

# **Solution Book of Mathematic**

*Senior 2 Part I*

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## Chapter 17

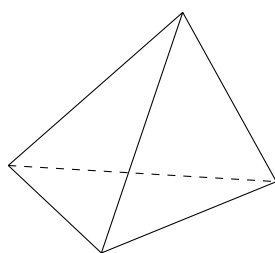
# Solid Geometry, Longitude and Latitude

### 17.1 Solid Geometry

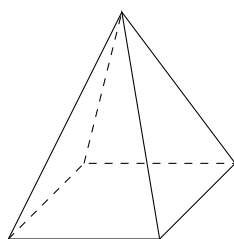
#### Polyhedron

A polyhedron is a solid bounded by a finite amount of flat polygon, and each side of the polygons must be the common edge of two polygons. Polyhedron can be classified into tetrahedron, pentahedron, hexahedron, etc. based on the number of flat surfaces, aka the *faces* of the polyhedron. The common side of two faces of a polyhedron is called an edge, and the common vertex of three edges is called an *apex*.

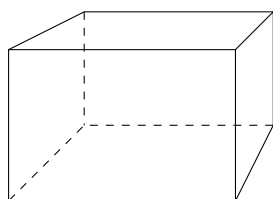
Besides, the angles formed by the faces intersecting at the same apex are called *polyhedral angles* or *solid angles*. The line segment connecting two apexes at different faces is called a *diagonal*.



Tetrahedron



Pentahedron

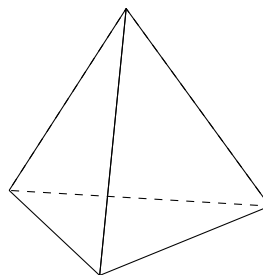


Hexahedron

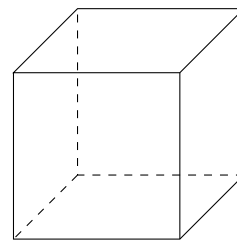
#### Regular Polyhedron

A *regular polyhedron* is a polyhedron with all faces being regular polygons, and all polyhedral angles being equal. The regular polyhedron can be classified into 5 types: *regular*

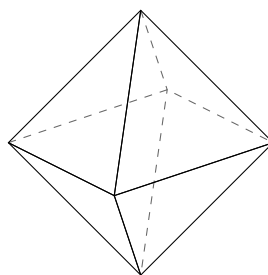
*tetrahedron, regular octahedron, regular hexahedron, regular dodecahedron and regular icosahedron.*



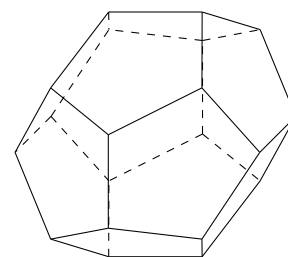
Regular Tetrahedron



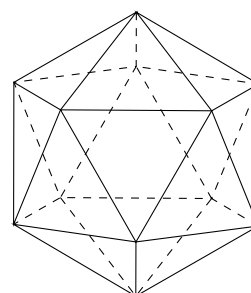
Regular Hexahedron



Regular Octahedron



Regular Dodecahedron

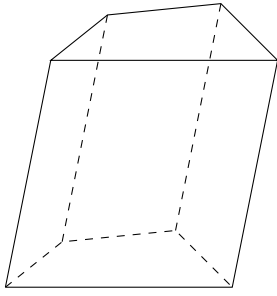


Regular Icosahedron

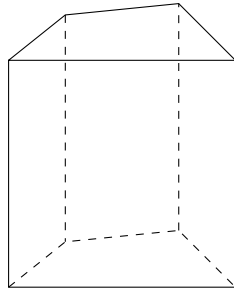
#### Prism

If two faces of a polyhedron are parallel, while the other faces intersect in sequence to form parallel lines, then the polyhedron is called a *prism*. The two faces which are parallel to each other are called the *bases of the prism*, and the other faces are called the *lateral faces of the prism*. The common sides that two adjacent lateral faces share is called the *lateral edges of the prism*. The distance between two bases is called the *height of the prism*.

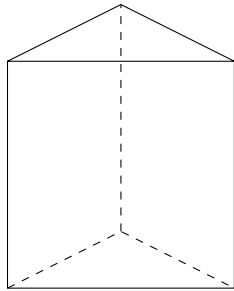
Prism with lateral edges that aren't parallel to each other are called *oblique prism*; prism with lateral edges that are parallel to each other are called *right prism*; regular prism with regular bases are called *regular prism*.



Oblique Prism

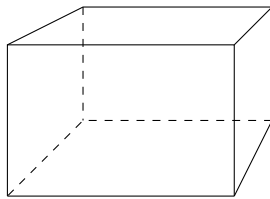


Right Prism

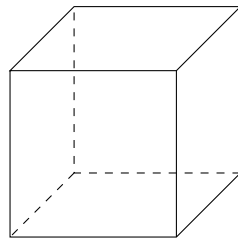


Regular Prism

Prism with bases of parallelogram are called *parallelepiped*. Parallelepiped with lateral edges that are parallel to each other are called *right parallelepiped*. Right parallelepiped with regular bases are called *cuboid*, and a cuboid with equal width, height, and depth is called a *cube*.



Cuboid

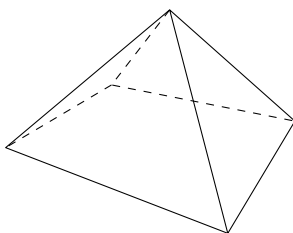


Cube

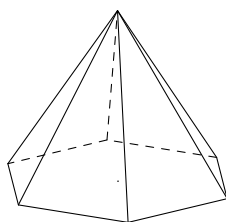
## Pyramid

If a polyhedron has a polygonal base and all its lateral faces are triangles that share a common apex, then the polyhedron is called a *pyramid*.

If the foot point of a pyramid is the centre of its base, then the pyramid is called a *right pyramid*. If the base of a right pyramid is a regular polygon, then the pyramid is called a *regular pyramid*.



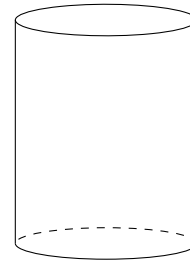
Right Pyramid



Regular Pyramid

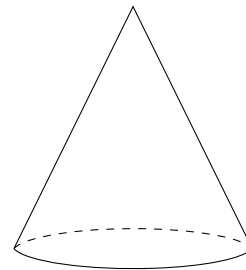
## Right Circular Cylinder

A *right circular cylinder* is the solid of revolution generated by rotating a rectangle about one of its sides.



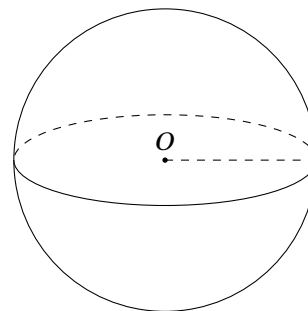
## Right Circular Cone

A *right circular cone* is the solid of revolution generated by rotating a right-angled triangle about one of its sides.



## Sphere

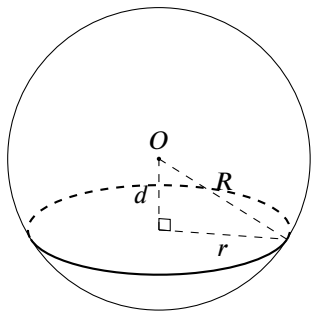
The surface of revolution generated by rotating a semicircle about its diameter is called a *spherical surface*, and the solid covered by it is called a *sphere*.



If the circle is cut with a plane, the plane has the following properties:

1. The line joining the centre of the sphere to the centre of the plane are perpendicular to the plane.
2. The distance of the plane from the centre of the sphere  $d$ , the radius of the sphere  $R$  and the radius of the plane  $r$  has the following relation:

$$r = \sqrt{R^2 - d^2}$$



The circle cut by a plane passing through the centre of the sphere is called a *great circle*; the circle cut by a plane that does not pass through the centre of the sphere is called a *small circle*.

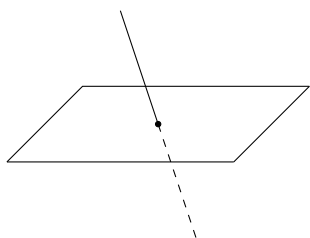
## 17.2 Angle Formed by Planes and Straight Lines

There are three types of positional relationship between a plane and a straight line:

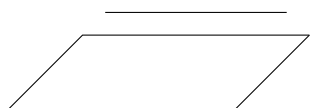
1. The line is on the plane



2. The line only intersects the plane at one point



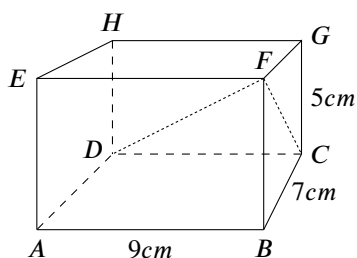
3. The line does not intersect the plane



The angle formed by a line and the orthoprojection of the line on the plane is called *the angle formed by the line and the plane*. This angle represents the inclination of the line with respect to the plane, thus it is called *the tilt angle of the line with respect to the plane*.

### 17.2.1 Practice 1

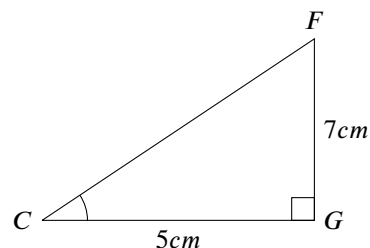
1. In the diagram below,  $AB = 9\text{cm}$ ,  $BC = 7\text{cm}$ ,  $CG = 5\text{cm}$ . Find:



- (a) The angle formed by line  $CF$  and plane  $GHDC$ .

**Sol.**

The angle formed by line  $CF$  and plane  $GHDC$  is  $\angle FCG$ .



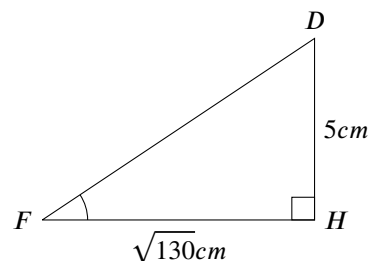
$$\begin{aligned}\tan \angle FCG &= \frac{FG}{CG} \\ &= \frac{7}{5} \\ \angle FCG &\approx 54.46^\circ\end{aligned}$$

- (b) The angle formed by line  $DF$  and plane  $EFGH$ .

**Sol.**

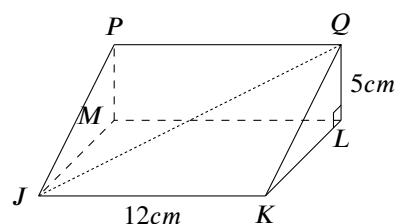
$$\begin{aligned}\text{In } EFGH, HF &= \sqrt{EF^2 + EH^2} \\ &= \sqrt{9^2 + 7^2} \\ &= \sqrt{130}\text{cm}\end{aligned}$$

The angle formed by line  $DF$  and plane  $EFGH$  is  $\angle DFH$ .



$$\begin{aligned}\tan \angle DFH &= \frac{DH}{FH} \\ &= \frac{5}{\sqrt{130}} \\ \angle DFH &\approx 23.68^\circ\end{aligned}$$

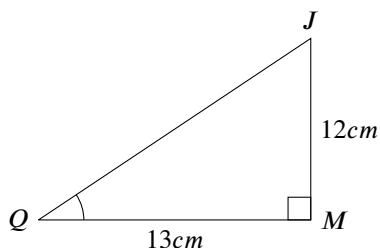
2. The diagram below shows a right prism, its base  $KQL$  is a right-angled triangle,  $JKLM$  is a square. Given that  $JK = 12\text{cm}$ ,  $LQ = 5\text{cm}$ , find the angle formed by line  $JQ$  and plane  $PQLM$ .



**Sol.**

$$\begin{aligned}\text{In } PQLM, QM &= \sqrt{JK^2 + KL^2} \\ &= \sqrt{12^2 + 5^2} \\ &= 13\text{cm}\end{aligned}$$

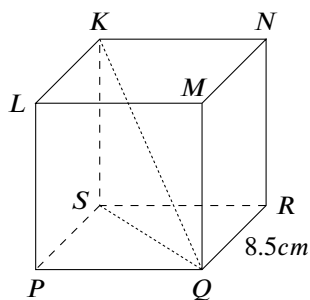
The angle formed by line  $JQ$  and plane  $PQLM$  is  $\angle JQM$ .



$$\begin{aligned}\tan \angle JQM &= \frac{JM}{QM} \\ &= \frac{12}{13} \\ \angle JQM &\approx 42.71^\circ\end{aligned}$$

### 17.2.2 Exercise 17.2

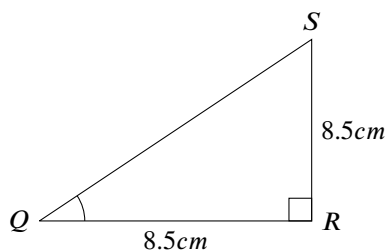
1. The diagram below shows a cube with side length of  $8.5\text{cm}$ . Find:



- (a) The angle formed by line  $QS$  and plane  $MNRQ$ .

