Spectral Clustering of Signed Graphs via Matrix Power Means

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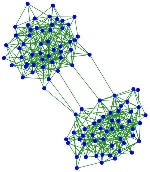
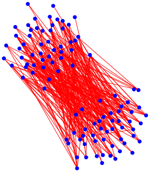
Poster #190

Our Goal: Extend Spectral Clustering to Graphs With Both Positive and Negative Edges

- **Positive Edges:** encode friendship, similarity, proximity, trust
- **Negative Edges:** encode enmity, dissimilarity, conflict, distrust

$G^{\pm} = ($  ,  $)$

A signed graph is the pair $G^{\pm} = (G^{+}, G^{-})$ where

$G^{+} = (V, W^{+})$ encodes **positive** relations, and

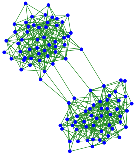
$G^{-} = (V, W^{-})$ encodes **negative** relations

Clustering of Signed Graphs

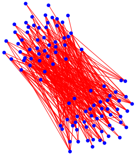
Given: an undirected signed graph $G^{\pm} = (G^{+}, G^{-})$

Goal : partition the graph such that

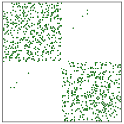
- edges **within** the same group have **positive** weights
- edges **between** different groups have **negative** weights



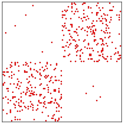
G^{+}



G^{-}



W^{+}



W^{-}

Our Goal: define an operator that <u>blends</u> the information of (G^{+}, G^{-}) such that the smallest eigenvectors are informative.

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State of the art approaches:

$$\mathbf{L_{SR}} = \mathbf{D}^+ - \mathbf{W}^+ + \mathbf{D}^- + \mathbf{W}^- \quad \text{(Kunegis, 2010)}$$
$$= \mathbf{L}^+ + \mathbf{Q}^-$$
$$\mathbf{L_{BR}} = \mathbf{D}^+ - \mathbf{W}^+ + \mathbf{W}^- \quad \text{(Chiang, 2012)}$$
$$= \mathbf{L}^+ + \mathbf{W}^-$$
$$\mathbf{H} = (\alpha - 1)\mathbf{I} - \sqrt{\alpha}(\mathbf{W}^+ - \mathbf{W}^-) + \mathbf{D}^+ + \mathbf{D}^- \quad \text{(Saade, 2015)}$$

Current methods are <u>arithmetic</u> means of Laplacians

The **power mean** of non-negative scalars a, b, and $p \in \mathbb{R}$:

$$m_p(a, b) = \left(\frac{a^p + b^p}{2}\right)^{1/p}$$

Particular cases of the scalar power mean are:

$p \to -\infty$	$p = -1$	$p \to 0$	$p = 1$	$p \to \infty$
$\min\{a, b\}$ minimum	$2\left(\frac{1}{a} + \frac{1}{b}\right)^{-1}$ harmonic mean	$\sqrt{ab}$ geometric mean	$(a + b)/2$ arithmetic mean	$\max\{a, b\}$ maximum

We introduce the **Signed Power Mean Laplacian** as an alternative to **blend the information** of the signed graph $G^{\pm}$:

$$\mathbf{L_p} = \left(\frac{\left(\mathbf{L}^{+}_{\mathbf{sym}}\right)^p + \left(\mathbf{Q}^{-}_{\mathbf{sym}}\right)^p}{2}\right)^{1/p}$$

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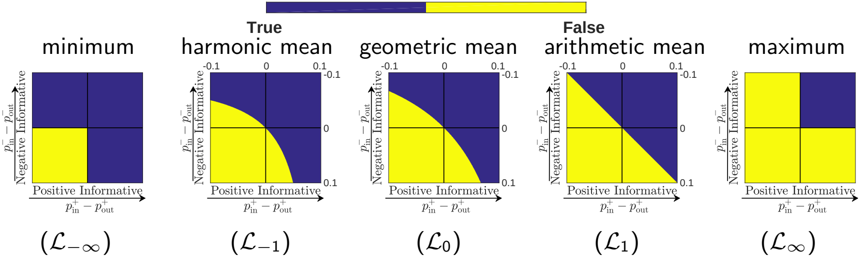
Analysis in the Stochastic Block Model

Theorem: The Signed Power Mean Laplacian L_p with $p \leq 0$ is better than arithmetic mean approaches in expectation.

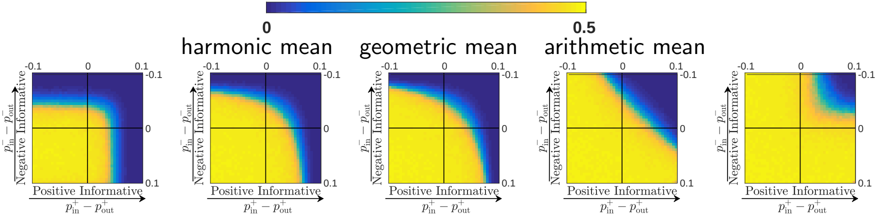
Recovery of Clusters in Expectation



Analysis in the Stochastic Block Model

Theorem: The Signed Power Mean Laplacian L_p with $p \leq 0$ is better than arithmetic mean approaches in expectation.

Average Clustering Error



(L_{-10}) (L_{-1}) (L_0) (L_1) (L_{10})

Theorem: with high probability eigenvalues and eigenvectors of L_p concentrate around those of the expected Signed Power Mean Laplacian $\mathcal{L}_p$