# Solution Book of Mathematic

*Ssnior 2 Part I*

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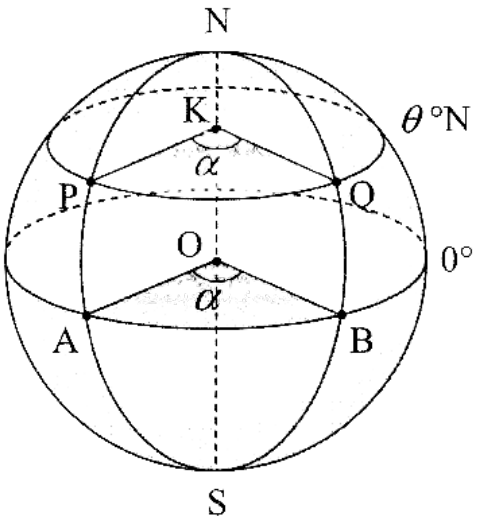
## 16.1 Distance of Two Locations on the Same Parallel of Latitude

The distance between two locations on the same parallel of latitude is the arc length on the parallel of latitude corresponding to the difference of their longitudes.




In the diagram above, $P$ and $Q$ are on the same parallel of latitude $\theta$, their difference of latitude is $\alpha$. $A$ and $B$ are locations on the equator.

Given that $\angle PKQ = \angle AOB = \alpha$. Let $R$ be the radius of the earth, $r$ be the radius of the parallel of latitude.

$$\begin{aligned}\frac{\overset{\frown}{PQ}}{\overset{\frown}{AB}} &= \frac{\frac{\alpha}{360^\circ} \times 2\pi r}{\frac{\alpha}{360^\circ} \times 2\pi R} \\ &= \frac{r}{R}\end{aligned}$$

From the radius of the the parallel of latitude $r = R\cos R$, we have $\frac{r}{R} = \cos\theta$.

$$\begin{aligned}\therefore \frac{\overset{\frown}{PQ}}{\overset{\frown}{AB}} &= \cos\theta \\ \overset{\frown}{PQ} &= \overset{\frown}{AB} \times \cos\theta \\ &= \alpha \times 60 \times \cos\theta NM \text{ or} \\ &= \alpha \times 60 \times \cos\theta \times 1.853km\end{aligned}$$

### 16.1.1 Practice 5

1. Fidn the distance of the following pairs of location on the same parallel of latitude (Express your answer in nautical miles):

   (a) $P(80^\circ N, 105^\circ W), Q(80^\circ N, 48^\circ W)$

   **Sol.**

   $$\begin{aligned}\overset{\frown}{PQ} &= (105 - 48) \times 60 \times \cos 80^\circ \\ &= 57 \times 60 \times \cos 80^\circ \\ &= 593.88NM\end{aligned}$$

   (b) $M(50^\circ S, 48^\circ E), N(50^\circ S, 100^\circ E)$

   **Sol.**

   $$\begin{aligned}\overset{\frown}{MN} &= (100 - 48) \times 60 \times \cos 50^\circ \\ &= 52 \times 60 \times \cos 50^\circ \\ &= 2005.50NM\end{aligned}$$

   (c) $X(40^\circ N, 28^\circ 15' E), Y(40^\circ N, 42^\circ 45' W)$

   **Sol.**

   $$\begin{aligned}\overset{\frown}{XY} &= (28.25 + 42.75) \times 60 \times \cos 40^\circ \\ &= 71 \times 60 \times \cos 40^\circ \\ &= 3263.35NM\end{aligned}$$

   (d) $K(20^\circ S, 160^\circ E), L(20^\circ S, 140^\circ W)$

   **Sol.**

   $$\begin{aligned}\overset{\frown}{KL} &= (360 - 160 - 140) \times 60 \times \cos 20^\circ \\ &= 60 \times 60 \times \cos 20^\circ \\ &= 3382.90NM\end{aligned}$$

2. Given that $A$ is located at the west of $B(46^\circ N, 72^\circ W)$ with a distance of $2350NM$. Find the longitude and latitude of $A$.

   **Sol.**

   $$\begin{aligned}\overset{\frown}{AB} &= \alpha \times 60 \times \cos 46 \\ 2350 &= \alpha \times 60 \times \cos 46 \\ \alpha &= \frac{2350}{60 \times \cos 46} \\ \alpha &= 56^\circ 23' \\ \text{Lon.}A &= (72^\circ + 56^\circ 23')W \\ &= 128^\circ 23' W\end{aligned}$$

   $$\therefore A(46^\circ N, 128^\circ 23' W)$$

### 16.1.2 Exercise 17.6

1. Find the distance of the following pairs of location on the same parallel of latitude (Express your answer in nautical miles):

   (a) $P(45^\circ S, 20^\circ E), Q(45^\circ S, 100^\circ E)$

   **Sol.**

   $$\begin{aligned}\overset{\frown}{PQ} &= (100 - 20) \times 60 \times \cos 45^\circ \\ &= 80 \times 60 \times \cos 45^\circ \\ &= 3394.11NM\end{aligned}$$

(b) $M(36°N, 45°W)$, $N(36°N, 105°W)$

**Sol.**

$$\begin{aligned}\overset{\frown}{MN} &= (105-45)\times 60\times\cos 36°\\ &= 60\times 60\times\cos 36°\\ &= 2192.46NM\end{aligned}$$

(c) $A(80°S, 130°E)$, $B(80°S, 165°E)$

**Sol.**

$$\begin{aligned}\overset{\frown}{AB} &= (165-130)\times 60\times\cos 80°\\ &= 35\times 60\times\cos 80°\\ &= 364.66NM\end{aligned}$$

(d) $K(70°N, 40°E)$, $L(70°N, 20°W)$

**Sol.**

$$\begin{aligned}\overset{\frown}{KL} &= (40+20)\times 60\times\cos 70°\\ &= 60\times 60\times\cos 70°\\ &= 1231.27NM\end{aligned}$$

(e) $T(0°, 128°W)$, $M(0°, 120°E)$

**Sol.**

$$\begin{aligned}\overset{\frown}{TM} &= (360-128-120)\times 60\times\cos 0°\\ &= 112\times 60\times\cos 0°\\ &= 6720NM\end{aligned}$$

2. Based on the following distances of location $P$ and $Q$ and the longitude and latitude of $P$, find the longitude and latitude of $Q$:

(a) $PQ = 800NM$, $Q$ is located at the west of $P(50°S, 100°W)$

**Sol.**

$$\begin{aligned}\overset{\frown}{PQ} &= \alpha\times 60\times\cos 50°\\ 800 &= \alpha\times 60\times\cos 50°\\ \alpha &= \frac{800}{60\times\cos 50°}\\ \alpha &= 20°45'\\ \text{Lon.}Q &= (100°-20°45')W\\ &= 120°45'W\end{aligned}$$

$$\therefore Q(50°S, 120°45'W)$$

(b) $PQ = 3400NM$, $Q$ is located at the east of $P(35°N, 68°E)$

**Sol.**

$$\begin{aligned}\overset{\frown}{PQ} &= \alpha\times 60\times\cos 35°\\ 3400 &= \alpha\times 60\times\cos 35°\\ \alpha &= \frac{3400}{60\times\cos 35°}\\ \alpha &= 69°11'\\ \text{Lon.}Q &= (68°+69°11')E\\ &= 137°11'E\end{aligned}$$

$$\therefore Q(35°N, 137°11'E)$$

(c) $PQ = 1450km$, $Q$ is located at the east of $P(42°N, 15°W)$

**Sol.**

$$\begin{aligned}\overset{\frown}{PQ} &= \alpha\times 60\times\cos 42°\\ \frac{1450}{1.853} &= \alpha\times 60\times\cos 42°\\ \alpha &= \frac{1450}{1.853\times 60\times\cos 42°}\\ \alpha &= 17°33'\\ \text{Lon.}Q &= |15°-17°33'|E\\ &= 2°33'E\end{aligned}$$

$$\therefore Q(42°N, 2°33'E)$$

3. Given that two places are on the parallel of latitude 60° north to the equator, and their difference of longitude is 160°. Find the distance of the two places. (Express your answer in kilometers)

**Sol.**

Let the two places are $A$ and $B$.

$$\begin{aligned}\overset{\frown}{AB} &= 160\times 60\times\cos 60°\\ &= 4800NM\\ &= 4800\times 1.853km\\ &= 8894.4km\end{aligned}$$

4. City $A$ and $B$ are on the parallel of latitude $5°30'$ north to the equator, their longitude are $100°15'E$ and $103°E$ respectively. Find the distance between two cities along the parallel of latitude.

