^{[i]} - \min_{p_j \in \mathcal{P}_j} L_j(\pi^{[j]}, p_j) - \min_{p_j \in \mathcal{P}_j} \left\{ L_j(\pi^{[i]}, p_j) - L_j(\pi^{[j]}, p_j) \right\} \tag{39b}$$

$$= d_j^{[i]} - D_j(\pi^{[j]}) + \max_{p_j \in \mathcal{P}_j} \left\{ L_j(\pi^{[j]}, p_j) - L_j(\pi^{[i]}, p_j) \right\} \tag{39c}$$

$$= \underbrace{d_j^{[i]} - d_j^{[j]}}_{(a)} + \underbrace{d_j^{[j]} - D_j(\pi^{[j]})}_{(b)} + \underbrace{\max_{p_j \in \mathcal{P}_j} \left\{ L_j(\pi^{[j]}, p_j) - L_j(\pi^{[i]}, p_j) \right\}}_{(c)}. \tag{39d}$$

(a) : Strict concavity of $J(\cdot)$ follows that $|J^{[i]}(k_i) - J^{[j]}(k_j)| \ge \sigma^2 \|z^{[i]}(k_i) - z^{[j]}(k_j)\|^2$ for some $\sigma > 0$ [21]. Theorem 1 indicates that $|J^{[i]}(k_i) - J^{[j]}(k_j)| < \epsilon^2$ holds when Condition 1 is satisfied for all processors in $\mathcal{N}$. Thus we have

$$|(a)| = |d_j^{[i]} - d_j^{[j]}| \le \|z^{[i]}(k_i) - z^{[j]}(k_j)\| < \frac{\epsilon}{\sigma}, \tag{40a}$$

$$\|\pi^{[j]} - \pi^{[i]}\| \le \|z^{[i]}(k_i) - z^{[j]}(k_j)\| < \frac{\epsilon}{\sigma}. \tag{40b}$$

(b) : From Condition 2 we have

$$(b) < \epsilon. \tag{41}$$

(c) : According to the charging model in subsection II-D, for any $i \in \mathcal{N}$, $p_i \in \mathbb{R}^{|\mathcal{T}|}$ is bounded in set $\mathcal{P}_i$ with $\|p_i\|^2 \le |\mathcal{T}|(p_i^{\max})^2$ where $p_i^{\max}$ is the maximum charging power of processor i. Thus there exists a finite positive number P such that $\max\{\|p_i\|, \|p_i - F\|\} \le P$ holds for all $i \in \mathcal{N}$ where $F \in \mathbb{R}^{|\mathcal{T}|}$ is the right-hand-side parameter of constraint (6b). Recalling the definition of $L_i(\cdot)$, together with the relationship in (40b), we have

$$(c) \le \|\pi^{[j]} - \pi^{[i]}\| P < \frac{\epsilon}{\sigma} P. \tag{42}$$

Combining (39d), (40a), (41) and (42), we have

$$d_j^{[i]} - D_j(\pi^{[i]}) < B\epsilon \tag{43}$$

where $B := 1 + \frac{1+P}{\sigma}$ is a constant greater than 1.

For the situation that $j = i$, since i satisfies Condition 2, $d_i^{[i]} - D_i(\pi^{[i]}) < \epsilon$. Thus $d_i^{[i]} - D_i(\pi^{[i]}) < B\epsilon$ holds by recalling that B is greater than 1. This completes the proof. □

Now we can prove Theorem 2.

Proof. Skip the k_i in $z^{[i]}(k_i)$, $d^{[i]}(k_i)$ and $\pi^{[i]}(k_i)$ for conciseness. According to Lemma 2, there exists a constant B which is greater than 1 such that $d_j^{[i]} - D_j(\pi^{[i]}) \le B\epsilon$ for any $j \in \mathcal{N}$. Then define a new vector $\delta z \in \mathbb{R}^{n+|\mathcal{T}|}$, the first $|\mathcal{T}|$ elements of which are zeros and the last n elements are $B\epsilon$.

$$\delta z := [\underbrace{0, \ldots, 0}_{|\mathcal{T}| \text{ elements}}, \underbrace{B\epsilon, \ldots, B\epsilon}_{n \text{ elements}}]^\mathsf{T} \tag{44}$$

We can construct an auxiliary solution with respect to $z^{[i]}$ as

$$\hat{z}^{[i]} := z^{[i]} - \delta z. \tag{45}$$

$\forall j \in \mathcal{N}$, $d_j^{[i]} - B\epsilon \le D_j(\pi^{[i]})$ holds according to Lemma 2. Thus we have $\hat{z}^{[i]} \in \cap_{j \in \mathcal{N}} \mathcal{Z}_j$, implying that $\hat{z}^{[i]}$ lies in the feasible region of (14), i.e., $\mathcal{S}^\mathrm{D}$.

Since D^* is the maximizer of $e^\mathsf{T} z$ in $\mathcal{S}^\mathrm{D}$ and $\hat{z}^{[i]} \in \mathcal{S}^\mathrm{D}$, we have $e^\mathsf{T} \hat{z}^{[i]} \le D^*$. Since $z^{[i]}$ is the maximizer of $e^\mathsf{T} z$ in $\mathcal{H}_{\mathrm{tmp}}^{[i]}$ and $\mathcal{S}^\mathrm{D} \subseteq \mathcal{H}_{\mathrm{tmp}}^{[i]}$, we have $D^* \le e^\mathsf{T} z^{[i]}$. Thus we get the following relationship:

$$e^\mathsf{T} \hat{z}^{[i]} \le D^* \le e^\mathsf{T} z^{[i]} \tag{46}$$

Look at the first and last items in (46):

$$e^\mathsf{T} z^{[i]} - e^\mathsf{T} \hat{z}^{[i]} = nB\epsilon. \tag{47}$$

From (46) and (47) we have $0 \le e^\mathsf{T} z^{[i]}(k_i) - D^* \le nB\epsilon$. Thus $|e^\mathsf{T} z^{[i]}(k_i) - D^*| \in \mathcal{O}(\epsilon)$, which completes the proof. □

3) Dual Feasibility: There is no guaranteed feasibility of truncated local solution sequence in cutting-plane based algorithms for dual problem (14). In other words, the obtainable consensus result within finite rounds could be outside $\mathcal{S}^\mathrm{D}$ even if it is very close to $\mathcal{S}^\mathrm{D}$. To justify the dual feasibility, we start with the situation that $z^{[i]}(k_i)$ is feasible, as stated in the following corollary coming from Theorem 2.

Corollary 1. Let Assumptions 1 and 2 hold. Assume Conditions 1 and 2 of the local criterion are satisfied for all processors in $\mathcal{N}$. For any $i \in \mathcal{N}$, if $z^{[i]}(k_i) \in \mathcal{S}^\mathrm{D}$, then $e^\mathsf{T} z^{[i]}(k_i) = D^*$.

Proof. According to the proof of Theorem 2, we have $D^* \le e^\mathsf{T} z^{[i]}(k_i)$. As D^* is the maximizer of $e^\mathsf{T} z$ in $\mathcal{S}^\mathrm{D}$, if $z^{[i]}(k_i) \in \mathcal{S}^\mathrm{D}$, then $D^* \ge e^\mathsf{T} z^{[i]}(k_i)$. Thus $e^\mathsf{T} z^{[i]}(k_i) = D^*$. □

Corollary 1 implies that if $z^{[i]}(k_i)$ is feasible, it must be optimal to (14). If unfortunately $z^{[i]}(k_i)$ lies outside the feasible region $\mathcal{S}^\mathrm{D}$, it is revealed in Corollary 2 that the distance from point $z^{[i]}(k_i)$ to the feasible set $\mathcal{S}^\mathrm{D}$ is bounded.

Corollary 2 (Dual Feasibility)**.** Assume Assumptions 1 and 2 hold. If Conditions 1 and 2 are satisfied for all processors in $\mathcal{N}$, then for each processor $i \in \mathcal{N}$, $\mathrm{dist}(z^{[i]}(k_i), \mathcal{S}^\mathrm{D}) \in \mathcal{O}(\epsilon^2)$.

Proof. Corollary 2 can be inferred from the proof of Theorem 2: If Conditions 1 and 2 hold for all processors in $\mathcal{N}$, then for any $i \in \mathcal{N}$ there exists an auxiliary solution $\hat{z}^{[i]}(k_i) \in \mathcal{S}^\mathrm{D}$ such that $\|z^{[i]}(k_i) - \hat{z}^{[i]}(k_i)\|^2 = nB^2\epsilon^2$. Thus we have

$$\mathrm{dist}(z^{[i]}(k_i), \mathcal{S}^\mathrm{D}) \le \|z^{[i]}(k_i) - \hat{z}^{[i]}(k_i)\|^2 = nB^2\epsilon^2 \tag{48}$$

Hence $\mathrm{dist}(z^{[i]}(k_i), \mathcal{S}^{\mathrm{D}}) \in \mathcal{O}(\epsilon^2)$, completing the proof. □

4) Primal Feasibility: For better clarity, here we adopt the following notations. For any $i \in \mathcal{N}$, we denote the polyhedron consisting local feasible set $\mathcal{P}_i$ by $A_i p_i \leq b_i$, and the corresponding dual multiplier by $\lambda_i \in \mathbb{R}^{m_i}$. Then an alternative local Lagrangian function is formulated as

$$\tilde{L}_i(\pi, \lambda_i, p_i) := L_i(\pi, p_i) + \lambda_i^{\mathsf{T}}(A_i p_i - b_i), \forall i \in \mathcal{N} \quad (49)$$

and the corresponding local dual function is

$$\tilde{D}_i(\pi, \lambda_i) := \min_{p_i} \tilde{L}_i(\pi, \lambda_i, p_i). \quad (50)$$

The unique optimal solution of the inner problem in dual function $\tilde{D}_i$ is denoted by

$$\tilde{p}_i(\pi, \lambda_i) := \arg\min_{p_i} \tilde{L}_i(\pi, \lambda_i, p_i). \quad (51)$$

Denote by $p_i(\pi)$ the unique optimal solution to the inner problem with dual function D_i, and by $\lambda_i(\pi)$ the dual multiplier associated with constraint $A_i p_i \leq b_i$. That is

$$(p_i(\pi), \lambda_i(\pi)) := \arg\min_{p_i} L_i(\pi, p_i) \quad (52a)$$

$$\text{s.t. } A_i p_i \leq b_i \text{ (dual:}\lambda_i\text{)} \quad (52b)$$

Then we have the following relationship.

Lemma 3. Let Assumption 2 hold, then for any $i \in \mathcal{N}$,

$$\tilde{p}_i(\pi, \lambda_i(\pi)) = p_i(\pi). \quad (53)$$

Proof. The KKT condition of (52) gives

$$\nabla f_i(p_i(\pi)) + \pi + A_i^{\mathsf{T}} \lambda_i(\pi) = 0 \quad (54a)$$

$$\lambda_i(\pi) \geq 0 \quad (54b)$$

$$\lambda_i(\pi)^{\mathsf{T}} (A_i p_i(\pi) - b_i) = 0 \quad (54c)$$

$$A_i p_i(\pi) \leq b_i \quad (54d)$$

From (54a) we have that $p_i(\pi)$ is the optimal solution to (51) when taking $\lambda_i = \lambda_i(\pi)$. Since the solutions to (52) and (51) are both unique under Assumption 2, $p_i(\pi) = \tilde{p}_i(\pi, \lambda_i(\pi))$, witch completes the proof. □

Denote by $\mathcal{S}^{\mathrm{P}}$ the feasible region of the CoC problem (6). Then we show that the primal solution retrieved from the local truncated dual solution $\{z^{[i]}(k_i)\}$ through (52) has bounded feasibility violation with respect to set $\mathcal{S}^{\mathrm{P}}$.

Theorem 3. Assume Assumptions 1 and 2 hold. If Conditions 1 and 2 of the local criterion are satisfied for all processors in $\mathcal{N}$, then $\forall i \in \mathcal{N}$,

$$\mathrm{dist}(p(\pi^{[i]}(k_i)), \mathcal{S}^{\mathrm{P}}) \in \mathcal{O}(\epsilon) \quad (55)$$

where

$$p(\pi^{[i]}(k_i)) := \begin{bmatrix} p_1(\pi^{[i]}(k_i)) \\ ... \\ p_n(\pi^{[i]}(k_i)) \end{bmatrix} \quad (56)$$

and $p_i(\cdot)$ in (56) denotes the unique optimal solution of the inner problem in dual function D_i, as stated in (52).

