Praktis 4
Permutation and Combination

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Praktis Formatif

4.1 Permutation

1. There are 3 types of fruits and 5 types of cupcakes in the kitchen. Determine the number of selections made by Aiman if

   (a) he can only choose one of the item,

   Sol.

   Using the addition rule, the number of selections made by Aiman is $3 + 5 = 8$. $\square$

   (b) he can choose one type of fruits and one type of cupcakes.

   Sol.

   Using the multiplication rule, the number of selections made by Aiman is $3 \times 5 = 15$. $\square$

2. During the school mid-year holidays, Zoe wants to do revision for her 3 Science subjects, 2 Mathematics subjects, 3 language subjects and 2 other core subjects. Find the number of ways the revision can be done if

   (a) Zoe revises one subject only,

   Sol.

   Using the addition rule, the number of ways the revision can be done is $3 + 2 + 3 + 2 = 10$. $\square$

   (b) Zoe wants to revise one science subject, one mathematics subject and one core subject in a day.

   Sol.

   Using the multiplication rule, there are $3 \times 2 \times 2 = 12$ ways to do the revision. $\square$

3. Ali plans to visit Zaleha who stays in Sarawak. Ali can choose to ride in either 3 of his friends's car or purchase a ticket from any of the 2 bus companies to Kuala Lumpur International Airport. From there, Ali can choose from any 3 flight companies to Sarawak. In how many ways can Ali go to Sarawak?

   Sol.

   To go to Kuala Lumpur International Airport, Ali can choose to ride in 4 of his friends's car or purchase a ticket from any of the 3 bus companies. Using the addition rule, there ara $3 + 2 = 5$ ways to go to Kuala Lumpur International Airport.

   To go to Sarawak, Ali can choose from any 3 flight companies. Using the multiplication rule, there are $5 \times 3 = 15$ ways to go to Sarawak. $\square$

4. During School's Entrepreneurship Carnival, Sandy is interested to participate in the Treasure Hunt game. The participants need to get two clues from Station A and Station B. There are 3 paths to reach station A. From Station A, there are 4 paths to Station B. All paths are of different distances. After obtaining two clues, every participant needs to return to the starting point to get his last clue. Determine the number of ways to travel to and from if Sandy

   (a) choose the same path,

   Sol.

   Since Sandy choose the same path to and from Station A and Station B, using the multiplication rule, the number of ways to travel to and from is $3 \times 4 = 12$. $\square$

   (b) does not use the same path.

   Sol.

   Since Sandy does not use the same path to and from Station A and Station B,

   From starting point to Station A, there are 3 paths.

   From Station A to Station B, there are 4 paths.

   From Station B back to Station A, there are 3 paths.

   From Station A back to starting point, there are 2 paths.

   Using the multiplication rule, there are $3 \times 4 \times 3 \times 2 = 72$ ways to travel to and from. $\square$

5. Chong wants to set a passcode to her safe deposit box. The passcode consists of a 4-digit number. Find the number of passcodes that can be formed by Chong.

   Sol.

   There are 10 digits. Since each digit can be used more than once, there are $10^4 = 10\,000$ passcodes that can be formed by Chong. $\square$

6. Solve each of the following situations.

   (a) Find the number of ways to rearrange all the letters in the word CUTE.

   Sol.

   There are 4 letters. There are $4! = 24$ ways to rearrange all the letters. $\square$

   (b) How many 4-digit numbers that are different that can be formed by using the digits 1, 2, 3, and 4 without repetition?

   Sol.

   There are 4 digits. Since each digit can be used only once, there are $4! = 24$ 4-digit numbers that are different that can be formed by using these digits. $\square$

   (c) How many ways can 5 people queue up to receive vaccinations in hospital?

   Sol.

   There are $5! = 120$ ways to queue up to receive vaccinations. $\square$

7. Find the number of arrangements in each of the following situations.

(a) Determine the number of ways to arrange 10 VIP guests to sit at a round table.

Sol.

There are $(10-1)! = 9! = 362\,880$ ways to arrange 10 VIP guests to sit at a round table. $\square$

(b) Find the number of ways fro 8 scouts to stand around a campfire.

Sol.

There are $(8-1)! = 7! = 5\,040$ ways to arrange 8 scouts to stand around a campfire. $\square$

(c) Find the number of ways to distribute 10 types of fruits to 10 students sitting at a round table.

Sol.

There are $(10-1)! = 9! = 362\,880$ ways to distribute 10 types of fruits to 10 students sitting at a round table. $\square$

8. Determine the number of arrangements for each of the following situations.

(a) How many ways to form a bracelet with 12 different pearls?

Sol.

Since the bracelet can be flipped, the clockwise and anticlockwise arrangements are the same.

Hence, there are $\frac{(12-1)!}{2} = \frac{11!}{2} = 19\,958\,400$ ways to form a bracelet. $\square$

(b) Find the number of ways to arrange 10 types of flowers to form a floral hoop.

Sol.

Since the floral hoop can be flipped, the clockwise and anticlockwise arrangements are the same.

Hence, there are $\frac{(10-1)!}{2} = \frac{9!}{2} = 181\,440$ ways to arrange 10 types of flowers to form a floral hoop. $\square$

9. Find the value of n for each of the following.

(a) ${}_nP_2 = 6$

Sol.

$$\begin{aligned}
{}_nP_2 &= 6\\
n(n-1) &= 6\\
n^2 - n &= 6\\
n^2 - n - 6 &= 0\\
(n-3)(n+2) &= 0\\
n &= 3 \quad (n>0) \quad \square
\end{aligned}$$

(b) ${}_{n+2}P_3 = 42n$

Sol.

$$\begin{aligned}
{}_{n+2}P_3 &= 42n\\
(n+2)(n+1)n &= 42n\\
n^2 + 3n + 2 &= 42\\
n^2 - 3n - 40 &= 0\\
(n-8)(n+5) &= 0\\
n &= 8 \quad (n>0) \quad \square
\end{aligned}$$

(c) $7({}_{n+1}P_2) = 5({}_{n+2}P_2)$

Sol.

$$\begin{aligned}
7({}_{n+1}P_2) &= 5({}_{n+2}P_2)\\
\frac{{}_{n+1}P_2}{{}_{n+2}P_2} &= \frac{5}{7}\\
\frac{(n+1)(n)}{(n+2)(n+1)} &= \frac{5}{7}\\
\frac{n}{n+2} &= \frac{5}{7}\\
7n &= 5n + 10\\
2n &= 10\\
n &= 5
\end{aligned}$$

(d) ${}_{2n}P_2 = 3({}_{n+1}P_2)$

Sol.

$$\begin{aligned}
{}_{2n}P_2 &= 3({}_{n+1}P_2)\\
\frac{{}_{2n}P_2}{{}_{n+1}P_2} &= 3\\
\frac{(2n)(2n-1)}{(n+1)n} &= 3\\
\frac{4n^2 - 2n}{n^2 + n} &= 3\\
4n^2 - 2n &= 3n^2 + 3n\\
n^2 - 5n &= 0\\
n(n-5) &= 0\\
n &= 5 \quad (n>0)
\end{aligned}$$

10. Determine the number of arrangements of the following situations.

(a) How many 3-digit numbers that are different that can be formed from digits 1, 2, 3, 4, and 5 without repetition?

Sol.

There are ${}_5P_3 = 60$ ways to arrange 4 digits from 5 digits. $\square$

(b) Find the number of ways to arrange 7 students in 4 chairs.

Sol.

There are ${}_7P_4 = 840$ ways to arrange 7 students in 4 chairs. $\square$

(c) How many ways can 4 cars park in 8 empty parking lots along the street as shown in the following diagram?

Sol.

There are ${}_8P_4 = 1\,680$ ways to arrange 4 cars in 8 empty parking lots. □

(d) There are 12 participants in a 100-m run. Find the number of possible results obtained if presents are only given to the champion, 1^{st} runner-up and 2^{nd} runner-up only.

Sol.

There are ${}_{12}P_3 = 1\,320$ results of having 3 winners in 12 participants. □

11. Determine the number of arrangements of r out of n different objects in a circle.

(a) Find the number of ways to arrange 10 students to sit in 6 chairs at a round table.

Sol.

There are $\dfrac{{}_{10}P_6}{6} = 25\,200$ ways to arrange 10 students to sit in 6 chairs at a round table. □

(b) If each of the children is given only a new school ag, find the number of ways to distribute 9 school bags to 5 children sitting in a circle.

Sol.

There are $\dfrac{{}_9P_5}{5} = 3\,024$ ways to distribute 9 school bags to 5 children sitting in a circle. □

(c) Determine the number of ways 5 people can sit in 7 empty chairs at a round table.

Sol.

There are $\dfrac{{}_7P_5}{7} = 360$ ways to arrange 5 people to sit

12. Solve each of the following questions.

(a) The following diagram shows a hula hoop that can be dismantled into 6 parts.The user can fix the hoop again by choosing the combination of colours desired.

How many ways can the hula hoop be fixed if Siti has 8 parts of different colours?

Sol.

Since the hoop can be flipped, the clockwise and anticlockwise arrangements are the same.

Hence, there are $\dfrac{{}_8P_6}{2 \cdot 6} = 1\,680$ ways to fix the hula hoop. □

(b) Find the number of ways to make a bracelet which contains 8 pearls chosen from 16 different pearls.

Sol.

Since the bracelet can be flipped, the clockwise and anticlockwise arrangements are the same.

Hence, there are $\dfrac{{}_{16}P_8}{2 \cdot 8} = 32\,432\,400$ ways to make a bracelet. □

13. Solve the following permutation questions involving identical objects.

(a) Find the number of ways to rearrange the letters from the word

i. LOOKOUT

Sol.

There are 3 Os.

Hence, there are $\dfrac{7!}{3!} = 840$ ways to rearrange the word. □

ii. MATHEMATICS

Sol.

There are 2Ms, 2T, and 2As.

Hence, there are $\dfrac{12!}{2!2!2!} = 4\,989\,600$ ways to rearrange the word. □

iii. MISSISSIPPI

Sol.

There are 4Is, 4Ss, and 2Ps.

Hence, there are $\dfrac{11!}{4!4!2!} = 34\,650$ ways to rearrange the word. □

(b) Find the number of ways to form a 5-digit number from cards that are labelled with 1, 2, 2, 2, 3, 4, 5, 6, 7, 8, and 8 if all the digit 2 must be used.

Sol.

Since all the digit 2 must be used, the 3 digit 2 cards can be treated as 1 card.

There are 2 card of 8.

If the number has no 8, there are ${}_6C_2 = 15$ ways to choose the 2 other digits.

Since 3 digit 2 cards and the 2 other digits can be arranged in any order, there are $\frac{5!}{3!} = 20$ ways to arrange them.

Hence, there are $15 \times 20 = 300$ ways to form a 5-digit number with no 8.

If the number has 1 8, there are ${}_6C_1 = 6$ ways to choose the 1 other digits.

Since 3 digit 2 cards and the 2 other digits can be arranged in any order, there are $\frac{5!}{3!} = 20$ ways to arrange them.

Hence, there are $6 \cdot 20 = 120$ ways to form a 5-digit number with 1 8.

If the number has 2 8s, there are only 1 way to choose the 2 other digits.

Since 3 digit 2 cards and the 2 digit 8 cards can be arranged in any order, there are $\frac{5!}{3!2!} = 10$ ways to arrange them.

Hence, there are $1 \cdot 10 = 10$ ways to form a 5-digit number with 2 8s.

Therefore, there are $300 + 120 + 10 = 430$ ways to form a 5-digit number from the 11 cards. □

14. Find the number of ways to rearrange all the letters in the word ENGLISH if

(a) vowels must be placed at both ends,

Sol.

There are 2 vowels.

There are $2! = 2$ ways to arrange the 2 vowels at both ends.

There are $5! = 120$ ways to arrange the 5 consonants in the middle.

Hence, there are $2 \times 120 = 240$ ways to rearrange all the letters. $\square$

(b) it must start with a consonant,

Sol.

There are 5 ways to choose the first letter.

There are $6! = 720$ ways to arrange the remaining letters.

Hence, there are $5 \times 720 = 3\,600$ ways to arrange them. $\square$

(c) vowels must be side by side,

Sol.

Since the vowels must be side by side, the vowels can be treated as 1 letter.

There are $6! = 720$ ways to arrange the 6 letters.

Since the vowels can be arranged in any order, there are $2! = 2$ ways to arrange them.

Hence, there are $720 \times 2 = 1\,440$ ways to arrange them. $\square$

(d) only 5 letters are arranged with the vowels placed side by side.

Sol.

Since the vowels must be side by side, the vowels can be treated as 1 letter.

Since the vowels can be arranged in any order, there are $2! = 2$ ways to arrange them.

There are ${}_5C_3 = 10$ ways to choose the other 3 letters.

There are $4! = 24$ ways to arrange the 4 letters (3 consonants and 2 vowel as 1 letter).

Hence, there are $2 \times 10 \times 24 = 480$ ways to arrange them. $\square$

(e) only 5 letters are arranged with N and G must be included.

Sol.

There are ${}_5C_3 = 10$ ways to choose the other 3 letters.

There are $5! = 120$ ways to arrange the 5 letters.

Hence, there are $10 \times 120 = 1\,200$ ways to arrange them. $\square$

15. Find the number of ways to arrange 7 family members at a round table if

(a) there are 5 vacant chairs and both parents must sit next to each other,

Sol.

Since both parents must sit next to each other, the parents can be treated as 1 person.

Since the parent can be arranged in any order, there are $2! = 2$ ways to arrange them.

There are ${}_5P_3 = 60$ ways to arrange the 3 other family members relative to the parent.

Hence, there are $2 \times 60 = 120$ ways to arrange them. $\square$

(b) there are 7 vacant chairs and both parents must sit next to each other,

Sol.

Since both parents must sit next to each other, the parents can be treated as 1 person.

Since the parent can be arranged in any order, there are $2! = 2$ ways to arrange them.

There are $(6 - 1)! = 120$ ways to arrange the 6 family members.

Hence, there are $2 \times 120 = 240$ ways to arrange them. $\square$

(c) there are 10 vacant chairs and both parents must sit next to each other.

Sol.

Since both parents must sit next to each other, the parents can be treated as 1 person.

Since the parent can be arranged in any order, there are $2! = 2$ ways to arrange them.

There are ${}_8P_5 = 6\,720$ ways to arrange the 5 other family members in the remaining 8 chairs relative to the parent.

Hence, there are $2 \times 6\,720 = 13\,440$ ways to arrange them. $\square$

16. How many ways can 4 male students and 2 female students be seated in a row if

(a) they can sit anywhere,

Sol.

There are $6! = 720$ ways to arrange the 6 students. $\square$

(b) 2 female students must sit together,

Sol.

Since two female students must sit together, they can be treated as one student.

Since the two female students can be arranged in any order, there are $2! = 2$ ways to arrange them.

There are $5! = 120$ ways to arrange 5 students.

Hence, there are $2 \times 120 = 240$ ways to arrange them. $\square$

(c) 2 female students must be separated.

Sol.

There are $720 - 240 = 480$ ways to arrange them. $\square$

17. Find the number of ways to arrange 7 different story books and 3 different magazines on a bookshelf if

(a) no condition is imposed,

Sol.

There are $10! = 3\,628\,800$ ways to arrange them. $\square$

(b) the magazines must be put together,

Sol.

Since the magazines must be put together, they can be treated as one book.

Since the magazines can be arranged in any order, there are $3! = 6$ ways to arrange them.

There are $8! = 40\,320$ ways to arrange the 8 books.

Hence, there are $6 \times 40\,320 = 241\,920$ ways to arrange them. $\square$

(c) the magazines cannot be put together.

Sol.

First, arrange the 7 story books in $7! = 5\,040$ ways.

Then, arrange the 3 magazines in 8 slots in between the story books, the beginning and the end of the bookshelf in ${}_8P_3 = 336$ ways.

Hence, there are $5\,040 \times 336 = 1\,693\,440$ ways to arrange them. $\square$

18. GIven cards that are labelled with the digits 0, 3, 4, 5, 6, and 7. Find the number of arrangements of those digits without repetition to form

(a) 4-digit odd numbers,

Sol.

Since the first digit cannot be 0, there are 5 digits to choose from.

Since the number is odd, there last digit can be either 3, 5, or 7. There are 3 digits to choose from.

If the first digit is odd, there are 2 digits to choose from.

If the first digit is even, there are 3 digits to choose from.

Hence, there are $3\times2+2\times3 = 12$ ways to arrange the first digit and the last digit.

There are ${}_4P_2 = 12$ ways to arrange the remaining 2 digits.

Hence, there are $12 \times 12 = 144$ ways to arrange them. $\square$

(b) 4-digit numbers that begin with an even digit,

Sol.

The first digit can be either 4 or 6. There are 2 digits to choose from.

There are ${}_5P_3 = 60$ ways to arrange the remaining 3 digits.

Hence, there are $2 \times 60 = 120$ ways to arrange them. $\square$

(c) 4-digit numbers with all the odd digits together,

Sol.

There are 3 odd digits. There are $3! = 6$ ways to arrange them. If the odd digits are at the beginning, the last digit can be chosen in 3 ways. If the odd digits are at the end, the first digit can be chosen in 2 ways.

Hence, there are $6\times3+6\times2 = 30$ ways to arrange them. $\square$

(d) 4-digit numbers with odd and even digits at the alternate positions,

Sol.

If the digits are arranged in the order 'odd, even, odd, even', there are ${}_3P_2 \times {}_3P_2 = 36$ ways to arrange them.

If the digits are arranged in the order 'even, odd, even, odd', there are ${}_2P_1 \times {}_2P_1 \times {}_3P_2 = 24$ ways to arrange them.

Hence, there are $36 + 24 = 60$ ways to arrange them. $\square$

(e) even numbers that are greater than $50\,000$.

Sol.

If the number is a 5-digit number:

Since the number is greater than $50\,000$, the first digit must be 5, 6, or 7. There are 3 digits to choose from.

Since the number is even, the last digit can be either 0, 4, or 6.

If the first digit is 6, there are 2 digits to choose from.

If the first digit is 5 or 7, there are 3 digits to choose from.

Hence, there are $2\times1+2\times3 = 8$ ways to arrange the first digit and the last digit.

There are ${}_4P_3 = 24$ ways to arrange the remaining 3 digits.

Hence, there are $8 \times 24 = 192$ ways to arrange them.

If the number is a 6-digit number:

Since the beginning of the number cannot be 0, there are 5 digits to choose from.

Since the number is even, the last digit can be either 0, 4, or 6.

If the first digit is 4, or 6, there are 2 digits to choose from.

If the first digit is 3, 5 or 7, there are 3 digits to choose from.

Hence, there are $2\times2+3\times3 = 13$ ways to arrange the first digit and the last digit.

There are $4! = 24$ ways to arrange the remaining 4 digits.

Hence, there are $13 \times 24 = 312$ ways to arrange them.

Therefore, there are $192 + 312 = 504$ ways to arrange them. $\square$

19. In every football match, the result may be win, lose or draw. Determine the number of possible outcomes obtained in a round of match that involves 12 teams (6 matches).

Sol.

There are 6 matches, each matches can yield 3 outcomes. Hence, there are $3^6 = 729$ possible outcomes. $\square$

20. The following diagram shows the seating arrangement in the meeting room of company X.

Determine the number of ways 9 workers can be seated during the meeting if

(a) no condition is imposed,

Sol.

There are 9 seats and 9 workers. Hence, there are $9! = 362\,880$ ways to arrange them. $\square$

(b) 3 particular workers must sit in the same row.

Sol.

There are $6! = 720$ ways to arrange the remaining 6 workers.

If the 3 workers seat at the top row, there are ${}_5P_3 = 60$ ways to arrange them.

If the 3 workers seat at the bottom row, there are ${}_4P_3 = 24$ ways to arrange them.

Hence there are $60 + 24 = 84$ ways to arrange them.

Therefore, there are $720 \times 84 = 60\,480$ ways to arrange them. $\square$

21. 8 teachers travel in 2 cars to attend a course. Only 5 teachers have driving licenses. Calculate the number of seating arrangements of the teachers in the 2 cars if each car can accommodate only 4 people.

Sol.

There are ${}_5P_2 = 20$ ways to arrange the drivers for the 2 cars.

There are $6! = 720$ ways to arrange the remaining 6 teachers in the 2 cars.

Hence, there are $20 \times 720 = 14\,400$ ways to arrange them. $\square$

22. Find the numbers of different arrangements using all the letters in the word COMMITMENT. Hence, determine the number of arrangements which

Sol.

There are 2Ts and 3Ms.

Hence, there are $\frac{12!}{2!3!} = 302\,400$ ways to arrange them. $\square$

(a) begin and end with the letter T,

Sol.

2 Ts are placed at the beginning and end of the word. There are only 1 way to arrange the Ts.

There are 3Ms.

There are $\frac{8!}{3!} = 6\,720$ ways to arrange the 8 letters except the 2 Ts.

Hence, there are $6\,720 \times 1 = 6\,720$ ways to arrange them. $\square$

(b) contain MMM,

Sol.

Since the Ms are placed side by side, we can treat them as a single letter.

There are 2 Ts.

Hence, there are $\frac{9!}{3!} = 60\,480$ ways to arrange them. $\square$

(c) do not contain TT.

Sol.

If the word contains TT, we treat it as a single letter.

There are 3Ms.

Hence, there are $302\,400 - 60\,480 = 241\,920$ ways to arrange them. $\square$

23. Determine the number of ways 4 doctors and 4 nurses can be seated at a round table if

(a) no condition is imposed,

Sol.

There are 8 seats and 8 people. Hence, there are $(8-1)! = 5\,040$ ways to arrange them. $\square$

(b) 2 nurses should not sit side by side,

Sol.

If the two nurses sit side by side, we can treat them as a single person.

Since the two nurses can switch their seats, there are 2 ways to arrange them.

There are 7 seats and 7 people. Hence, there are $(7-1)! = 720$ ways to arrange them.

Hence, there are $720 \times 2 = 1440$ ways to arrange them.

Therefore, there are $5\,040 - 1\,440 = 3\,600$ ways to arrange them such that the 2 nurses do not sit side by side. $\square$

(c) the doctors and nurses sit in alternate positions.

Sol.

First, arrange the doctors. There are $(4-1)! = 6$ ways to arrange them.

Then, arrange the nurses in between the doctors. There are $4! = 24$ ways to arrange them.

Hence, there are $6 \times 24 = 144$ ways to arrange them. $\square$

4.2 Combination

24. (a) Find the number of ways to choose 2 out of 5 story books on the bookshelf.

Sol.

There are ${}_5C_2 = 10$ ways to choose 2 books. $\square$

(b) Determine the number of ways to select 4 representatives from 10 students to participate in the national debate competition.

Sol.

There are ${}_{10}C_4 = 210$ ways to choose them. $\square$

25. Calculate the number of ways to form groups in which 9 students are divided into

(a) 2 groups of 4 and 5 students respectively,

Sol.

Choose 4 students from 9 students. There are ${}_9C_4 = 126$ ways to choose them.

Choose 5 students from 5 students. There are ${}_5C_5 = 1$ way to choose

Hence, there are $126 \times 1 = 126$ ways to arrange them. □

(b) 3 groups of 2, 3, and 4 students respectively,

Sol.

Choose 2 students from 9 students. There are ${}_9C_2 = 36$ ways to choose them.

Choose 3 students from 7 students. There are ${}_7C_3 = 35$ ways to choose them.

Choose 4 students from 4 students. There are ${}_4C_4 = 1$ way to choose them.

Hence, there are $36 \times 35 \times 1 = 1\,260$ ways to arrange them. □

(c) 2 groups in which the difference between the group members must be at least 3 people.

Sol.

Two group of 1 and 8 students respectively, there are ${}_9C_1 \cdot {}_8C_8 = 9$ ways to arrange them.

Two group of 2 and 7 students respectively, there are ${}_9C_2 \cdot {}_7C_7 = 36$ ways to arrange them.

Two group of 3 and 6 students respectively, there are ${}_9C_3 \cdot {}_6C_6 = 84$ ways to arrange them.

Hence, there are $9 + 36 + 84 = 129$ ways to arrange them. □

26. Four letters are selected from the word HITUNG. How many different selections that are possible? From the selections, determine the number of selections that

Sol.

There are ${}_6C_4 = 15$ ways to choose 4 letters from 6 letters.

(a) do not contain the vowel U,

Sol.

There are 5 letters without U. Hence, there are ${}_5C_4 = 5$ ways to choose them. □

(b) contains the vowel U.

Sol.

There are ${}_5C_3 = 10$ ways to choose the other 3 letters. □

27. The following diagram shows 8 cards of 1-digit number.

Find the number of selections of one card if

(a) the chosen number is a multiple of 2,

Sol.

There are 4 cards of even digit. Hence, there are ${}_4C_1 = 4$ ways to choose them. □

(b) a prime number is chosen,

Sol.

There are 4 cards of prime digit. Hence, there are ${}_4C_1 = 4$ ways to choose them. □

(c) the number that is less than 6 is chosen.

Sol.

There are 4 cards of digit less than 6. Hence, there are ${}_4C_1 = 4$ ways to choose them. □

28. A team of 4 members is selected from 4 men and 6 women. Find the number of ways the team can be formed if

(a) no condition is imposed,

Sol.

There are ${}_{10}C_4 = 210$ ways to form the team. □

(b) the team consists of 1 man and 3 women,

Sol.

Choose 1 man from 4 men, there are ${}_4C_1 = 4$ ways to do so.

Choose 3 women from 6 women, there are ${}_6C_3 = 20$ ways to do so.

Hence, there are $4 \times 20 = 80$ ways to form the team. □

(c) the number of men in the team is at least 2 people.

Sol.

According to the condition imposed, the team with all women or only one man is not allowed.

The team with all women, there are ${}_4C_0 \times {}_6C_4 = 15$ way to form the team.

The team with only one man, there are ${}_4C_1 \times {}_6C_3 = 80$ way to form the team.

Hence, there are $15 + 80 = 95$ ways to form the team with less than two men.

Therefore, there are $210 - 95 = 115$ ways to form the team in which at least 2 men are chosen. □

29. The following diagram shows points that can be connected to form a geometrical shape.

Find the possible number of ways to form

(a) a triangle,

Sol.

The triangle can be either upright or upside down.

For upright triangle, choose one point from the top row and two points from the bottom row. There are ${}_4C_1 \times {}_5C_2 = 40$ ways to do so.