**Sol.**

$$\begin{aligned}\overset{\frown}{AB} &= (103°-100°15')\times 60\times\cos 5°30'\\ &= 2°45'\times 60\times\cos 5°30'\\ &= 164.24NM\end{aligned}$$

5. Find the circumference of the parallel of latitude $35°30'S$.

**Sol.**

$$\begin{aligned} C &= 360 \times 60 \times \cos 35°30' \\ &= 17584.90NM \end{aligned}$$

6. Find the radius of the parallel of latitude $60° N$.

   **Sol.**

$$\begin{aligned} r &= \frac{360 \times 60 \times \cos 60°}{2\pi} \\ &= 1718.87NM \\ &= 1718.87 \times 1.853km \\ &= 3185.10km \end{aligned}$$

7. A ship set sail from $P(20° E)$ and sail $600NM$ due east along $42° N$ parallel of latitude. Find the longitude and latitude of the destination.

8. A ship sails from port $P(48° N, 12° W)$ $1000NM$ due west to another port $Q$, find the longitude and latitude of $Q$.

9. Given that $A$ is located at the east of Paris$(49° N, 2°30' E)$ with a distance of $2200km$. Find the longitude and latitude of $A$.

10. A plane flies from $X(40° N, 2°30' E)$ $9265km$ due east to $Y$, find the longitude and latitude of $Y$.

11. Given that the earth takes $24hrs$ to rotate once. Find the speed of Kuala Lumpur$(3°15' N, 102° E)$ to rotate once. (Express your answer in $NM/hr$)

12. Given that the longitude of $P$ and $Q$ are $50°$ and $100c°$ respectively. If $P$ and $Q$ both located at the west of $R(55° S)$ and $PR = PQ$, find:

    (a) The longitude of $R$.

    (b) Th distance between $Q$ and $R$ along the parallel of latitude.

13. A plane flies from $F(50° S, 50° E)$ due west to $H(50° S, 45° W)$, then flies from $H$ due north $4800NM$ to $K$. Given that the average speed of the plane is $480NM/hr$ throughout the journey, find:

    (a) The latitude of $K$.

    (b) The distance between $F$ and $H$ along the parallel of latitude.

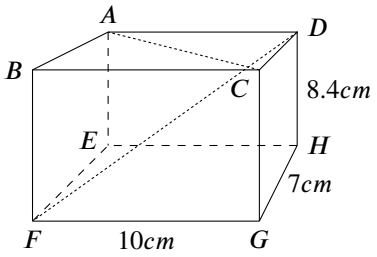
    (c) The flight duration for the whole journey.

## 16.2 Revision Exercise 17

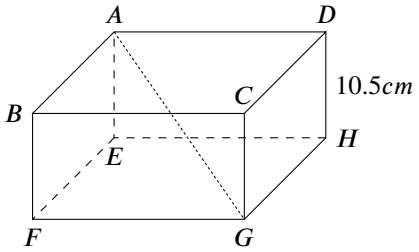
1. In the cuboid shown below, $FG = 10cm$, $GH = 7cm$, $DH = 8.4cm$, find:

   (a) The angle formed by angle $AC$ and plane $BFGC$.

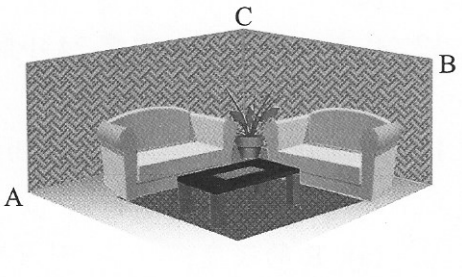
   (b) The angle formed by angle $FD$ and plane $EFGH$.



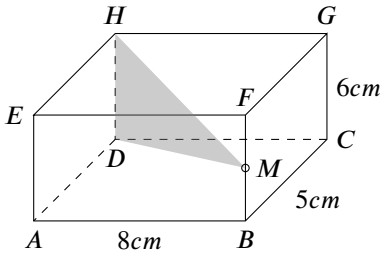

2. The diagram below shows a cuboid with volume of $400cm^3$, height of $10.5cm$, $AD = 2DC$. Find the angle formed by angle $AG$ and plane $ADHE$.




3. The diagram below shows a reception room with a square floor with side length of $6m$. Given that the elevation angle of corner $C$ measured from corner $A$ is $30°$, find the angle formed by the line connecting corner $A$ and $B$ with the floor.



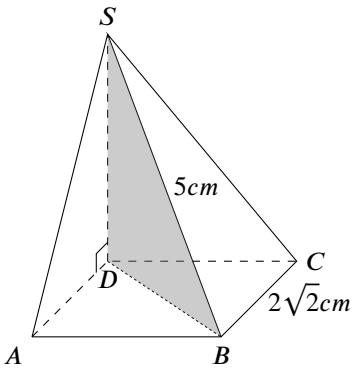

4. The diagram below shows a cuboid with length of $8cm$, width of $5cm$ and height of $6cm$, $M$ is the midpoint of $BF$. Find the angle formed by plane $HDM$ and plane $ADHE$.