**Sol.**

The angle formed by line  $QS$  and plane  $MNRQ$  is  $\angle SQR$ .



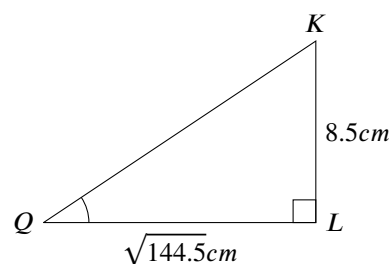
$$\begin{aligned}\tan \angle SQR &= \frac{SR}{QR} \\ &= \frac{8.5}{8.5} \\ &= 1 \\ \angle SQR &= 45^\circ\end{aligned}$$

- (b) The angle formed by line  $KQ$  and plane  $PQML$ .

**Sol.**

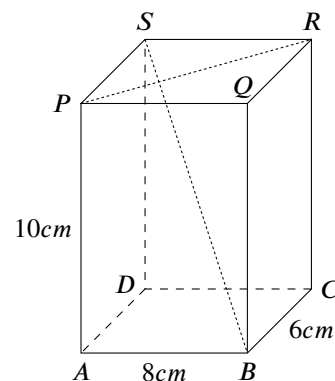
$$\begin{aligned}\text{In } KLMN, KM &= \sqrt{8.5^2 + 8.5^2} \\ &= \sqrt{144.5}\text{cm}\end{aligned}$$

The angle formed by line  $KQ$  and plane  $PQML$  is  $\angle KQL$ .



$$\begin{aligned}\tan \angle KQL &= \frac{KL}{QL} \\ &= \frac{8.5}{\sqrt{144.5}} \\ \angle KQL &\approx 35.26^\circ\end{aligned}$$

2. The diagram below shows a cuboid,  $AB = 8\text{cm}$ ,  $BC = 6\text{cm}$ ,  $AP = 10\text{cm}$ . Find:



- (a) The length of PR.

**Sol.**

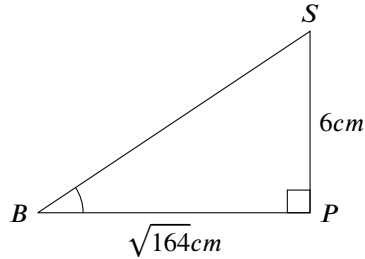
$$\begin{aligned}PR &= \sqrt{PQ^2 + QR^2} \\ &= \sqrt{8^2 + 6^2} \\ &= 10\text{cm}\end{aligned}$$

- (b) The angle formed by line  $SB$  and plane  $APQB$ .

**Sol.**

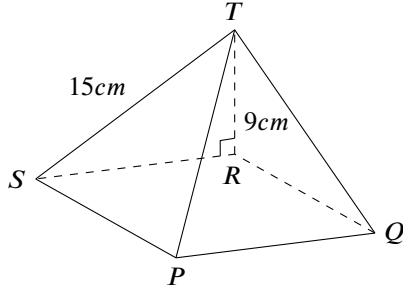
$$\begin{aligned}\text{In } APQB, PB &= \sqrt{PA^2 + AB^2} \\ &= \sqrt{10^2 + 8^2} \\ &= \sqrt{164} \text{ cm}\end{aligned}$$

The angle formed by line  $SB$  and plane  $APQB$  is  $\angle SBP$ .



$$\begin{aligned}\tan \angle SBP &= \frac{SP}{BP} \\ &= \frac{6}{\sqrt{164}} \\ \angle SBP &\approx 25.10^\circ\end{aligned}$$

3. The diagram below shows a pyramid. Given that its base  $PQRS$  is a square,  $TR$  is perpendicular to the base,  $TS = 15\text{cm}$ ,  $TR = 9\text{cm}$ . Find:



- (a) The length of  $RS$ .

**Sol.**

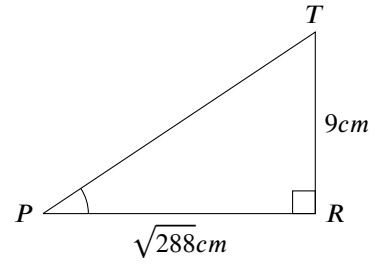
$$\begin{aligned}RS &= \sqrt{ST^2 - TR^2} \\ &= \sqrt{15^2 - 9^2} \\ &= 12 \text{ cm}\end{aligned}$$

- (b) The angle formed by line  $PT$  and plane  $PQRS$ .

**Sol.**

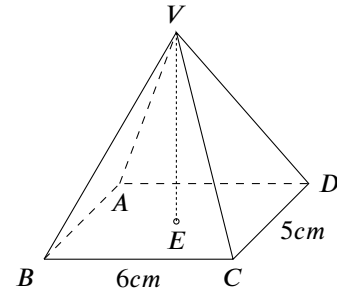
$$\begin{aligned}\text{In } PQRS, PR &= \sqrt{PQ^2 + RQ^2} \\ &= \sqrt{12^2 + 12^2} \\ &= \sqrt{288} \text{ cm}\end{aligned}$$

The angle formed by line  $PT$  and plane  $PQRS$  is  $\angle TPR$ .



$$\begin{aligned}\tan \angle TPR &= \frac{TR}{PR} \\ &= \frac{9}{\sqrt{288}} \\ \angle TPR &\approx 27.94^\circ\end{aligned}$$

4. The diagram below shows a right pyramid with height of  $8\text{cm}$ , its base is a rectangle,  $E$  is the foot point from  $V$  to the base. Given that  $CD = 5\text{cm}$ ,  $BC = 6\text{cm}$ . Find:

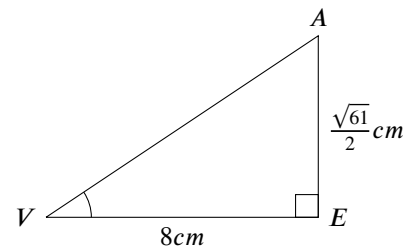


- (a) The angle formed by line  $VA$  and line  $VE$ .

**Sol.**

$$\begin{aligned}\text{In } ABCD, AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{5^2 + 6^2} \\ &= \sqrt{61} \text{ cm} \\ AE &= \frac{AC}{2} \\ &= \frac{\sqrt{61}}{2} \text{ cm}\end{aligned}$$

The angle formed by line  $VA$  and line  $VE$  is  $\angle AVE$ .



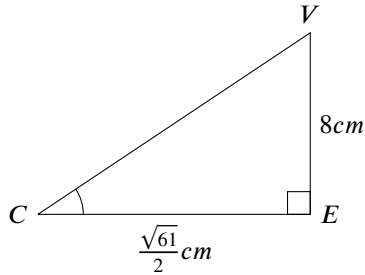
$$\begin{aligned}\tan \angle AVE &= \frac{AE}{VE} \\ &= \frac{\frac{\sqrt{61}}{2}}{8} \\ \angle AVE &\approx 26.02^\circ\end{aligned}$$

- (b) The angle formed by line  $VC$  and plane  $ABCD$ .

**Sol.**

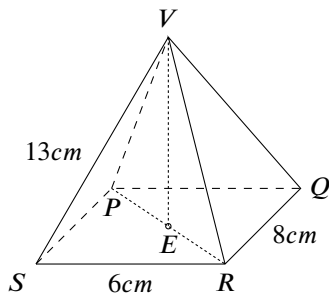
$$\begin{aligned}\text{In } ABCD, EC &= \frac{AC}{2} \\ &= \frac{\sqrt{61}}{2} \text{ cm}\end{aligned}$$

The angle formed by line  $VC$  and plane  $ABCD$  is  $\angle VCE$ .



$$\begin{aligned}\tan \angle VCE &= \frac{VE}{CE} \\ &= \frac{8}{\frac{\sqrt{61}}{2}} \\ \angle VCE &\approx 63.98^\circ\end{aligned}$$

5. The diagram below shows a right pyramid, its base  $PQRS$  is a rectangle. Given that  $SR = 6\text{cm}$ ,  $QR = 8\text{cm}$ ,  $VS = 13\text{cm}$ . Find:



- (a) The length of  $PR$ .

**Sol.**

$$\begin{aligned}PR &= \sqrt{SR^2 + SP^2} & (1) \\ &= \sqrt{6^2 + 8^2} & (2) \\ &= 10\text{cm} & (3)\end{aligned}$$

- (b) The height of the pyramid.

**Sol.**

Let the foot point of the pyramid be  $E$ .

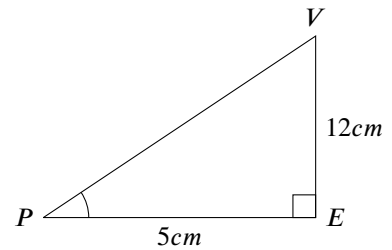
**Sol.**

$$\begin{aligned}\text{In } PQRS, PE &= \frac{PR}{2} \\ &= \frac{10}{2} \\ &= 5\text{cm}\end{aligned}$$

$$\begin{aligned}VE &= \sqrt{VP^2 - PE^2} \\ &= \sqrt{13^2 - 5^2} \\ &= 12\text{cm}\end{aligned}$$

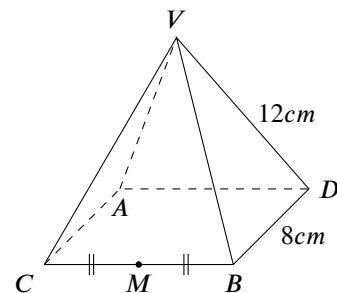
- (c) The angle of the line  $VP$  and plane  $PQRS$ .

**Sol.** The angle of the line  $VP$  and plane  $PQRS$  is  $\angle VPE$ .



$$\begin{aligned}\tan \angle VPE &= \frac{VE}{PE} \\ &= \frac{12}{5} \\ \angle VPE &\approx 67.38^\circ\end{aligned}$$

6. The diagram below shows a regular pyramid, the length of its lateral edge is  $12\text{cm}$ , its base  $ABCD$  is a square with side length of  $8\text{cm}$ ,  $M$  is the midpoint of  $BC$ . Find:



- (a) The angle formed by the lateral edge and the base of the pyramid.

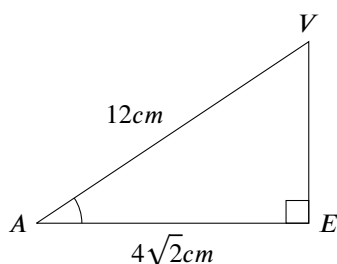
**Sol.**



Let the foot point of the pyramid be  $E$ .

$$\begin{aligned}
 \text{In } ABCD, AB &= \sqrt{AD^2 + BD^2} \\
 &= \sqrt{8^2 + 8^2} \\
 &= \sqrt{128} \text{ cm} \\
 &= 8\sqrt{2} \\
 AE &= \frac{AB}{2} \\
 &= 4\sqrt{2} \text{ cm}
 \end{aligned}$$

The angle formed by the lateral edge and the base of the pyramid is  $\angle VAE$ .



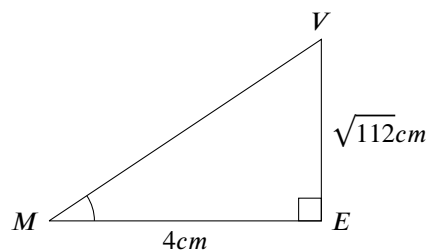
$$\begin{aligned}
 \cos \angle VAE &= \frac{AE}{AV} \\
 &= \frac{4\sqrt{2}}{12} \\
 &= \frac{\sqrt{2}}{3} \\
 \angle VAE &= 61.87^\circ
 \end{aligned}$$

- (b) The angle formed by line  $VM$  and the base of the pyramid.

**Sol.**

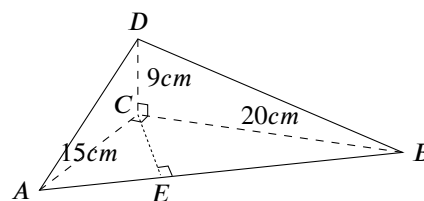
$$\begin{aligned}
 EM &= \frac{BD}{2} \\
 &= \frac{8}{2} \\
 &= 4 \text{ cm} \\
 VE &= \sqrt{AV^2 - AE^2} \\
 &= \sqrt{12^2 - (4\sqrt{2})^2} \\
 &= \sqrt{144 - 32} \\
 &= \sqrt{112}
 \end{aligned}$$

The angle formed by line  $VM$  and the base of the pyramid is  $\angle VME$ .



$$\begin{aligned}
 \cos \angle VME &= \frac{VE}{VM} \\
 &= \frac{\sqrt{112}}{4} \\
 \angle VME &= 69.30^\circ
 \end{aligned}$$

7. In the pyramid shown below,  $\triangle ABC$  is a right-angled triangle,  $CD$  is perpendicular to plane  $ABC$ ,  $CE$  is perpendicular to  $AB$ . Given that  $AC = 15 \text{ cm}$ ,  $BC = 20 \text{ cm}$  and  $CD = 9 \text{ cm}$ . Find:



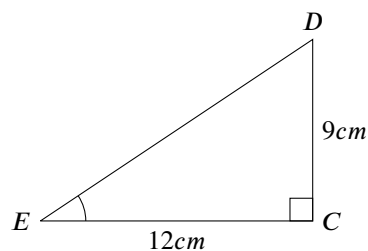
- (a) The length of  $CE$ .