For upside down triangle, choose one point from the bottom row and two points from the top row. There are ${}_5C_1 \times {}_4C_2 = 30$ ways to do so.

Hence, there are $40 + 30 = 70$ ways to form a triangle. □

(b) a quadrilateral,

Sol.

Choose 2 points from the top row and 2 points from the bottom row to form a quadrilateral. There are ${}_4C_2 \times {}_5C_2 = 60$ ways to do so. □

(c) a triangle in which point A or point C but not both and point F must be used.

Sol.

The first vertex of the triangle is point F.

The second vertex can be either point A or point C. There are 2 ways to choose it.

The third vertex can be any other point. There are ${}_6C_1 = 6$ ways to choose it.

Hence, there are $2 \times 6 = 12$ ways to form a triangle in which point A or point C but not both and point F must be used. $\square$

30. A tennis team of 4 men and 4 women is to be selected from 6 men and 7 women.

(a) Find the number of selections to form the team.

Sol.

Choose 4 men from 6 men. There are ${}_6C_4 = 15$ ways to choose them.

Choose 4 women from 7 women. There are ${}_7C_4 = 35$ ways to choose them.

Hence, there are $15 \times 35 = 525$ ways to form the team. $\square$

(b) Determine the number of formations of the team if 2 out of 7 women must be selected together or not selected at all.

If the two women are selected together, there are ${}_5C_2 = 10$ ways to select the other two female members.

If the two women are not selected at all, there are ${}_5C_4 = 5$ ways to choose the 4 female members.

Hence, there are $10 + 5 = 15$ ways to choose the 4 female members.

Therefore, there are $15 \times 15 = 225$ ways to form the team. $\square$

31. During a meeting, 3 executive officers, 3 managers and 4 workers are seated at a round table. Determine the number of ways they are seated if

(a) no condition is imposed,

Sol.

There are $(10 - 1)! = 362,880$ ways to seat them. $\square$

(b) a particular executive officer must sit between a manager and a worker,

Sol.

Choose one manager from 3 managers. There are ${}_3C_1 = 3$ ways to do so.

Choose one worker from 4 workers. There are ${}_4C_1 = 4$ ways to do so.

Treat the executive officer, the manager and the worker as one person.

Since the manager and the worker can be seated in any order, there are $2! = 2$ ways to do so.

There are $(8 - 1)! = 40,320$ ways to seat the 8 people.

Hence, there are $3 \times 4 \times 2 \times 5,040 = 129\,960$ ways to seat them.

(c) 3 executive officers sit separately.

Sol.

First, arrange 3 managers and 4 workers to sit. There are $(7 - 1)! = 720$ ways to do so.

Then, arrange the 3 executive officers to sit in between the managers and the workers. There are ${}_7P_3 = 210$ ways to do so.

Hence, there are $720 \times 210 = 151,200$ ways to seat them. $\square$

32. A quiz team of 10 players is to be chosen from a class of 8 boys and 12 girls. Find

(a) the number of different teams that can be formed if the number of boys is equal to the number of girls,

Sol.

Choose 5 boys from 8 boys. There are ${}_8C_5 = 56$ ways to choose them.

Choose 5 girls from 12 girls. There are ${}_{12}C_5 = 792$ ways to choose them.

Hence, there are $56 \times 792 = 44,352$ ways to form the team. $\square$

(b) the number of different teams that can be formed if the number of girls is more than the number of boys,

Sol.

Choosing 1 boy and 9 girls, there are ${}_8C_1 \times {}_{12}C_9 = 1\,760$ ways to form the team.

Choosing 2 boys and 8 girls, there are ${}_8C_2 \times {}_{12}C_8 = 13\,860$

Choosing 3 boys and 7 girls, there are ${}_8C_3 \times {}_{12}C_7 = 44\,352$

Choosing 4 boys and 6 girls, there are ${}_8C_4 \times {}_{12}C_6 = 64\,680$

Hence, there are $1\,760 + 13\,860 + 44\,352 + 64\,680 = 124\,652$ ways to form the team. $\square$

(c) the number of different teams that can be formed if Kamal and Ali as well as Fatimah and Mei Mei must be chosen.

Sol.

Since 4 people must be chosen, there are 6 positions remaining.

Choosing 6 people from the rest of 14 people, there are ${}_{16}C_6 = 8\,008$ ways to do so. $\square$

33. During the National Mathematics Olympiad Competition 2021, all the 100 participants gather in a hall for a briefing. After the briefing all participants are divided equally into 5 groups. All the participants in each group are instructed to take their seats in their respective classrooms. Before the competition begins, the participants in Group A shake hands with each other. Find

(a) the number of handshakes made between the participants in Group A,

Sol.

There are 20 participants in Group A.

Choose 2 participants from 20 participants to shake hands. There are ${}_{20}C_2 = 190$ hand shakes made. ∎

(b) the number of handshakes made if Afiq, Ben and Cathy do not shake hands.

Sol.

Since these 3 participants do not shake hands, there are ${}_{3}C_2 = 3$ hand shakes that do not take place.

Hence, there are $190 - 3 = 187$ hand shakes made. ∎

Praktis Summatif

4.1 Kertas 1

1. (a) State the values of r if ${}_7C_r = 1$.

   Sol.

   $$\begin{aligned} {}_7C_r &= 1 \\ {}_7C_r &= {}_7C_0 = {}_7C_7 = 1 \\ r &= 0,\ 7 \quad \square \end{aligned}$$

   (b) Express s in terms of t and u if ${}_sC_t = {}_sC_u$

   Sol.

   $$\begin{aligned} {}_sC_t &= {}_sC_u \\ s &= t + u \quad \square \end{aligned}$$

2. (a) Show that ${}_nC_r = {}_nC_{n-r}$ where n and r are positive integers and $n > r$.

