Exercise 1a

Use mathematical induction to prove the following statements (1 - 7).

1. $1+2+3+\cdots+n=\dfrac{n(n+1)}{2}$

2. $1\cdot 2+2\cdot 3+3\cdot 4+\cdots+n(n+1)=\dfrac{1}{3}n(n+1)(n+2)$

3. $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \dfrac{n(4n^2-1)}{3}$

4. $1 \cdot 4 + 2 \cdot 7 + 3 \cdot 10 + \cdots + n(3n+1) = n(n+1)^2$

5. $2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + (n+1) \cdot 2^n = n \cdot 2^{n+1}$

6. $\dfrac{1}{1 \cdot 3} + \dfrac{1}{3 \cdot 5} + \dfrac{1}{5 \cdot 7} + \cdots + \dfrac{1}{(2n-1)(2n+1)} = \dfrac{n}{2n+1}$

Exercise 1b

Prove the following statements using the method of mathematical induction:

1. $-1 + 3 - 5 + \cdots + (-1)^n(2n - 1) = (-1)^n \cdot n$

2. $\sum(5n - 1) = \dfrac{n(5n + 3)}{2}, n \in \mathbb{N}$

3. $\sum 3^{n-1} = \dfrac{3^n - 1}{2}, n \in \mathbb{N}$

4. $2^n > n^2, n > 4$ and $n \in \mathbb{N}$

5. $2^n + 2 > n^2, n \in \mathbb{N}$

6. The sum of the interior angles of a polygon with n sides is $(n-2)\pi$, $n \geq 3$.

7. $(a^n - b^n)$ is divisible by $(a - b)$.

8. $x^{n+2} + (x+1)^{2n+1}$ is divisible by $x^2 + x + 1$, $n \geq 0$ and $n \in \mathbb{Z}$.

9. $x^n + 5n$ ($n \in \mathbb{N}$) is divisible by 6.

10. The sum of the cube of three consecutive integers is divisible by 9.

11. For all natural number n, $9^n - 8n - 1$ is a multiple of 64, $n \geq 2$.

12. Determine the general formula for the following, and prove it using the method of mathematical induction.

$$\begin{aligned}
1 &= 1 \\
3 + 5 &= 8 \\
7 + 9 + 11 &= 27 \\
13 + 15 + 17 + 19 &= 64 \\
21 + 23 + 25 + 27 + 29 &= 125
\end{aligned}$$