**Sol.**

$$\begin{aligned}
 \text{In } \triangle ABC, \tan \angle CBA &= \frac{AC}{AB} \\
 &= \frac{15}{20} \\
 &= \frac{3}{4} \\
 \angle CBA &= 36.87^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{In } \triangle CBE, \sin \angle CBE &= \frac{CE}{CB} \\
 \sin 36.87^\circ &= \frac{CE}{20} \\
 CE &= 20 \sin 36.87^\circ \\
 &= 12 \text{ cm}
 \end{aligned}$$

- (b)  $\angle DEC$ .

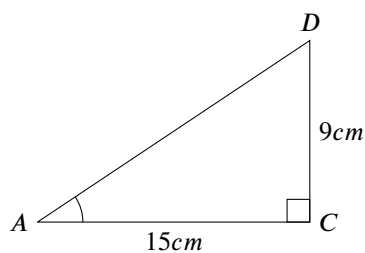


$$\begin{aligned}\cos \angle DEC &= \frac{DC}{EC} \\ &= \frac{9}{12} \\ &= \frac{3}{4} \\ \angle DEC &= 36.87^\circ\end{aligned}$$

(c) The angle formed by line  $AD$  and plane  $ABC$ .

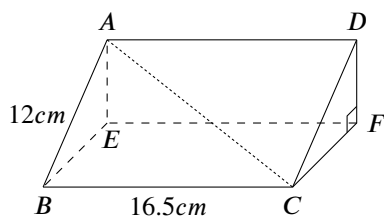
**Sol.**

The angle formed by line  $AD$  and plane  $ABC$  is  $\angle DAC$ .



$$\begin{aligned}\cos \angle DAC &= \frac{DC}{AC} \\ &= \frac{9}{15} \\ &= \frac{3}{5} \\ \angle DAC &= 30.96^\circ\end{aligned}$$

8. The diagram below shows a right prism, its base  $CDF$  is a right-angled triangle. Given that  $BC = 16.5\text{cm}$  and  $AB = 12\text{cm}$ . Assume that  $CF = 2DF$ , find:

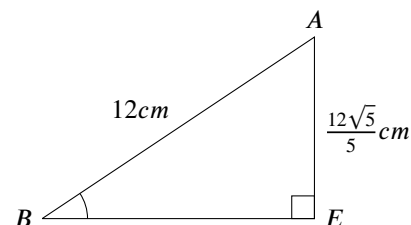


(a) The angle formed by line  $AB$  and plane  $BCFE$ .

**Sol.**

$$\begin{aligned}CF &= 2DF \\ DF^2 + (2DF)^2 &= 12^2 \\ DF^2 + 4DF^2 &= 144 \\ 5DF^2 &= 144 \\ DF^2 &= \frac{144}{5} \\ DF &= \frac{12}{\sqrt{5}} \\ &= \frac{12\sqrt{5}}{5}\text{cm}\end{aligned}$$

The angle formed by line  $AB$  and plane  $BCFE$  is  $\angle ABE$ .



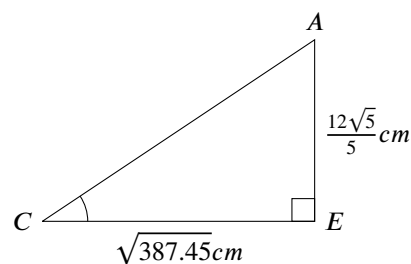
$$\begin{aligned}\sin \angle ABE &= \frac{AE}{AB} \\ &= \frac{\frac{12\sqrt{5}}{5}}{12} \\ &= \frac{\sqrt{5}}{5} \\ \angle ABE &= 26.57^\circ\end{aligned}$$

(b) The angle formed by line  $AC$  and plane  $BCFE$ .

**Sol.**

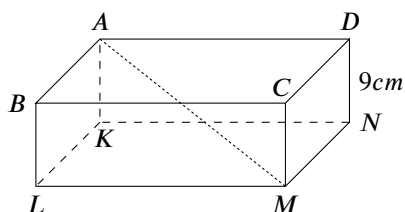
$$\begin{aligned}CF &= 2DF \\ &= \frac{24}{\sqrt{5}} \\ \text{In } BCFE, EC &= \sqrt{BC^2 + BE^2} \\ &= \sqrt{16.5^2 + \frac{24^2}{5}} \\ &= \sqrt{387.45}\end{aligned}$$

The angle formed by line  $AC$  and plane  $BCFE$  is  $\angle ACE$ .



$$\begin{aligned}\sin \angle ACE &= \frac{AE}{EC} \\ &= \frac{\frac{12\sqrt{5}}{5}}{\sqrt{387.45}} \\ \angle ACE &= 15.25^\circ\end{aligned}$$

9. The diagram below shows a cuboid with volume of  $300\text{cm}^3$ . Given that  $AD = 2DC$  and  $DN = 9\text{cm}$ . Find the angle formed by line  $AM$  and plane  $KLMN$ .



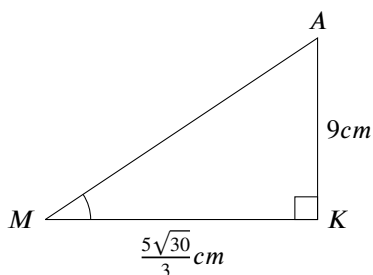
**Sol.**

$$\begin{aligned}
 AD &= 2DC \\
 AD \times DC \times DN &= 300 \\
 2DC \times DC \times 9 &= 300 \\
 2DC^2 \times 9 &= 300 \\
 2DC^2 &= \frac{100}{3} \\
 DC^2 &= \frac{50}{3} \\
 DC &= \frac{5\sqrt{2}}{\sqrt{3}} \\
 &= \frac{5\sqrt{6}}{3} \\
 AD &= 2DC \\
 &= \frac{10\sqrt{6}}{3}
 \end{aligned}$$

In  $KLMN$ ,  $KM = \sqrt{MN^2 + KN^2}$

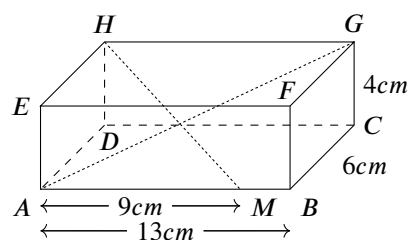
$$\begin{aligned}
 &= \sqrt{\left(\frac{5\sqrt{6}}{3}\right)^2 + \left(\frac{10\sqrt{6}}{3}\right)^2} \\
 &= \sqrt{\frac{50}{3} + \frac{200}{3}} \\
 &= \frac{5\sqrt{30}}{3}
 \end{aligned}$$

The angle formed by line  $AM$  and plane  $KLMN$  is  $\angle AMK$ .



$$\begin{aligned}
 \tan \angle AMK &= \frac{AK}{MK} \\
 &= \frac{9}{\frac{5\sqrt{30}}{3}} \\
 \angle AMK &\approx 44.59^\circ
 \end{aligned}$$

10. The diagram below shows a cuboid. Given that  $AB = 13cm$ ,  $BC = 6cm$ ,  $CG = 4cm$ .  $M$  is a point on  $AB$ ,  $AM = 9cm$ . Find:

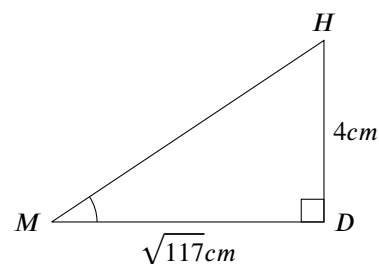


- (a) The angle formed by line  $HM$  and plane  $ABCD$ .

**Sol.**

$$\begin{aligned}
 \text{In } ABCD, DM &= \sqrt{AM^2 + AD^2} \\
 &= \sqrt{9^2 + 6^2} \\
 &= \sqrt{117}cm
 \end{aligned}$$

The angle formed by line  $HM$  and plane  $ABCD$  is  $\angle HMD$ .



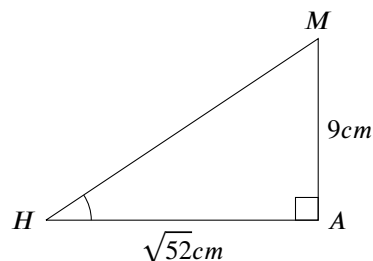
$$\begin{aligned}
 \tan \angle HMD &= \frac{HD}{MD} \\
 &= \frac{4}{\sqrt{117}} \\
 \angle HMD &\approx 20.29^\circ
 \end{aligned}$$

- (b) The angle formed by line  $HM$  and plane  $HDAE$ .

**Sol.**

$$\begin{aligned}
 \text{In } HDAE, HA &= \sqrt{AD^2 + HD^2} \\
 &= \sqrt{6^2 + 4^2} \\
 &= \sqrt{52}cm
 \end{aligned}$$

The angle formed by line  $HM$  and plane  $HDAE$  is  $\angle MHA$ .

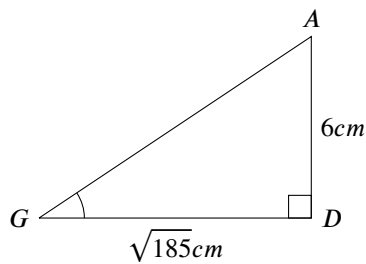


$$\begin{aligned}\tan \angle MHA &= \frac{MA}{HA} \\ &= \frac{9}{\sqrt{52}} \\ \angle MHA &\approx 51.30^\circ\end{aligned}$$

- (c) The angle formed by line  $AG$  and plane  $CDHG$ .  
**Sol.**

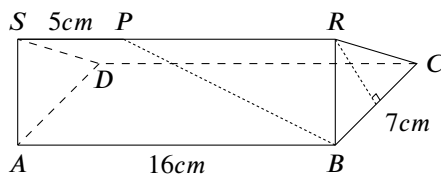
$$\begin{aligned}\text{In } CDHG, DG &= \sqrt{DC^2 + GC^2} \\ &= \sqrt{13^2 + 4^2} \\ &= \sqrt{185}cm\end{aligned}$$

The angle formed by line  $AG$  and plane  $CDHG$  is  $\angle AGD$ .



$$\begin{aligned}\tan \angle AGD &= \frac{AD}{GD} \\ &= \frac{6}{\sqrt{185}} \\ \angle AGD &\approx 23.80^\circ\end{aligned}$$

11. The diagram below shows a regular prism, its bases  $ADS$  and  $BCR$  are equilateral triangles. Given that  $AB = 16cm$ ,  $BC = 7cm$ ,  $SP = 5cm$ . Find:



- (a) The length of  $BP$ .

**Sol.**

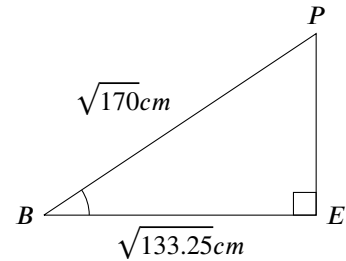
$$\begin{aligned}PR &= SR - SP \\ &= 16 - 5 \\ &= 11cm \\ BP &= \sqrt{BR^2 + PR^2} \\ &= \sqrt{7^2 + 11^2} \\ &= \sqrt{170} \\ &\approx 13.04cm\end{aligned}$$

- (b) The angle formed by line  $BP$  and plane  $ABCD$ .  
**Sol.**

Let the foot point of  $P$  be  $E$ .

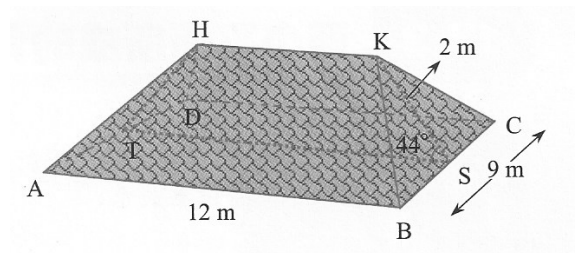
$$\begin{aligned}EB &= \sqrt{3.5^2 + 11^2} \\ &= \sqrt{133.25}cm\end{aligned}$$

The angle formed by line  $BP$  and plane  $ABCD$  is  $\angle PBE$ .



$$\begin{aligned}\cos \angle PBE &= \frac{BE}{BP} \\ &= \frac{\sqrt{133.25}}{\sqrt{170}} \\ \angle PBE &\approx 27.71^\circ\end{aligned}$$

12. The diagram below shows a roof,  $HK$  is the ridge of the roof, its edges  $HA$ ,  $HD$ ,  $KB$ ,  $KC$  are equal in length. Both of the planes  $HAD$  and  $KBC$  form a  $44^\circ$  angle with plane  $ABCD$ . Given that  $S$  and  $T$  are the midpoints of  $BC$  and  $AD$  respectively. Find:



- (a) The distance from line  $HK$  to plane  $ABCD$ .

