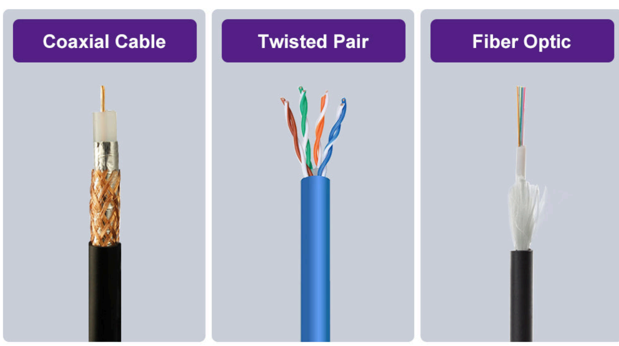
Lecture 03 - Physical Layer: Guided Media and Digital Signaling

Networks transmit electromagnetic energy:
- Electrical signals on copper wire
- Light pulses in optical fiber
- Radio waves through air

Medium	Bandwidth	Distance	Cost	Use Case
Twisted Pair	1-1000 MHz	100 m	Low	Ethernet, phone
Coaxial	1-1000 MHz	500 m	Medium	Cable TV
Optical Fiber	>10 THz	100 km+	High	Backbone

--> each on for a different application



Twisted Pair Cable → link channel / low pass filter

Structure:
- 2 insulated copper wires twisted together
- Twisting reduces electromagnetic interface (EMI)
- Multiple pairs bundled in cable

Parameters:
- Characteristic impedance: Z_0= 100Ω (for ethernet)
- attenuation : ~2-3 cB per 100 m at 100 Mhz
    - Attenuation in wire is the reduction of signal strength (amplitude) as it travels through a conductor, caused by electrical resistance, dielectric loss, and interference
- Propagation speed: v~ 0.64c (c = speed of light)

Coaxial cable:

structure:
- Central conductor (copper wire) (conductorL allow electricity to flow freely)
- Dielectric insulator (dielectric, insulating material or a very poor conductor of electric current)
- Outer conductor (shield / braid)

- Protective jacket

Types:

- RG-6 (75 ohms) → TV
- RG-58 (50 ohms) → ethernet

Key parameters:

- Attenuation: 0.5 - 2 dB per 100 m (lower than twisted)
- Bandwidth: DC to 1+ GHz
- Modern use: cable internet

<u>Optical fiber:</u>

- light propagates via total internal reflection
- Core has higher refractive index than cladding
- Critical angle Oc = arcsin (n2/n1)

2 types:

- Single mode (SMF)
- Multi mode (MMF)

Parameters:

Attenuation: (ULTRA LOW)

- 1550 nm: ~0.2 db
- 1310 nm: ~ 0.4 dB/km
- …
- Fiber has 100x less attenuation

Bandwidth: ENORMOUS

- Single diver: > 10 THz spectrum
- WDM: 100+ wavelengths

Propagation speed: v~ 0.67c

<u>Decibels (dB)</u>:

Why use dB? → multiplication becomes addition

Power ratio in dB:

$$dB = 10 \log_{10} (P_{out}/P_{in})$$

Absolute power levels:

- $dBm = 10 \log_{10} (P_{out}/1\ mW)$
- $dBW = 10 \log_{10} (P_{out}/1W)$

Memorize:

Power Ratio	dB	Meaning
2	3 dB	Double
10	10 dB	10×
100	20 dB	100×
1/2	-3 dB	Half
1/10	-10 dB	0.1×

Link budget:

$$P_{rx}(\text{dBm}) = P_{tx}(\text{dBm}) - L_{cable}(\text{dB}) + G_{tx} + G_{rx}$$

A link budget is a detailed accounting of all power gains and losses in a communication system, from the transmitter to the receiver, to determine if the signal will arrive strong enough for reliable communication

P_{rx}: dBm → received power → ultimately what you care about
P_{tx}: dBm → transmitted power → how much power is sent out
cable: dB → cable loss → always positive measuring power loss in cable
G_{tx}: dBi/ dB → transmit antenna gain → how well the transmit antenna focuses energy
G_{rx}: dBi/ dB → receive antenna gain → how well the receive antenna captures incoming energy

Shannon Hartley theorem:

$$\boxed{C = B \log_2\left(1 + \frac{S}{N}\right) \text{ bits/s}}$$

C = channel capacity
B = Bandwidth (Hz)
S/N = Signal to Noise ratio (linear, not dB)

The Shannon-Hartley theorem defines the absolute maximum rate (channel capacity) at which error-free digital data can be transmitted over a communication channel of a specific bandwidth (\(B\)) in the presence of noise (\(N\)). It establishes that capacity increases with higher bandwidth or higher signal-to-noise ratio (\(S/N\)).

