

Comprehensive Technical Report: Dynamic Modeling and Simulation of Electrical Machines (Machine Dynamics) by Melvin Chibudom Amadi-Duru

Part 1: Dynamic Modeling of a Separately Excited DC Motor

1.1 Introduction and System Specifications

The objective of this study was to implement and analyze the dynamic behavior of a separately excited DC motor under varying load conditions. The mathematical model was developed using the MATLAB/SIMULINK environment to simulate transient and steady state responses.

System Parameters:

- Armature Resistance (R_a): 0.5Ω
- Armature Inductance (L_a): 0.003 H
- Back-EMF Constant (K_b): 0.8 V/rad/sec
- Inertia (J): $0.0167 \text{ kg} \cdot \text{m}^2$
- Viscous Friction (B_1): $0.01 \text{ N} \cdot \text{m/rad/sec}$
- Supply Voltage: 220 V DC

Mathematical Framework

The system is modeled using two coupled differential equations representing the electrical and mechanical domains.

1.2.1 Electrical Domain Equation

The voltage equation for the armature circuit is derived from Kirchhoff's Voltage Law: $V_{in} = E_a + R_a i_a + L_a \frac{di_a}{dt}$

Where E_a is the Back Electromotive Force (EMF), defined as $E_a = K_e \omega_m$. Rearranging for the rate of change of current: $\frac{di_a}{dt} = \frac{1}{L_a} (V_{in} - K_e \omega_m - R_a i_a)$

1.2.2 Mechanical Domain Equation

The mechanical torque balance is governed by Newton's Second Law for rotation: $J \frac{d\omega_m}{dt} + B_1 \omega_m = T_e - T_L$. Where T_e is the electromagnetic torque ($T_e = K_t i_a$) and T_L is the load torque. Rearranging for acceleration: $\frac{d\omega_m}{dt} = \frac{1}{J} (K_t i_a - T_L - B_1 \omega_m)$

1.3 Simulink Implementation

The model was constructed using standard integrator blocks to solve the differential equations.

- **Current Loop:** The input voltage (V_{in}) and Back-EMF ($K_e \omega_m$) are summed and divided by inductance (L_a) to obtain di/dt , which is integrated to solve for armature current (i_a)
- **Speed Loop:** The electromagnetic torque ($K_t i_a$) and load torque (T_L) are summed and divided by inertia (J) to obtain angular acceleration, which is integrated to solve for rotor speed (ω_m)

1.4 Results and Discussion

1.4.1 No-Load Characteristics Under no-load conditions ($T_L = 0$),

The motor accelerates from a standstill.

- **Current Response:** The armature current initially spikes to generate the necessary starting torque but decays to zero as the motor reaches steady state. This confirms that without a mechanical load, the steady-state current is negligible.
- **Speed Response:** The speed rises smoothly and settles at the rated speed where the Back-EMF balances the input voltage.

1.4.2 Loaded Characteristics (100 Nm)

When a starting torque of 100 Nm is applied:

- **Transient Overshoot:** A significant current overshoot is observed during startup. This is physically required because the electromagnetic torque (T_e) must exceed the load torque (T_L) to accelerate the rotor inertia (J).
- **Steady-State Current:** Unlike the no-load case, the armature current stabilizes at a positive non-zero value. Because the field current is constant, T_e is directly proportional to i_a . Therefore, a constant load torque demands a constant holding current.
- **Speed Regulation:** The motor stabilizes at a lower speed compared to the no-load condition. This speed reduction is consistent with power conservation principles; for a constant power output, an increase in torque load necessitates a reduction in speed.

Part 2: Dynamic Modeling of a Hydro-Turbine Synchronous Generator

2.1 Introduction and System Specifications

This section details the modeling of a high-power hydro-turbine synchronous generator to analyze its transient stability during load perturbations. The system includes stator windings, field windings, and damper windings modeled in the $q - d$ reference frame.

