

# **Comprehensive Technical Report: Dynamic Modeling and Simulation of Electrical Machines (Machine Dynamics) by Melvin Chibudom Amadi-Duru**

## **Part 1: Dynamic Modeling of a Separately Excited DC Motor**

### **1.1 Introduction and System Specifications**

The objective of this study was to implement and analyze the dynamic behavior of a separately excited DC motor under varying load conditions. The mathematical model was developed using the MATLAB/SIMULINK environment to simulate transient and steady state responses.

#### **System Parameters:**

- Armature Resistance ( $R_a$ ): 0.5 Ω
- Armature Inductance ( $L_a$ ): 0.003 H
- Back-EMF Constant ( $K_b$ ): 0.8 V/rad/sec
- Inertia ( $J$ ): 0.0167  $kg \cdot m^2$
- Viscous Friction ( $B_1$ ): 0.01 N · m/rad/sec
- Supply Voltage: 220 V DC
- Supply Frequency: 50 Hz

### **Mathematical Framework**

The system is modeled using two coupled differential equations representing the electrical and mechanical domains.

#### **1.2.1 Electrical Domain Equation**

The voltage equation for the armature circuit is derived from Kirchhoff's Voltage Law:  $V_{in} = E_a + R_a i_a + L_a \frac{di_a}{dt}$

Where  $E_a$  is the Back Electromotive Force (EMF), defined as  $E_a = K_e \omega_m$ . Rearranging for the rate of change of current:  $\frac{di_a}{dt} = \frac{1}{L_a} (V_{in} - K_e \omega_m - R_a i_a)$

#### **1.2.2 Mechanical Domain Equation**

The mechanical torque balance is governed by Newton's Second Law for rotation:  $J \frac{d\omega_m}{dt} + B_1 \omega_m = T_e - T_L$ . Where  $T_e$  is the electromagnetic torque ( $T_e = K_t i_a$ ) and  $T_L$  is the load torque. Rearranging for acceleration:  $\frac{d\omega_m}{dt} = \frac{1}{J} (K_t i_a - T_L - B_1 \omega_m)$

### **1.3 Simulink Implementation**

The model was constructed using standard integrator blocks to solve the differential equations.

- Current Loop: The input voltage ( $V_{in}$ ) and Back-EMF ( $K_e \omega_m$ ) are summed and divided by inductance ( $L_a$ ) to obtain  $di/dt$ , which is integrated to solve for armature current ( $i_a$ )
- Speed Loop: The electromagnetic torque ( $K_t i_a$ ) and load torque ( $T_L$ ) are summed and divided by inertia ( $J$ ) to obtain angular acceleration, which is integrated to solve for rotor speed ( $\omega_m$ )

## 1.4 Results and Discussion

### 1.4.1 No-Load Characteristics Under no-load conditions ( $T_L = 0$ ),

The motor accelerates from a standstill.

- Current Response: The armature current initially spikes to generate the necessary starting torque but decays to zero as the motor reaches steady state. This confirms that without a mechanical load, the steady-state current is negligible.
- Speed Response: The speed rises smoothly and settles at the rated speed where the Back-EMF balances the input voltage.

### 1.4.2 Loaded Characteristics (100 Nm)

When a starting torque of 100 Nm is applied:

- Transient Overshoot: A significant current overshoot is observed during startup. This is physically required because the electromagnetic torque ( $T_e$ ) must exceed the load torque ( $T_L$ ) to accelerate the rotor inertia ( $J$ ).
- Steady-State Current: Unlike the no-load case, the armature current stabilizes at a positive non-zero value. Because the field current is constant,  $T_e$  is directly proportional to  $i_a$ . Therefore, a constant load torque demands a constant holding current.
- Speed Regulation: The motor stabilizes at a lower speed compared to the no-load condition. This speed reduction is consistent with power conservation principles; for a constant power output, an increase in torque load necessitates a reduction in speed.

## Part 2: Dynamic Modeling of a Hydro-Turbine Synchronous Generator

### 2.1 Introduction and System Specifications

This section details the modeling of a high-power hydro-turbine synchronous generator to analyze its transient stability during load perturbations. The system includes stator windings, field windings, and damper windings modeled in the  $q - d$  reference frame.