Key Insights:

1. Capacity $\propto$ bandwidth (linear)
2. Capacity $\propto$ log(SNR) (diminishing returns)
3. **Bandwidth is precious!**
4. Can trade SNR for bandwidth

Shannon capacity example:

Telephone channel: $B = 3$ kHz, SNR $= 30$ dB (1000:1)

$$C = 3000 \times \log_2(1001) \approx 30 \text{ kbps}$$

Ethernet link: $B = 100$ MHz, SNR $= 20$ dB (100:1)

$$C = 10^8 \times \log_2(101) \approx 666 \text{ Mbps}$$

Note: Practical systems achieve 50-90% of Shannon capacity

<u>Propagation speed →</u> Propagation speed in computers is the physical speed at which a signal (bit) travels through a transmission medium, such as copper wire or fiber optic cable

Speed of EM waves in medium :

$$v = \frac{c}{\sqrt{\epsilon_r}}$$

Typical Values:

- Vacuum: $v = c = 3 \times 10^8$ m/s
- Air: $v \approx 0.997c$
- Twisted pair: $v \approx 0.64c$ ($\epsilon_r \approx 2.3$)
- Coax: $v \approx 0.66c$
- Optical fiber: $v \approx 0.67c$ ($n \approx 1.5$)

<u>Communication system</u>

[encoder] → [modulator] → [channel] → [demodulator] → [decorder]

Simplified physical layer model:

- Transmitter generates signal x(t)
- Channel modifies signal (attenuation/ delay)
- Noise adds random disturbance n(t)
- Receiver thus gets → y(t) = h(t)*x(t) + n(t)

<u>AWGN channel</u> : y(t) = h(t)*x(t) + n(t)
Additive White Gaussian Noise

Noise characteristics

- Additive: noise adds to signal
- White: equal power at all frequencies
- Gaussian: amplitude follows gaussian distribution

Thermal Noise:

johnson -Nyquist noise: Random electron motion in conductors

Available noise power: $N = k_B TB$

$k_B = 1.38 \times 10^{-23}$ J/K (Boltzmann constant)
T = temperature (kevlin)
B = Bandwidth (Hz)

Noise power is proportional to bandwidth !!!!!!
The equation N=kbTB defines the maximum available thermal noise power (\(N\)) in Watts produced by a matched resistive source, dependent only on Boltzmann's constant (\(k=1.38\times 10^{-23}\) J/K), absolute temperature (\(T\) in Kelvin), and bandwidth (\(B\) in Hertz)

Noise Figure
NF = measures SNR degradation by component

$$\mathrm{NF\ (dB)} = 10\log_{10}\left(\frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}\right)$$

Friis formula (cascade components)

$$F_{\mathrm{total}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

- Keep first stage NF as low as possible
- High first stage gain reduces impact of later stages

Signal to Noise Ratio (SNR)

Definition:

$$\mathrm{SNR} = \frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}} = \frac{S}{N}$$

In dB:

$$\mathrm{SNR\ (dB)} = 10\log_{10}\left(\frac{S}{N}\right) = P_s(\mathrm{dBm}) - P_n(\mathrm{dBm})$$

Typical values:
- voice = 10-15 dB

- Music = 40 - 60 dB
- Digital communication = 10 - 30 dB

Bits as Signals:

Goal: Transmit digital data (bits) over physical channel

General transmitted signal:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT)$$

A_k = symbol values from data bits
g(t) = pulse shape which determines bandwidth
T = symbol period (symbol rate = 1/T)

Binary Signaling Schemes:
Unipolar (On-Off Keying)

- Bit 0: $a_k = 0$
- Bit 1: $a_k = A$
- Simple but waste energy

Polar (bipolar) NRZ:

- Bit 0: $a_k = -A$
- Bit 1: $a_k = +A$
- Balanced

Pulse Shape

- Rectangular pulse

$$g(t) = \begin{cases} 1 & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

  - Bandwidth: $B \approx 1/T$
  - Simple to generate
  - Causes intersymbol interference

- Raised cosine pulse
  - Bandwidth: $B = \frac{1+\alpha}{2T}$ (α = roll-off factor)
  - Zero ISI at sampling points
  - Controllable bandwidth
  - Used in WiFi, cellular, satellite

Energy Per Bit

Energy per bit:

$$E_b = \int_0^T x^2(t)\,dt$$

For bipolar NRZ: $E_b = A^2 T$

Key performance metric: E_b/N_0

$$\frac{E_b}{N_0} = \frac{\text{Energy per bit}}{\text{Noise spectral density}}$$

Why use E_b/N_0 instead of SNR?

- Normalizes for data rate
- Independent of bandwidth
- Universal metric across modulation schemes

Optimal Receiver Design

Goal: minimize bit error probability in AWGN

Received signal:

$$y(t) = a_k g(t - kT) + n(t)$$

Matched filter theorem: to maximize SNR at sampling instant, use filter matched to pulse:

$$h_{\mathrm{MF}}(t) = g(T - t)$$

Receiver structure:

y(t) → [Matched Filter] → [Sample at t=T] → [Decide] → $\hat{a}_k$

Decision Device

For binary signaling (ak ∈ {−A, +A})

Decision rule:

If z > 0: decide ˆak = +A (bit 1)

If z < 0: decide ˆak = −A (bit 0)

This is the maximum likelihood (ML) decision for equally likely bits.

Threshold:

- Symmetric signaling (bipolar): threshold = 0
- Asymmetric (OOK): threshold = (A0 + A1)/2

Probability of error

Received sample after matched filter: $z = a_k + n_s$, $n_s \sim N(0, \sigma^2)$ with $\sigma^2 = N_0/2$

For bipolar NRZ:

$$P(\text{error}) = Q\left(\frac{A}{\sigma}\right)$$

Error if noise pushes sample across threshold:

Bit error rate: $\boxed{P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$

The Q Function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2}\, du$$

→ small SNR increase → huge BER decrease

BER vs E_b/N_0

E_b/N_0 (dB)	Argument	BER
6 dB	$Q(2)$	2.3×10^{-2}
8 dB	$Q(2.52)$	6×10^{-3}
10 dB	$Q(3.16)$	8×10^{-4}
12 dB	$Q(4)$	3×10^{-5}
15 dB	$Q(5.6)$	10^{-8}

M-ary PAM

Motivation: Send more bits per symbol ⇒ higher data rate

M-ary Pulse Amplitude Modulation:

- M = 2 → Binary (1 bit/symbol)
- M = 4→ 4-PAM (2 bits/symbol)
- M = 8→ 8-PAM (3 bits/symbol)
- M = 16→ 16-PAM (4 bits/symbol)

Multi Level PAM: Performance

Minimunum distance: $d_{min} = 2A$

$$P_s \approx 2\left(1 - \frac{1}{M}\right) Q\left(\frac{d_{\min}}{2\sigma}\right)$$

Probability of symbol error: --> doubling M requires around 6 dB more E_b/N_0 for the same BER

Trade off:

- Higher M → better bandwidth efficiency but requires more power (SNR)

Bandwidth Efficiency

Symbol rate: $R_s = 1/T$ symbols/sec
Bit rate: $R_b = R_s \log_2(M)$ bits/sec

Spectral efficiency: $\eta = R_b/B$ (bits/s/Hz)

M	**bits/symbol**	η **(bits/s/Hz)**
2	1	1.33
4	2	2.67
8	3	4.0
16	4	5.33

Fundamental tradeoffs

1. Power vs. Data Rate (fixed bandwidth)
   a. Higher data rate requires more power
   b. Shannon: C = B log2 (1 + SNR)
2. Bandwidth vs. Data Rate (fixed power)
   a. Higher data rate requires more bandwidth
3. Error Rate vs. SNR
   a. Lower BER requires higher E_b/N_0
   b. Exponential improvement (Q-function)
4. Multi-level signaling
   a. More bits/symbol → better bandwidth efficiency
   b. requires higher SNR for same BER