

1. Solve the following system of linear equations:

$$\begin{aligned}x - 2y + 3z &= 8 \cdots (1) \\x + y - 3z &= -10 \cdots (2) \\2x + y - 2z &= -12 \cdots (3)\end{aligned}$$

Solution:

$$\begin{aligned}(1) \times 2 : 2x - 4y + 6z &= 16 \cdots (4) \\(2) \times 2 : 2x + 2y - 6z &= -20 \cdots (5) \\(4) - (5) : -6y + 12z &= 36 \\-y + 2z &= 6 \cdots (6) \\(3) - (5) : -y + 4z &= 8 \cdots (7) \\(6) - (7) : -2z &= -2 \\z &= 1\end{aligned}$$

Substituting $z = 1$ into equation (6),

$$\begin{aligned}-y + 2(1) &= 6 \\-y + 2 &= 6 \\-y &= 4 \\y &= -4\end{aligned}$$

Substituting $y = -4$ and $z = 1$ into equation (1),

$$\begin{aligned}x - 2(-4) + 3(1) &= 8 \\x + 8 + 3 &= 8 \\x + 11 &= 8 \\x &= -3\end{aligned}$$

Therefore, $x = -3$, $y = -4$ and $z = 1$.

2. The first term and the common ratio of a geometric progression is 3. The first term and the common difference of an arithmetic progression are also 3. A new sequence is formed by adding the corresponding terms of the two progressions. Find

- (a) the first four terms of the new sequence,

Solution:

The first four terms of the new sequence are

$$\begin{aligned}T_1 &= (1 \times 3) + (3^1) = 3 + 3 = 6, \\T_2 &= (2 \times 3) + (3^2) = 6 + 9 = 15, \\T_3 &= (3 \times 3) + (3^3) = 9 + 27 = 36, \\T_4 &= (4 \times 3) + (3^4) = 12 + 81 = 93\end{aligned}$$

- (b) the n^{th} term of the new sequence,

Solution:

The general formula of the arithmetic progression is

$$\begin{aligned}T_n &= a + (n - 1)d, \\&= 3 + (n - 1)3, \\&= 3 + 3n - 3, \\&= 3n\end{aligned}$$

The general formula of the geometric progression is

$$\begin{aligned}T_n &= ar^{n-1}, \\&= 3(3)^{n-1}\end{aligned}$$

The n^{th} term of the new sequence is

$$T_n = 3n + 3(3)^{n-1}$$

- (c) the sum of the first 10 terms of the new sequence.

Solution:

$$\begin{aligned}S_{10} &= \frac{10}{2} [2(3) + 9(3)] + \frac{3(3^{10} - 1)}{3 - 1} \\&= 5(6 + 27) + \frac{3(59048)}{2} \\&= 5(33) + 88572 \\&= 165 + 88572 \\&= 88737\end{aligned}$$

3. (a) Solve the equation:

$$2^{x+3} - 2^{x+2} = \frac{1}{2}$$

Solution:

$$\begin{aligned}2^{x+3} - 2^{x+2} &= \frac{1}{2} \\2^x \times 2^3 - 2^x \times 2^2 &= \frac{1}{2} \\8 \times 2^x - 4 \times 2^x &= \frac{1}{2} \\4 \times 2^x &= \frac{1}{2} \\2^x &= \frac{1}{8} = 2^{-3} \\x &= -3\end{aligned}$$

(b) It is given that $4^x = p$ and $3^y = p$. Express each of the following in terms of x and/or y .

i. $\log_4 4p$,

Solution:

$$4^x = p \implies x = \log_4 p$$

$$3^y = p \implies y = \log_3 p$$

$$\begin{aligned}\log_4 4p &= \log_4 4 + \log_4 p \\ &= 1 + x\end{aligned}$$

ii. $\log_p 48$.

