QC Mentorship Project

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1 Introduction

Regarding Task 4, we solved the problem using two methods. One of these method is a new way that we thought of. We really don't know if this method is already done, but the idea is so obvious that for sure someone did it. Regarding the VQE method mentioned on your website, we solved the problem in two ways. One way is that we solved it after simplifying the problem trivially to a 2D problem, and the second is by solving the problem in 4D as you wanted; just to show our skills. This document is mainly for explaining the second method since the VQE is already explained very well on your blog.

2 VQE

2.1 2D Problem

Since all the elements on the boundary are zeros, we can simply take our main problem as the four elements in the middle of the matrix, which can be decomposed to an X gate and an identity matrix. The method itself, as i said, is explained well in your blog, so we won't explain it again.

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (1)$$

2.2 4D problem

In the 4D problem, we decomposed our matrix $U = (XX + YY + ZZ - 1)/2$. The ansatz used will be the one suggested by the hints. The only proble we face is to construct the circuit that changes the basis of XX and YY. For XX, we apply a controlled-X gate, in which the second qubit is the control qubit, then hadamard on the second, and finally a controlled-x, where the first qubit is the control qubit. For the YY, we apply a controlled-x gate, where the second qubit is the control, then hadamard on the second, and finally an X gate on the second also. Then, we measure our states and perform tha same post-processing as the first simple version of the problem.

3 Time Evolution Method

3.1 Motivation

First of all the matrix we got is hermitian, which can be thought of as the Hamiltonian of an unknown quantum system. Our goal is to simulate this quantum system on a quantum circuit. To do this, we need to have a unitary matrix that can be done by a quantum circuit. This unitary matrix can be got easily from the time evolution operator:e^{-iHt}.

But how can we use such a thing to find the lowest eigenvalue? Well, first we will need to get the eigenvectors, which will be then used in a classical process to compute the eigenvalues. But how can we get the eigenvectors? Well, remember that according to quantum mechanics, an eigenstate of a hamiltonian will always be in that eigenstate and will not change forever. Thus, if the unitary operator acted on an eigenstate, the state of the particle will remain in this eigenstate. We will use this simple concept to find the eigenvctors.

3.2 Algorithm

We will explain our algorithm here and apply it on the problem you wrote.

3.2.1 First Step

We will get unitary matrix by using the time evolution operator. The time parameter in the operator can be used always to simplify the matrix by using certain nice guessed values for the time. In our problem we used $t = \pi/2$, and the result unitary matrix was the X gate.

3.2.2 Second Step

As we said, if a certain vector was an eigenvector, then it will not change or transform after applying the unitary gate.So, we will apply our ansatz on the unitary matrix and then measure the difference of the probability of the 0's and 1's. We will compare this difference with the difference of our ansatz before applying the unitary matrix. If it is an eigenvector, then both differences will be the same. If it wasn't an eigenvector, then the vector will change; as a result, the difference of the probability of the ansatz after and before applying the matrix will be different. In our problem, we tried different angles to find our eigenvector.

To ried differebe h we tonest, there is a shortage or a disadvantage in this method, which I didn't think of a solution to it. The problem is that sometimes a state, which isn't an eigenvector, will change to a state with the same probabilities for its 0's and 1's; thus, we will think that the ansatz is an eigenvector, which isn't true. Here is a simple example. We have a state $|\psi\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$ that will change after a unitary matrix to $|\psi\rangle = 1/\sqrt{2}(|0\rangle + i\,|1\rangle)$. As you can see here, both have the same probability, so we may think that our ansatz was an eigenvector.

3.2.3 Third Step: Classical Processing

After plotting our differences of our ansatz before and after applying the unitary matrix, we will identify the intersections between both graphs, and these intersection are the angles that will give us our eigenvectors. In our problem, we plotted the graphs and found that the angles were $\pi/2$ and $3\pi/2$. We have to test both of these ansatz to find the eigenvalues, and then we pick the lowest value.