   Sol.

   $$\begin{aligned} {}_nC_{n-r} &= \frac{n!}{(n-r)![n-(n-r)]!} \\ &= \frac{n!}{(n-r)!r!} \\ &= {}_nC_r \quad \text{(shown)} \quad \square \end{aligned}$$

   (b) Find the value of r if $\dfrac{{}_nP_r}{{}_nC_r} = 120$.

   Sol.

   $$\begin{aligned} \frac{{}_nP_r}{{}_nC_r} &= 120 \\ \frac{n!}{(n-r)!} \cdot \frac{(n-r)!r!}{n!} &= 120 \\ r! &= 120 \\ r &= 5 \quad \square \end{aligned}$$

3. A committee of 7 students is selected from 7 male students and 10 female students find the number of ways the committee can be formed if

   (a) no condition is imposed,

   Sol.

   Choose 7 students from 17 students. There are ${}_{17}C_7 = 19\,448$ ways to do so. $\square$

   (b) there are 3 male students and 4 female students.

   Sol.

   Choose 3 male students from 7 male students, there are ${}_7C_3 = 35$ ways to do so.

   Choose 4 female students from 10 female students, there are ${}_{10}C_4 = 210$ ways to do so.

   Hence, there are $35 \times 210 = 7\,350$ ways to form the committee. $\square$

   (c) The number of female students must be more than the number of male students.

   Sol.

   Choose 1 male student and 9 female students, there are ${}_7C_1 \times {}_{10}C_6 = 1\,470$ ways to do so.

   Choose 2 male student and 8 female students, there are ${}_7C_2 \times {}_{10}C_5 = 5\,292$ ways to do so.

   Choose 3 male student and 7 female students, there are ${}_7C_3 \times {}_{10}C_4 = 7\,350$ ways to do so.

   Hence, there are $1\,470 + 5\,292 + 7\,350 = 14\,112$ ways to form the committee. $\square$

4. During the school year end dinner, every round table must be seated with 10 people. The VIP table has only 6 seats. 6 teachers are selected from 5 mathematics teachers and 5 science teachers to fill in the vacant seats at the VIP table. Find a number of ways to seat the teachers if

   (a) no condition is imposed,

   Sol.

   Choose 6 teachers from 10 teachers. There are ${}_{10}C_6 = 210$ ways to do so.

   Arranging these 6 teachers in the VIP table, there are $(6-1)! = 120$ ways to do so.

   Hence, there are $210 \times 120 = 25\,200$ ways to seat the teachers. $\square$

   (b) the mathematics teachers cannot sit next to each other.

   Sol.

   Since the mathematics teacher cannot sit next to each other, the number of mathematics teacher not be greater than the number of science teacher.

   Choose 3 mathematics teachers and 3 science teachers, there are ${}_5C_3 \times {}_5C_3 = 100$ ways to do so.

   First, arrange the three science teacher in the VIP table, there are $(3-1)! = 2$ ways to do so.

   Then, arrange all the three mathematics teacher to sit between the 3 science teacher, there are ${}_3P_3 = 6$ ways to do so.

   Hence, there are $2 \times 6 \times 100 = 1\,200$ ways to seat the teachers.

   Choose 2 mathematics teachers and 4 science teachers, there are ${}_5C_2 \times {}_5C_4 = 50$ ways to do so.

   First, arrange the four science teacher in the VIP table, there are $(4-1)! = 6$ ways to do so.

   Then, arrange all the two mathematics teacher to sit between the 4 science teacher, there are ${}_4P_2 = 12$ ways to do so.

   Hence, there are $6 \times 12 \times 50 = 3\,600$ ways to seat the teachers.

Choose 1 mathematics teacher and 5 science teachers, there are ${}_5C_1 \times {}_5C_5 = 5$ ways to do so.

First, arrange the five science teacher in the VIP table, there are $(5-1)! = 24$ ways to do so.

Then, arrange all the one mathematics teacher to sit between the 5 science teacher, there are ${}_5P_1 = 5$ ways to do so.

Hence, there are $24 \times 5 \times 5 = 600$ ways to seat the teachers.

Therefore, there are $1\,200 + 3\,600 + 600 = 5\,400$ ways to seat the teachers. ∎

5. Diagram below shows 8 cards that are labelled with letters.

   (a) Find the number of arrangements if

      i. all the letters are used without repetition,

         Sol.

         There are 2 Is.

         Hence, there are $\frac{8!}{2!} = 20\,160$ ways to arrange the cards. ∎

      ii. all the letters are used and the vowels must be side by side.

         Sol.

         There are 4 vowels.

         Treat the vowels as a single letter.

         Hence, there are $5! = 120$ ways to arrange the five letters.

         There are 2 Is.

         Since each vowels can be arranged in any order, there are $\frac{4!}{2!} = 12$ ways to arrange the vowels.

         Hence, there are $120 \times 12 = 1\,440$ ways to arrange the cards. ∎

   (b) Find the number of different ways to select 5 cards in which S and T needs to be chosen.

      Sol.

      There are 2 Is.

      If no I is chosen, there are ${}_4C_3 = 4$ ways to choose the three letters.

      If one I is chosen, there are ${}_4C_2 = 6$ ways to choose the other two letters.