**Sol.**

Let the foot point of  $K$  on plane  $ABCD$  be  $P$ .

$$\begin{aligned}\text{In } \triangle KPS, \sin \angle KSP &= \frac{KP}{KS} \\ \sin 44^\circ &= \frac{KP}{2} \\ KP &= 2 \sin 44^\circ \\ &\approx 1.39m\end{aligned}$$

- (b) The length of  $HK$ .

**Sol.**

$$\begin{aligned}\cos \angle KSP &= \frac{PS}{KS} \\ \cos 44^\circ &= \frac{PS}{2} \\ PS &= 2 \cos 44^\circ \\ &\approx 1.44m \\ HK &\approx 12 - 2PS \\ &\approx 12 - 2.88 \\ &\approx 9.12m\end{aligned}$$

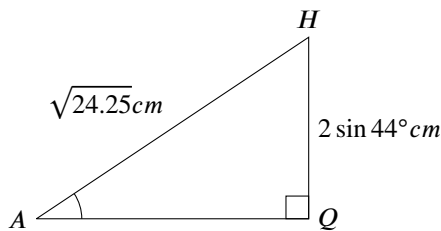
- (c) The angle formed by line  $HA$  and plane  $ABCD$ .

**Sol.**

Let the foot point of  $H$  on plane  $ABCD$  be  $Q$ .

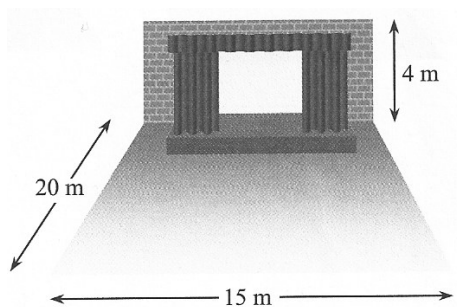
$$\begin{aligned}HA &= \sqrt{HT^2 + AT^2} \\ &= \sqrt{2^2 + 4.5^2} \\ &= \sqrt{24.25}cm\end{aligned}$$

The angle formed by line  $HA$  and plane  $ABCD$  is  $\angle HAQ$ .



$$\begin{aligned}\sin \angle HAQ &= \frac{HQ}{HA} \\ \sin \angle HAQ &= \frac{2 \sin 44^\circ}{\sqrt{24.25}} \\ \angle HAQ &\approx 16.38^\circ\end{aligned}$$

13. The length, width and height of a hall are  $20m$ ,  $15m$ , and  $4m$  respectively. Find:



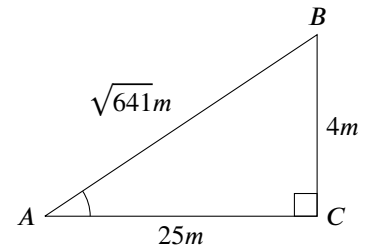
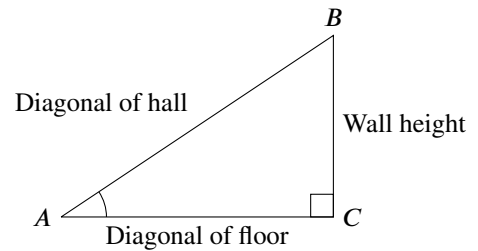
- (a) The length of the diagonal of the hall.

**Sol.**

$$\begin{aligned}\text{Diagonal of floor} &= \sqrt{20^2 + 15^2} \\ &= \sqrt{625}m \\ &= 25m \\ \text{Diagonal of hall} &= \sqrt{4^2 + 25^2} \\ &= \sqrt{641}m \\ &= 25.32m\end{aligned}$$

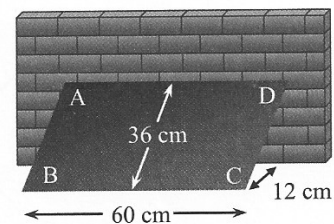
- (b) The angle formed by the diagonal and the floor of the hall.

**Sol.**



$$\begin{aligned}\tan \angle BAC &= \frac{4}{25} \\ \angle BAC &\approx 9.09^\circ\end{aligned}$$

14. In the diagram below,  $ABCD$  represents a rectangular plank with length and width of  $60cm$  and  $36cm$  respectively, its base  $BC$  is on the ground and the top of it lies on the wall. Assume that the distance between  $BC$  and the corner of the wall is  $12cm$ , find the angle formed by the diagonal  $BD$  of the plank and the ground.

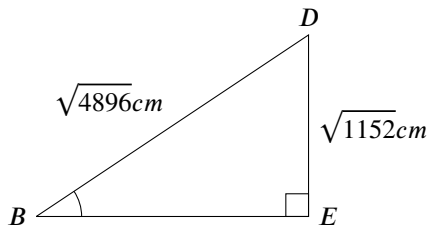


**Sol.**

Let the footpoint of  $D$  on the ground be  $E$ .

$$\begin{aligned} BD &= \sqrt{BC^2 + CD^2} \\ &= \sqrt{60^2 + 36^2} \\ &= \sqrt{4896} \text{ cm} \\ DE &= \sqrt{DC^2 - CE^2} \\ &= \sqrt{36^2 - 12^2} \\ &= \sqrt{1152} \text{ cm} \end{aligned}$$

The angle formed by the diagonal  $BD$  and the ground is  $\angle DBE$ .



$$\begin{aligned} \sin \angle DBE &= \frac{\sqrt{1152}}{\sqrt{4896}} \\ \angle DBE &\approx 29.02^\circ \end{aligned}$$

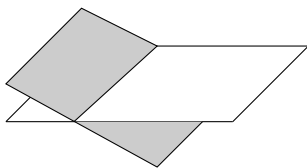
### 17.3 Angle Formed by Two Planes

There are three types positional relationship between two planes:

1. Two planes coincide with each other.



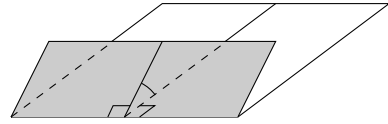
2. Two planes intersect with each other at a line.



3. Two planes are parallel to each other and do not intersect with each other.

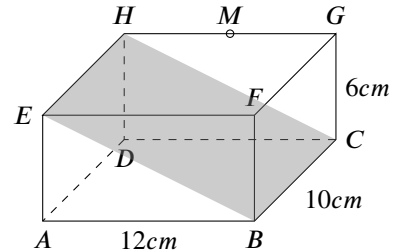


Two non-parallel planes intersect with each other at a line, the line is called the *common edge*. At any point on the common edge, draw a line perpendicular to the common edge on each plane, the acute angles formed by these two perpendicular lines are called *the angle formed by the two planes*.



#### 17.3.1 Practice 2

1. The diagram below shows a cuboid with length of  $12\text{ cm}$ , width of  $10\text{ cm}$  and height of  $6\text{ cm}$ .

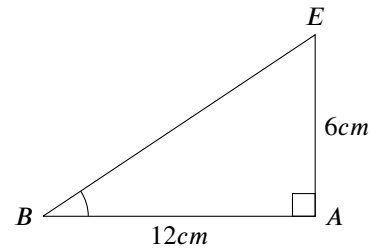


- (a) Find the angle formed by plane  $EBCH$  and plane  $ABCD$ .

**Sol.**

$\because BC$  is the common edge of plane  $EBCH$  and plane  $ABCD$ ,  $AB \perp BC$  and  $EB \perp BC$ .

$\therefore$  The angle formed by plane  $EBCH$  and plane  $ABCD$  is  $\angle EBA$ .



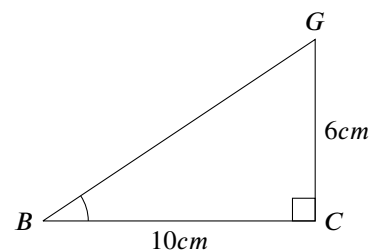
$$\begin{aligned} \tan \angle EAB &= \frac{6}{12} \\ &= \frac{1}{2} \\ \angle EAB &\approx 26.57^\circ \end{aligned}$$

- (b) Assume that  $M$  is a point on  $HG$ , find the angle formed by plane  $MAB$  and plane  $ABCD$ .

**Sol.**

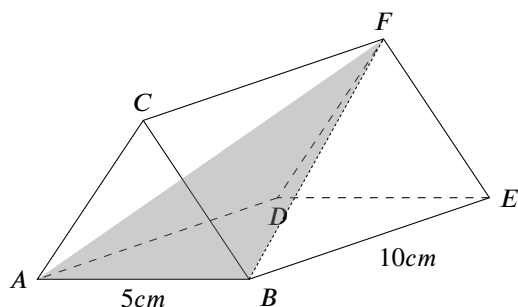
$\because AB$  is the common edge of plane  $MAB$  and plane  $ABCD$ ,  $M$  is on  $HG$ ,  $HG \perp AB$ ,  $BC \perp AB$ .

$\therefore$  The angle formed by plane  $MAB$  and plane  $ABCD$  is  $\angle GBC$ .



$$\begin{aligned}\tan \angle GBC &= \frac{6}{10} \\ &= \frac{3}{5} \\ \angle GBC &\approx 30.96^\circ\end{aligned}$$

2. The diagram below shows a regular prism, its bases  $ABC$  and  $DEF$  are equilateral triangles with side length of  $5\text{cm}$ . Given that the height of the prism is  $10\text{cm}$ , find:



- (a) The length of  $BF$ .

**Sol.**

$$\begin{aligned}BF &= \sqrt{EF^2 + BE^2} \\ &= \sqrt{10^2 + 5^2} \\ &= \sqrt{125} \\ &\approx 11.18\text{cm}\end{aligned}$$

- (b) The angle formed by plane  $ABF$  and plane  $ABC$ .

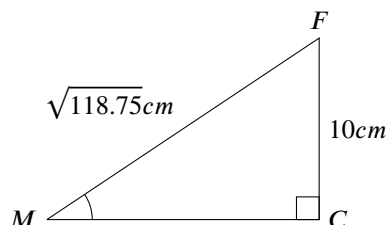
**Sol.**

Let the midpoint of  $AB$  be  $M$ .

$$\begin{aligned}MF &= \sqrt{FB^2 - BM^2} \\ &= \sqrt{125 - 2.5^2} \\ &= \sqrt{118.75}\text{cm}\end{aligned}$$

$\because AB$  is the common edge of plane  $ABF$  and plane  $ABC$ ,  $MF \perp AB$ ,  $CF \perp AB$ .

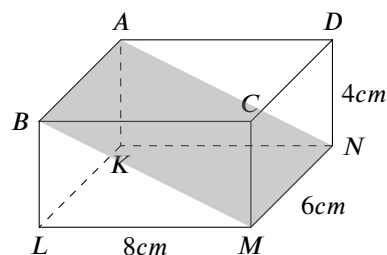
$\therefore$  The angle formed by plane  $ABF$  and plane  $ABC$  is  $\angle FMC$ .



$$\begin{aligned}\sin \angle FMC &= \frac{FC}{MF} \\ &= \frac{10}{\sqrt{118.75}} \\ \angle FMC &\approx 66.59^\circ\end{aligned}$$

### 17.3.2 Exercise 17.3

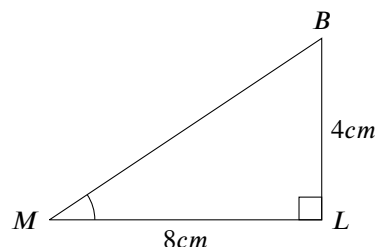
1. The diagram below shows a cuboid with length of  $8\text{cm}$ , width of  $6\text{cm}$  and height of  $4\text{cm}$ . Find the angle formed by plane  $ABMN$  and  $KLMN$ .



**Sol.**

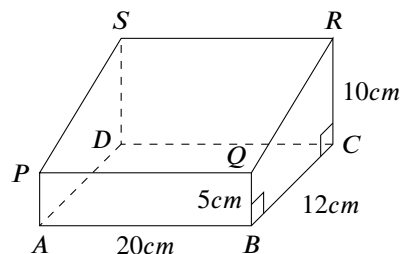
$\because MN$  is the common edge of  $ABMN$  and  $KLMN$ ,  $LM \perp MN$  and  $BM \perp MN$ .

$\therefore$  The angle formed by plane  $ABMN$  and  $KLMN$  is  $\angle BML$ .



$$\begin{aligned}\tan \angle BML &= \frac{BL}{LM} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \\ \angle BML &\approx 26.57^\circ\end{aligned}$$

2. In the right prism shown below,  $ABCD$  is a rectangle with length of  $20\text{cm}$  and width of  $12\text{cm}$ ,  $BCRQ$  is a trapezoid,  $\angle QBC$  and  $\angle RCB$  are both right angles,  $BQ = 5\text{cm}$ ,  $CR = 10\text{cm}$ . Find the angle formed by plane  $PQRS$  and plane  $ABCD$ .