Generator Specifications:

- Power Rating: 325 MVA 25
- Voltage: 20 kV (Line-to-Line) 26
- Speed: 112.5 RPM (64 Poles) 27
- Inertia (J): $35.1 \times 10^6 J \cdot s^2$ 28
- Reactances (pu): $X_d = 0.850$, $X_q = 0.480$, $X_{ls} = 0.12029$

2.2 Mathematical Framework and Park's Transformation Modeling AC machines involves complex time-varying inductances. To simplify this, "Park's Transformation" is applied to convert the stator variables (a, b, c) onto a rotor-reference frame ($q, d, 0$).

2.2.1 Transformation Matrix (K_s) The transformation converts sinusoidal voltages into time-invariant DC quantities: $[V_{qdo}] = K_s [V_{abc}]$ This simplifies the analysis, decoupling the magnetic flux (d-axis) from the torque production (q-axis).

2.2.2 Flux Linkage Equations The magnetic coupling between the stator and rotor windings is represented by a flux linkage matrix (λ): $[\lambda] = [L] \times [i]$ The inductance matrix $[L]$ includes self-inductances (L_d, L_q) and mutual inductances (L_{md}, L_{mq}) between the stator and the rotor damper windings (k_{q1}, k_{q2}, k_d).

2.2.3 Torque and Speed Equations The electromagnetic torque is derived from the flux linkages and currents: $T_{em} = \frac{3P}{2} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds})$ The rotor speed is determined by the swing equation, integrating the difference between mechanical load torque (T_L) and electromagnetic torque (T_{em}):
$$\omega_r = \int \frac{1}{J} (T_L + T_{em} - T_f) dt$$

2.3 Simulation Results and Analysis The generator was simulated under a sudden step increase in input torque from 0 to $27.6 \times 10^6 N \cdot m$.

2.3.1 Speed and Torque Transient Response Immediately following the torque step, the rotor speed increased, peaking at approximately 380 electrical rad/sec before oscillating and settling. This oscillation occurs because the accelerating torque temporarily exceeds the load torque, causing the rotor to "swing" before finding a new equilibrium.

2.3.2 Current Analysis (i_d, i_q) The simulation accurately calculated the steady-state RMS current to be 9.04 kA, which aligns closely with the theoretical calculation of 9.37 kA³⁷. The transient period (0 to 4 seconds) showed significant oscillation in the i_q and i_d components as the machine reacted to the load change.

2.3.3 Torque Angle (δ) Verification The torque angle is a critical stability metric.

- Theoretical Calculation: $\delta = 18^\circ 39$.
- Simulated Result: $\delta \approx 16^\circ 40$.

The close agreement verifies the model's accuracy. The negative sign observed in the simulation confirms the machine is operating in generating mode, exporting power to the grid.

2.4 Conclusion

This project successfully developed and validated dynamic models for both DC and Synchronous machines.

1. **DC Machine:** The model demonstrated the direct proportionality between load torque and armature current in the steady state, validating the electromechanical governing equations.
2. **Synchronous Machine:** The use of Park's transformation effectively linearized the complex AC dynamics. The model accurately predicted the steady-state current (96.5% accuracy) and torque angle (89% accuracy), confirming its robustness for analyzing transient stability in power systems.

