MIGUEL BLANCO GODÓN

PRÁCTICA 4 MNI

PROBLEMA 1:

Esquema explícito:

$$\begin{cases} y_{n+1} = y_n + \Delta t \cdot f_1(y_n, v_n, t_n) \\ v_{n+1} = v_n + \Delta t \, f_2(y_n, v_n, t_n) \end{cases}$$

$$\begin{cases} \dfrac{dy}{dt} = v = f_1(y, v, t) \\ \dfrac{dv}{dt} = -w^2 y - 2\xi v = f_2(y, v, t) \end{cases}$$

$$\begin{cases} v_n = v_{n-1} + \Delta t \left(-w^2 \cdot y_{n-1} - 2\xi v_{n-1}\right) \\ y_n = y_{n-1} + \Delta t \, v_{n-1} \end{cases}$$

Esquema implícito:

$$v_n = v_{n-1} + \Delta t \left(-w^2 y_n - 2\xi v_n\right)$$

$$v_n + \Delta t \, 2\xi v_n = v_{n-1} - \Delta t w^2 y_n$$

$$v_n (1 + \Delta t \, 2\xi) = v_{n-1} - w^2 y_n \Delta t$$

$$\begin{cases} v_n = \dfrac{v_{n-1} - w^2 y_{n-1} \Delta t}{1 + 2\xi \Delta t} \\ y_n = y_{n-1} + \Delta t \, v_n \end{cases}$$

Escaneado con CamScanner

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PROBLEMA 2:

$$\frac{\partial u}{\partial t} - \frac{\sigma^2}{2}x^2\frac{\partial^2 u}{\partial x^2} - rx\frac{\partial u}{\partial x} + ru = 0 \Rightarrow \frac{u_i^{n+1} - u_i^n}{\Delta t} +$$

$$- \frac{\sigma^2 x^2}{2}\left[\theta\left(\frac{u_{i-1}^{n+1} - 2u_i^{n+1} + u_{i+1}^{n+1}}{(\Delta x)^2}\right) + (1-\theta)\left(\frac{u_{i-1}^n - 2u_i^n + u_{i+1}^n}{(\Delta x)^2}\right)\right]$$

$$-rx\left[\theta\left(\frac{u_{i+1}^{n+1} - u_i^{n+1}}{\Delta x}\right) + (1-\theta)\left(\frac{u_{i+1}^n - u_i^n}{\Delta x}\right)\right] + r\left[\theta u_i^{n+1} + (1-\theta)u_i^n\right] = 0$$

$$\frac{u_i^{n+1}}{\Delta t} - \frac{u_i^n}{\Delta t} - \frac{\sigma^2 x^2\theta u_{i-1}^{n+1}}{2(\Delta x)^2} + \frac{2\sigma^2 x^2 \theta u_i^{n+1}}{2(\Delta x)^2} - \frac{\sigma^2 x^2 \theta u_{i+1}^{n+1}}{2(\Delta x)^2} - \frac{\sigma^2 x^2(1-\theta)u_{i-1}^n}{2(\Delta x)^2}$$

$$+ \frac{2\sigma^2 x^2(1-\theta)u_i^n}{2(\Delta x)^2} - \frac{\sigma^2 x^2(1-\theta)u_{i+1}^n}{2(\Delta x)^2} \frac{-rx\theta u_{i+1}^{n+1}}{\Delta x} + \frac{rx\theta u_i^{n+1}}{\Delta x} + \Rightarrow$$

$$- \frac{(1-\theta)rx u_{i+1}^n}{\Delta x} + rx\frac{(1-\theta)u_i^n}{\Delta x} + r\theta u_i^{n+1} + r(1-\theta)u_i^n = 0$$

$$u_i^{n+1}\left[\frac{1}{\Delta t} + \frac{\sigma^2 x^2 \theta}{(\Delta x)^2} + \frac{rx\theta}{\Delta x} + r\theta\right] + u_{i-1}^{n+1}\left[-\frac{\sigma^2 x^2\theta}{2(\Delta x)^2}\right] + u_{i+1}^{n+1}\left[-\frac{\sigma^2 x^2 \theta}{2(\Delta x)^2} \frac{-rx\theta}{\Delta x}\right] =$$

$$-u_i^n\left[-\frac{1}{\Delta t} + \frac{\sigma^2 x^2(1-\theta)}{(\Delta x)^2} + \frac{rx(1-\theta)}{\Delta x} + r(1-\theta)\right] - u_{i-1}^n\left[-\frac{\sigma^2 x^2(1-\theta)}{2(\Delta x)^2}\right] +$$

$$- u_{i+1}^n\left[-\frac{\sigma^2 x^2(1-\theta)}{2(\Delta x)^2} - \frac{(1-\theta)rx}{\Delta x}\right]$$