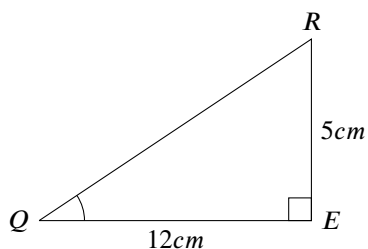


**Sol.**

Let the midpoint of  $RC$  and  $SD$  be  $E$  and  $F$  respectively.

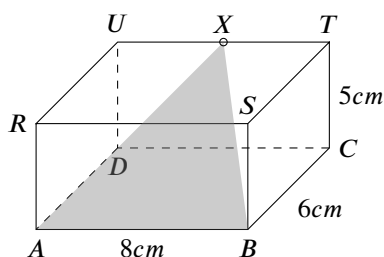
$\because PQEF \parallel ABCD$ ,  $PQ$  is the common edge of  $PQRS$  and  $PQER$ ,  $PQ \perp QE$ , and  $PQ \perp QR$ .

$\therefore$  The angle formed by plane  $PQRS$  and  $ABCD$  is  $\angle RQE$ .



$$\begin{aligned}\tan \angle RQE &= \frac{RE}{QE} \\ &= \frac{5}{12} \\ \angle RQE &\approx 22.62^\circ\end{aligned}$$

3. The diagram below shows a cuboid,  $AB = 8\text{cm}$ ,  $BC = 6\text{cm}$ ,  $CT = 5\text{cm}$ ,  $X$  is the midpoint of  $TU$ . Find:



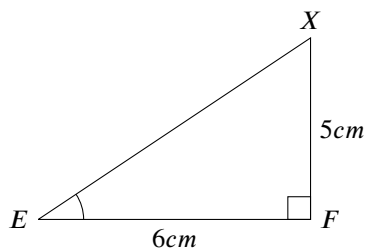
- (a) The angle formed by plane  $XAB$  and plane  $ABCD$ .

**Sol.**

Let the midpoint of  $AB$  and  $CD$  be  $E$  and  $F$  respectively.

$\therefore AB$  is the common edge of  $ABCD$  and  $XAB$ ,  $AB \perp XE$ , and  $AB \perp EF$ .

$\therefore$  The angle formed by plane  $ABCD$  and  $XAB$  is  $\angle XEF$ .



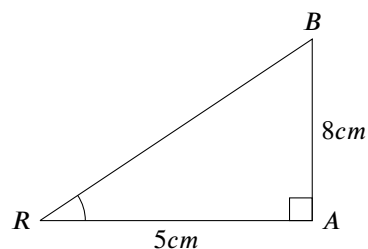
$$\begin{aligned}\tan \angle XEF &= \frac{XF}{EF} \\ &= \frac{5}{6} \\ \angle XEF &\approx 39.81^\circ\end{aligned}$$

- (b) The angle formed by plane  $BCUR$  and plane  $ADUR$ .

**Sol.**

$\therefore UR$  is the common edge of  $BCUR$  and  $ADUR$ ,  $UR \perp RB$ , and  $UR \perp AR$ .

$\therefore$  The angle formed by plane  $BCUR$  and  $ADUR$  is  $\angle BRA$ .



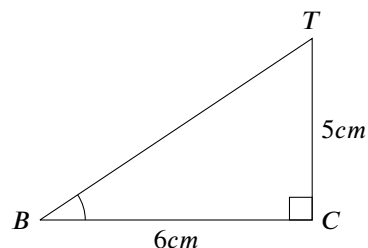
$$\begin{aligned}\tan \angle BRA &= \frac{BA}{RA} \\ &= \frac{8}{5} \\ \angle BRA &\approx 57.99^\circ\end{aligned}$$

- (c) The angle formed by plane  $ABTU$  and plane  $ABCD$ .

**Sol.**

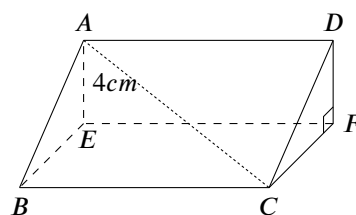
$\therefore AB$  is the common edge of  $ABTU$  and  $ABCD$ ,  $AB \perp TB$ , and  $AB \perp BC$ .

$\therefore$  The angle formed by plane  $ABTU$  and  $ABCD$  is  $\angle TBC$ .



$$\begin{aligned}\tan \angle TBC &= \frac{TC}{BC} \\ &= \frac{5}{6} \\ \angle TBC &\approx 39.81^\circ\end{aligned}$$

4. The diagram below shows a right pyramid, its bases  $ABE$  and  $DCF$  are right-angled triangles. Given that  $AE = 4\text{cm}$ ,  $BE = \frac{2}{3}EF$ ,  $EF = 4DF$ , find the angle formed by plane  $ABCD$  and plane  $BCFE$ .



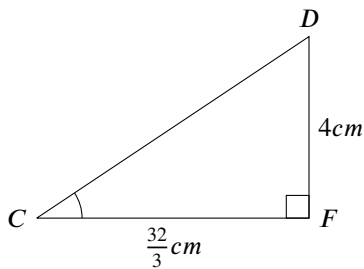


**Sol.**

$$\begin{aligned}
 EF &= 4DF \\
 &= 4 \times 4 \\
 &= 16\text{cm} \\
 BE &= \frac{2}{3}EF \\
 &= \frac{32}{3}\text{cm}
 \end{aligned}$$

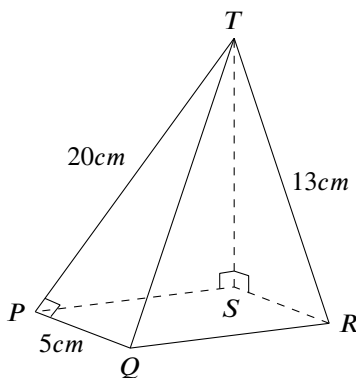
$\therefore BC$  is the common edge of  $ABCD$  and  $BCFE$ ,  $BC \perp CD$ , and  $BC \perp CF$ .

$\therefore$  The angle formed by plane  $ABCD$  and  $BCFE$  is  $\angle DCF$ .



$$\begin{aligned}
 \tan \angle DCF &= \frac{DF}{CF} \\
 &= \frac{4}{\frac{32}{3}} \\
 \angle DCF &\approx 20.56^\circ
 \end{aligned}$$

5. In the pyramid shown below,  $PQT$ ,  $SPT$ , and  $SRT$  are all right-angled triangles,  $PQRS$  is a triangle. Given that  $PQ = 5\text{cm}$ ,  $RT = 13\text{cm}$ ,  $PT = 20\text{cm}$ . Find:



- (a) The height of the prism.

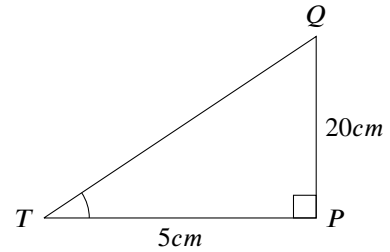
**Sol.**

$$\begin{aligned}
 \text{Height of the prism} &= TS \\
 &= \sqrt{TR^2 - RS^2} \\
 &= \sqrt{13^2 - 5^2} \\
 &= 12\text{cm}
 \end{aligned}$$

- (b) The angle formed by line  $TQ$  and plane  $PST$ .

**Sol.**

The angle formed by line  $TQ$  and plane  $PST$  is  $\angle QTP$ .



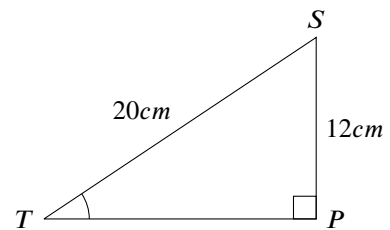
$$\begin{aligned}
 \tan \angle QTP &= \frac{PQ}{PT} \\
 &= \frac{20}{5} \\
 &= 4 \\
 \angle QTP &\approx 76.10^\circ
 \end{aligned}$$

- (c) The angle formed by plane  $RST$  and  $PQT$ .

**Sol.**

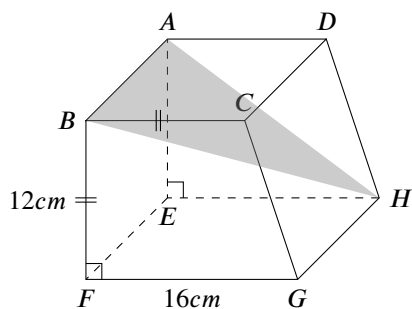
The angle formed by plane  $RST$  and  $PQT$  is  $\angle STP$ .

$$\begin{aligned}
 \text{In } \triangle TRS, TS &= \sqrt{TR^2 - SR^2} \\
 &= \sqrt{13^2 - 5^2} \\
 &= 12\text{cm}
 \end{aligned}$$



$$\begin{aligned}
 \cos \angle STP &= \frac{TS}{TP} \\
 &= \frac{12}{20} \\
 &= \frac{3}{5} \\
 \angle STP &\approx 53.13^\circ
 \end{aligned}$$

6. The diagram below shows a right prism, its base  $BCGF$  is a trapezoid,  $BC = BF = 12\text{cm}$ ,  $FG = 16\text{cm}$ . The lateral face  $EFGH$  is a square, and is perpendicular to another lateral face  $ABFE$ . Find:



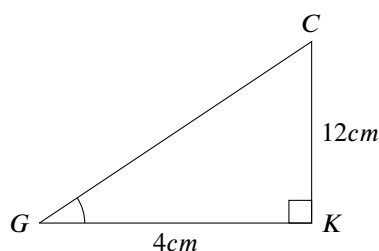
- (a) The angle formed by plane  $CDHG$  and plane  $EFGH$ .

**Sol.**

Let the foot point of  $C$  be  $K$ .

$\therefore GH$  is the common edge of the plane  $CDHG$  and plane  $EFGH$ ,  $CG \perp GH$ , and  $KG \perp GH$ .  
 $\therefore$  The angle formed by plane  $CDHG$  and plane  $EFGH$  is  $\angle CGK$ .

$$\begin{aligned} KG &= FG - FK \\ &= 16 - 12 \\ &= 4cm \end{aligned}$$

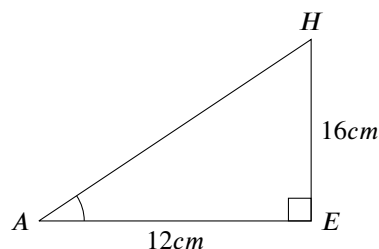


$$\begin{aligned} \tan \angle CGK &= \frac{CK}{KG} \\ &= \frac{12}{4} \\ &= 3 \\ \angle CGK &\approx 71.57^\circ \end{aligned}$$

- (b) The angle formed by plane  $ABH$  and plane  $ABFE$ .

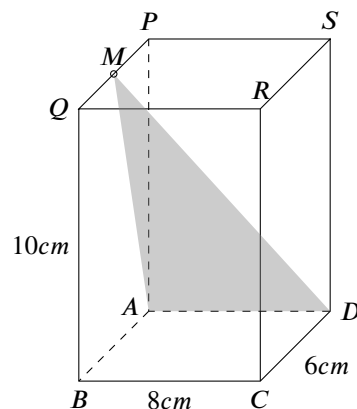
**Sol.**

$\therefore AB$  is the common edge of the plane  $ABH$  and plane  $ABFE$ ,  $AB \perp AH$  and  $AB \perp AE$ .  
 $\therefore$  The angle formed by plane  $ABH$  and plane  $ABFE$  is  $\angle HAE$ .



$$\begin{aligned} \tan \angle HAE &= \frac{HE}{AE} \\ &= \frac{16}{12} \\ &= \frac{4}{3} \\ \angle HAE &\approx 53.13^\circ \end{aligned}$$

7. In the cuboid shown below,  $BC = 8cm$ ,  $CD = 6cm$ ,  $BQ = 10cm$ . Given that  $M$  is the midpoint of  $PQ$ . Find:

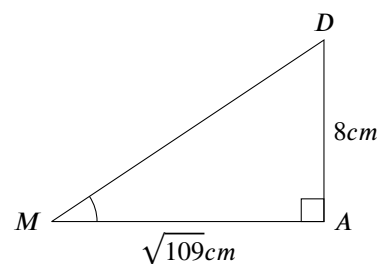


- (a) The angle formed by line  $MD$  and plane  $PQBA$ .

**Sol.**

The angle formed by line  $MD$  and plane  $PQBA$  is  $\angle DMA$ .

$$\begin{aligned} \text{In } \triangle MPA, MA &= \sqrt{PA^2 + MP^2} \\ &= \sqrt{10^2 + 3^2} \\ &= \sqrt{109}cm \end{aligned}$$



$$\begin{aligned} \tan \angle DMA &= \frac{DA}{MA} \\ &= \frac{8}{\sqrt{109}} \\ \angle DMA &\approx 37.46^\circ \end{aligned}$$

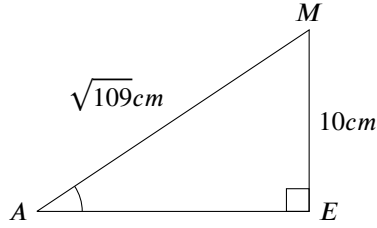
- (b) The angle formed by plane  $AMD$  and plane  $ABCD$ .