We give two lemmas before proving Theorem 3.

Lemma 4. Assume Assumption 2 holds. If Conditions 1 and 2 are satisfied for all processors in $\mathcal{N}$, then for any $\pi \geq 0$ and $i \in \mathcal{N}$, there is

$$\frac{\sigma_i}{2} \|p_i(\pi) - p_i^*\|^2 \leq L_i(\pi, p_i^*) - D_i(\pi) \quad (57)$$

where p_i^* is the primal optimum given in Definition 1.

Proof. Since f_i is σ_i-strongly convex as assumed in Assumption 2, $\tilde{L}_i(\pi, \lambda_i, p_i)$ is also σ_i-strongly convex in p_i. Thus

$$\begin{aligned} \frac{\sigma_i}{2} \|p_i(\pi) - p_i^*\|^2 \leq \tilde{L}_i(\pi, \lambda_i(\pi), p_i^*) \\ - \tilde{L}_i(\pi, \lambda_i(\pi), \tilde{p}_i(\pi, \lambda_i(\pi))). \end{aligned} \quad (58)$$

Recalling the fact that

$$\tilde{L}_i(\pi, \lambda_i, p_i) = L_i(\pi, p_i) + \lambda_i^{\mathsf{T}}(A_i p_i - b_i) \quad (59)$$

and the relationship that $\tilde{p}_i(\pi, \lambda_i(\pi)) = p_i(\pi)$ given in Lemma 3, we have

$$\tilde{L}_i(\pi, \lambda_i(\pi), \tilde{p}_i(\pi, \lambda_i(\pi))) \quad (60a)$$

$$= \tilde{L}_i(\pi, \lambda_i(\pi), p_i(\pi)) \quad (60b)$$

$$= L_i(\pi, p_i(\pi)) + \lambda_i(\pi)^{\mathsf{T}}(A_i p_i(\pi) - b_i). \quad (60c)$$

Since $p_i(\pi), \lambda_i(\pi)$ satisfies the KKT condition (54c),

$$\lambda_i(\pi)^{\mathsf{T}}(A_i p_i(\pi) - b_i) = 0. \quad (61)$$

Thus

$$\tilde{L}_i(\pi, \lambda_i(\pi), \tilde{p}_i(\pi, \lambda_i(\pi))) = L_i(\pi, p_i(\pi)) = D_i(\pi) \quad (62)$$

Similarly, we have

$$\tilde{L}_i(\pi, \lambda_i(\pi), p_i^*) = L_i(\pi, p_i^*) + \lambda_i(\pi)^{\mathsf{T}}(A_i p_i^* - b_i). \quad (63)$$

Since $\lambda_i(\pi) \geq 0$ according to the KKT condition (54b), and $A_i p_i^* - b_i \leq 0$, we know that $\lambda_i(\pi)^{\mathsf{T}}(A_i p_i^* - b_i) \leq 0$. Thus

$$\tilde{L}_i(\pi, \lambda_i(\pi), p_i^*) \leq L_i(\pi, p_i^*). \quad (64)$$

Combining (58), (62) and (64), we have

$$\frac{\sigma_i}{2} \|p_i(\pi) - p_i^*\|^2 \leq L_i(\pi, p_i^*) - D_i(\pi) \quad (65)$$

which completes the proof. □

Lemma 5. Assume Assumptions 1 and 2 hold. If Conditions 1 and 2 are satisfied for all processors $i \in \mathcal{N}$, then there is

$$\|p(\pi^{[i]}(k_i)) - p^*\|^2 \in \mathcal{O}(\epsilon) \quad (66)$$

where p^* is the primal optimum given in Definition 1.

