Shaum's Outline
Chapter 35: Improper Integrals

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17. Evaluate the given integrals:

(a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}} = 2$

Sol.

$$\begin{aligned}\int_0^1 \frac{dx}{\sqrt{x}} &= \lim_{a\to 0}\int_a^1 \frac{dx}{\sqrt{x}} \\ &= \lim_{a\to 0} 2\sqrt{x}\Big]_a^1 \\ &= \lim_{a\to 0}\left(2-2\sqrt{a}\right) \\ &= 2\end{aligned}$$

(b) $\displaystyle\int_0^4 \frac{1}{4-x}dx = +\infty$

Sol.

$$\begin{aligned}\int_0^4 \frac{1}{4-x}dx &= \lim_{a\to 4}\int_0^a \frac{1}{4-x}dx \\ &= \lim_{a\to 4} -\ln(4-x)\Big]_0^a \\ &= \lim_{a\to 4}\left(-\ln(4-a)+\ln 4\right) \\ &= +\infty\end{aligned}$$

(c) $\displaystyle\int_0^4 \frac{1}{\sqrt{4-x}}dx = 4$

Sol.

$$\begin{aligned}\int_0^4 \frac{1}{\sqrt{4-x}}dx &= \lim_{a\to 4}\int_0^a \frac{1}{\sqrt{4-x}}dx \\ &= \lim_{a\to 4} -2\sqrt{4-x}\Big]_0^a \\ &= \lim_{a\to 4}\left(-2\sqrt{4-a}+4\right) \\ &= 4\end{aligned}$$

(d) $\displaystyle\int_0^4 \frac{1}{(4-x)^{3/2}}dx = +\infty$

(e) $\displaystyle\int_{-2}^2 \frac{1}{\sqrt{4-x}}dx = \pi$

(f) $\displaystyle\int_{-1}^8 \frac{1}{x^3}dx = \frac{9}{2}$

(g) $\displaystyle\int_0^4 \frac{dx}{(x-2)^{2/3}} = 6\sqrt[3]{2}$

(h) $\int_{-1}^{1} \frac{dx}{x^4} = +\infty$

(i) $\int_{0}^{1} \ln x dx = -1$

(j) $\int_{0}^{1} x \ln x dx = -\frac{1}{4}$