Generator Specifications:

- Power Rating: 325 MVA 25
- Voltage: 20 kV (Line-to-Line) 26
- Speed: 112.5 RPM (64 Poles) 27
- Inertia ( $J$ ):  $35.1 \times 10^6 J \cdot s^2$  28
- Reactances (pu):  $X_d = 0.850$ ,  $X_q = 0.480$ ,  $X_{ls} = 0.12029$

2.2 Mathematical Framework and Park's Transformation Modeling AC machines involves complex time-varying inductances. To simplify this, "Park's Transformation" is applied to convert the stator variables ( $a, b, c$ ) onto a rotor-reference frame ( $q, d, 0$ ).

2.2.1 Transformation Matrix ( $K_s$ ) The transformation converts sinusoidal voltages into time-invariant DC quantities:  $[V_{qdo}] = K_s[V_{abc}]$  This simplifies the analysis, decoupling the magnetic flux (d-axis) from the torque production (q-axis).

2.2.2 Flux Linkage Equations The magnetic coupling between the stator and rotor windings is represented by a flux linkage matrix ( $\lambda$ ):  $[\lambda] = [L] \times [i]$  The inductance matrix  $[L]$  includes self-inductances ( $L_d, L_q$ ) and mutual inductances ( $L_{md}, L_{mq}$ ) between the stator and the rotor damper windings ( $k_{q1}, k_{q2}, k_d$ ).

2.2.3 Torque and Speed Equations The electromagnetic torque is derived from the flux linkages and currents:  $T_{em} = \frac{3}{2} \frac{P}{2} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds})$  33 The rotor speed is determined by the swing equation, integrating the difference between mechanical load torque ( $T_L$ ) and electromagnetic torque ( $T_{em}$ ):  
$$\omega_r = \int \frac{1}{J} (T_L + T_{em} - T_f) dt$$

2.3 Simulation Results and Analysis The generator was simulated under a sudden step increase in input torque from 0 to  $27.6 \times 10^6 N \cdot m$ .

2.3.1 Speed and Torque Transient Response Immediately following the torque step, the rotor speed increased, peaking at approximately 380 electrical rad/sec before oscillating and settling. This oscillation occurs because the accelerating torque temporarily exceeds the load torque, causing the rotor to "swing" before finding a new equilibrium.

2.3.2 Current Analysis ( $i_d, i_q$ ) The simulation accurately calculated the steady-state RMS current to be 9.04 kA, which aligns closely with the theoretical calculation of 9.37 kA 37. The transient period (0 to 4 seconds) showed significant oscillation in the  $i_q$  and  $i_d$  components as the machine reacted to the load change.

2.3.3 Torque Angle ( $\delta$ ) Verification The torque angle is a critical stability metric.

- Theoretical Calculation:  $\delta = 18^\circ 39$ .
- Simulated Result:  $\delta \approx 16^\circ 40$ .

The close agreement verifies the model's accuracy. The negative sign observed in the simulation confirms the machine is operating in generating mode, exporting power to the grid.

## 2.4 Conclusion

This project successfully developed and validated dynamic models for both DC and Synchronous machines.

1. **DC Machine:** The model demonstrated the direct proportionality between load torque and armature current in the steady state, validating the electromechanical governing equations.
2. **Synchronous Machine:** The use of Park's transformation effectively linearized the complex AC dynamics. The model accurately predicted the steady-state current (96.5% accuracy) and torque angle (89% accuracy), confirming its robustness for analyzing transient stability in power systems.