Condición de contorno

$$u(x=4E,t) = 0 \Rightarrow u_n = 0$$

$$\frac{\partial u}{\partial x}(x=0,t) = \theta\frac{(u_1^{n+1} - u_0^{n+1})}{\Delta x} + \frac{(1-\theta)(u_1^n - u_0^n)}{\Delta x} = -1$$

$$\frac{\theta u_1^{n+1}}{\Delta x} - \frac{\theta u_0^{n+1}}{\Delta x} = -1 - \frac{(1-\theta)u_1^n}{\Delta x} + \frac{(1-\theta)u_0^n}{\Delta x}$$

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PROBLEMA 3:

$$-\operatorname{div}\left[a(x,y)\nabla u\right] + \vec{b}(x,y)\nabla u + c(x,y)u = f(x,y,t) \Rightarrow$$

$$-\frac{\partial a(x,y)}{\partial x}\cdot\frac{\partial u}{\partial x} - a(x,y)\frac{\partial^2 u}{\partial x^2} - \frac{\partial a(x,y)}{\partial y}\cdot\frac{\partial u}{\partial y} - a(x,y)\frac{\partial^2 u}{\partial y^2} - 3y\frac{\partial u}{\partial x}$$

$$+ 3x\frac{\partial u}{\partial y} + c(x,y)u = -0{,}1x\frac{\partial u}{\partial x} - a_{i,j}\frac{\partial^2 u}{\partial x^2} - 0{,}004y\frac{\partial u}{\partial y} - a_{ij}\frac{\partial^2 u}{\partial y^2}$$

$$-3y\frac{\partial u}{\partial x} + 3x\frac{\partial u}{\partial y} + c_{i,j}u_{ij} = -0{,}1x\left(\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x}\right) - a_{ij}\left(\frac{u_{i-1,j} - 2u_{ij} + u_{i+1,j}}{(\Delta x)^2}\right)$$

$$-0{,}004\left(\frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y}\right) - a_{i,j}\left(\frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{(\Delta y)^2}\right) - 3y\left(\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x}\right)$$

$$+ 3x\left(\frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y}\right) + c_{i,j}u_{ij} \Rightarrow$$

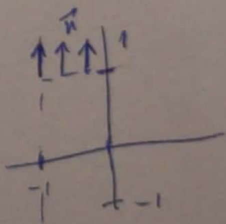
$$u_{i-1,j}\cdot\left(\frac{0{,}1x}{2\Delta x} - \frac{a_{i,j}}{(\Delta x)^2} + \frac{3y}{2\Delta x}\right) \longrightarrow p(k,k-1)$$

$$u_{i+1,j}\cdot\left(\frac{-0{,}1x}{2\Delta x} - \frac{a_{ij}}{(\Delta x)^2} - \frac{3y}{2\Delta x}\right) \longrightarrow p(k,k+1)$$

$$u_{i,j}\cdot\left(\frac{+2a_{i,j}}{(\Delta x)^2} + \frac{2a_{i,j}}{(\Delta y)^2} + c_{ij}\right) \longrightarrow p(k,k)$$

$$u_{i,j-1}\left(\frac{0{,}004}{2\Delta y} - \frac{a_{ij}}{(\Delta y)^2} - \frac{3x}{2\Delta y}\right) \longrightarrow p(k,k-(N-2))$$

$$u_{i,j+1}\left(\frac{-0{,}004}{2\Delta y} - \frac{a_{ij}}{(\Delta y)^2} + \frac{3x}{2\Delta y}\right) \longrightarrow p(k,k+(N-2))$$



Contorno (apartado a): $\partial\Omega = 0$;

Contorno (apartado b):

$$\nabla u\cdot\vec{n} = 0{,}08 \Rightarrow \begin{pmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial u}{\partial y}\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{\partial u}{\partial y} = 0{,}08$$

$$\frac{u_{i,j+1} - u_{i,j-1}}{\Delta y} = 0{,}08 \Rightarrow \begin{cases} p(k,k) = \frac{-1}{\Delta y} \\ p(k,k+(N-2)) = \frac{1}{\Delta y}\end{cases}$$