Solution:

$$\begin{aligned}\log_p 48 &= \frac{\log_4(4^2 \times 3)}{\log_4 p} \\ &= \frac{2\log_4 4 + \log_4 3}{\log_4 p} \\ &= \frac{2 + \frac{\log_p 3}{\log_p 4}}{\log_p p} \\ &= \frac{2 + \frac{\log_4 p}{\log_3 p}}{\log_3 p} \\ &= \frac{2 + \frac{x}{y}}{x} \\ &= \frac{2y + x}{xy}\end{aligned}$$

4. Given a triangle ABC such that $\overrightarrow{AB} = 3\vec{i} - 2\vec{j}$ and $\overrightarrow{AC} = 6\vec{i} + 3\vec{j}$. R lies on BC such that $BR = \frac{1}{2}BC$.

(a) \overrightarrow{BC}

Solution:

$$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{AC} - \overrightarrow{AB} \\ &= (6\vec{i} + 3\vec{j}) - (3\vec{i} - 2\vec{j}) \\ &= 6\vec{i} + 3\vec{j} - 3\vec{i} + 2\vec{j} \\ &= 3\vec{i} + 5\vec{j}\end{aligned}$$

- (b) the unit vector in the direction of \overrightarrow{BC} ,

Solution:

$$\begin{aligned}\overrightarrow{BC} &= 3\vec{i} + 5\vec{j} \\ \text{Let } \vec{u} &= \frac{\overrightarrow{BC}}{|\overrightarrow{BC}|} \\ &= \frac{3\vec{i} + 5\vec{j}}{\sqrt{3^2 + 5^2}} \\ &= \frac{3\vec{i} + 5\vec{j}}{\sqrt{34}}\end{aligned}$$

- (c) \overrightarrow{AR} .

Solution:

$$\begin{aligned}\overrightarrow{AR} &= \overrightarrow{AB} + \overrightarrow{BR} \\ &= 3\vec{i} - 2\vec{j} + \frac{1}{2}\overrightarrow{BC} \\ &= 3\vec{i} - 2\vec{j} + \frac{1}{2}(3\vec{i} + 5\vec{j}) \\ &= 3\vec{i} - 2\vec{j} + \frac{3}{2}\vec{i} + \frac{5}{2}\vec{j} \\ &= \frac{9}{2}\vec{i} + \frac{1}{2}\vec{j}\end{aligned}$$

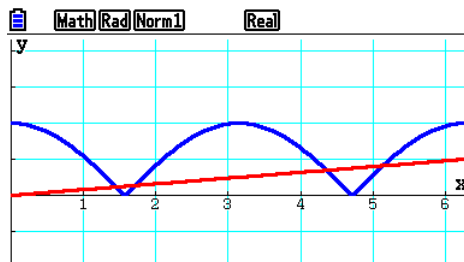
5. (a) Prove that $\sin 2x = \tan x + \tan x \cos 2x$.

Proof:

$$\begin{aligned}\text{R.H.S.} &= \tan x + \tan x \cos 2x \\ &= \tan x(1 + \cos 2x) \\ &= \frac{\sin x}{\cos x}(1 + 2\cos^2 x - 1) \\ &= \frac{\sin x}{\cos x}(2\cos^2 x) \\ &= 2\sin x \cos x \\ &= \sin 2x \\ &= \text{L.H.S.}\end{aligned}$$

- (b) i. Sketch the graph of $y = 2|\cos x|$ for $0 \leq x \leq 2\pi$.

Solution:



- ii. Hence, using the same axes, sketch a suitable straight line to determine the number of solutions for the equation $4\pi|\cos x| - x = 0$ for $0 \leq x \leq 2\pi$. State the number of solutions.

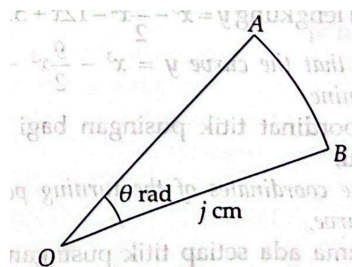
Solution:

$$4\pi|\cos x| = x$$

$$2|\cos x| = \frac{x}{2\pi}$$

From the graph, the straight line intersects the curve at 4 points. Therefore, the number of solutions is 4.