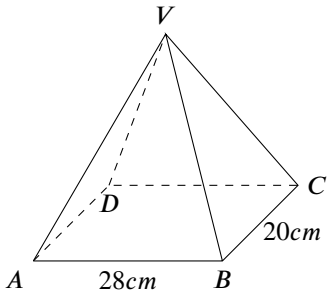

5. The diagram below shows a pyramid with a square base, its lateral edge $SD$ is perpendicular to its base. Given that $BC = 2\sqrt{2}cm$, $SB = 5cm$. Find:

   (a) The angle formed by plane $SAD$ and plane $SBD$.

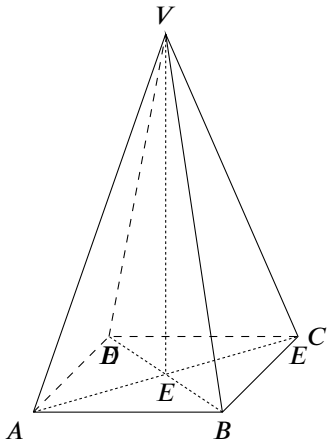
   (b) The angle formed by lateral edge $SA$ and base $ABCD$.




6. The diagram below shows a right prism with a rectangular base $ABCD$ with length of $28cm$ and width of $20cm$. Assume that plane $VBC$ and the base of the pyramid forms a 60° angle. Find the angle formed by plane $VAB$ and the base.




7. The diagram below shows a regular cuboid with a square base. Given that $VE = \frac{5}{2}AD$. Find:
   (a) The angle formed by the angle $VA$ and the base $ABCD$.
   (b) The angle formed by plane $VAD$ and the base.




8. Find the distance from the Panama City$(9°N, 79°30'W)$ to Toronto $(43°45'N, 79°30'W)$. (Express your answer in nautical miles)

9. Tokyo and Adelaide are located at the same longitude, their latitude are $35°45'N$ and $35°S$ respectively. Find the distance between two cities along the parallel of latitude.

10. A plane flies $2000NM$ along the equator, Find the difference of longitude between the point of departure and the destination.

11. Location $M$ and $N$ are both located at the parallel of latitude 45° north to the equator with a difference in longitude of 20°. Find the distance between $M$ and $N$ along the parallel of latitude. (Express your answer in nautical miles)

12. Location $X$ and $Y$ are on the parallel of latitude 20° north to the equator, their longitude are $45°E$ and $80°E$ respectively. Find the distance between location $X$ and $Y$ along the parallel of latitude. (Express your answer in nautical miles)

13. A plane flies from $A(42°E)$ to $B(20°E)$ along the equator, then it flies from $B$ due north to $C(30°N)$. Find the distance the plane flies in total.

14. Assume that $A$ is located $1000NM$ due north of the equator, $600NM$ due east of the Greenwich Meridian, find the longitude and latitude of $A$.

15. A plane flies from $P(15°N, 30°E)$ $2000NM$ due south to $B$, find the longitude and latitude of $B$. Another plane flies from $P$ $3000NM$ due east to $C$, find the longitude and latitude of $C$.

16. A plane flies from $A(130°E)$ along the equator to $B(120°30'E)$ along the equator, then flies from $B$ due north to $C(20°45')$. Assume that the average speed of the plane is $300NM/hr$ throughout the journey, find the flight duration for the whole journey.

17. A plane flies from $A(50°N, 10°E)$ due east to $B(45°E)$.
   (a) Find the flight distance of the plane. (Express your answer in nautical miles)
   (b) Assume that the speed of the plane is $420NM/hr$ in average, find the flight duration of the plane.

18. Given that three locations $P$, $Q$ and $R$ are located on the same parallel of latitude 40° north to the equator, The longitude of $P$ and $R$ are $10°30'W$ and $4°30'E$, $Q$ is located at the middle of $P$ and $R$.
   (a) Find the difference of longitude between $P$ and $R$.
   (b) Find the longitude of $Q$.
   (c) Find the distance between $P$ and $R$ along the parallel of latitude.
   (d) A ship sails from $P$ to $Q$ along the parallel of latitude with a speed of $18NM/hr$, find the sailing duration of the ship.