Praktis 3
Integration

Melvin Chia

March 16, 2023

Praktis Formatif

3.1 Integration as the Inverse of Differentiation

1. (a) Given $\frac{d}{dx}(2x^3 + 5x^2 - 7x) = 6x^2 + 10x - 7$, find $\int 6x^2 + 10x - 7\,dx$.

Sol.

$$\int 6x^2 + 10x - 7\,dx = 2x^3 + 5x^2 - 7x \quad \square$$

(b) Given $\frac{d}{dx}(5x^4 + 3x^2 + x) = 20x^3 + 6x + 1$, find $\int 20x^3 + 6x + 1\,dx$.

Sol.

$$\int 20x^3 + 6x + 1\,dx = 5x^4 + 3x^2 + x \quad \square$$

2. (a) Given $\frac{d}{dx}(4x - 5x^2 + 2x^3) = 4 - 10x + 6x^2$, find $\int 2 - 5x + 3x^2\,dx$.

Sol.

$$\begin{aligned}\int 2 - 5x + 3x^2\,dx &= \frac{2}{2}\int 2 - 5x + 3x^2\,dx\\ &= \frac{1}{2}\int 4 - 10x + 6x^2\,dx\\ &= \frac{1}{2}(4x - 5x^2 + 2x^3)\\ &= 2x - \frac{5}{2}x^2 + x^3 \quad \square\end{aligned}$$

(b) Given $\frac{d}{dx}\left(2x - \frac{3}{x^4}\right) = 2 + \frac{12}{x^5}$, find $\int 6 + \frac{36}{x^5}\,dx$

Sol.

$$\begin{aligned}\int 6 + \frac{36}{x^5}\,dx &= 6\int 1 + \frac{6}{x^5}\,dx\\ &= 3\int 2 + \frac{12}{x^5}\,dx\\ &3\left(2x - \frac{3}{x^4}\right)\\ &= 6x - \frac{9}{x^4} \quad \square\end{aligned}$$

(c) Given $f(x) = \frac{d}{dx}[g(x)]$, find $\int 2f(x)\,dx$.

Sol.

$$\begin{aligned}\int 2f(x)\,dx &= 2\int f(x)\,dx\\ &= 2g(x) \quad \square\end{aligned}$$

(d) Differentiate $\frac{2x^2}{3x-1}$ with respect to x and hence, find $\int \frac{6x(3x-2)}{(3x-1)^2}\,dx$.

Sol.

$$\begin{aligned}\frac{d}{dx}\left(\frac{2x^2}{3x-1}\right) &= \frac{4x(3x-1) - 3(2x^2)}{(3x-1)^2}\\ &= \frac{12x^2 - 4x - 6x^2}{(3x-1)^2}\\ &= \frac{6x^2 - 4x}{(3x-1)^2}\\ &= \frac{2x(3x-2)}{(3x-1)^2}\end{aligned}$$

$$\begin{aligned}\int \frac{6x(3x-2)}{(3x-1)^2}\,dx &= 3\int \frac{2x(3x-2)}{(3x-1)^2}\,dx\\ &= 3\left(\frac{2x^2}{3x-1}\right)\\ &= \frac{6x^2}{3x-1} \quad \square\end{aligned}$$

3. The daily production of bread of a bakery shop is given b y the function $R(x) = -50(x^2 - 12x)$, where x represents the number of bakers who work in the shop with condition x is not more than 6.

(a) Find the rate of daily production of bread in terms of x.

Sol.

$$R'(x) = -100x + 600 \quad \square$$

(b) If the rate of daily production of bread becomes $300 - 50x$ on a particular day, calculate the revenue of the bakery shop if all the loaves of bread baked by three bakers on that day are sold out at a price of RM5.50 for each loaf.

Sol.

$$\begin{aligned}\int 300 - 50x\,dx &= \frac{1}{2}\int (600 - 100x)\,dx\\ &= \frac{1}{2}(-50x^2 + 600x)\\ &= -25x^2 + 300x\\ R(3) &= -25(3)^2 + 300(3)\\ &= -225 + 900\\ &= 675\end{aligned}$$

$$\begin{aligned}\text{Revenue} &= 675 \times 5.50\\ &= \text{RM}3712.50 \quad \square\end{aligned}$$

4. Given $f(x) = x^4 - 2x^3$ and $f'(x) = 4x^3 - 6x^2$. Express $f'(x)\int f'(x)\,dx$ in factored form.

Sol.

$$\begin{aligned}f'(x)\int f'(x)\,dx &= (4x^3 - 6x^2)(x^4 - 2x^3)\\ &= 2x^5(2x-3)(x-2) \quad \square\end{aligned}$$

5. Given $y = \dfrac{2x-6}{x}$.

   (a) Find $\dfrac{dy}{dx}$.

   Sol.

$$\begin{aligned}\frac{dy}{dx} &= \frac{2x-2x-6}{x^2}\\ &= -\frac{6}{x^2} \quad \square\end{aligned}$$

   (b) Solve $4 + \displaystyle\int \left(\frac{dy}{dx}\right)\, dx = 0$.

   Sol.

$$\begin{aligned}4 + \int \left(\frac{dy}{dx}\right)\, dx &= 0\\ 4 + \int \left(-\frac{6}{x^2}\right)\, dx &= 0\\ 4 + \frac{2x-6}{x} &= 0\\ 4x + 2x - 6 &= 0\\ 6x &= 6\\ x &= 1 \quad \square\end{aligned}$$

6. Given $f'(x) = g(x)$. Find $\dfrac{3f(x)}{\displaystyle\int g(x)dx}$.

   Sol.

$$\begin{aligned}f'(x) &= g(x)\\ f(x) &= \int g(x)dx\\ \frac{3f(x)}{\displaystyle\int g(x)dx} &= \frac{3f(x)}{f(x)}\\ &= 3 \quad \square\end{aligned}$$

7. The population of town A is given by a function $P(t) = \frac{5}{6}(2.72^{1.2t}) - t^2 + 1495$ and the population continues to increase at the rate of $2.72^{1.2t} - 2t$ people per year where t is the number of years. Given that the population of town b increases at twice the rate of the population of town A based on the same model, find, to the nearest integer,

   (a) the rate of increase of the population of town B at $t = 5$ years.

   Sol.

$$\begin{aligned}P_B'(5) &= 2[2.72^{1.2(5)} - 2(5)]\\ &= 2[404.96 - 10]\\ &= 2(394.96)\\ &= 789.92\\ &= 790 \text{ people per year} \quad \square\end{aligned}$$

   (b) the population of town B after 5 years.

   Sol.

$$\begin{aligned}P_B(5) &= 2\left[\frac{5}{6}(2.72^{1.2\cdot 5}) - (5)^2 + 1495\right]\\ &= \frac{5}{3}(2.72^6) - 50 + 2990\\ &= 3614.93\\ &= 3615 \text{ people} \quad \square\end{aligned}$$

3.2 Indefinite Integral

8. By using the indefinite integral formula, find the integral of each of the following constants or algebraic functions.

   (a) $\displaystyle\int 3\, dx$

   Sol.

$$\int 3\, dx = 3x + C \quad \square$$

   (b) $\displaystyle\int 24x\, dx$

   Sol.

$$\int 24x\, dx = 12x^2 + C \quad \square$$

   (c) $\displaystyle\int 6x^2\, dx$

   Sol.

$$\int 6x^2\, dx = 2x^3 + C \quad \square$$

   (d) $\displaystyle\int 3x^2 + 4x\, dx$

   Sol.

$$\int 3x^2 + 4x\, dx = x^3 + 2x^2 + C \quad \square$$

   (e) $\displaystyle\int \frac{2}{x^4}\, dx$

   Sol.

$$\int \frac{2}{x^4}\, dx = -\frac{2}{x^3} + C \quad \square$$

   (f) $\displaystyle\int x^2(x-3)\, dx$

   Sol.

$$\begin{aligned}\int x^2(x-3)\, dx &= \int x^3 - 3x^2\, dx\\ &= \frac{1}{4}x^4 - x^3 + C \quad \square\end{aligned}$$

   (g) $\displaystyle\int (x+2)(2x^4-1)\, dx$

   Sol.

$$\begin{aligned}&\int (x+2)(2x^4-1)\, dx\\ &= \int 2x^5 - x + 4x^4 - 2\\ &= \frac{1}{3}x^6 + \frac{4}{5}x^5 - \frac{1}{2}x^2 - 2x + C \quad \square\end{aligned}$$

(h) $\displaystyle\int \frac{x^2+3x+2}{x+2}\,dx$

Sol.

$$\begin{aligned}\int \frac{x^2+3x+2}{x+2}\,dx &= \int \frac{(x+2)(x+1)}{x+2}\,dx \\ &= \int x+1\,dx \\ &= \frac{1}{2}x^2+x+C \quad \square\end{aligned}$$

9. Find the indefinite integral for each of the following by using

(a) the substitution method.

(b) the indefinite integral formula.

i. $\displaystyle\int \frac{2}{(x+2)^5}\,dx$

Sol.

(a) Let $v=(x+2)$.

$$\begin{aligned}\int \frac{2}{(x+2)^5}\,dx &= \int \frac{2}{v^5}\,dv \\ &= \int 2v^{-5}\,dv \\ &= -\frac{1}{2}v^{-4}+C \\ &= -\frac{1}{2v^4}+C \\ &= -\frac{1}{2(x+2)^4}+C \quad \square\end{aligned}$$

(b)

$$\begin{aligned}\int \frac{2}{(x+2)^5}\,dx &= \int 2(x+2)^{-5}\,dx \\ &= 2\int (x+2)^{-5}\,dx \\ &= 2\left[\frac{(x+2)^{-4}}{-4}\right]+C \\ &= -\frac{1}{2(x+2)^4}+C \quad \square\end{aligned}$$

ii. $\displaystyle\int \frac{3}{5}(3x+2)^8\,dx$

Sol.

(a) Let $v=3x+2$, $\dfrac{dv}{dx}=3$.

$$\begin{aligned}\int \frac{3}{5}(3x+2)^8\,dx &= \int \frac{3}{5}v^8\,dv \\ &= \int \frac{3}{5}v^8\frac{dv}{3} \\ &= \int \frac{1}{5}v^8\,dv \\ &= \frac{1}{45}v^9+C \\ &= \frac{(3x+2)^9}{45}+C \quad \square\end{aligned}$$

(b)

$$\begin{aligned}\int \frac{3}{5}(3x+2)^8\,dx &= \frac{3}{5}\int (3x+2)^8\,dx \\ &= \frac{3}{5}\left[\frac{(3x+2)^9}{27}\right]+C \\ &= \frac{(3x+2)^9}{45}+C \quad \square\end{aligned}$$

10. Determine the equation of a curve based on the following information.

(a) The gradient function of the curve is $\dfrac{dy}{dx}=3x^2+x-2$ and it passes through the point $p(2,15)$.

Sol.

$$\begin{aligned}\frac{dy}{dx} &= 3x^2+x-2 \\ y &= \int 3x^2+x-2\,dx \\ &= x^3+\frac{x^2}{2}-2x+C\end{aligned}$$

When $x=2$, $y=15$,

$$\begin{aligned}15 &= 2^3+\frac{2^2}{2}-2(2)+C \\ 15 &= 8+2-4+C \\ 15 &= 6+C \\ C &= 9\end{aligned}$$

Hence, the equation of the curve is $y=x^3+\dfrac{x^2}{2}-2x+9$. $\square$

(b) The gradient function of the curve is $f'(x)=2x+9$ and $f(3)=21$.

Sol.

$$\begin{aligned}f'(x) &= 2x+9 \\ f(x) &= \int 2x+9\,dx \\ &= x^2+9x+C \\ f(3) &= 3^2+9(3)+C \\ 21 &= 9+27+C \\ C &= -15\end{aligned}$$

Hence, the equation of the curve is $f(x)=x^2+9x-15$. $\square$

(c) The gradient function of the curve is given by $g(t)=\dfrac{5t^2-6t+1}{t^3(t-1)}$ and it passes through the point $(1,3)$.

Sol.

$$\begin{aligned} g(t) &= \frac{5t^2-6t+1}{t^3(t-1)} \\ &= \frac{(5t-1)(t-1)}{t^3(t-1)} \\ &= \frac{5t-1}{t^3} \\ &= \frac{5}{t^2}-\frac{1}{t^3} \\ &= 5t^{-2}-t^{-3} \\ f(t) &= \int 5t^{-2}-t^{-3}\,dt \\ &= -\frac{5}{t}+\frac{1}{2t^2}+C \end{aligned}$$

When $t=1$, $f(1)=3$,

$$\begin{aligned} 3 &= -5+\frac{1}{2}+C \\ 3 &= -\frac{9}{2}+C \\ C &= \frac{15}{2} \end{aligned}$$

Hence, the equation of the curve is $f(t)=-\frac{5}{t}+\frac{1}{2t^2}+\frac{15}{2}$. ▯

11. Tommy moves in his roller skates at the rate of change in displacement, $\frac{ds}{dt}=t^2+9$ metres per second, where t is the time in seconds. At $t=3$ seconds, Tommy is 4 metres away from his starting place. Find the displacement, s metres, when $t=10$ seconds.

Sol.

$$\begin{aligned} \frac{ds}{dt} &= t^2+9 \\ s &= \int t^2+9\,dt \\ &= \frac{t^3}{3}+9t+C \end{aligned}$$

When $t=3$, $s=4$,

$$\begin{aligned} 4 &= \frac{3^3}{3}+9(3)+C \\ 4 &= 9+27+C \\ 4 &= 54+C \\ C &= -32 \\ s &= \frac{t^3}{3}+9t-32 \end{aligned}$$

When $t=10$,

$$\begin{aligned} s &= \frac{10^3}{3}+9(10)-32 \\ &= 333+90-32 \\ &= 391\frac{1}{3}m \quad ▯ \end{aligned}$$

12. Given the gradient function of a curve is $\frac{dy}{dx}=kx^2+2x$ where k is a constant. The curve passes through point $A(1,6)$ and point $B(-2,0)$. Determine the equation of the curve.

Sol.

$$\begin{aligned} \frac{dy}{dx} &= kx^2+2x \\ y &= \int kx^2+2x\,dx \\ &= \frac{kx^3}{3}+x^2+C \end{aligned}$$

When $x=1$, $y=6$,

$$\begin{aligned} 6 &= \frac{k}{3}+1+C \\ k+3C &= 15 \quad (1) \end{aligned}$$

When $x=-2$, $y=0$,

$$\begin{aligned} 0 &= -\frac{8k}{3}+4+C \\ 8k-3C &= 12 \quad (2) \end{aligned}$$

$$\begin{aligned} (2)+(1)\cdot 8:\; 9k &= 27 \\ k &= 3 \\ C &= 4 \end{aligned}$$

Hence, the equation of the curve is $y=x^3+x^2+4$ ▯

3.3 Definite Integral

13. Calculate each of the following.

(a) $\int_2^1 \left(\sqrt{x}+\frac{1}{x}\right)$

Sol.

$$\begin{aligned} \int_2^1 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right) &= \int_1^2 \left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right) \\ &= \left[\frac{2\sqrt{x^3}}{3}+2\sqrt{x}\right]_1^2 \\ &= \left[\frac{4\sqrt{2}}{3}+2\sqrt{2}\right]-\left[\frac{2}{3}+2\right] \\ &= \frac{10\sqrt{2}}{3}-\frac{8}{3} \\ &= \frac{10\sqrt{2}-8}{3} \\ &\approx 2.0474 \quad ▯ \end{aligned}$$

(b) $\int_0^3 \left(\frac{x^4+3x}{x}\right)\,dx$

Sol.

$$\begin{aligned}\int_0^3 \left(\frac{x^4+3x}{x}\right) dx &= \int_0^3 \left(x^3+3\right) dx \\ &= \left[\frac{1}{4}x^4+3x\right]_0^3 \\ &= \left[\frac{1}{4}(3^4)+3\cdot 3\right]-0 \\ &= \frac{81}{4}+9 \\ &= \frac{117}{4} \\ &= 29.25 \quad \square\end{aligned}$$

(c) $\int_{-2}^{-1} \left(\frac{(4-x)(3-x)}{x^5}\right) dx$

Sol.

$$\begin{aligned}&\int_{-2}^{-1} \left(\frac{(4-x)(3-x)}{x^5}\right) dx \\ &= \int_{-2}^{-1} \left(\frac{x^2-7x+12}{x^5}\right) dx \\ &= \int_{-2}^{-1} \left(\frac{1}{x^3}-\frac{7}{x^4}+\frac{12}{x^5}\right) dx \\ &= \left[-\frac{1}{2x^2}+\frac{7}{3x^3}-\frac{3}{x^4}\right]_{-2}^{-1} \\ &= \left[-\frac{1}{2}-\frac{7}{3}-3\right]-\left[-\frac{1}{8}-\frac{7}{24}-\frac{3}{16}\right] \\ &= -\frac{35}{6}+\frac{29}{48} \\ &= -5\frac{11}{48} \quad \square\end{aligned}$$

14. Given $\int_a^b f(x)\,dx = 5$, $\int_b^c f(x)\,dx = 8$ and $\int_a^b g(x)\,dx = 2$. Find each of the following. [answer can be in terms of a and/or b.]

(a) $\int_a^b 3f(x)\,dx$

Sol.

$$\begin{aligned}\int_a^b 3f(x)\,dx &= 3\int_a^b f(x)\,dx \\ &= 3\cdot 5 \\ &= 15 \quad \square\end{aligned}$$

(b) $\int_a^c f(x)\,dx$

Sol.

$$\begin{aligned}\int_a^c f(x)\,dx &= \int_a^b f(x)\,dx + \int_b^c f(x)\,dx \\ &= 5+8 \\ &= 13 \quad \square\end{aligned}$$

(c) $\int_a^b [f(x)+g(x)]\,dx$

Sol.

$$\begin{aligned}\int_a^b [f(x)+g(x)]\,dx &= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx \\ &= 5-2 \\ &= 3 \quad \square\end{aligned}$$

(d) $\int_c^a f(x)\,dx$

Sol.

$$\begin{aligned}\int_c^a f(x)\,dx &= -\int_a^c f(x)\,dx \\ &= -13 \quad \square\end{aligned}$$

(e) $\int_a^b [g(x)+3]\,dx$

Sol.

$$\begin{aligned}\int_a^b [g(x)+3]\,dx &= \int_a^b g(x)\,dx + \int_a^b 3\,dx \\ &= 2+3(b-a) \\ &= 3b-3a+2 \quad \square\end{aligned}$$

(f) $\int_a^a f(x)\,dx$

Sol.

$$\int_a^a f(x)\,dx = 0 \quad \square$$

(g) The value of k such that $\int_b^a [f(x)+kx]\,dx = 25$ if $a=1$ and $b=4$.

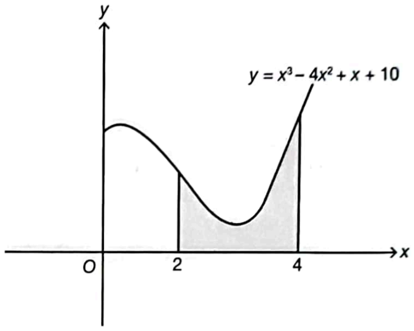
Sol.

$$\begin{aligned}\int_b^a [f(x)+kx]\,dx &= \int_b^a f(x)\,dx + \int_b^a kx\,dx \\ &= -5+\int_b^a kx\,dx\end{aligned}$$

$$\begin{aligned}-5+\int_1^4 kx\,dx &= 25 \\ \int_4^1 kx\,dx &= 30 \\ k\left[\frac{x^2}{2}\right]_4^1 &= 30 \\ k\left(\frac{1}{2}-\frac{16}{2}\right) &= 30 \\ -\frac{15k}{2} &= 30 \\ -15k &= 60 \\ k &= -4 \quad \square\end{aligned}$$

15. Find the area of the shaded region for each of the following diagrams.

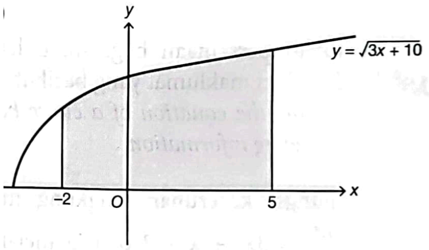
(a)



Sol.

$$
\begin{aligned}
A &= \int_2^4 (x^3 - 4x^2 + x + 10)\, dx \\
&= \left[\frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 + 10x\right]_2^4 \\
&= \left(64 - \frac{256}{3} + 8 + 40\right) - \left(4 - \frac{32}{3} + 2 + 20\right) \\
&= \frac{80}{3} - \frac{46}{3} \\
&= 11\frac{1}{3} \text{ units}^2 \quad □
\end{aligned}
$$

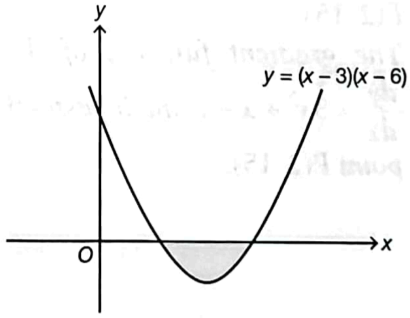
(b)



Sol.

$$
\begin{aligned}
A &= \int_{-2}^{5} \sqrt{3x + 10}\, dx \\
&= \int_{-2}^{5} (3x + 10)^{\frac{1}{2}}\, dx \\
&= \left[\frac{2(3x + 10)^{\frac{3}{2}}}{9}\right]_{-2}^{5} \\
&= \frac{2(25)^{\frac{3}{2}}}{9} - \frac{2(4)^{\frac{3}{2}}}{9} \\
&= \frac{250}{9} - \frac{16}{9} \\
&= \frac{234}{9} \\
&= 26 \text{ units}^2 \quad □
\end{aligned}
$$

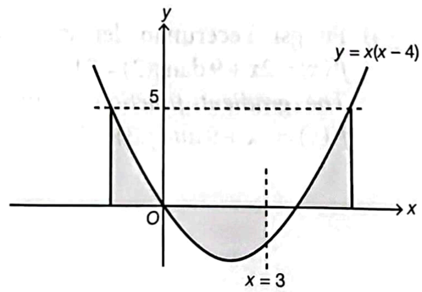
(c)



Sol.

$$
\begin{aligned}
A &= \left|\int_3^6 (x - 3)(x - 6)\, dx\right| \\
&= \left|\int_3^6 (x^2 - 9x + 18)\, dx\right| \\
&= \left|\left[\frac{1}{3}x^3 - \frac{9}{2}x^2 + 18x\right]_3^6\right| \\
&= \left|(72 - 162 + 108) - \left(9 - \frac{81}{2} + 54\right)\right| \\
&= \left|18 - \frac{45}{2}\right| \\
&= 4.5 \text{ units}^2 \quad □
\end{aligned}
$$

(d)



Sol.

When $y = 5$,

$$
\begin{aligned}
x(x - 4) &= 5 \\
x^2 - 4x - 5 &= 0 \\
(x - 5)(x + 1) &= 0 \\
x &= -1 \text{ or } x = 5
\end{aligned}
$$

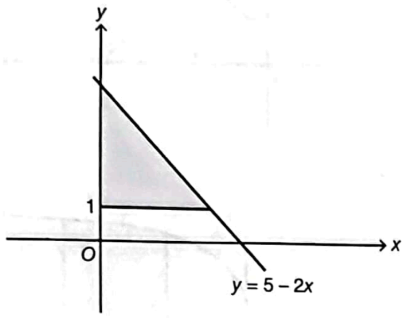
When $y = 0$,

$$
\begin{aligned}
x(x - 4) &= 0 \\
x &= 0 \text{ or } x = 4
\end{aligned}
$$

$$
\begin{aligned}
A &= \int_{-1}^{0} x(x - 4)\, dx + \left|\int_0^3 x(x - 4)\, dx\right| \\
&\quad + \int_4^5 x(x - 4)\, dx \\
&= \int_{-1}^{0} (x^2 - 4x)\, dx + \left|\int_0^3 (x^2 - 4x)\, dx\right| \\
&\quad + \int_4^5 (x^2 - 4x)\, dx \\
&= \left[\frac{1}{3}x^3 - 2x^2\right]_{-1}^{0} + \left|\left[\frac{1}{3}x^3 - 2x^2\right]_0^3\right| \\
&\quad + \left[\frac{1}{3}x^3 - 2x^2\right]_4^5 \\
&= 0 - \left(-\frac{1}{3} - 2\right) + |(9 - 18) - 0| \\
&\quad + \left(\frac{125}{3} - 50\right) - \left(\frac{64}{3} - 32\right) \\
&= 13\frac{2}{3} \text{ units}^2 \quad □
\end{aligned}
$$

16. Determine the area bounded by the curve, the horizontal line(s) and the y-axis.

(a)



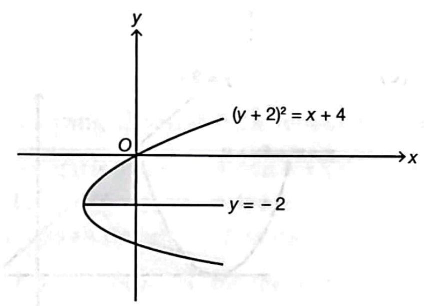
Sol. When $x = 0$,

$$\begin{aligned} y &= 5 - 2x \\ y &= 5 - 2(0) \\ &= 5 \end{aligned}$$

$$\begin{aligned} y &= 5 - 2x \\ 2x &= 5 - y \\ x &= \frac{5 - y}{2} \end{aligned}$$

$$\begin{aligned} A &= \int_1^5 \frac{5 - y}{2}\, dy \\ &= \int_1^5 \left(\frac{5}{2} - \frac{1}{2}y\right) dy \\ &= \left[\frac{5}{2}y - \frac{1}{4}y^2\right]_1^5 \\ &= \left(\frac{25}{2} - \frac{25}{4}\right) - \left(\frac{5}{2} - \frac{1}{4}\right) \\ &= \frac{25}{4} - \frac{9}{4} \\ &= 4 \text{ units}^2 \quad \square \end{aligned}$$

(b)

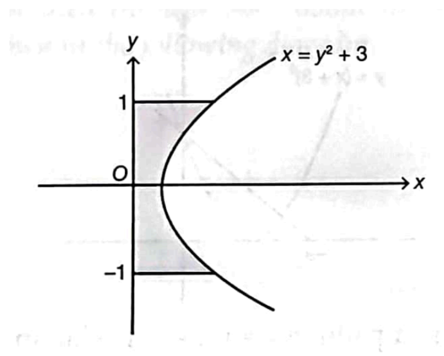


Sol.

$$\begin{aligned} (y + 2)^2 &= x + 4 \\ x &= (y + 2)^2 - 4 \\ &= y^2 + 4y + 4 - 4 \\ &= y^2 + 4y \end{aligned}$$

$$\begin{aligned} A &= \int_{-2}^{0} (y^2 + 4y)\, dy \\ &= \left[\frac{1}{3}y^3 + 2y^2\right]_{-2}^{0} \\ &= 0 - \left(-\frac{8}{3} + 8\right) \\ &= 5\frac{1}{3} \text{ units}^2 \quad \square \end{aligned}$$

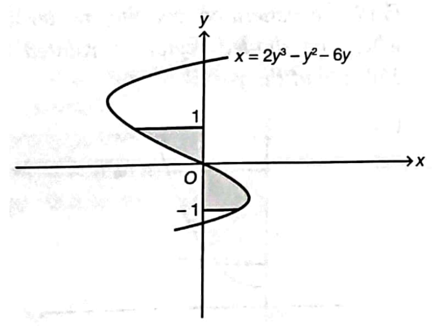
(c)



Sol.

$$\begin{aligned} A &= \int_{-1}^{1} (y^2 + 3)\, dy \\ &= \left[\frac{1}{3}y^3 + 3y\right]_{-1}^{1} \\ &= \left(\frac{1}{3} + 3\right) - \left(-\frac{1}{3} - 3\right) \\ &= 6\frac{2}{3} \text{ units}^2 \quad \square \end{aligned}$$

(d)

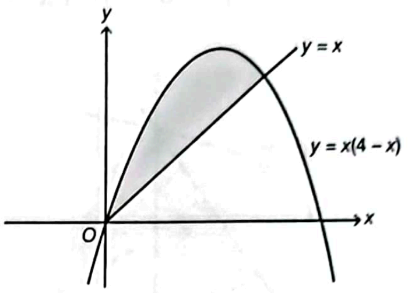


Sol.

$$\begin{aligned} A &= \int_{-1}^{0} (2y^3 - y^2 - 6y)\, dy + \left|\int_0^1 (2y^3 - y^2 - 6y)\, dy\right| \\ &= \left[\frac{1}{2}y^4 - \frac{1}{3}y^3 - 3y^2\right]_{-1}^{0} + \left|\left[\frac{1}{2}y^4 - \frac{1}{3}y^3 - 3y^2\right]_0^1\right| \\ &= 0 - \left(\frac{1}{2} + \frac{1}{3} - 3\right) + \left|\left(\frac{1}{2} - \frac{1}{3} - 3\right) - 0\right| \\ &= \frac{13}{6} + \frac{17}{6} \\ &= 5 \text{ units}^2 \quad \square \end{aligned}$$

17. Find the area of the shaded region for each of the following.

(a)

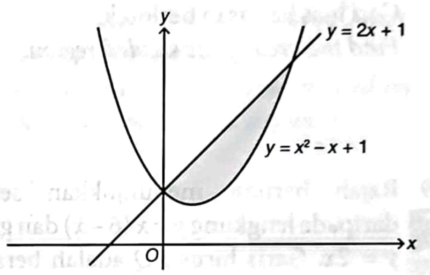


Sol.

$$\begin{aligned} x &= x(4-x) \\ x &= 4x - x^2 \\ x^2 - 3x &= 0 \\ x(x-3) &= 0 \\ x = 0 &\text{ or } x = 3 \end{aligned}$$

$$\begin{aligned} A &= \int_0^3 [x(4-x) - x] \, dx \\ &= \int_0^3 \left[4x - x^2 - x\right] \, dx \\ &= \int_0^3 \left[3x - x^2\right] \, dx \\ &= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3 \\ &= \left(\frac{27}{2} - 9\right) - 0 \\ &= 4.5 \text{ units}^2 \quad \square \end{aligned}$$

(b)

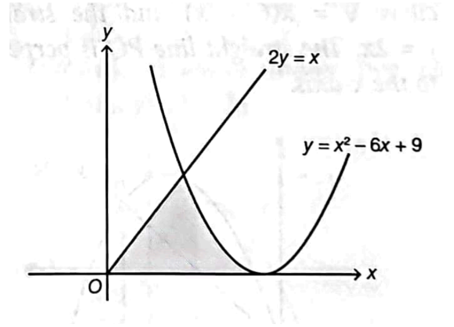


Sol.

$$\begin{aligned} 2x + 1 &= x^2 - x + 1 \\ x^2 - 3x &= 0 \\ x(x-3) &= 0 \\ x = 0 &\text{ or } x = 3 \end{aligned}$$

$$\begin{aligned} A &= \int_0^3 \left[2x + 1 - x^2 + x - 1\right] \, dx \\ &= \int_0^3 \left[-x^2 + 3x\right] \, dx \\ &= \left[-\frac{1}{3}x^3 + \frac{3}{2}x^2\right]_0^3 \\ &= \left(-9 + \frac{27}{2}\right) - 0 \\ &= 4.5 \text{ units}^2 \quad \square \end{aligned}$$

(c)

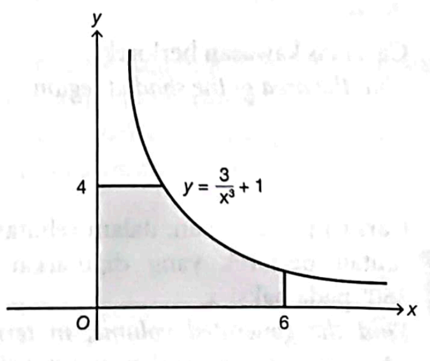


Sol.

$$\begin{aligned} 2y &= x \\ y &= \frac{1}{2}x \\ \frac{1}{2}x &= x^2 - 6x + 9 \\ x &= 2x^2 - 12x + 18 \\ 2x^2 - 13x + 18 &= 0 \\ (2x-9)(x-2) &= 0 \\ x = 2 &\text{ or } x = 4.5 \\ x^2 - 6x + 9 &= 0 \\ (x-3)^2 &= 0 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} A &= \int_0^2 \frac{x}{2} \, dx + \int_2^3 (x^2 - 6x + 9) \, dx \\ &= \left[\frac{1}{4}x^2\right]_0^2 + \left[\frac{1}{3}x^3 - 3x^2 + 9x\right]_2^3 \\ &= 1 - 0 + (9 - 27 + 27) - \left(\frac{8}{3} - 12 + 18\right) \\ &= 10 - \frac{26}{3} \\ &= 1\frac{1}{3} \text{ units}^2 \quad \square \end{aligned}$$

(d)

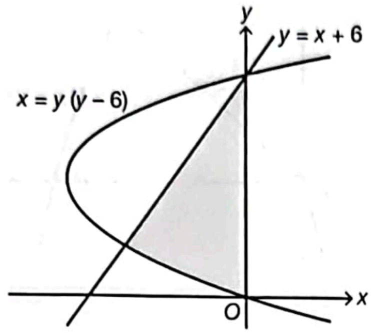


Sol.

$$\begin{aligned} \frac{3}{x^3} + 1 &= 4 \\ \frac{3}{x^3} &= 3 \\ x^3 &= 1 \\ x &= 1 \end{aligned}$$

$$\begin{aligned}
A &= 1 \cdot 4 + \int_1^6 \left(\frac{3}{x^3} + 1\right) dx \\
&= 4 + \left[-\frac{3}{2x^2} + x\right]_1^6 \\
&= 4 + \left(-\frac{1}{24} + 6\right) - \left(-\frac{3}{2} + 6\right) \\
&= 4 + \frac{143}{24} + \frac{9}{2} \\
&= 14\frac{11}{24} \text{ units}^2 \quad \square
\end{aligned}$$

18. The following diagram shows a part of the curve $x = y(y-6)$ and the straight line $y = x + 6$.



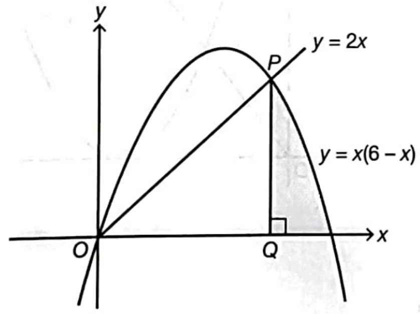
Find the area of the shaded region.

Sol.

$$\begin{aligned}
y &= x + 6 \\
x &= y - 6 \\
y - 6 &= y(y-6) \\
&= y^2 - 6y \\
y^2 - 7y + 6 &= 0 \\
(y-6)(y-1) &= 0 \\
y = 6 &\text{ or } y = 1 \\
1 &= x + 6 \\
x &= -5
\end{aligned}$$

$$\begin{aligned}
A &= \left|\int_0^1 y(y-6)\,dy\right| + \frac{1}{2}(5)(5) \\
&= \left|\int_0^1 (y^2 - 6y)\,dy\right| + \frac{25}{2} \\
&= \left|\left[\frac{1}{3}y^3 - 3y^2\right]_0^1\right| + \frac{25}{2} \\
&= \left|\frac{1}{3} - 3 - 0\right| + \frac{25}{2} \\
&= \frac{8}{3} + \frac{25}{2} \\
&= 15\frac{1}{6}\ \textit{units}^2 \quad \square
\end{aligned}$$

19. The following diagram shows a part of the curve $y = x(6-x)$ and a straight line $y = 2x$. The straight line PQ is perpendicular to the x-axis.



Find the area of the shaded region.

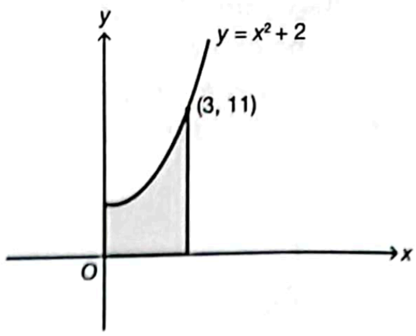
Sol.

$$\begin{aligned}
x(6-x) &= 0 \\
x = 0 &\text{ or } x = 6 \\
2x &= x(6-x) \\
2x &= 6x - x^2 \\
x^2 - 4x &= 0 \\
x(x-4) &= 0 \\
x = 0 &\text{ or } x = 4
\end{aligned}$$

$$\begin{aligned}
A &= \int_4^6 x(6-x)\,dx \\
&= \int_4^6 (6x - x^2)\,dx \\
&= \left[3x^2 - \frac{1}{3}x^3\right]_4^6 \\
&= (108 - 72) - \left(48 - \frac{64}{3}\right) \\
&= 36 - \frac{80}{3} \\
&= 9\frac{1}{3}\ \textit{units}^2 \quad \square
\end{aligned}$$

20. Find the generated volume, in terms of π, when the shaded region is rotated through $360°$ about the x-axis.

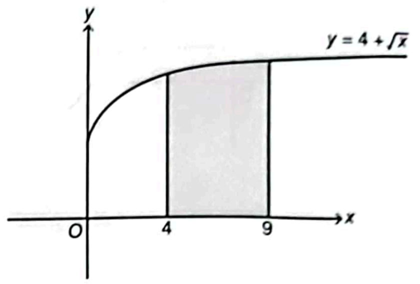
(a)



Sol.

$$\begin{aligned}
V_x &= \int_0^3 \pi y^2 \, dx \\
&= \pi \int_0^3 (x^2+2)^2 \, dx \\
&= \pi \int_0^3 (x^4 + 4x^2 + 4) \, dx \\
&= \pi \left[\frac{1}{5}x^5 + \frac{4}{3}x^3 + 4x \right]_0^3 \\
&= \left[\left(\frac{243}{5} + 36 + 12 \right) - 0 \right] \pi \\
&= 96.6\pi \textit{ units}^3 \quad \square
\end{aligned}$$

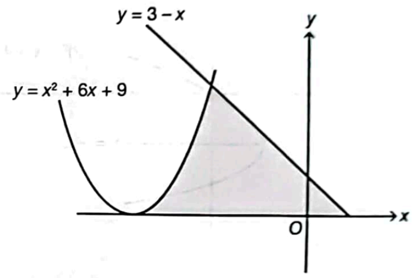
(b)



Sol.

$$\begin{aligned}
V_x &= \int_4^9 \pi (4+\sqrt{x})^2 \, dx \\
&= \pi \int_4^9 (16 + 8\sqrt{x} + x) \, dx \\
&= \pi \left[16x + \frac{16x^{\frac{3}{2}}}{3} + \frac{1}{2}x^2 \right]_4^9 \\
&= \pi \left[\left(144 + 144 + \frac{81}{2} \right) - \left(64 + \frac{128}{3} + 8 \right) \right] \\
&= 213.83\pi \textit{ units}^3 \quad \square
\end{aligned}$$

(c)

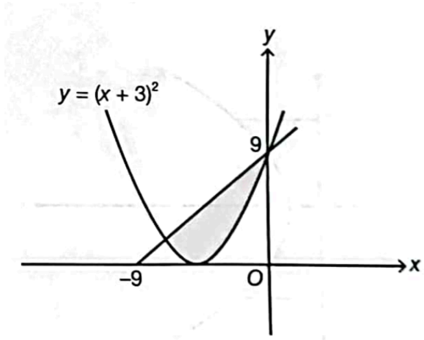


Sol.

$$\begin{aligned}
x^2 + 6x + 9 &= 3 - x \\
x^2 + 7x + 6 &= 0 \\
(x+6)(x+1) &= 0 \\
x &= -6 \text{ or } x = -1 \\
x^2 + 6x + 9 &= 0 \\
(x+3)^2 &= 0 \\
x &= -3 \\
0 &= 3 - x \\
x &= 3
\end{aligned}$$

$$\begin{aligned}
V_x &= \int_{-3}^{-1} \pi (x+3)^4 \, dx + \int_{-1}^{3} \pi (3-x)^2 \, dx \\
&= \pi \int_{-3}^{-1} (x+3)^4 \, dx + \pi \int_{-1}^{3} (3-x)^2 \, dx \\
&= \pi \left\{ \left[\frac{(x+3)^5}{5} \right]_{-3}^{-1} + \left[-\frac{(3-x)^3}{3} \right]_{-1}^{3} \right\} \\
&= \pi \left[\left(\frac{32}{5} - 0 \right) + \left(0 + \frac{64}{3} \right) \right] \\
&= 27\frac{11}{15}\pi \textit{ units}^3 \quad \square
\end{aligned}$$

(d)



Sol.

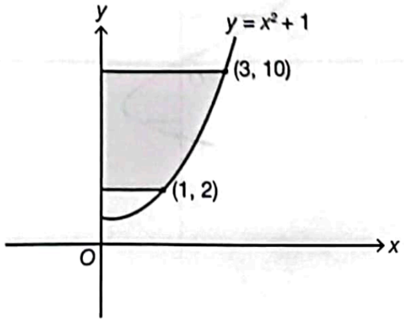
Let the line be l. Since l passes through $(-9, 0)$ and $(0, 9)$,

$$\begin{aligned}
m_l &= \frac{9}{9} = 1 \\
y - 9 &= 1(x - 0) \\
y &= x + 9 \\
x + 9 &= (x+3)^2 \\
&= x^2 + 6x + 9 \\
x^2 + 5x &= 0 \\
x(x+5) &= 0 \\
x &= 0 \text{ or } x = -5
\end{aligned}$$

$$\begin{aligned}
V_x &= \int_{-5}^{0} \pi (x+9)^2 \, dx - \int_{-5}^{0} \pi (x+3)^4 \, dx \\
&= \pi \int_{-5}^{0} (x+9)^2 \, dx - \pi \int_{-5}^{0} (x+3)^4 \, dx \\
&= \pi \left\{ \left[\frac{(x+9)^3}{3} \right]_{-5}^{0} - \left[\frac{(x+3)^5}{5} \right]_{-5}^{0} \right\} \\
&= \pi \left[\left(243 - \frac{64}{3} \right) - \left(\frac{243}{5} + \frac{32}{5} \right) \right] \\
&= 116\frac{2}{3}\pi \textit{ units}^3 \quad \square
\end{aligned}$$

21. Find the generated volume, in terms of π, when the shaded region is rotated through 360° about the y-axis.

(a)

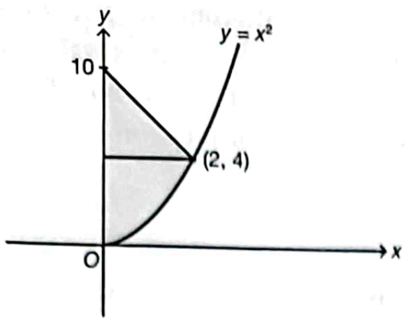


Sol.

$$\begin{aligned} y &= x^2 + 1 \\ x^2 &= y - 1 \\ x - \sqrt{y-1} &= 0 \end{aligned}$$

$$\begin{aligned} V_y &= \int_2^{10} \pi(\sqrt{y-1})^2\,dy \\ &= \pi \int_2^{10} y - 1\,dy \\ &= \pi \left[\frac{y^2}{2} - y\right]_2^{10} \\ &= \pi\left[(50-10) - (2-2)\right] \\ &= 40\pi \textit{ units}^3 \quad \square \end{aligned}$$

(b)

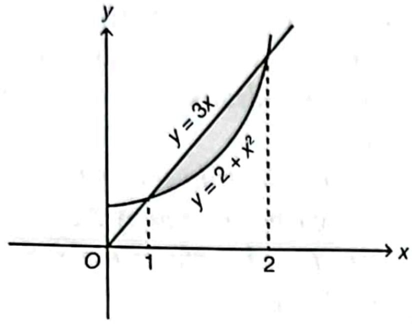


Sol.

$$\begin{aligned} y &= x^2 \\ x &= \sqrt{y} \end{aligned}$$

$$\begin{aligned} V_y &= \int_0^4 \pi(\sqrt{y})^2\,dy + \frac{1}{3}\pi \cdot 4 \cdot 6 \\ &= \pi \int_0^4 y\,dy + 8\pi \\ &= \pi\left[\frac{y^2}{2}\right]_0^4 + 8\pi \\ &= 8\pi + 8\pi \\ &= 16\pi \textit{ units}^3 \quad \square \end{aligned}$$

(c)

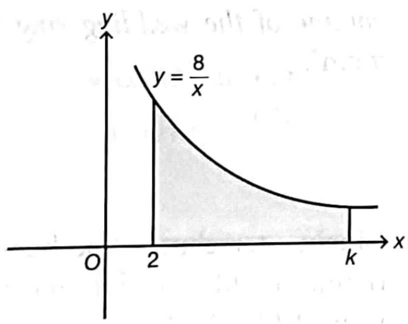


Sol.

$$\begin{aligned} y &= 3x \\ x &= \frac{y}{3} \\ y &= 2 + x^2 \\ x^2 &= y - 2 \\ x &= \sqrt{y-2} \\ x = 1,\ y &= 3(1) = 3 \\ x = 2,\ y &= 3(2) = 6 \end{aligned}$$

$$\begin{aligned} V_y &= \int_3^6 \pi\left(\sqrt{y-2}\right)^2 dy - \int_3^6 \pi\left(\frac{y}{3}\right)^2 dy \\ &= \pi \int_3^6 (y-2)\,dy - \pi \int_3^6 \frac{y^2}{9}\,dy \\ &= \pi\left[\frac{y^2}{2} - 2y\right]_3^6 - \pi\left[\frac{y^3}{27}\right]_3^6 \\ &= \pi\left\{\left[(18-12) - \left(\frac{9}{2} - 6\right)\right] - (8-1)\right\} \\ &= \pi\left(\frac{15}{2} - 7\right) \\ &= \frac{1}{2}\pi \textit{ units}^3 \quad \square \end{aligned}$$

22. The region bounded by the curve $y = \dfrac{8}{x}$, the x-axis, and the straight line $x = 2$ and $x = k$ is rotated through $360°$ about the x-axis as shown in the following diagram.

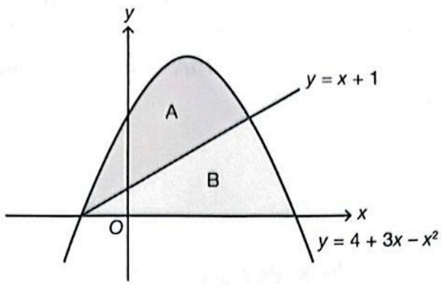


Express the volume generated by the region in terms of k. If the value of k becomes extremely large, deduce the nearest value of volume.

$$
\begin{aligned}
V_x &= \int_2^k \pi\left(\frac{8}{x}\right)^2 dx \\
&= \pi\int_2^k \frac{64}{x^2}\,dx \\
&= \pi\left[-\frac{64}{x}\right]_2^k \\
&= -\frac{64\pi}{k} + 32\pi \quad ▯
\end{aligned}
$$

$$
\begin{aligned}
&k \to \infty \Rightarrow \frac{1}{k} \approx 0 \\
&\therefore V_x \approx 32\ \textit{units}^3 \quad ▯
\end{aligned}
$$

23. The following diagram shows a part of the curve $y = 4 + 3x - x^2$ and the straight line $y = x + 1$.



Find the ratio of the area of the shaded region A to the area of the shaded region B.

Sol.

$$
\begin{aligned}
x + 1 &= 4 + 3x - x^2 \\
-x^2 + 2x + 3 &= 0 \\
x^2 - 2x - 3 &= 0 \\
(x-3)(x+1) &= 0 \\
x = 3 &\text{ or } x = -1 \\
x = 3,\ y &= 3 + 1 = 4
\end{aligned}
$$

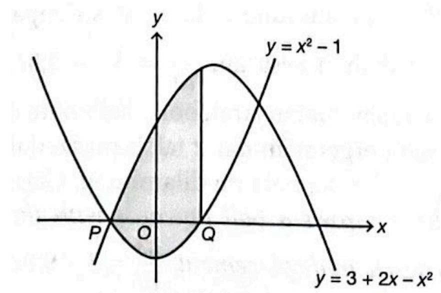
$$
\begin{aligned}
A_A &= \int_{-1}^3 (4 + 3x - x^2)\,dx - \frac{1}{2}\cdot 4\cdot 4 \\
&= \left[4x + \frac{3}{2}x^2 - \frac{1}{3}x^3\right]_{-1}^3 - 8 \\
&= \left(12 + \frac{27}{2} - 9\right) - \left(-4 + \frac{3}{2} + \frac{1}{3}\right) - 8 \\
&= \frac{33}{2} + \frac{13}{6} - 8 \\
&= \frac{32}{3}\ \textit{units}^2
\end{aligned}
$$

$$
\begin{aligned}
4 + 3x - x^2 &= 0 \\
x^2 - 3x - 4 &= 0 \\
(x-4)(x+1) &= 0 \\
x = 4 &\text{ or } x = -1
\end{aligned}
$$

$$
\begin{aligned}
A_B &= \int_3^4 (4 + 3x - x^2)\,dx + \frac{1}{2}\cdot 4\cdot 4 \\
&= \left[4x + \frac{3}{2}x^2 - \frac{1}{3}x^3\right]_3^4 + 8 \\
&= \left(16 + 24 - \frac{64}{3}\right) - \left(12 + \frac{27}{2} - 9\right) + 8 \\
&= \frac{56}{3} - \frac{33}{2} + 8 \\
&= \frac{61}{6}\ \textit{units}^2
\end{aligned}
$$

$$
\begin{aligned}
\therefore\ A_A : A_B &= \frac{32}{3} : \frac{61}{6} \\
&= \frac{64}{61} \\
&= 64 : 61 \quad ▯
\end{aligned}
$$

24. The following diagram shows two curves $y = x^2 - 1$ and $y = 3 + 2x - x^2$.



Find the coordinate of the points P and Q. Hence, calculate the area of the shaded region.

Sol.

$$
\begin{aligned}
x^2 - 1 &= 0 \\
(x+1)(x-1) &= 0 \\
x = -1 &\text{ or } x = 1
\end{aligned}
$$

$$
\begin{aligned}
3 + 2x - x^2 &= 0 \\
(x-3)(x+1) &= 0 \\
x = 3 &\text{ or } x = -1
\end{aligned}
$$

Since $y = x^2 - 1$ and $y = 3 + 2x - x^2$ intersect at $x = -1$, $P(-1, 0)$. ▯

Since another root of $y = x^2 - 1$ is 1, $Q(1, 0)$. ▯

$$
\begin{aligned}
A &= \left|\int_{-1}^1 (x^2 - 1)\,dx\right| + \int_{-1}^1 (3 + 2x - x^2)\,dx \\
&= \left|\left[\frac{1}{3}x^3 - x\right]_{-1}^1\right| + \left[3x + x^2 - \frac{1}{3}x^3\right]_{-1}^1 \\
&= \left|\left(\frac{1}{3} - 1\right) - \left(-\frac{1}{3} + 1\right)\right| + \left(3 + 1 - \frac{1}{3}\right) \\
&\quad - \left(-3 + 1 + \frac{1}{3}\right) \\
&= \frac{4}{3} + \frac{11}{3} + \frac{5}{3} \\
&= 6\frac{2}{3}\ \textit{units}^2 \quad ▯
\end{aligned}
$$

25. Calculate the volume generated, in terms of π, when the region bound by the curve $y = 3x^2$, the straight line $x = 1$ and $x = 4$ and the x-axis is rotated through two right angles about the x-axis.

Sol.

$$\begin{aligned}
V_x &= \frac{1}{2}\int_1^4 \pi(3x^2)^2\,dx \\
&= \frac{1}{2}\pi\int_1^4 (9x^4)\,dx \\
&= \frac{1}{2}\pi\left[\frac{9}{5}x^5\right]_1^4 \\
&= \frac{1}{2}\pi\left(\frac{9216}{5} - \frac{9}{5}\right) \\
&= 920\frac{7}{10}\pi \textit{ units}^3 \quad \square
\end{aligned}$$

3.4 Application of Integration

27. A container is filled with water. After t seconds, the height of the water, hcm, in the containers increases at the rate of $0.56t cms^{-1}$. Given that the container is empty when $t = 0$, find the value of t when $h = 28$.

Sol.

$$\begin{aligned}
\frac{dh}{dt} &= 0.56t \\
\int \frac{dh}{dt}\,dt &= \int 0.56t\,dt \\
h &= 0.28t^2 + C \\
\because\ t &= 0 \text{ when } h = 0 \\
\therefore\ C &= 0 \\
h &= 0.28t^2 \\
0.28t^2 &= 28 \\
0.01t^2 &= 1 \\
t^2 &= 100 \\
t &= 10s \quad (t > 0) \quad \square
\end{aligned}$$

28. Raja throws a ball upwards with the rate of change in displacement, $\frac{ds}{dt} = 3 - 9.8t$, where s is the displacement, in m, of the ball from the initial point and t is the time, in seconds, the moment the ball is thrown upwards. Find

Sol.

$$\begin{aligned}
\frac{ds}{dt} &= 3 - 9.8t \\
\int \frac{ds}{dt}\,dt &= \int (3 - 9.8t)\,dt \\
s &= 3t - 4.9t^2 + C \\
\because\ t &= 0,\ s = 0 \\
\therefore\ C &= 0 \\
s &= 3t - 4.9t^2
\end{aligned}$$

(a) the maximum height, in m, achieved by the ball.

Sol. $\frac{ds}{dt} = 0$ when the ball is at its maximum height.

$$\begin{aligned}
3 - 9.8t &= 0 \\
9.8t &= 3 \\
t &= \frac{15}{49}s \\
s &= 3\left(\frac{15}{49}\right) - 4.9\left(\frac{15}{49}\right)^2 \\
&= \frac{45}{98} \quad \square
\end{aligned}$$

(b) the time taken, in seconds, for the ball to return to its initial point.

Sol.

$$\begin{aligned}
s &= 0 \\
3t - 4.9t^2 &= 0 \\
t(3 - 4.9t) &= 0 \\
t = 0 \text{ or } t &= \frac{30}{49}s
\end{aligned}$$

Therefore, the time taken for the ball to return to its initial point is $\frac{30}{49}$ seconds.

29. The following diagram shows the cross section of a wedding ring ordered by Azmin. The inner and outer curved surfaces are represented by the quadratic equations of $x = 0.8 - 0.5y^2$ and $x = 1 - y^2$ respectively.

(a) By using the calculus method, find the volume of the wedding ring in terms of πcm^3.

Sol.

$$\begin{aligned}
V_x &= \int_{-0.2}^{0.2} \pi\left[(1 - y^2)^2 - (0.8 - 0.5y^2)^2\right]\,dy \\
&= \pi\int_{-0.2}^{0.2} \left[(1 - 2y^2 + y^4) - (0.64 - 0.8y^2\right. \\
&\quad \left. + 0.25y^4)\right]\,dy \\
&= \pi\int_{-0.2}^{0.2} \left[0.36 - 1.2y^2 + 0.75y^4\right]\,dy \\
&= \pi\int_{-\frac{1}{5}}^{\frac{1}{5}} \left[\frac{9}{25} - \frac{6}{5}y^2 + \frac{3}{4}y^4\right]\,dy \\
&= \pi\left[\frac{9}{25}y - \frac{2}{5}y^3 + \frac{3}{20}y^5\right]_{-\frac{1}{5}}^{\frac{1}{5}} \\
&= \pi\left[\left(\frac{9}{125} - \frac{2}{625} + \frac{3}{62500}\right) - \left(-\frac{9}{125} + \frac{2}{625}\right.\right. \\
&\quad \left.\left. - \frac{3}{62500}\right)\right] \\
&= \pi\left(\frac{4303}{62500} + \frac{4303}{62500}\right) \\
&= 0.1377\pi cm^3 \quad \square
\end{aligned}$$

(b) The ring is made of titanium. If the rate of price of titanium is RM153.49 per gram and the rate of titanium mass per unit volume is $4.51gcm^{-3}$, calculate the price needed to be paid by Azmin. (Use $\pi = 3.142$).

Sol.

$$\begin{aligned}\text{Mass} &= 4.51gcm^{-3} \times 0.1377\pi cm^3 \\ &= 1.9513g \\ \text{Price} &= \text{RM}153.49g^{-1} \times 1.9513g \\ &= \text{RM}299.51 \quad \square\end{aligned}$$

Praktis Summatif

3.1 Kertas 1

1. Given that $\int_m^2 (2x+3)\,dx = -8$ where $m > 0$, find the value of m.
2. Given that $\frac{dy}{dx} = 10(5x+3)^2$ and $y = 4$ when $x = 0$. Express y in terms of x.
3. Given $\int_5^m f(t)\,dx = \frac{7}{3}$, find
   (a) $\int_m^5 3f(t)\,dx = \frac{7}{3}$.
   (b) the value of m, where $\int_5^m [4 - f(t)]\,dx = 7$.
4. Given that $\int \frac{3}{(3x-2)^n}\,dx = a(3x-2)^{1-n} + C$,
   (a) State the impossible value of n.
   (b) Hence, express n in terms of a.
5. Diagram below shows a curve $y = f(x)$.
   Given area of region B is three times the area of region A and $\int_0^b f(x)\,dx = 20$, find the area of region B.

3.2 Kertas 2

1. Differentiate $2x^4\sqrt{4x-3}$ with respect to x. Hence, find $\int \frac{3x^4 - 2x^3}{\sqrt{4x-3}}\,dx$.
2. The number of customers in a restaurant on a certain day changes at a rate of $\frac{dB}{dt} = 70 - 10t$ people per hour. When $t = 2$, the number of customers in the restaurant is 120. Find,
   (a) the number of customers when $t = 10$.
   (b) the maximum number of customers at a certain time on that day. Hence, find the income of the restaurant at that moment if each customer spends an average of RM25.
3. The gradient function of a curve is given by $\frac{dy}{dx} = kx - 6$, where k is a constant. The gradient of normal to the curve at point $(2, -5)$ is $\frac{1}{2}$. Find the equation of the curve.
4. The curve with gradient function $f'(x) = 3x^2 + mx + n$ where m and n are constants, has stationary points at $(1, -3)$ and $(-3, 29)$. Find
   (a) the values of m and n.
   (b) the equation of the curve.
5. Diagram below shows two regions labelled as A and B respectively. Region A is bounded by the curve $y = \left(\frac{x}{a}\right)^3$, the straight line $x = a$ and the x-axis whereas region B is bounded by the curve $y = \left(\frac{a}{x}\right)^3$, the straight lines $x = a$ and $x = b$, and the x-axis.
   (a) Find the area of the region A in terms of a.
   (b) Find the area of the region B in terms of a and b.
   (c) Show that the area of region $A > \frac{1}{2}$*area of region* B for all values of a and b where $0 < a < b$.
6. Diagram below shows the cross-section of an anti-heat bowl which is made of stainless steel. The bowl has two layers in which the space between the two layers is a vacuum which functions as a heat insulator.
   THe inner and the outer layers of the bowl are parabolic in shape which are represented by the equations $y = ax^2 + b$ and $y = \frac{32}{289}x^2$ respectively.
   (a) Find the values of a and b.
   (b) Anis wants to pour 1.5 litres of milk into the bowl. Identify whether the bowl can hold 1.5 litres of milk. Justify your answer.
7. Diagram below shows parts of the curves $y = x^2 + 5x + 4$ and $y = 4 - x^2$.
   Find
   (a) the points of intersection P and S.
   (b) the coordinates of the points Q and R.
   (c) the area of the shaded region.