

DISMATH101: Solved Problems in Discrete Math

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Chapter 1

Logic

1.1 Propositional Logic

1.2 Logical connectives

1.3 Truth table

1.4 Logical Equivalences

1.5 Predicate Logic

1.6 Quantifiers

1.7 Rules of Inference

Exercise 1.

The following statements are propositions, EXCEPT

A. Euclid of Alexandria is human.

C. $5 > 8$

B. $1 + 1 = 2$

D. Who am I?

Definition 1 (Proposition). *a statement that is either true or false (but not both).*

Solution: “D. Who am I?” is not a proposition because interrogative statements have no truth value.

Exercise 2.

What is the question?

Solution: “I thoroughly disapprove of duels. If a man should challenge me, I would take him kindly and forgivingly by the hand and lead him to a quiet place and kill him.”

Chapter 2

Sets

2.1 Cardinality of Sets

2.2 Set Operations

2.3 Set Identities

2.4 Set Representation

2.5 Venn Diagram

2.6 Cartesian Product

Chapter 3

Proofs

3.1 Direct Proof

3.2 Proof by Contraposition

3.3 Proof by Contradiction

3.4 Proof by Equivalence

3.5 Mathematical Induction

Chapter 4

Functions and Relations

4.1 Basic Types of Functions

4.2 Composition of Functions

4.3 Graphs of Functions

4.4 Relation Properties

4.5 Closures of Relations

4.6 Equivalence Relations

Chapter 5

Arrays and Matrices

5.1 Indexing

5.2 Sequences and Summation

5.3 Matrix Arithmetic

5.4 Transpose

5.5 Powers of Matrices

5.6 Determinants

Chapter 6

Combinatorics

6.1 Basics of Counting

6.2 Pigeonhole Principle

6.3 Permutation and Combination

6.4 Binomial Theorem

Chapter 7

Graphs

7.1 Graph Representation

7.2 Euler and Hamilton Paths

7.3 Shortest Path

7.4 Planar Graphs

7.5 Graph Coloring

Chapter 8

Trees

8.1 Tree Traversal

8.2 Spanning Trees

8.3 Binomial Trees

Chapter 9

Algorithms

9.1 Searching

9.2 Sorting

9.3 Algorithm Paradigms

9.4 Complexity of Algorithms

9.5 Applications

Chapter 10

Computation Models

10.1 Finite State Machines

10.2 Turing Machine

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