6. Diagram 1 shows the sector AOB of a circle with centre O such that $\angle AOB$ is an acute angle. Given the perimeter and the area of the sector AOB are 12 cm and 5 cm^2 respectively.



- (a) Form two equations that relate j and θ based on the above information.

Solution:

$$\text{Perimeter} = 2j + \theta j = 12 \dots (1)$$

$$\text{Area} = \frac{1}{2}\theta j^2 = 5$$

$$\theta j^2 = 10$$

$$\theta = \frac{10}{j^2} \dots (2)$$

- (b) Hence, find the value of j and of θ .

Solution: Substituting equation (2) into equation (1),

$$\begin{aligned} 2j + \frac{10}{j^2} \cdot j &= 12 \\ 2j + \frac{10}{j} &= 12 \\ 2j^2 + 10 &= 12j \\ 2j^2 - 12j + 10 &= 0 \\ j^2 - 6j + 5 &= 0 \\ (j - 5)(j - 1) &= 0 \\ j &= 5 \text{ or } j = 1 \text{ (rejected)} \\ \theta &= \frac{10}{5^2} = \frac{10}{25} \\ &= \frac{2}{5} \\ &= 0.4 \text{ rad} \end{aligned}$$

Therefore, $j = 5$ and $\theta = 0.4$ rad.

7. (a) Determine the number of ways to form 4-digit odd numbers from the digits 4, 5, 7 and 9 if the number must be less than 7000.

Solution:

First, choose the thousands place. Since the number must be less than 7000, the thousands place can only be 4 or 5.

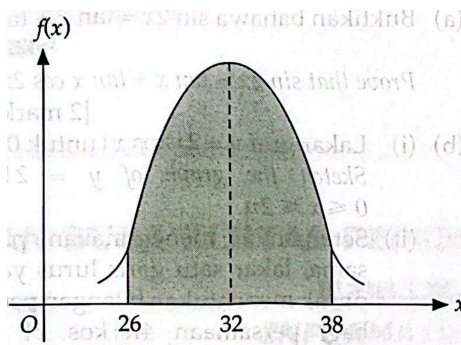
If the thousands place is 4, then the units place can be filled in 3 ways.

If the thousands place is 5, then the units place can be filled in 2 ways.

The tens and hundreds place can be filled in ${}_2P_2 = 2$ ways.

The total number of ways to form the 4-digit odd numbers is $(3 + 2)2 = 10$.

- (b) Diagram 2 shows a normal distribution graph which is symmetrical at $X = 32$.



- i. State the mean, μ .

Solution:

The mean, $\mu = 32$.

- ii. Express the shaded region in probability notation.

Solution:

The shaded region is $P(26 < X < 38)$.

- iii. If the probability of the shaded region is 0.68, find $P(X < 26)$.

Solution:

$$\begin{aligned} P(X < 26) &= 0.5 - \frac{0.68}{2} \\ &= 0.5 - 0.34 \\ &= 0.16 \end{aligned}$$

8. (a) Find the equation of the tangent and normal to the curve $f(x) = x^3 - 3x^2 + 6$ at point $A(3, 6)$.

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f'(3) &= 3(3)^2 - 6(3) \\ &= 27 - 18 \\ &= 9 \end{aligned}$$

The gradient of the tangent is 9. The equation of the tangent is

$$\begin{aligned} y - 6 &= 9(x - 3) \\ y &= 9x - 27 + 6 \\ y &= 9x - 21 \end{aligned}$$

The gradient of the normal is $-\frac{1}{9}$. The equation of the normal is

$$\begin{aligned} y - 6 &= -\frac{1}{9}(x - 3) \\ y &= -\frac{1}{9}x + \frac{1}{3} + 6 \\ y &= -\frac{1}{9}x + \frac{19}{3} \end{aligned}$$

- (b) Given that the curve $y = x^3 - \frac{9}{2}x^2 - 12x + 5$. Determine

- i. the coordinates of the turning point of the curve,

Solution:

$$\begin{aligned} y' &= 3x^2 - 9x - 12 = 0 \\ x^2 - 3x - 4 &= 0 \\ (x - 4)(x + 1) &= 0 \\ x &= 4 \text{ or } x = -1 \end{aligned}$$

When $x = 4$, $y = 4^3 - \frac{9}{2}(4)^2 - 12(4) + 5 = -51$.