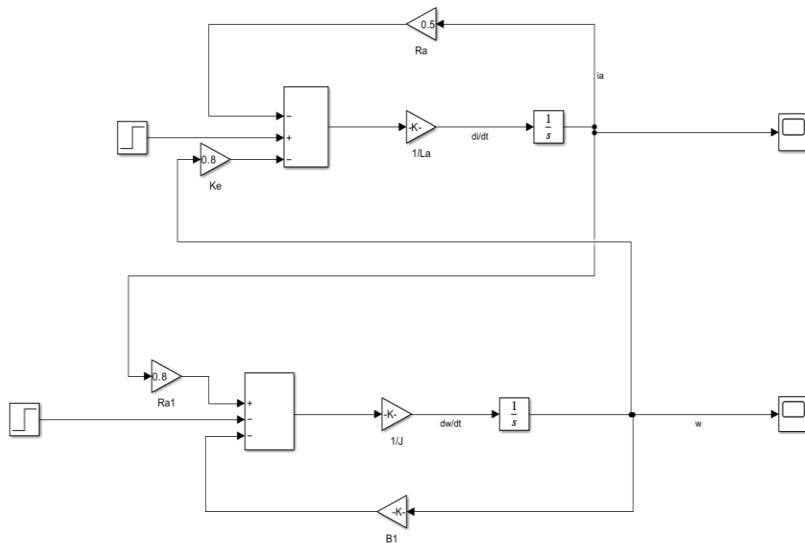


Fig. 1: Simulink Model of a DC Electric Machine

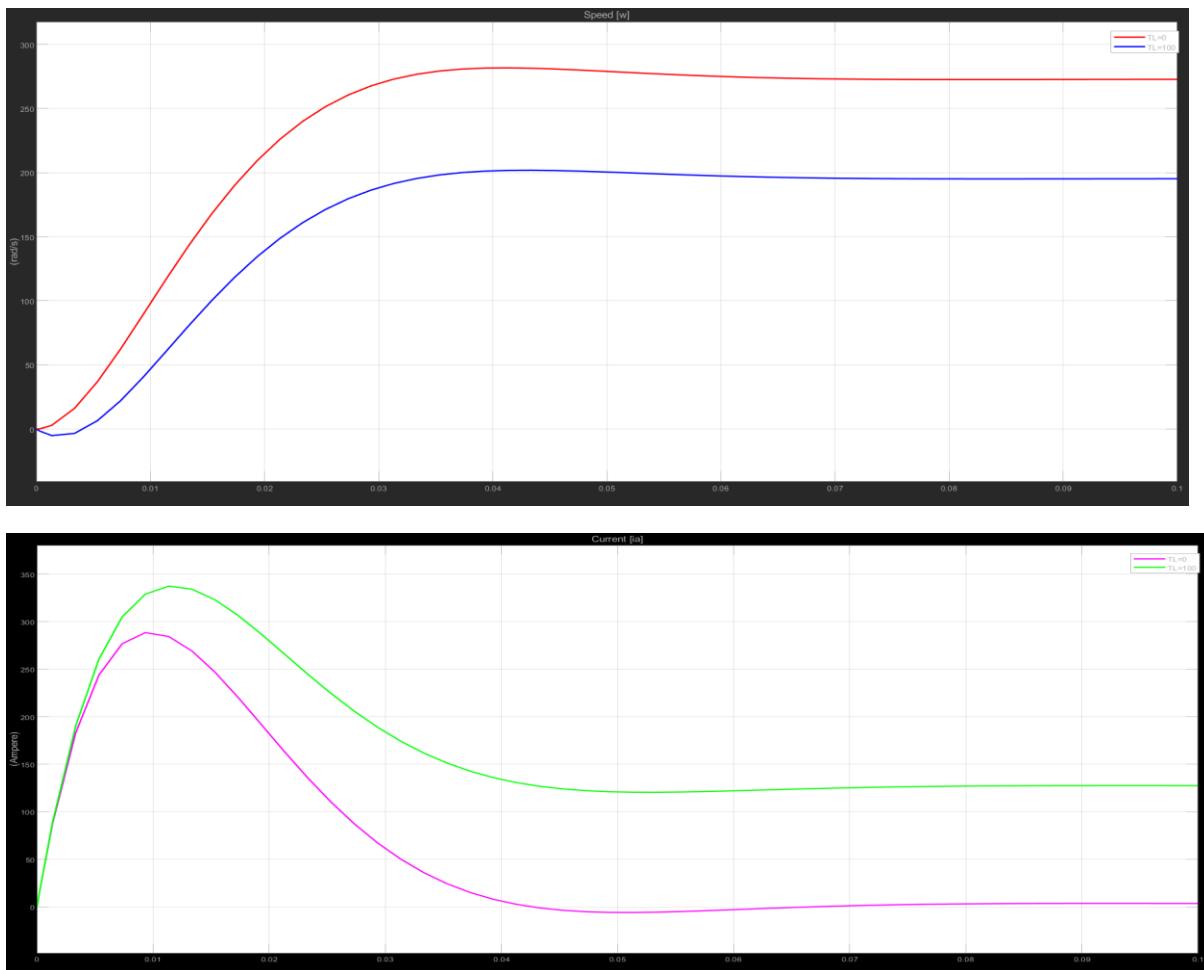
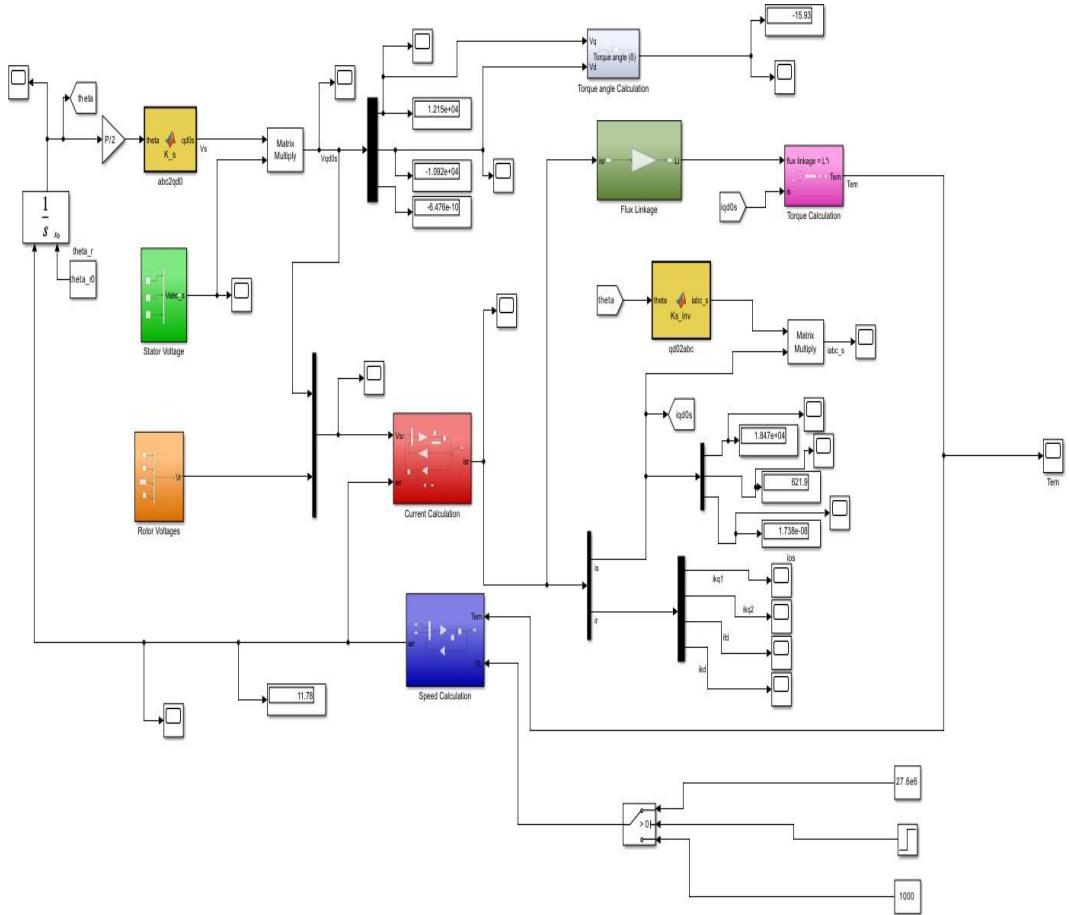


Fig. 2: Combined Simulation Results



**Fig 3: Simulink Model of a Synchronous Generator**

## References

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