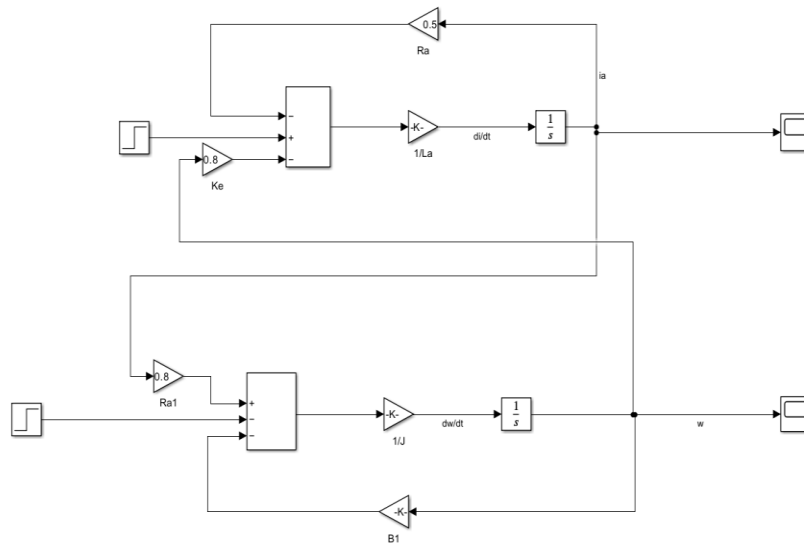


Fig. 1: Simulink Model of a DC Electric Machine

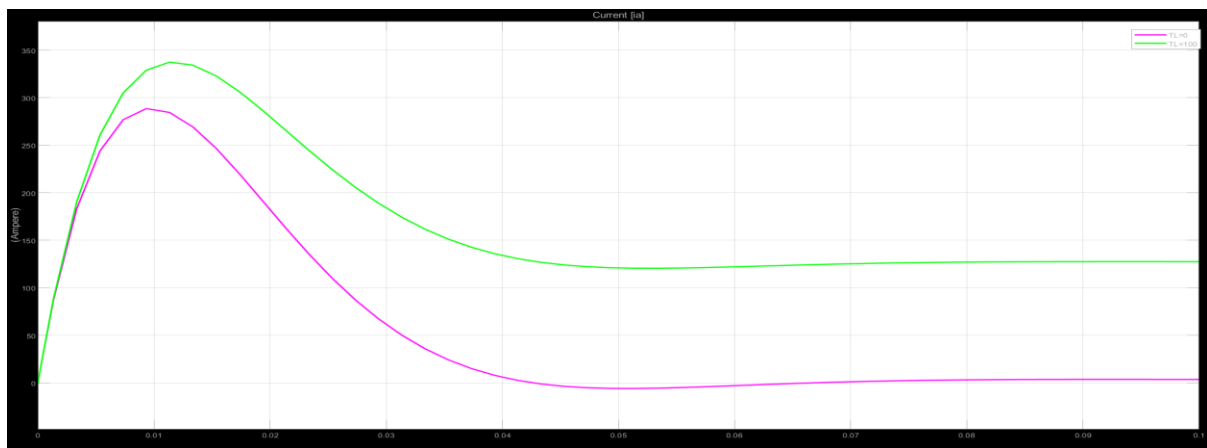
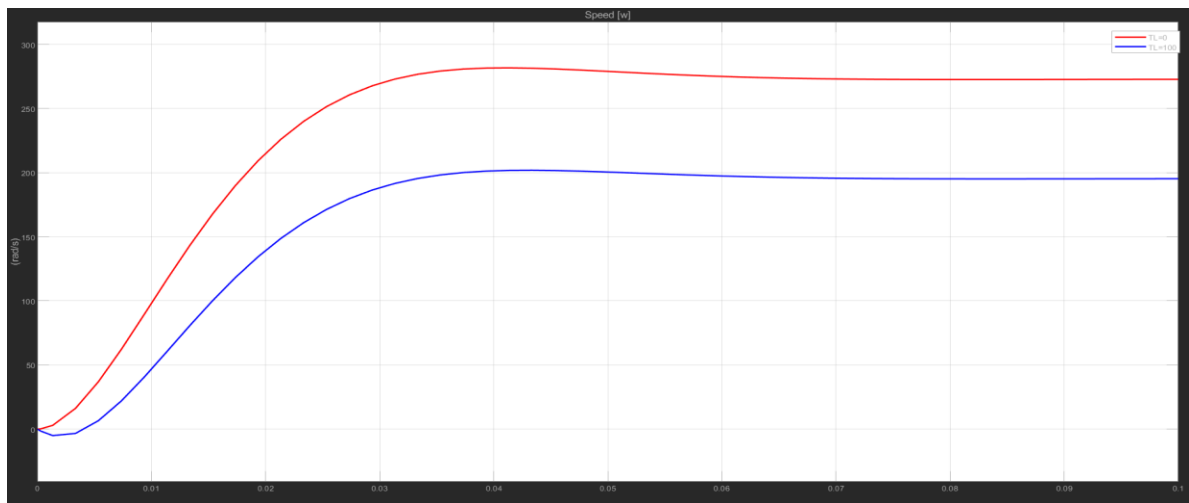