Proof. Recalling the structure of $p(\pi^{[i]}(k_i))$ in (56), we have

$$\|p(\pi^{[i]}(k_i)) - p^*\|^2 = \sum_{j \in \mathcal{N}} \|p_j(\pi^{[i]}(k_i)) - p_j^*\|^2. \quad (67)$$

Combining with the fact that $\sigma_f := \min_{j \in \mathcal{N}} \sigma_j$, we have

$$\frac{\sigma_f}{2} \|p(\pi^{[i]}(k_i)) - p^*\|^2 = \frac{\sigma_f}{2} \sum_{j \in \mathcal{N}} \|p_j(\pi^{[i]}(k_i)) - p_j^*\|^2 \quad (68)$$

$$\leq \sum_{j \in \mathcal{N}} \frac{\sigma_j}{2} \|p_j(\pi^{[i]}(k_i)) - p_j^*\|^2. \quad (69)$$

According to Lemma 4, $\forall j \in \mathcal{N}$, there is

$$\begin{aligned} \frac{\sigma_j}{2} \|p_j(\pi^{[i]}(k_i)) - p_j^*\|^2 \\ \leq L_j(\pi^{[i]}(k_i), p_j^*) - D_j(\pi^{[i]}(k_i)). \end{aligned} \quad (70)$$

Thus

$$\begin{aligned} \frac{\sigma_f}{2} \|p(\pi^{[i]}(k_i)) - p^*\|^2 \\ \leq \sum\nolimits_{j \in \mathcal{N}} L_j(\pi^{[i]}(k_i), p_j^*) - \sum\nolimits_{j \in \mathcal{N}} D_j(\pi^{[i]}(k_i)) \end{aligned} \quad (71)$$

Invoking the definition of $L_j(\cdot)$ in (8b), we have

$$\begin{aligned} \sum\nolimits_{j \in \mathcal{N}} L_j(\pi^{[i]}(k_i), p_j^*) \\ = \sum\nolimits_{j \in \mathcal{N}} f_j^* + \pi^{[i]}(k_i)^{\mathsf{T}} (\sum\nolimits_{j \in \mathcal{N}} p_j^* - F) \end{aligned} \quad (72)$$

By noting the facts that $\sum_{j \in \mathcal{N}} p_j^* \leq F$ and $\pi^{[i]}(k_i) \geq 0$, we have $\pi^{[i]}(k_i)^{\mathsf{T}} (\sum_{j \in \mathcal{N}} p_j^* - F) \leq 0$. Thus

$$\sum\nolimits_{j \in \mathcal{N}} L_j(\pi^{[i]}(k_i), p_j^*) \leq \sum\nolimits_{j \in \mathcal{N}} f_j^* = D^* \quad (73)$$

since strong duality holds under Assumption 2.

Combining (71) with (73), we have

$$\frac{\sigma_f}{2}\|p(\pi^{[i]}(k_i)) - p^*\|^2 \leq D^* - \sum\nolimits_{j\in\mathcal{N}} D_j(\pi^{[i]}(k_i)). \quad (74)$$

According to Lemma 2, $\forall j \in \mathcal{N}$,

$$-D_j(\pi^{[i]}(k_i)) \leq B\epsilon - d_j^{[i]}(k_i). \quad (75)$$

where B is a positive constant. Thus

$$\frac{\sigma_f}{2}\|p(\pi^{[i]}(k_i)) - p^*\|^2 \leq D^* + nB\epsilon - \sum_{j\in\mathcal{N}} d_j^{[i]}(k_i) \quad (76)$$

$$= D^* + nB\epsilon - e^{\mathsf{T}} z^{[i]}(k_i) \quad (77)$$

$$\leq nB\epsilon + |e^{\mathsf{T}} z^{[i]}(k_i) - D^*|. \quad (78)$$

According to Theorem 2, $|e^{\mathsf{T}} z^{[i]}(k_i) - D^*| \in \mathcal{O}(\epsilon)$, thus

$$\frac{\sigma_f}{2}\|p(\pi^{[i]}(k_i)) - p^*\|^2 \in \mathcal{O}(\epsilon) \quad (79)$$

which completes the proof. □

Then we can prove Theorem 3 by applying Lemma 5.