**Sol.**

Let the midpoint of  $AB$  be  $E$ .

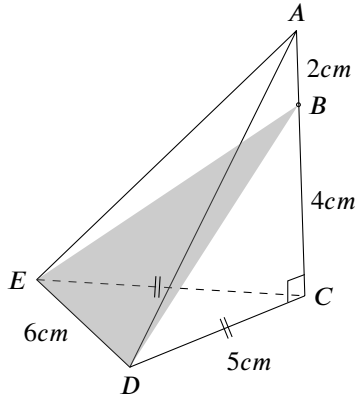
$\therefore AD$  is the common edge of plane  $AMD$  and plane  $ABCD$ ,  $AM \perp AD$ , and  $EA \perp AD$ .

∴ The angle formed by plane  $AMD$  and plane  $ABCD$  is  $\angle MAE$ .



$$\begin{aligned}\tan \angle MAE &= \frac{ME}{MA} \\ &= \frac{10}{\sqrt{109}} \\ \angle MAE &\approx 73.30^\circ\end{aligned}$$

8. The diagram below shows a pyramid with an isosceles triangle base. Given that  $CD = CE = 5\text{cm}$ ,  $ED = 6\text{cm}$ ,  $ACD$  is a right-angled triangle,  $B$  is a point on  $AC$ ,  $AD = 2\text{cm}$ ,  $BC = 4\text{cm}$ . Find:



- (a) The angle formed by plane  $BDE$  and plane  $CDE$ .

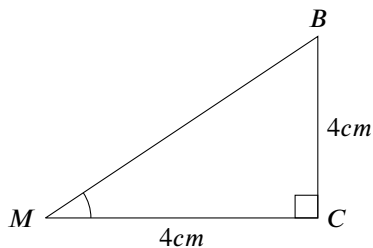
**Sol.**

Let the midpoint of  $ED$  be  $M$ .

∵  $DE$  is the common edge of plane  $BDE$  and plane  $CDE$ ,  $BM \perp DE$ , and  $CM \perp DE$ .

∴ The angle formed by plane  $BDE$  and plane  $CDE$  is  $\angle BMC$ .

$$\begin{aligned}MC &= \sqrt{DC^2 - DM^2} \\ &= \sqrt{5^2 - 3^2} \\ &= 4\text{cm}\end{aligned}$$



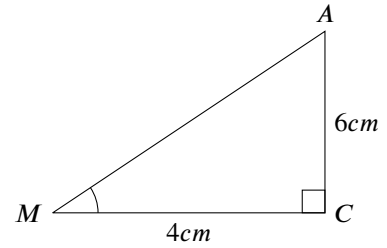
$$\begin{aligned}\tan \angle BMC &= \frac{BC}{CM} \\ &= \frac{4}{4} \\ &= 1 \\ \angle BMC &= 45^\circ\end{aligned}$$

- (b) The angle formed by the plane  $ADE$  and  $CDE$ .

**Sol.**

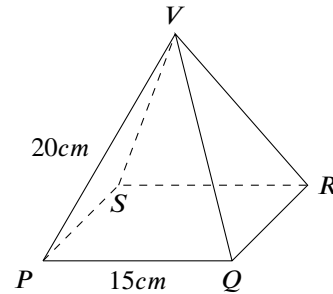
∵  $DE$  is the common edge of plane  $ADE$  and plane  $CDE$ ,  $CM \perp DE$ , and  $AM \perp DE$ .

∴ The angle formed by plane  $ADE$  and plane  $CDE$  is  $\angle AMC$ .



$$\begin{aligned}\tan \angle AMC &= \frac{AC}{CM} \\ &= \frac{6}{4} \\ &= \frac{3}{2} \\ \angle AMC &= 56.31^\circ\end{aligned}$$

9. The diagram below shows a regular pyramid with a square base. Given that  $PQ = 15\text{cm}$ ,  $PV = 20\text{cm}$ . Find:



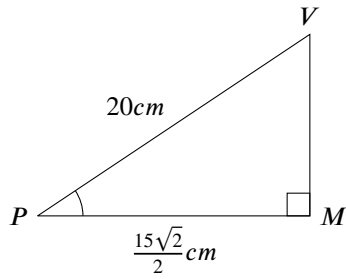
- (a) The angle formed by line  $PV$  and plane  $PQRS$ .

**Sol.**

Let the footpoint of  $V$  be  $M$ .

The angle formed by line  $PV$  and plane  $PQRS$  is  $\angle VPM$ .

$$\begin{aligned}PR &= \sqrt{PQ^2 + QR^2} \\ &= \sqrt{15^2 + 15^2} \\ &= 15\sqrt{2}\text{cm} \\ PM &= \frac{PR}{2} \\ &= \frac{15\sqrt{2}}{2}\end{aligned}$$



$$\begin{aligned}\cos \angle VPM &= \frac{PM}{PV} \\ &= \frac{\frac{15\sqrt{2}}{2}}{20} \\ &= \frac{3\sqrt{2}}{8} \\ \angle VPM &= 57.97^\circ\end{aligned}$$

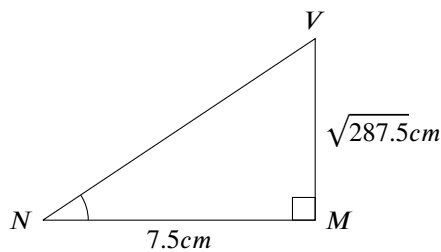
- (b) The angle formed by the lateral faces and the base of the pyramid.

**Sol.**

$$\begin{aligned}VM &= \sqrt{VP^2 - PM^2} \\ &= \sqrt{20^2 - \left(\frac{15\sqrt{2}}{2}\right)^2} \\ &= \sqrt{287.5}cm\end{aligned}$$

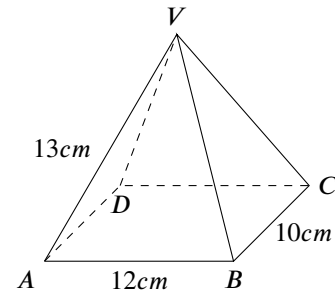
Let the midpoint of  $PQ$  be  $N$ .

The angle formed by the lateral faces and the base of the pyramid is  $\angle VNM$ .



$$\begin{aligned}\tan \angle VNM &= \frac{VM}{NM} \\ &= \frac{\sqrt{287.5}}{7.5} \\ \angle VNM &= 66.14^\circ\end{aligned}$$

10. The diagram below shows a right pyramid with lateral edges of  $13cm$ . Its base  $ABCD$  is a rectangle with length of  $12cm$  and width of  $10cm$ . Find:



- (a) The angle formed by plane  $VBC$  and plane  $ABCD$ .

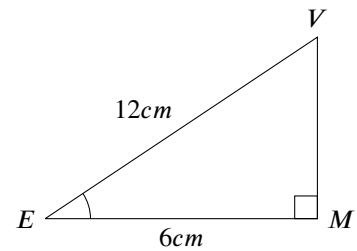
**Sol.**

Let the midpoint of  $BC$  be  $E$ , and the footpoint of  $V$  be  $M$ .

$\therefore BC$  is the common edge of plane  $VBC$  and plane  $ABCD$ ,  $VE \perp BC$ , and  $ME \perp BC$ .

$\therefore$  The angle formed by plane  $VBC$  and plane  $ABCD$  is  $\angle VEM$ .

$$\begin{aligned}VE &= \sqrt{VB^2 - BE^2} \\ &= \sqrt{13^2 - 5^2} \\ &= 12cm\end{aligned}$$



$$\begin{aligned}\cos \angle VEM &= \frac{ME}{VE} \\ &= \frac{1}{2} \\ \angle VEM &= 60^\circ\end{aligned}$$

- (b) The angle formed by plane  $VCD$  and plane  $ABCD$ .

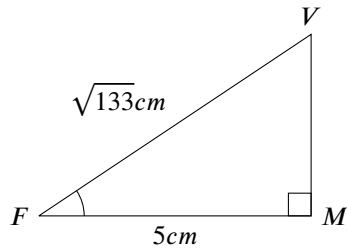
**Sol.**

Let the midpoint of  $CD$  be  $F$

$\therefore CD$  is the common edge of plane  $VCD$  and plane  $ABCD$ ,  $VF \perp CD$ , and  $MF \perp CD$ .

$\therefore$  The angle formed by plane  $VCD$  and plane  $ABCD$  is  $\angle VFM$ .

$$\begin{aligned}VF &= \sqrt{VD^2 - DF^2} \\ &= \sqrt{13^2 - 6^2} \\ &= \sqrt{133}cm\end{aligned}$$



$$\begin{aligned}\cos \angle VEM &= \frac{MF}{VF} \\ &= \frac{5}{\sqrt{133}} \\ \angle VEM &= 64.31^\circ\end{aligned}$$

RIGHT HERE LMAO

## 17.4 Longitude and Latitude

The earth is approximately spherical in shape, its radius is about  $6,370\text{km}$ , and its axis is a line that passes through the north ( $N$ ) and south ( $S$ ) poles. The earth rotating around its axis once is called a day, and the earth rotating around the sun once is called a year.

Any point on the earth's surface can be identified by two angles, the first is the angle between the point and the equator, called the *latitude* of the point, and the second is the angle between the point and the prime meridian, called the *longitude* of the point.

### Longitude and Lines of Longitude

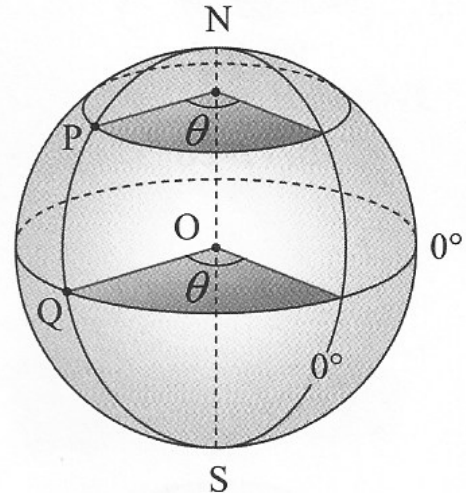
The two semicircles that are formed by the intersection of the earth's surface with the plane that passes through the north and south poles are called the *lines of longitude*, also called *meridians*. The lines of longitude that passes through the *Greenwich Observatory* in England are considered as  $0^\circ$  longitude, called the *Greenwich Meridian* or *prime meridian*.



Prime meridian

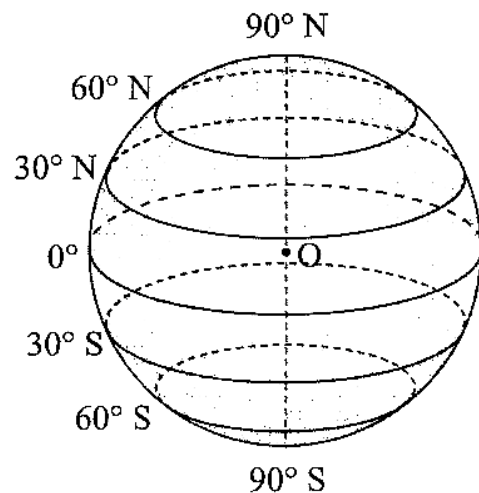
The angle between the Greenwich Meridian and the line of longitude that passes through the point  $P$  is called the *lon-*

*gitude* of  $P$ . There are 360 degrees of longitude ( $+180^\circ$  eastward and  $-180^\circ$  westward.). The prime meridian divides the world into the Eastern Hemisphere and the Western Hemisphere.  $180^\circ E$  and  $180^\circ W$  coincide with each other at the same line of longitude, called the  $180^{\text{th}}$  Meridian or *Antimeridian*.

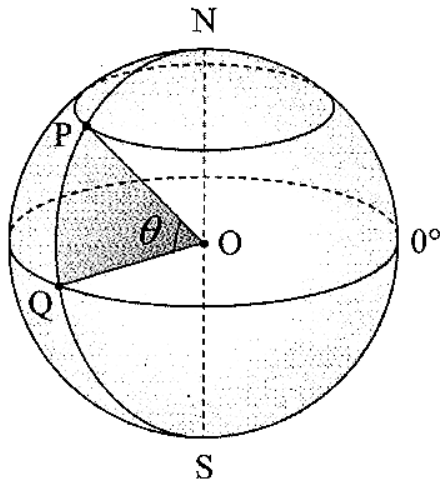


### Latitude and Parallels of Latitude

The lines of latitude are the circles that are perpendicular to the plane that passes through the north and south poles. The *equator* is the one and only great circle among the parallels of latitude.

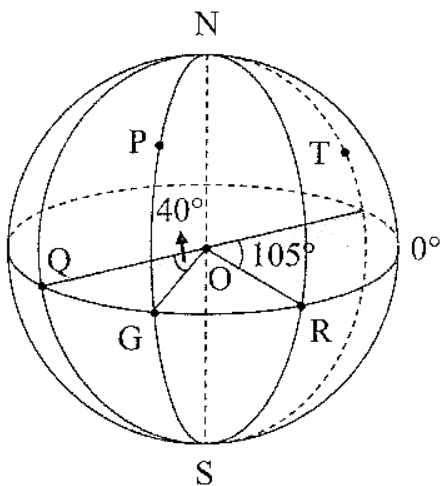


The angle between the equator and the line of latitude that passes through the point  $P$  is called the *latitude* of  $P$ . There are 180 degrees of latitude ( $+90^\circ$  northward and  $-90^\circ$  southward). The equator divides the world into the Northern Hemisphere and the Southern Hemisphere.

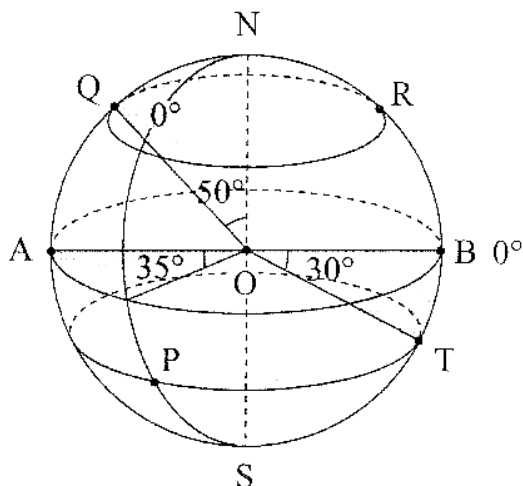


### 17.4.1 Practice 3

1. In the diagram below,  $NGS$  is the prime meridian,  $O$  is the centre of the earth. Find the longitude of locations  $P$ ,  $Q$ ,  $R$  and  $T$ .

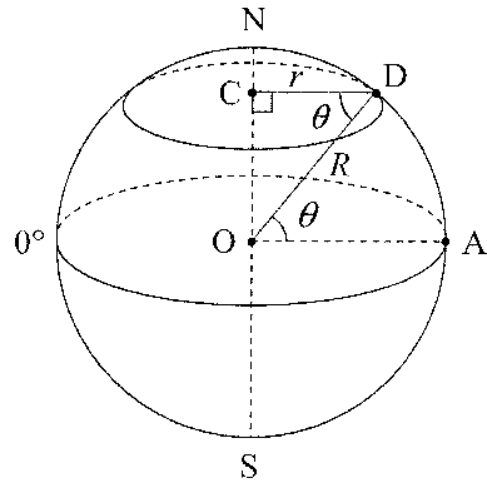


2. In the diagram below,  $O$  is the centre of the earth, location  $A$  and  $B$  are on the equator. Find the location of  $P$ ,  $Q$ ,  $R$  and  $T$ .



### Radius of the Parallel of Latitude

Let  $R$  be the radius of the earth,  $r$  be the radius of latitude  $\theta$ , then  $r = R \cos \theta$ .



### Nautical Miles

The arc length corresponding to  $1'$  ( $= \frac{1}{60}^\circ$ ) of the great circle on earth is called a *nautical mile* ( $1NM$ ), that is,  $1NM = \frac{1}{60 \times 360} \times 2\pi \times 6370km = 1.853km$ .

### Time Difference and Longitude

The time is calculated by the rotation of the earth around its axis. The earth rotates around its axis from west to east once in  $24h$ . That is, the earth rotates  $15^\circ$  in  $1h$ . Thus, the time difference between two locations on the earth is equal to the difference of their longitudes. Thus, the time difference is  $1hr$  per  $15^\circ$  of longitude difference.

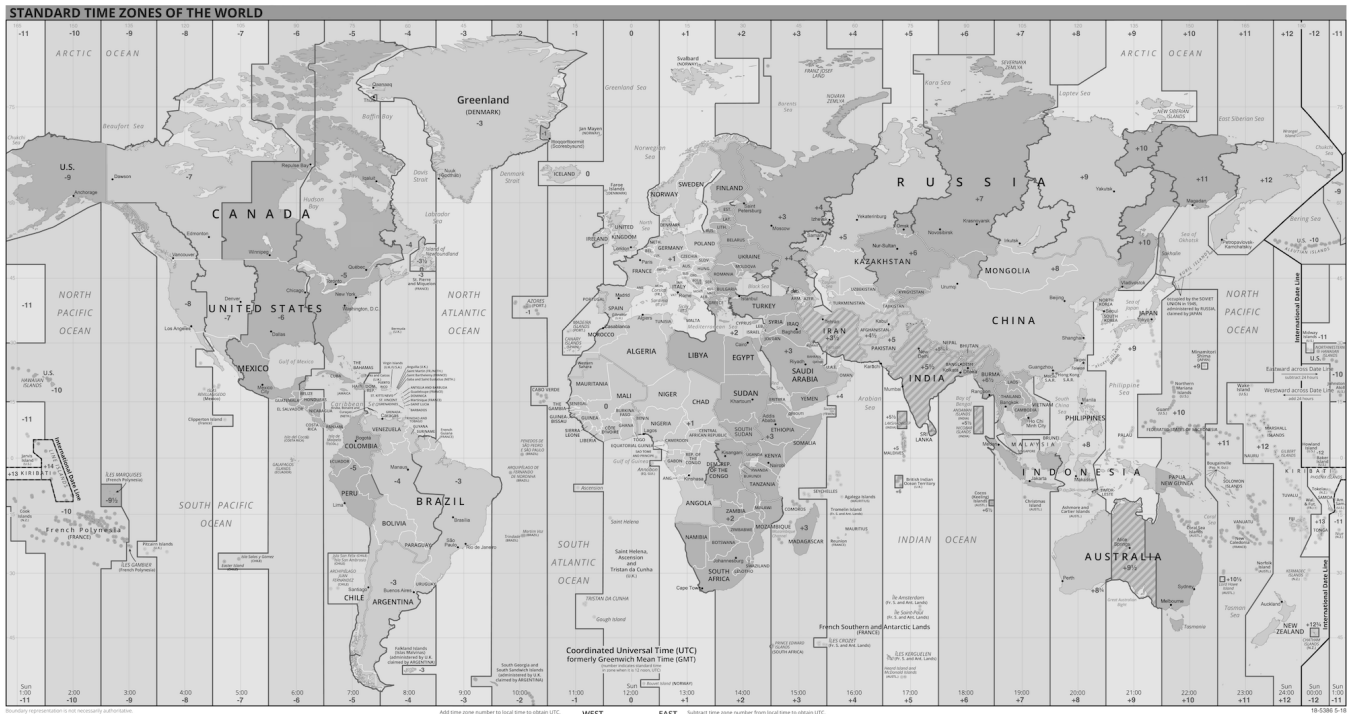
#### 1. Local Time

The local time is the time at a location on the earth. The local time for any location on the same line of longitude is the same.

#### 2. Standard Time

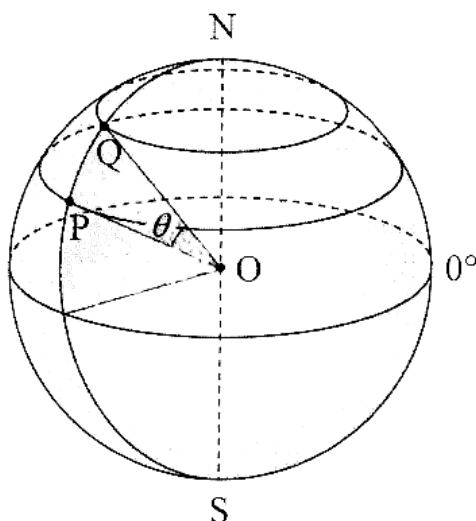
Back in the year 1844, International Meridian Conference was held in Washington DC. The conference decided to divide the world into 24 time zones based on the Greenwich Meridian, called the *Greenwich Meridian Time (GMT)*. There is zero time offset  $7.5^\circ$  eastward and  $7.5^\circ$  westward of the Greenwich Meridian. The time offset is  $1hr$  per  $15^\circ$  of longitude difference. All places in the same time zone share the same local time with the location located on the line of longitude that passes through the centre of the time zone, called the *standard time* or *zone time*.

When entering a new time zone from the east, the local time is advanced by  $1hr$  per  $15^\circ$  of longitude difference. When entering a new time zone from the west, the local time is delayed by  $1hr$  per  $15^\circ$  of longitude difference.



## 17.5 Distance of Two Locations on the Same Line of Longitude

The distance of two location on the same line of longitude is the arc length corresponding to the difference of their latitudes. Given two location  $P$  and  $Q$  on the same line of longitude, according to the definition of nautical mile, the distance between  $P$  and  $Q$  can be acquired by the arc length of  $PQ$ . That is,  $PQ = \theta \times 60\text{NM}$ , where  $\theta$  is the difference of their latitudes.



### 17.5.1 Practice 4

- Given that location  $A$  and  $B$  are on the same line of longitude. Based on the following longitude, find the distance between  $A$  and  $B$  (Express your answer in nautical miles):

- $A(50^\circ N), B(75^\circ N)$
- $A(0^\circ), B(42^\circ S)$
- $A(43^\circ N), B(38^\circ S)$

- Given that location  $P$  and  $Q$  are on the same line of longitude. The distance between two locations is  $1000\text{NM}$ ,  $P$  is located at  $7^\circ 30'$  north of the equator. Based on the following criteria, find the latitude of  $Q$ :

- $Q$  is located at the north of  $P$
- $Q$  is located at the south of  $P$

### 17.5.2 Exercise 17.5

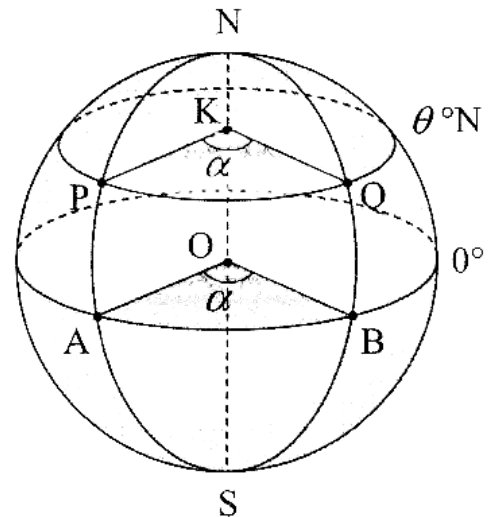
- Given that  $A$  and  $B$  are on the same line of longitude. Based on the following difference of latitude of two locations, find the distance between  $A$  and  $B$  (Express your answer in nautical miles):
  - $\theta = 39^\circ$
  - $\theta = 80^\circ 30'$
  - $\theta = 64^\circ 20'$
- Given that  $A$  and  $B$  are on the same line of longitude. Based on the following distance between two locations, find the difference of latitude of  $A$  and  $B$  (Round your answer to the nearest minute):
  - $700\text{NM}$
  - $318\text{NM}$
  - $3450\text{NM}$
- Find the distance between two locations along the same line of longitude:
  - $A(21^\circ S, 110^\circ E), B(33^\circ S, 110^\circ E)$
  - $X(38^\circ N, 40^\circ W), Y(19^\circ N, 40^\circ W)$
  - $E(34^\circ 45' S, 80^\circ E), F(0^\circ, 80^\circ E)$

- (d)  $P(18^\circ 15' N, 90^\circ W)$ ,  $Q(43^\circ 30' N, 90^\circ W)$   
 (e)  $T(15^\circ 30' N, 120^\circ E)$ ,  $M(24^\circ 30' N, 120^\circ E)$

- Location  $X$  and  $Y$  are on the same line of longitude, the distance between them is  $400NM$ . Find the difference of latitude of  $X$  and  $Y$ .
- Location  $P$  and  $Q$  are on the same line of longitude, and their distance along the line of longitude is  $600NM$ , find the difference between their latitude.
- $X$  city and  $Y$  city are on the same line of longitude, the latitude of  $X$  city is  $2^\circ 15'$  north of the equator, the latitude of  $Y$  city is  $6^\circ$  north of the equator. Find the distance between  $X$  city and  $Y$  city (Express your answer in kilometers).
- A plane is flying  $1000km$  due north from airport  $A(15^\circ N, 115^\circ E)$  to airport  $B$ . Find the longitude and latitude of airport  $B$ .
- A plane is flying  $1500km$  due south from airport  $A(5^\circ N, 100^\circ E)$  to airport  $B$ . Find the longitude and latitude of airport  $B$ .
- Find the distance from  $A(18^\circ 30' S)$  to the north pole along the same line of longitude.
- The distance between location  $C$  and  $D$  is  $700NM$ ,  $C$  is located at  $5^\circ 30'$  north of the equator. Find the latitude of  $D$ .
- A plane takes off from  $P(60^\circ N, 60^\circ E)$  and flies pass north pole along the great circle route to  $Q(50^\circ N, 120^\circ W)$ . Find the flying distance.
- A ship sails from  $P(50^\circ S, 160^\circ E)$  due north to another port  $Q(30^\circ N, 160^\circ E)$ . The sailing time is 10 days. Find the average speed of the ship. (Express your answer in  $NM/hr$ )
- Given that  $PQ$  is the diameter of the parallel of latitude  $35^\circ S$ . A plane takes off from location  $P$ , flies pass the south pole along the line of longitude, and lands at location  $Q$  after  $13hrs40mins$ . Find the average speed of the plane for the whole flight duration. (Express your answer in  $NM/hr$ )

## 17.6 Distance of Two Locations on the Same Parallel of Latitude

The distance between two locations on the same parallel of latitude is the arc length on the parallel of latitude corresponding to the difference of their longitudes.



