

DISMATH Q1

Name: _____ Section: _____ Start time: _____ End: _____

Instructions:

- (1) Bring **ONLY** your test booklet and pencil/pen in your test area.
- (2) Place your bags and other things, not included in (1), in the **front** of the room.
Note: You should **KEEP** your valuables, such as wallets, smartphones, etc. with you.
- (3) Turn **OFF** all your mobile devices, i.e. smartphones, tablets, laptops, etc. for the whole duration of your test. Violating this rule will be considered as cheating!
- (4) Going to the Rest room is allowed **BUT** only at a **ONE-AT-A-TIME** basis or if somebody from the test room is outside, wait for the person to return. Leave your smartphone in your test area (**OFF** state) or into your bag in the front.
- (5) Utilize your test booklet for the solutions, but write your final answers in the blanks provided in this questionnaire. Note: **"No Solution, No Credit. Wrong Solution, No Credit!"**
- (6) **Neighborhood Map**: Write the full name of the person in your Front, Left, Right, and Back.
Use **NONE** for nobody. Inaccurate neighborhood map is considered cheating!

Name: _____

Front

_____:Left **YOU** Right:_____

Back

0. Let a sequence $(x_i)_{i \in \mathbb{N}}$ be determined by the recurrence relation

$$x_{n+2} = \frac{1 + x_{n+1}}{x_n}$$

and the initial conditions $x_0 = a$ and $x_1 = b$ with $a, b \neq 0$. Find x_{10} .

1. Prove (or disprove) the validity of the ff. statement: (Solution in the booklet)

Either Binay is guilty or Duterte is guilty.

Either Duterte is guilty or Pacquiao is guilty.

Therefore, either Binay is guilty or Pacquiao is guilty.

Is this argument valid? _____ (Yes/No)

2. Prove (or disprove) the ff. theorem: (Solution in the booklet)

"Suppose x and y are real numbers. If $0 < x < y$ then $x^2 < y^2$."

Is this theorem correct? _____ (Yes/No)

3. Prove (or disprove) the validity of the ff. argument: (Solution in the booklet)

Poe and Duterte are not both innocent.

Either Poe is lying or Duterte is innocent.

Therefore, Poe is either lying or not innocent.

Is this argument valid? _____ (Yes/No)

4. Simplify: $\neg(Q \wedge \neg P) \vee P$ (in 4 steps)

- a. _____ (_____ law)
- b. _____ (_____ law)
- c. _____ (_____ law)
- d. _____ (_____ law)

5. Simplify: $\neg(P \vee (Q \wedge \neg R)) \wedge Q$ (in 7 steps)

- a. _____ (_____ law)
- b. _____ (_____ law)
- c. _____ (_____ law)
- d. _____ (_____ law)
- e. _____ (_____ law)
- f. _____ (_____ law)
- g. _____ (_____ law)

6. Prove (or disprove) the ff. theorem: (Solution in the booklet)

"Suppose a , b , and c are real numbers and $a > b$. If $ac \leq bc$, then $c \leq 0$."

Is this theorem correct? _____ (Yes/No)

7. Determine the truth value of the ff. statements, where the universe of discourse in each case is the set of all natural numbers, N .

- | | | | |
|----------------------------------|-------|----------------------------------|-------|
| a. $\forall x \exists y (x < y)$ | _____ | d. $\forall y \exists x (x < y)$ | _____ |
| b. $\exists y \forall x (x < y)$ | _____ | e. $\exists x \exists y (x < y)$ | _____ |
| c. $\exists x \forall y (x < y)$ | _____ | f. $\forall x \forall y (x < y)$ | _____ |

8. Prove (or disprove) the validity of the ff. statement: (Solution in the booklet)

Either Juan isn't stupid and he is lazy, or he's stupid.

Juan is stupid.

Therefore, Juan isn't lazy.

Is this argument valid? _____ (Yes/No)

9. Determine the truth value of $\exists y \forall x (x + y = 9)$, where the universe of discourse for both x and y is the set of all integer numbers, \mathbb{Z} .

Truth value: _____; Why?: _____

10. Prove (or disprove) the ff. theorem: (Solution in the booklet)

"Suppose x and y are real numbers and $x + y = 10$. Then x is not equal to 3 and y is not equal to 8." Is this theorem correct? _____ (Yes/No)

11. Prove (or disprove) the ff. theorem: (Solution in the booklet)

"Suppose that x and y are real numbers. If $x^2 + y = 13$ and y is not equal to 4 then x is not equal to 3." Is this theorem correct? _____ (Yes/No)

12. Prove (or disprove) the ff. theorem: (Solution in the booklet)

"Suppose that x and y are real numbers and x is not equal to 3. If $x^2 y = 9y$ then $y = 0$."

Is this theorem correct? _____ (Yes/No)

13. Use truth table to prove show that disjunctive syllogism is a valid rule of inference.

14. Prove (or disprove) by mathematical induction that for all positive integers n :

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$$

15. Prove (or disprove) by mathematical induction: $1^3 + 2^3 + \dots + n^3 = \frac{1}{4} n^2(n+1)^2$

16. Given a Fibonacci sequence $f_1, f_2, f_3, f_4, f_5, \dots, f_n = 1, 1, 2, 3, 5, \dots$

Prove (or disprove): $f_1 + f_2 + f_3 + \dots + f_n = f_n f_{n+1}$

17. Verify the program segment

if $a < b$ then

$a := b$

with respect to the initial assertion T and the final assertion $a \leq b$.

18. Verify the program segment; `int x; int y; int result; result = x XOR ((x XOR y) AND NOT(x < y));` with respect to the initial assertion T and the final assertion, $result = \max(x, y)$.

***** END *****