Proof. Since $p^* \in \mathcal{S}^{\mathrm{P}}$, $\mathrm{dist}(p(\pi^{[i]}(k_i)), \mathcal{S}^{\mathrm{P}}) \leq \|p(\pi^{[i]}(k_i)) - p^*\|^2$. According to Lemma 5, $\|p(\pi^{[i]}(k_i)) - p^*\|^2 \in \mathcal{O}(\epsilon)$. Thus $\mathrm{dist}(p(\pi^{[i]}(k_i)), \mathcal{S}^{\mathrm{P}}) \in \mathcal{O}(\epsilon)$, completing the proof. □

VI. Case Studies

In this section, we test the performance of the proposed method and compare it with the celebrated ADMM algorithm. Simulations are carried out on the IEEE 37- and IEEE 123-node feeders [28], with MATLAB on a laptop with Intel(R) Core(TM) i5-5200U 2.20GHz CPU and 4GB of RAM.

A. Setup

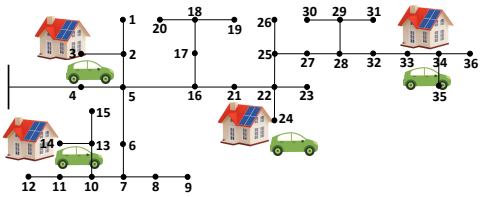

Fig. 3. Distributed network diagram of IEEE 37-node Test Feeder.

1) Physical and Communication Networks: We consider two typical radial residential distribution networks: the IEEE 37-bus and 123-bus test feeders. The topology of the former system is depicted in Fig.3, while the latter is omitted for space limitation. For both the systems, suppose that load of one household and one PEV is located at each bus. The feeder head (bus 4) is the only power supplier and its maximal capacity is set as the peak load without PEVs. We use the proposed method to coordinate PEVs' charging to avoid overload. The communication topology of charging PEVs is chosen similar to the power network topology for simplicity[4]. We use real data from the hourly residential load profile of Los Angeles [29] as the baseline load and scale it to match the household numbers. The information on hourly day-ahead electricity prices comes from California ISO [30].

[4]This setting is made solely for the clarity of presentation. Theoretically, the communication topology can be arbitrary provided Assumption 1 holds.

2) PEV Specifications: The parameters of PEVs are given: Battery capacities lie in a uniform distribution between 18 kW.h to 20 kW.h [31]. The scheduling horizon is from 5:00 pm to 9:00 am in the next day and is divided into 16 time slots hour-by-hour. Accordingly, we assume that the arrival and departure time of PEVs in the test cases lie in 5:00 pm-9:00 am and their hourly probability distributions are determined according to [2]. Initial and designated SOC are uniformly distributed in $[0.3, 0.5]$ and $[0.7, 0.9]$ respectively [4]. The maximum charge power is set as 3.3 kW for Level II charger. A charging efficiency of 0.9 is considered.

B. Optimality

In this case, Algorithm 1 is applied to the IEEE 37-node test feeder. Results are presented in Fig.4, compared with two common uncoordinated charging modes:

- *Mode (i)*: PEVs start charging immediately when they arrive and until the designated SOC is reached.
- *Mode (ii)*: PEVs optimize charging cost on their own without coordinating with others.

A total charging cost of \$109.12, \$130.60 and \$105.06 is achieved under coordinated charging, uncoordinated mode (i) and (ii) respectively. As shown in Fig.4, under mode (i), a new peak load is imposed on the baseload profile around evening rush hours, which gives rise to a great burden on the feeder head. Moreover, mode (i) costs the most because of charging during high price periods, which is uneconomical. As for mode (ii), a minimal charging cost is achieved, however, with there being a new peak load around 2 am to 3 am. The aggregation charging behavior during low price period around midnight also threatens the system security, although it is in off-peak time for non-EV baseload. The proposed coordinated charging meets system constraints at minimum cost, striving a balance between security and economic efficiency. It tries to schedule PEV load to off-peak time as well as avoids overload on the feeder head. An observation is made that the strategy plays a role in 'valley-filling' of total load profile as a consequence when PEV load and baseload are roughly on the same scale. The outcomes of the distributed algorithm are coincident with that of the centralized method, which is solved by CPLEX.