In the diagram above,  $P$  and  $Q$  are on the same parallel of latitude  $\theta$ , their difference of longitude is  $\alpha$ .  $A$  and  $B$  are locations on the equator.

Given that  $\angle PKQ = \angle AOB = \alpha$ . Let  $R$  be the radius of the earth,  $r$  be the radius of the parallel of latitude.

$$\frac{\widehat{PQ}}{\widehat{AB}} = \frac{\frac{\alpha}{360^\circ} \times 2\pi r}{\frac{\alpha}{360^\circ} \times 2\pi R} = \frac{r}{R}$$

From the radius of the parallel of latitude  $r = R \cos \theta$ , we have  $\frac{r}{R} = \cos \theta$ .

$$\begin{aligned} \therefore \frac{\widehat{PQ}}{\widehat{AB}} &= \cos \theta \\ \widehat{PQ} &= \widehat{AB} \times \cos \theta \\ &= \alpha \times 60 \times \cos \theta NM \text{ or} \\ &= \alpha \times 60 \times \cos \theta \times 1.853 km \end{aligned}$$

### 17.6.1 Practice 5

- Find the distance of the following pairs of location on the same parallel of latitude (Express your answer in nautical miles):
  - $P(80^\circ N, 105^\circ W)$ ,  $Q(80^\circ N, 48^\circ W)$
  - $M(50^\circ S, 48^\circ E)$ ,  $N(50^\circ S, 100^\circ E)$
  - $X(40^\circ N, 28^\circ 15' E)$ ,  $Y(40^\circ N, 42^\circ 45' W)$
  - $K(20^\circ S, 160^\circ E)$ ,  $L(20^\circ S, 140^\circ W)$
- Given that  $A$  is located at the west of  $B(46^\circ N, 72^\circ W)$  with a distance of  $2350NM$ . Find the longitude and latitude of  $A$ .

### 17.6.2 Exercise 17.6

- Find the distance of the following pairs of location on the same parallel of latitude (Express your answer in



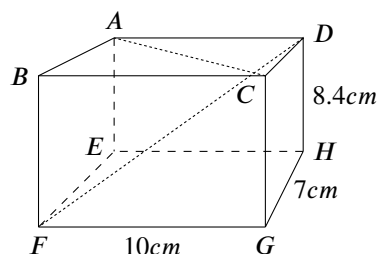
nautical miles):

- $P(45^\circ S, 20^\circ E), Q(45^\circ S, 100^\circ E)$
  - $M(36^\circ N, 45^\circ W), N(36^\circ N, 105^\circ W)$
  - $A(80^\circ S, 130^\circ E), B(80^\circ S, 165^\circ E)$
  - $K(70^\circ N, 40^\circ E), L(70^\circ N, 20^\circ W)$
  - $T(0^\circ, 128^\circ W), M(0^\circ, 120^\circ E)$
- Based on the following distances of location  $P$  and  $Q$  and the longitude and latitude of  $P$ , find the longitude and latitude of  $Q$ :
    - $PQ = 800NM$ ,  $Q$  is located at the west of  $P(50^\circ S, 100^\circ W)$
    - $PQ = 3400NM$ ,  $Q$  is located at the east of  $P(35^\circ N, 68^\circ E)$
    - $PQ = 1450NM$ ,  $Q$  is located at the east of  $P(42^\circ N, 150^\circ W)$
  - Given that two places are on the parallel of latitude  $60^\circ$  north to the equator, and their difference of longitude is  $160^\circ$ . Find the distance of the two places. (Express your answer in kilometers)
  - City  $A$  and  $B$  are on the parallel of latitude  $5^\circ 30'$  north to the equator, their longitude are  $100^\circ 15' E$  and  $103^\circ E$  respectively. Find the distance between two cities along the parallel of latitude.
  - Find the circumference of the parallel of latitude  $35^\circ 30' S$ .
  - Find the radius of the parallel of latitude  $60' N$ .
  - A ship set sail from  $P(20^\circ E)$  and sail  $600NM$  due east along  $42^\circ N$  parallel of latitude. Find the longitude and latitude of the destination.
  - A ship sails from port  $P(48^\circ N, 12^\circ W)$   $1000NM$  due west to another port  $Q$ , find the longitude and latitude of  $Q$ .
  - Given that  $A$  is located at the east of Paris( $49^\circ N, 2^\circ 30' E$ ) with a distance of  $2200km$ . Find the longitude and latitude of  $A$ .
  - A plane flies from  $X(40^\circ N, 2^\circ 30' E)$   $9265km$  due east to  $Y$ , find the longitude and latitude of  $Y$ .
  - Given that the earth takes  $24hrs$  to rotate once. Find the speed of Kuala Lumpur( $3^\circ 15' N, 102^\circ E$ ) to rotate once. (Express your answer in  $NM/hr$ )
  - Given that the longitude of  $P$  and  $Q$  are  $50^\circ$  and  $100^\circ$  respectively. If  $P$  and  $Q$  both located at the west of  $R(55^\circ S)$  and  $PR = PQ$ , find:
    - The longitude of  $R$ .
    - The distance between  $Q$  and  $R$  along the parallel of latitude.

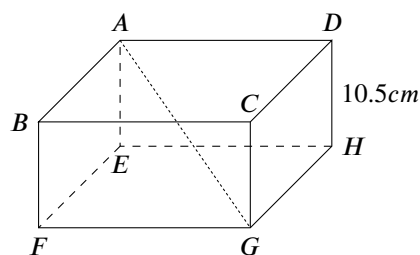
- A plane flies from  $F(50^\circ S, 50^\circ E)$  due west to  $H(50^\circ S, 45^\circ W)$ , then flies from  $H$  due north  $4800NM$  to  $K$ . Given that the average speed of the plane is  $480NM/hr$  throughout the journey, find:
  - The latitude of  $K$ .
  - The distance between  $F$  and  $H$  along the parallel of latitude.
  - The flight duration for the whole journey.

## 17.7 Revision Exercise 17

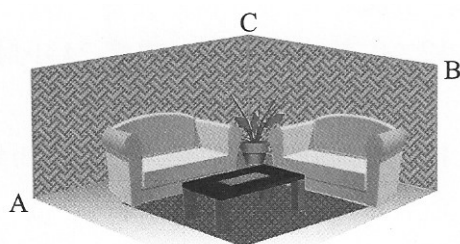
- In the cuboid shown below,  $FG = 10cm$ ,  $GH = 7cm$ ,  $DH = 8.4cm$ , find:
  - The angle formed by angle  $AC$  and plane  $BFGC$ .
  - The angle formed by angle  $FD$  and plane  $EFGH$ .



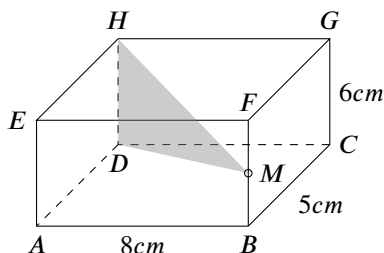
- The diagram below shows a cuboid with volume of  $400cm^3$ , height of  $10.5cm$ ,  $AD = 2DC$ . Find the angle formed by angle  $AG$  and plane  $ADHE$ .



- The diagram below shows a reception room with a square floor with side length of  $6m$ . Given that the elevation angle of corner  $C$  measured from corner  $A$  is  $30^\circ$ , find the angle formed by the line connecting corner  $A$  and  $B$  with the floor.

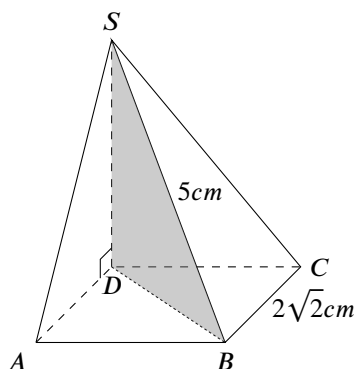


4. The diagram below shows a cuboid with length of  $8\text{cm}$ , width of  $5\text{cm}$  and height of  $6\text{cm}$ ,  $M$  is the midpoint of  $BF$ . Find the angle formed by plane  $HDM$  and plane  $ADHE$ .

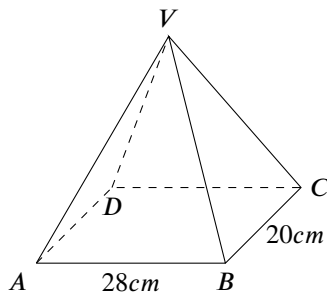


5. The diagram below shows a pyramid with a square base, its lateral edge  $SD$  is perpendicular to its base. Given that  $BC = 2\sqrt{2}\text{cm}$ ,  $SB = 5\text{cm}$ . Find:

- The angle formed by plane  $SAD$  and plane  $SBD$ .
- The angle formed by lateral edge  $SA$  and base  $ABCD$ .

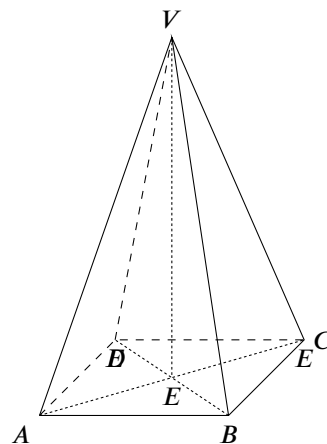


6. The diagram below shows a right prism with a rectangular base  $ABCD$  with length of  $28\text{cm}$  and width of  $20\text{cm}$ . Assume that plane  $VBC$  and the base of the pyramid forms a  $60^\circ$  angle. Find the angle formed by plane  $VAB$  and the base.



7. The diagram below shows a regular cuboid with a square base. Given that  $VE = \frac{5}{2}AD$ . Find:

- The angle formed by the angle  $VA$  and the base  $ABCD$ .
- The angle formed by plane  $VAD$  and the base.



- Find the distance from the Panama City ( $9^\circ N, 79^\circ 30' W$ ) to Toronto ( $43^\circ 45' N, 79^\circ 30' W$ ). (Express your answer in nautical miles)
- Tokyo and Adelaide are located at the same longitude, their latitude are  $35^\circ 45' N$  and  $35^\circ S$  respectively. Find the distance between two cities along the parallel of latitude.
- A plane flies  $2000\text{NM}$  along the equator, Find the difference of longitude between the point of departure and the destination.
- Location  $M$  and  $N$  are both located at the parallel of latitude  $45^\circ$  north to the equator with a difference in longitude of  $20^\circ$ . Find the distance between  $M$  and  $N$  along the parallel of latitude. (Express your answer in nautical miles)
- Location  $X$  and  $Y$  are on the parallel of latitude  $20^\circ$  north to the equator, their longitude are  $45^\circ E$  and  $80^\circ E$  respectively. Find the distance between location  $X$  and  $Y$  along the parallel of latitude. (Express your answer in nautical miles)
- A plane flies from  $A(42^\circ E)$  to  $B(20^\circ E)$  along the equator, then it flies from  $B$  due north to  $C(30^\circ N)$ . Find the distance the plane flies in total.
- Assume that  $A$  is located  $1000\text{NM}$  due north of the equator,  $600\text{NM}$  due east of the Greenwich Meridian, find the longitude and latitude of  $A$ .
- A plane flies from  $P(15^\circ N, 30^\circ E)$   $2000\text{NM}$  due south to  $B$ , find the longitude and latitude of  $B$ . Another plane flies from  $P$   $3000\text{NM}$  due east to  $C$ , find the longitude and latitude of  $C$ .
- A plane flies from  $A(130^\circ E)$  along the equator to  $B(120^\circ 30' E)$  along the equator, then flies from  $B$  due north to  $C(20^\circ 45')$ . Assume that the average speed of the plane is  $300\text{NM/hr}$  throughout the journey, find the flight duration for the whole journey.
- A plane flies from  $A(50^\circ N, 10^\circ E)$  due east to  $B(45^\circ E)$ .
  - Find the flight distance of the plane. (Express your answer in nautical miles)

- (b) Assume that the speed of the plane is  $420\text{NM/hr}$  in average, find the flight duration of the plane.
18. Given that three locations  $P$ ,  $Q$  and  $R$  are located on the same parallel of latitude  $40^\circ$  north to the equator, The longitude of  $P$  and  $R$  are  $10^\circ 30' W$  and  $4^\circ 30' E$ ,  $Q$  is located at the middle of  $P$  and  $R$ .
- (a) Find the difference of longitude between  $P$  and  $R$ .
- (b) Find the longitude of  $Q$ .
- (c) Find the distance between  $P$  and  $R$  along the parallel of latitude.
- (d) A ship sails from  $P$  to  $Q$  along the parallel of latitude with a speed of  $18\text{NM/hr}$ , find the sailing duration of the ship.