

DISMATH Q2

Name: _____

Section: _____

1. Use the Insert sort algorithm to sort alphabetically the following list.

S, P, T, W, N

Write the state of the **full list** for every assignment operation or when an element changes value during the execution of the algorithm. Use asterisk (*) to denote elements that are already sorted. (Solution in the test booklet). How many comparisons are needed in this case? _____

What is the time complexity (big-Theta notation) of Insert sort? _____

2.

Let $f : \mathbb{N} \longrightarrow \mathbb{R}$ be defined by

$$f(n) = \frac{n^4 + \log_2 n}{n^2 + 1}.$$

a. Θ (_____) b. Upper-bound witnesses: _____ c. Lower-bound witnesses: _____

3. If $A = \{1, 2, 3\}$, $B = \{2, 4, 6, 8\}$ and the universal set $U = \{1, 2, 3, \dots\}$ find

a. $A - B = \{ \text{_____} \}$ b. $B' = \{ \text{_____} \}$

c. Power set of $A = \{ \text{_____} \}$

4. Give a proof of or a counterexample to the following statement: (Solution in the test booklet)

$$A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$$

5. Construct a table showing the interchanges that occur at each step when bubble sort is applied to the following list: 6, 4, 5, 7, 3

steps	lists

Continue the solution to the booklet if necessary.

6. Given the following function:

$$f(x) = (x^2 + 5x + 3)(x + 2 \log x)$$

a. $f(x)$ is $O(x^4)$: True/ False _____

d. $f(x)$ is $O(x^3 \log x)$: True/ False _____

b. $f(x)$ is $O(x^3)$: True/ False _____

e. $f(x)$ is $O(x^2 \log x)$: True/ False _____

c. $f(x)$ is $O(x^2)$: True/ False _____

7. Given:

Procedure A (n : positive integer)

$s := 0$

for $i := 1$ **to** n

for $j := 1$ **to** i

$s := s + j$

return s

a. Suppose that procedure A is started with input $n = 4$. Then what number is returned by the algorithm?

b. The worst-case time complexity of procedure A is: _____

8. (a) How many functions are there from $\{1,2\}$ to $\{a,b,c\}$?

(b) How many of these functions are one-to-one?

(c) How many of these functions are onto?

(d) How many of these functions are bijective?

9. Count the number of comparisons for the ff. algorithm: _____

What is its time complexity? _____

Require: $\{a_1, a_2, \dots, a_i, \dots, a_n\} \neq \in \mathbb{Z}$, where $a_1 < a_2 < \dots < a_n$; $x \in \mathbb{Z}$

Ensure: $result = k$, where $(a_k = x)$ and $k \in \{1, \dots, n\}$ if the element is found; otherwise $k = -1$

$i \leftarrow 1$

$j \leftarrow n$

while $i < j$ **do**

$mid \leftarrow \left\lfloor \frac{i+j}{2} \right\rfloor$

if $x > a_{mid}$ **then**

$i \leftarrow mid + 1$

else

$j \leftarrow mid$

end if

end while

if $x == a_i$ **then**

$result \leftarrow i$

else

$result \leftarrow -1$

end if

10. Given a set of two-dimensional points, $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$, in the Cartesian plane.

a. Write a pseudocode to find the farthest pair of points by computing the distances between all pairs of the n points and determining the largest distance.

b. Write a pseudocode to sort the points according to the abscissa of the corresponding points.

c. Write a pseudocode to sort the points according to the ordinate of the corresponding points.

d. Give the time complexity estimate (Big Theta) for each of the previous algorithms.

*** END ***