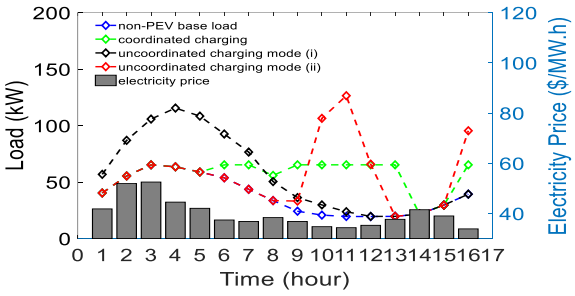

Fig. 4. Total load profile under coordinated/uncoordinated charging modes.

C. Convergence

Fig.5 shows the iterative process of several selected nodes, converging to the optimal J^*. An observation is made on the different initial values of the optimization objective for individual processors. This is because they independently choose

their M_i uniformly in $[150, 200]$ to generate their initial cutting plane set. Also, it is observed that $J^{[i]}(k)$ of each processor is monotonously nonincreasing along with the communication round k, in accordance with theoretical analysis.

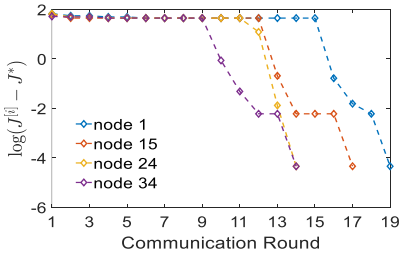

Fig. 5. Evolution of the objective value.

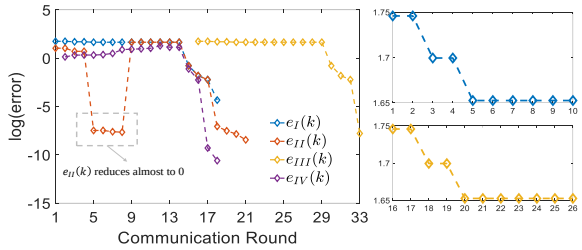

Fig. 6. Performance of different criterion.

To demonstrate performance of the different criterion, we show the evolution of four alternative errors: (i) $e_{\mathrm{I}}(k) = \max_i |J^{[i]}(k) - J^*|$; (ii) $e_{\mathrm{II}}(k)$ which is exactly the *global criterion* in (21); (iii) $e_{\mathrm{III}}(k) = \max_i\{J^{[i]}(k-K) - J^{[i]}(k)\}$ where $K = 15$ in this case, and (iv) $e_{\mathrm{IV}}(k) = \max_i\{u_i^{[i]}(k) - \mathcal{U}_i(\pi^{[i]}(k))\}$, as in Fig.6. $e_{\mathrm{I}}(k)$ essentially characterizes the convergence performance of the algorithm, however, entails the optimal J^* being known a priori. Note that the $e_{\mathrm{III}}(k)$ associated with Condition 1 and $e_{\mathrm{IV}}(k)$ associated with Condition 2 together give a sketch of the proposed *local criterion*. As shown in Fig.6, the algorithm converges after about 18 rounds of iteration, with $e_{\mathrm{I}}(k)$ reduced to less than 10^{-3}. Notice that the *global criterion* fails in this case, since the $e_{\mathrm{II}}(k)$ reduces almost to 0 at round 5 whereas achieving consensus on a non-optimal objective value. The *local criterion* with $\epsilon = 10^{-3}$ is met at round 33 with $e_{\mathrm{III}}(k) < \epsilon$ (namely, Condition 1 satisfied by all processors) and $e_{\mathrm{IV}}(k) < \epsilon$ (namely, Condition 2 fulfilled by all). Though the *local criterion* may be conservative due to Condition 1, it provides guaranteed optimality of the consensus outturn, as opposed to the *global criterion*.

We adopt the ADMM algorithm in [22], which is also center-free, for comparison. Also, case of IEEE 123-node feeder is tested to showcase the scalability to large problems. The error tolerance is set as $\epsilon = 10^{-3}$. Table II compares the convergence performances of the two methods. It is observed that the proposed algorithm needs only half of the communication rounds of ADMM to converge, showing a better convergence performance.

D. Performance of Asynchronous Charging

In this subsection, the robustness of the algorithm to communication delay, packet loss, and communication topology changes are tested in IEEE 37-node feeder.

TABLE I
CONTRAST OF COMMUNICATION ROUNDS ($\epsilon = 10^{-3}$)

Cases	The Proposed Method			ADMM
	$e_{\mathrm{I}} < \epsilon$	$e_{\mathrm{III}} < \epsilon$ (Cond.1)	$e_{\mathrm{IV}} < \epsilon$ (Cond.2)	$e_{\mathrm{I}} < \epsilon$
IEEE 37	18	33	16	51
IEEE 123	42	66	42	113

1) Communication Delay and Packet Losses: We assume that in each time step, each communication link has a delay of one time step with a probability of 10%. Also we suppose that the packet loss probability is 10% for the transmitted cutting-planes. Results given in Fig.7 evidently illustrate a satisfactory performance of the proposed algorithm even with communication delay and package losses.

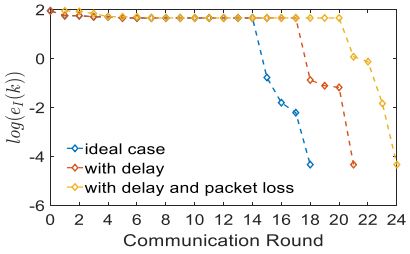

Fig. 7. Performance of the algorithm characterized by e_{I} under communication delay and packet loss.

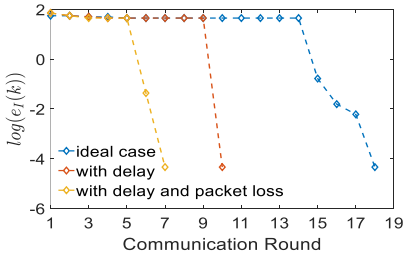

Fig. 8. Performance of the algorithm characterized by e_{I} under different communication topologies.

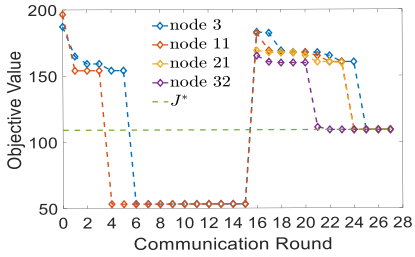

Fig. 9. Evolution of local objective value of selected nodes under plug and play setting.

2) Topology Varying of Communication Network : The original IEEE 37-node feeder topology has a diameter of 15. Consider another topology with a diameter of 10. Cases with different communication topologies are tested and the comparison of convergence performances are shown in Fig.8. It is observed that a static graph with a bigger diameter calls for more iteration rounds to converge. The rationale for this observation is that the diameter of the communication topology determines the longest time needed for passing cutting-planes from one node to another indirectly. Surprisingly, when alternating the two topologies in coordinated charging (with Assumption 1 hold always), the algorithm converges even

faster than the cases with static topologies. It shows that, with the proposed algorithm, topology varying may even accelerate the cutting-plane exchanging process, which could facilitate the convergence.

E. Plug-and-Play Operation

The proposed algorithm is tested on the IEEE 37-node system in a plug-and-play operation. Classify the 36 nodes into two parts $\mathcal{N}_1 = \{1, 2, \ldots, 20\}$ and $\mathcal{N}_2 = \{21, 22, \ldots, 36\}$. At the beginning, only PEVs in $\mathcal{N}_1$ coordinate on charging. Nodes in $\mathcal{N}_2$ participate at the 16 round. Results are shown in Fig.9, showing that new players can join in at any time, which well supports the plug-and-play operation.

VII. Conclusion

In this paper, we derived a cutting-plane based method to fulfill an optimal distributed coordination of PEV charging under local and global constraints. The proposed method strives the minimal overall charging cost without violating feeder head capacity, which is in accordance with the result of centralized global optimization. During the PEV charging, private information of individual PEVs can be well protected. It also performs resiliently under various kinds of asynchrony in practice such as time delays, packet losses, and topology changes. We hope this work can promote an alternative path toward a more robust protocol for PEV charging and other distributed coordination problems. As an initial study, uncertainties of renewable resources and inaccuracy of baseload forecast have not been considered in this work. Extending the proposed cutting-plane based distributed coordination framework to incorporate uncertainties remains our ongoing works.

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