      If two Is are chosen, there are ${}_4C_1 = 4$ ways to choose the other two letters.

      Hence, there are $4 + 6 + 4 = 14$ ways to select 5 cards in which S and T needs to be chosen. ∎

   (c) Find the number of different 5-letter codes that begin with a vowel and end with a consonant could be formed.

      Sol.

      There are 3 ways to choose the first letter.

      If the first letter is not I, there are $\frac{{}_6P_3}{2!} = 60$ ways to choose the middle three letters.

      If the first letter is I, there are ${}_6P_3 = 120$ ways to choose the middle three letters.

      Hence, there are $2 \cdot 60 + 1 \cdot 120 = 240$ ways to choose the first 4 letters.

      There are 4 ways to choose the last letter.

      Hence, there are $240 \times 4 = 960$ ways to form the 5-letter codes. ∎

6. Agnes decorates her hat with 18 artificial flowers. She uses the same number and the same type of artificial flowers, but of a smaller size to form a bracelet as shown in the diagram below.

   Given that the ratio of the number of roses to the number of morning glories to the number of sunflowers on the hat is $3:2:1$.

   (a) Find the number of roses used.

      Sol.

$$\begin{aligned}\text{Number of roses} &= \frac{18}{3+2+1} \times 3 \\ &= 9 \quad ∎\end{aligned}$$

   (b) If the morning glories have to be side by side on both the hat and bracelet find the total number of ways to arrange the artificial flowers.

      Sol.

      Since the morning glories have to be side by side, we treat the morning glories as a single flower.

      For the hat, there are $\frac{(13-1)!}{9!3!} = 220$ ways to arrange the flowers.

      For the bracelet, since the bracelet can be flipped, the clockwise and the anticlockwise arrangement are considered as the same arrangement.

      Hence, there are $\frac{(13-1)!}{2 \times 9!3!} = 110$ ways to arrange the flowers.

      Therefore, there are $220 + 110 = 330$ ways to arrange the artificial flowers. ∎

7. Diagram below shows 8 cards where 3 cards are labelled with letters and 4 cards are labelled with digits.

   Find the number of different arrangements that can be done if

   (a) no condition is imposed,

      Sol.

      There are $8! = 40\,320$ ways to arrange the cards. ∎

   (b) the arrangements begin with R and ends with an even digit.

      Sol.

      There are 4 ways to choose the last card.

      There are $6! = 720$ ways to arrange the remaining 6 cards.

      Hence, there are $4 \times 720 = 2\,880$ ways to arrange the cards. ∎

8. 10 participants successfully enter the final round of a competition. The score is used to determine the champion, 1^{st} runner up, 2^{nd} runner up, and 3^{rd} runner up.

(a) Find the number of different results that are possible to be obtained.

Sol.

There are ${}_{10}P_3 = 5\,040$ possible results. □

(b) If Ben Hong and Haikal are two of the 10 participants, find the number of different results obtained if

i. neither Ben Hong nor Haikal wins the competition,

Sol.

There are ${}_{8}P_4 = 1\,680$ possible results of the 4 winners. □

ii. Ben Hong and Haikal win the competition.

Sol.

There are ${}_{8}C_2 = 28$ possible results of the other 2 winners.

There are $4! = 24$ possible arrangement of the 4 winners

Hence, there are $28 \times 24 = 672$ possible results. □

9. Diagram below shows cards that are labelled with digits.

Find the number of 4-digit numbers that can be formed if the digit are used without repetition. From the numbers formed, find the number of 4-digit numbers that are

Sol.

There are ${}_{6}P_4 = 360$ ways to form the 4-digit numbers.

(a) greater than $6\,000$,

Sol.

Since the number is greater than $6\,000$, the first digit must be 6.

There are ${}_{5}P_3 = 60$ ways to form the remaining 3 digits.

Hence, there are $1 \times 60 = 60$ ways to form the 4-digit numbers. □

(b) odd numbers and greater than $6\,000$.

Sol.

Since the number is greater than $6\,000$, the first digit must be 6.

Since the number is odd, the last digit must be 1, 3, or 5. There are 3 ways to choose the last digit.

There are ${}_{4}P_2 = 12$ ways to form the remaining 2 digits.

Hence, there are $3 \times 12 = 36$ ways to form the 4-digit numbers. □

10. Diagram below shows the arrangement of tables in an exhibition room. A few panels of partition board are arranged in the middle of the room to create a one-way path.

After the visiting hour to the exhibition, the worker uses pieces of cloth to cover the exhibition objects on each table. It is given that the worker brings three pieces of red cloth, three pieces of green cloth, two pieces of blue cloth and a piece of yellow cloth. Find the number of ways to cover the tables with cloth if

(a) the worker chooses the cloth at random,

Sol.

There are $\dfrac{(7-1)!}{3!3!2!1!} = 560$ ways to choose the cloth. □

(b) the pieces of green cloth are used side by side,

Sol.

Treat the green cloth as a single piece of cloth.

There are $\dfrac{(7-1)!}{3!2!1!} = 60$ ways to choose the cloth. □

(c) the yellow cloth must be used to cover table A and the blue cloth cannot be used to cover its adjacent table.

Sol.

There are $\dfrac{6!}{3!3!} = 20$ ways to cover the adjacent table of table A.

There are $\dfrac{{}_{7}P_2}{2!} = 21$ ways to cover the remaining tables.

Hence, there are $20 \times 21 = 420$ ways to cover the tables. □