Praktis 8
Vectors

Melvin Chia

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Praktis Formatif

8.1 Vectors

1. Vectors $\underset{\sim}{x}$ and $\underset{\sim}{y}$ are non-zero and not parallel. Given that $(2m+3)\underset{\sim}{x}+(5-n)\underset{\sim}{y}=0$, find the value of m and n.

2. Given that points R, S and T lie on a straight line. If $|\overrightarrow{RS}| = 15$ *units* and $|\overrightarrow{RT}| = 40$ *units*, express $\overrightarrow{RS}$ in terms of $\overrightarrow{ST}$.

3. Given that $\overrightarrow{EF} = (p-4)\underset{\sim}{a}+6q\underset{\sim}{b}$ and $\overrightarrow{GH} = \underset{\sim}{a}+2\underset{\sim}{b}$. If line EF and line GH are parallel, express p in terms of q.

4. GIven that $\overrightarrow{AB} = 9\underset{\sim}{p}-12\underset{\sim}{q}$ and $\overrightarrow{BC} = 6\underset{\sim}{p}+(5-m)\underset{\sim}{q}$ where m is a constant. If points A, B, and C are collinear, find

   (a) the value of m,

   (b) the ratio of $AB : BC$.

8.2 Addition and Subtraction of Vectors

5. In the diagram in the answer space, $\overrightarrow{OX} = \underset{\sim}{x}$ and $\overrightarrow{OY} = \underset{\sim}{y}$. On the same diagram,

   (a) draw the vector $\overrightarrow{OU}$ such that $\overrightarrow{OU} = 3\underset{\sim}{x}-2\underset{\sim}{y}$,

   (b) mark point V such that $\overrightarrow{UV} = 4\underset{\sim}{y}-\underset{\sim}{x}$.

6. Given that $\underset{\sim}{a} = 3\underset{\sim}{p}+4\underset{\sim}{q}$, $\underset{\sim}{b} = 2\underset{\sim}{p}-\underset{\sim}{q}$, and $\underset{\sim}{c} = m\underset{\sim}{p}+(m-n)\underset{\sim}{q}$, where m and n are constants. Find the value of m and n when $\underset{\sim}{c} = 4\underset{\sim}{a}-2\underset{\sim}{b}$.

7. In the following diagram, $OPQR$ is a trapezium where PQ is parallel to OR adn $4PQ = 3OR$.

   Given that $\overrightarrow{OP} = \underset{\sim}{p}$, and $\overrightarrow{OR} = 4\underset{\sim}{r}$, express in terms of $\underset{\sim}{p}$ and $\underset{\sim}{r}$,

   (a) $\overrightarrow{PR}$,

   (b) $\overrightarrow{RQ}$.

8. The following diagram shows a regular hexagon $OPQRST$ with origin O, $\overrightarrow{OQ} = \underset{\sim}{q}$ and $\overrightarrow{OR} = \underset{\sim}{r}$.

   Find in terms of $\underset{\sim}{q}$ and $\underset{\sim}{r}$,

   (a) $\overrightarrow{RQ}$,

   (b) $\overrightarrow{OS}$.

9. The following diagram shows a trapezium where PQ is parallel to SR.

   Given that $\overrightarrow{PQ} = h\underset{\sim}{a}$, $\overrightarrow{RS} = k\underset{\sim}{a}$, $\overrightarrow{SP} = h\underset{\sim}{b}$, and $\overrightarrow{RQ} = 2\underset{\sim}{a}+(k+8)\underset{\sim}{b}$. Find the value of h and k.

10. The following diagram shows a triangle OPR and the point Q lies on the straight line PR.

    It is given that $\overrightarrow{PQ} : \overrightarrow{QR} = 1 : 2$. Express $\overrightarrow{OQ}$, int erms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

11. The following diagram shows a rectangle $ABCD$ and BED is a straight line.

    Given that $\overrightarrow{AB} = 10\underset{\sim}{p}$, $\overrightarrow{BC} = 6\underset{\sim}{q}$, and $\overrightarrow{BE} = 3\overrightarrow{ED}$. Express each of the following vectors in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$.

    (a) $\overrightarrow{BD}$,

    (b) $\overrightarrow{EC}$,

12. The following diagram shows a triangle OAB.

    Given that $CS = 2OC$, D is the midpoint of AB, $OE : ED = 2 : 1$, $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

    (a) Express $\overrightarrow{OD}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

    (b) Find the ratio of $CE : OB$.

13. Given that $\overrightarrow{OP} = -5\underset{\sim}{x}+10\underset{\sim}{y}$, $\overrightarrow{OQ} = 5\underset{\sim}{x}+8\underset{\sim}{y}$, and $\overrightarrow{OR} = (m-1)\underset{\sim}{x}+7\underset{\sim}{y}$, where m is a constant.

    (a) Find

        i. $\overrightarrow{PQ}$, in terms of $\underset{\sim}{x}$ and $\underset{\sim}{y}$,

        ii. $\overrightarrow{PR}$, in terms of m, $\underset{\sim}{x}$ and $\underset{\sim}{y}$.

    (b) If the points P, Q, and R are collinear, find hte value of m.

8.3 Vectors in a Cartesian Plane

14. The following diagram shows two vectors, PO and QO.

    Given that $\underset{\sim}{QP} = m\underset{\sim}{i}+n\underset{\sim}{j}$. Find the value of m and n.

15. The following diagram shows a parallelogram $ABCD$ drawn on a Cartesian plane where E is the midpoint of BD.

    Given that $\overrightarrow{AB} = 3\underset{\sim}{i}+2\underset{\sim}{j}$ and $\underset{\sim}{BC} = 7\underset{\sim}{i}-6\underset{\sim}{j}$. Find

    (a) $\overrightarrow{ED}$,

    (b) $|\overrightarrow{EC}|$.

16. Given that points $A(2,-1)$ and $B(5,3)$ lie on a Cartesian plane.

    (a) Express $\overrightarrow{AB}$ in the form of $\begin{pmatrix} x \\ y \end{pmatrix}$.

    (b) Find the unit vector in the direction of $\overrightarrow{AB}$.

17. Given that $\overrightarrow{OM} = \begin{pmatrix} -5 \\ k \end{pmatrix}$ and $\overrightarrow{ON} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, find the possible values of k if $|\overrightarrow{MN}| = 10$ *units*.

18. Given the vectors $\underset{\sim}{a} = -7\underset{\sim}{i}-m\underset{\sim}{j}$, $\underset{\sim}{b} = 8\underset{\sim}{i}-\underset{\sim}{j}$ and $\underset{\sim}{c} = -10\underset{\sim}{i}+6\underset{\sim}{j}$. If vector $\underset{\sim}{a}-\underset{\sim}{b}$ is parallel to vector $\underset{\sim}{c}$, find the value of the constant m.

19. Given $A(2,-5)$, $B(3,4)$ and $C(p,q)$. Find the value of p and q such that $\overrightarrow{AB}-2\overrightarrow{BC} = 9\underset{\sim}{i}-5\underset{\sim}{j}$.

20. Given the vectors $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\overrightarrow{OR} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, and $\overrightarrow{OS} = \begin{pmatrix} m \\ 2 \end{pmatrix}$, find

(a) vector $\overrightarrow{QR}$,

(b) the value of m when $\overrightarrow{PS}$ is parallel to $\overrightarrow{QR}$.

(c) the values of m such that $|\overrightarrow{OS}| = 2|\overrightarrow{QR}|$.