Math 131ABH Notes

Mealaud Mokhtarzad

This set of notes is very informal and tries its best to simplify often hard to digest ideas. Hopefully it's useful in your learning of the material.

FINISH: put figures in right places

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test

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2.1 Topological spaces

We start with the titular definition.

Definition 2.1 (Topology). A *topology* on a set X is a collection $\mathcal{T}$ of subsets of X such that

(1) $\varnothing, X \in$;

(2) Any union of sets in is in ; and

(3) Any finite intersection of sets in is in .

The elements of are called *open sets*. A topological space is a pair $(X,)$.

Remark. A topology is a prescription for which sets are open.

Let's relate this back to something more familiar and intuitive: open sets in metric spaces. Recall the following properties of open sets in metric spaces: the empty set and the whole space are open; any union of open sets is open; and any finite intersection of open sets is open. I think of our definition of topology as a generalization of that.

Now let's get to some examples of topologies. (Some are gross, which seems to be a common theme in topology. Things can get pretty pathological at times.)

Example 2.2. (1) If X is a metric space, the collection of open sets is a topology called the *metric topology*.

(2) If X is any set, the family $\mathcal{T} \coloneqq \{\varnothing, X\}$ forms a topology called the *indiscrete topology*.

(3) If X is any set, the family $\mathcal{T} \coloneqq (X)$ (the collection of all subsets) forms the *discrete topology*.

(4) Let $X \coloneqq \{a, b\}$. Then $\mathcal{T} \coloneqq \{\varnothing, \{a\}, X\}$ is a topology.