How to Prove It: A Structured Approach, Second Edition

Exercises for Section 1.2

1. 1. Make truth tables for the following formulas:

   (a) $\neg P \vee Q$.

   Solution.

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

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   (b) $(S \vee G) \wedge (\neg S \vee \neg G)$.

   Solution.

S	G	$\neg S$	$\neg G$	$S \vee G$	$\neg S \vee \neg G$	$(S \vee G) \wedge (\neg S \vee \neg G)$
T	T	F	F	T	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	T	F

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2. Make truth tables for the following formulas:

   (a) $\neg[P \wedge (Q \vee \neg P)]$.

   Solution.

P	Q	$\neg P$	$Q \vee \neg P$	$P \wedge (Q \vee \neg P)$	$\neg[P \wedge (Q \vee \neg P)]$
T	T	F	T	T	F
T	F	F	F	F	T
F	T	T	T	F	T
F	F	T	T	F	T

■

   (b) $(P \vee Q) \wedge (\neg P \vee R)$.

   Solution.

P	Q	R	$\neg P$	$P \vee Q$	$\neg P \vee R$	$(P \vee Q) \wedge (\neg P \vee R)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	F
F	F	F	T	F	T	F

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3. In this exercise we will use the symbol + to mean exclusive or. In other words, $P + Q$ means " P or Q, but not both."

   (a) Make a truth table for $P + Q$.

   Solution.

P	Q	$P + Q$
T	T	F
T	F	T
F	T	T
F	F	F

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   (b) Find a formula using only the connectives $\wedge, \vee$, and $\neg$ that is equivalent to $P + Q$. Justify your answer with a truth table.

   Solution.

$$P + Q \equiv (P \vee Q) \wedge \neg(P \wedge Q)$$

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$P \vee Q$	$(P \vee Q) \wedge \neg(P \wedge Q)$	$P + Q$
T	T	T	F	T	F	F
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	F	F	F

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4. Find a formula using only the connectives $\wedge$ and $\neg$ that is equivalent to $P \vee Q$. Justify your answer with a truth table.

   Solution.

$$P \vee Q \equiv \neg(\neg P \wedge \neg Q)$$

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$	$P \vee Q$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

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5. Some mathematicians use the symbol $\downarrow$ to mean nor. In other words, $P \downarrow Q$ means "neither P nor Q."

   (a) Make a truth table for $P \downarrow Q$.

   Solution.

P	Q	$P \downarrow Q$
T	T	F
T	F	F
F	T	F
F	F	T

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   (b) Find a formula using only the connectives $\wedge, \vee$, and $\neg$ that is equivalent to $P \downarrow Q$.

   Solution.

$$P \downarrow Q \equiv \neg(P \vee Q)$$

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$P \downarrow Q$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

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   (c) Find formulas using only the connective $\downarrow$ that are equivalent to $\neg P$, $P \vee Q$, and $P \wedge Q$.

   Solution.

$$\neg P \equiv \neg(P \wedge P) \equiv P \downarrow P$$
$$P \vee Q \equiv \neg(P \downarrow Q) \equiv (P \downarrow Q) \downarrow (P \downarrow Q)$$
$$P \wedge Q \equiv \neg\neg(P \wedge Q) \equiv \neg(P \downarrow Q) \equiv (P \downarrow Q) \downarrow (P \downarrow Q)$$

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6. Some mathematicians write $P \mid Q$ to mean " P and Q are not both true." (This connective is called nand, and is used in the study of circuits in computer science.)

   (a) Make a truth table for $P \mid Q$.

   Solution.

P	Q	$P \mid Q$
T	T	F
T	F	T
F	T	T
F	F	T

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(b) Find a formula using only the connectives $\wedge, \vee$, and $\neg$ that is equivalent to $P \mid Q$.

Solution.

$$P \mid Q \equiv \neg(P \wedge Q)$$

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(c) Find formulas using only the connective $\mid$ that are equivalent to $\neg P$, $P \vee Q$, and $P \wedge Q$.

Solution.

$$\neg P \equiv P \mid P$$

$$P \vee Q \equiv \neg P \mid \neg Q \equiv (P \mid P) \mid (Q \mid Q)$$

$$P \wedge Q \equiv \neg(P \mid Q) \equiv (P \mid Q) \mid (P \mid Q)$$

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7. Use truth tables to determine whether or not the arguments in exercise 7 of Section 1.1 are valid.

8. Use truth tables to determine which of the following formulas are equivalent to each other:

   (a) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$.

   (b) $\neg P \vee Q$.

   (c) $(P \vee \neg Q) \wedge (Q \vee \neg P)$.

   (d) $\neg(P \vee Q)$.

   (e) $(Q \wedge P) \vee \neg P$.

9. 9. Use truth tables to determine which of these statements are tautologies, which are contradictions, and which are neither: