

Praktis 3 Integration

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Praktis Summatif

3.1 Kertas 1

1. Given that $\int_m^2 (2x + 3) dx = -8$ where $m > 0$, find the value of m .

Sol.

$$\begin{aligned}\int_m^2 (2x + 3) dx &= [x^2 + 3x]_m^2 \\ &= 4 + 6 - m^2 - 3m \\ &= 10 - 3m - m^2\end{aligned}$$

$$10 - 3m - m^2 = -8$$

$$m^2 + 3m - 10 = 8$$

$$m^2 + 3m - 18 = 0$$

$$(m + 6)(m - 3) = 0$$

$$m = 3 \quad (m > 0) \quad \square$$

2. Given that $\frac{dy}{dx} = 10(5x + 3)^2$ and $y = 4$ when $x = 0$. Express y in terms of x .

3. Given $\int_5^m f(t) dx = \frac{7}{3}$, find

(a) $\int_m^5 3f(t) dx = \frac{7}{3}$.

(b) the value of m , where $\int_5^m [4 - f(t)] dx = 7$.

4. Given that $\int \frac{3}{(3x - 2)^n} dx = a(3x - 2)^{1-n} + C$,

(a) State the impossible value of n .

(b) Hence, express n in terms of a .

5. Diagram below shows a curve $y = f(x)$.

Given area of region B is three times the area of region

A and $\int_0^b f(x) dx = 20$, find the area of region B .

3.2 Kertas 2

1. Differentiate $2x^4\sqrt{4x-3}$ with respect to x . Hence, find $\int \frac{3x^4 - 2x^3}{\sqrt{4x-3}} dx$.

Sol.

$$\frac{d}{dx} [2x^4\sqrt{4x-3}] =$$

2. The number of customers in a restaurant on a certain day changes at a rate of $\frac{dB}{dt} = 70 - 10t$ people per hour. When $t = 2$, the number of customers in the restaurant is 120. Find,

(a) the number of customers when $t = 10$.

(b) the maximum number of customers at a certain time on that day. Hence, find the income of the restaurant at that moment if each customer spends an average of RM25.

3. The gradient function of a curve is given by $\frac{dy}{dx} = kx - 6$, where k is a constant. The gradient of normal to the curve at point $(2, -5)$ is $\frac{1}{2}$. Find the equation of the curve.

4. The curve with gradient function $f'(x) = 3x^2 + mx + n$ where m and n are constants, has stationary points at $(1, -3)$ and $(-3, 29)$. Find

(a) the values of m and n .

(b) the equation of the curve.

5. Diagram below shows two regions labelled as A and B respectively. Region A is bounded by the curve $y = \left(\frac{x}{a}\right)^3$, the straight line $x = a$ and the x -axis whereas region B is bounded by the curve $y = \left(\frac{a}{x}\right)^3$, the straight lines $x = a$ and $x = b$, and the x -axis.

(a) Find the area of the region A in terms of a .

(b) Find the area of the region B in terms of a and b .

(c) Show that the area of region $A > \frac{1}{2}$ area of region B for all values of a and b where $0 < a < b$.

6. Diagram below shows the cross-section of an anti-heat bowl which is made of stainless steel. The bowl has two layers in which the space between the two layers is a vacuum which functions as a heat insulator.

The inner and the outer layers of the bowl are parabolic in shape which are represented by the equations $y = ax^2 + b$ and $y = \frac{32}{289}x^2$ respectively.

(a) Find the values of a and b .

(b) Anis wants to pour 1.5 litres of milk into the bowl. Identify whether the bowl can hold 1.5 litres of milk. Justify your answer.

7. Diagram below shows parts of the curves $y = x^2 + 5x + 4$ and $y = 4 - x^2$.

Find

(a) the points of intersection P and S .

(b) the coordinates of the points Q and R .

(c) the area of the shaded region.