Fig. 2: Combined Simulation Results

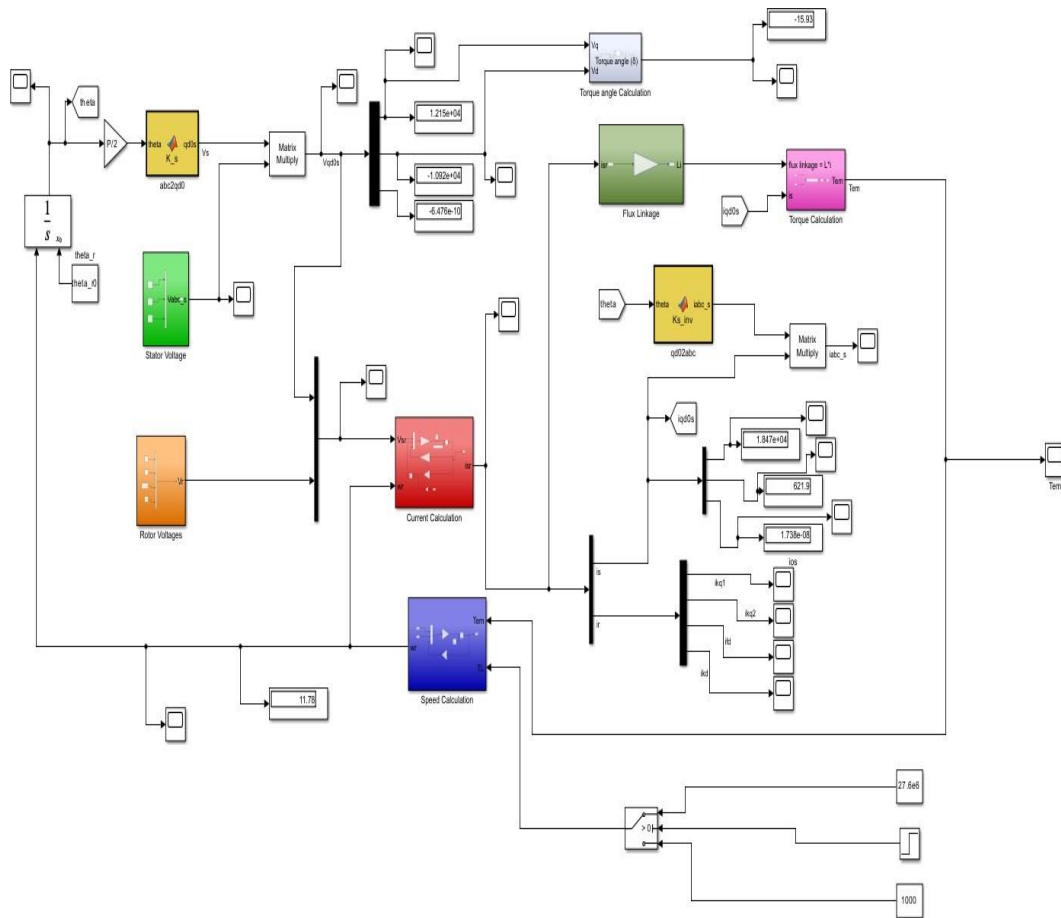


Fig 3: Simulink Model of a Synchronous Generator

References

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