When $x = -1$, $y = (-1)^3 - \frac{9}{2}(-1)^2 - 12(-1) + 5 = \frac{23}{2}$.

The coordinates of the turning points are $(4, -51)$ and $\left(-1, \frac{23}{2}\right)$.

- ii. whether each turning point is a maximum or minimum point.

Solution:

$$y''(x) = 6x - 9$$

$$y''(4) = 6(4) - 9 = 15$$

$$y''(-1) = 6(-1) - 9 = -15$$

The turning point $(4, -51)$ is a minimum point and the turning point $\left(-1, \frac{23}{2}\right)$ is a maximum point.

9. Use graph paper to answer this question.

Table 1 shows the values of two variables, x and y , obtained from an experiment. The variables x and y are related by the equation $py = x^2 + qx$, where p and q are constants.

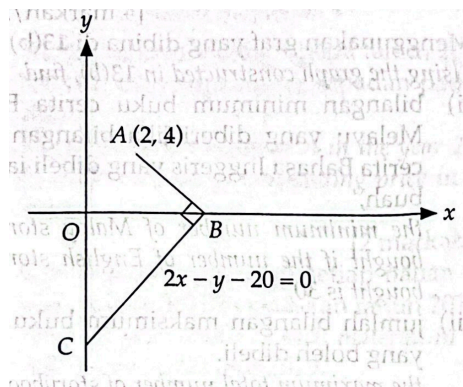
x	1	2	3	4	5	6
y	2	5	9	14	20	27

- (a) Plot $\frac{y}{x}$ against x by using a scale of 2 cm to 1 unit on the x -axis and 2 cm to 0.5 unit on the $\frac{y}{x}$ -axis. Hence, draw the line of best fit.
- (b) Using the graph in 9(a), find the value of
- p ,
 - q .

Solution:

Lazy to do this. =)

10. Diagram 3 shows a straight line AB which intersects a straight line BC at point B . The point C lies on the y -axis.



(a) Find

- i. the equation of the straight line AB ,

Solution:

$$2x - y - 20 = 0$$

$$y = 2x - 20$$

$$m = 2$$

$\therefore AB$ is perpendicular to the line BC .

\therefore the gradient of AB is $-\frac{1}{2}$.

$$y - 4 = -\frac{1}{2}(x - 2)$$

$$2y - 8 = -x + 2$$

$$x + 2y = 10$$

- ii. the coordinates of point B .

Solution:

Point B is touching the x -axis. Therefore, the y -coordinate of point B is 0.

When $y = 0$,

$$0 = 2x - 20$$

$$x = 10$$

Therefore, the coordinates of point B are $(10, 0)$.

- (b) The straight line AB is extended to point D such that $AB : BD = 1 : 3$. Find the coordinates of point D .

Solution:

Let the coordinates of point D be (x, y) .

$$\left(\frac{3(2) + 1(x)}{4}, \frac{3(4) + 1(y)}{4} \right) = (10, 0)$$

$$\left(\frac{6 + x}{4}, \frac{12 + y}{4} \right) = (10, 0)$$

$$\frac{6 + x}{4} = 10$$

$$6 + x = 40$$

$$x = 34$$

$$\frac{12 + y}{4} = 0$$

$$12 + y = 0$$

$$y = -12$$

Therefore, the coordinates of point D are $(34, -12)$.

- (c) Point P moves such that its distance from point A is always 5 units. Find the equation of the locus of point P .

Solution:

The locus of point P is a circle with centre $(2, 4)$ and radius 5.

$$\begin{aligned}(x - 2)^2 + (y - 4)^2 &= 5^2 \\ x^2 - 4x + 4 + y^2 - 8y + 16 &= 25 \\ x^2 + y^2 - 4x - 8y - 5 &= 0\end{aligned}$$

- (d) Determine whether point $(2, 6)$ lies in the locus of point P .

Solution: When $x = 2$ and $y = 6$,

$$\begin{aligned}2^2 + 6^2 - 4(2) - 8(6) - 5 &= 4 + 36 - 8 - 48 - 5 \\ &= -21 \neq 0\end{aligned}$$

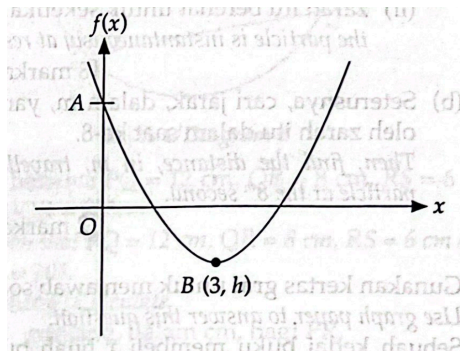
Therefore, point $(2, 6)$ does not lie in the locus of point P .

11. (a) Find the range of values of x if $(x - 2)^2 > 16 - 3x$.

Solution:

$$\begin{aligned}(x - 2)^2 &> 16 - 3x \\ x^2 - 4x + 4 &> 16 - 3x \\ x^2 - x - 12 &> 0 \\ (x - 4)(x + 3) &> 0 \\ x &< -3 \text{ or } x > 4\end{aligned}$$

- (b) Diagram 4 shows the curve of a quadratic function $f(x) = x^2 + mx + 8$. The curve has a minimum point $B(3, h)$ and intersects the $f(x)$ -axis at point A .



- i. Find the coordinates of point A .

Solution:

When the curve intersects the $f(x)$ -axis, $x = 0$.

$$\begin{aligned}f(0) &= 0^2 + m(0) + 8 \\ &= 8\end{aligned}$$

Therefore, the coordinates of point A are $(0, 8)$.

- ii. By using the method of completing the square, find the value of m and of h .

Solution:

$$\begin{aligned} f(x) &= x^2 + mx + 8 \\ &= \left(x^2 + mx + \frac{m^2}{4}\right) - \frac{m^2}{4} + 8 \\ &= \left(x + \frac{m}{2}\right)^2 - \frac{m^2}{4} + 8 \\ &= \left(x + \frac{m}{2}\right)^2 - \frac{m^2 - 32}{4} \end{aligned}$$

The minimum point is $\left(-\frac{m}{2}, \frac{m^2 - 32}{4}\right) = (3, h)$.

$$\begin{aligned} -\frac{m}{2} &= 3 \\ m &= -6 \end{aligned}$$

$$\begin{aligned} h &= \frac{m^2 - 32}{4} \\ &= \frac{(-6)^2 - 32}{4} \\ &= \frac{36 - 32}{4} \\ &= 1 \end{aligned}$$

Therefore, $m = -6$ and $h = 1$.

- iii. Determine the range of values of x if $f(x) < 8$.

Solution:

$$\begin{aligned} f(x) &< 8 \\ x^2 - 6x + 8 &< 8 \\ x^2 - 6x &< 0 \\ x(x - 6) &< 0 \\ 0 &< x < 6 \end{aligned}$$

12. A particle moves along a straight line and passes through a fixed point O with a velocity of 14 m s^{-1} . Its acceleration, a ms^{-2} , at t seconds after passing through O is given by $a = 5 - 2t$.

- (a) Find the instantaneous displacement, in m , of the particle when

- i. $t = 3$

Solution:

$$\begin{aligned} a &= 5 - 2t \\ v &= \int a \, dt \end{aligned}$$

$$\begin{aligned}
&= \int (5 - 2t) dt \\
&= 5t - t^2 + C
\end{aligned}$$

When $t = 0$, $v = 14$.

$$\begin{aligned}
14 &= 5(0) - (0)^2 + C \\
C &= 14
\end{aligned}$$

Therefore, $v = 5t - t^2 + 14$.

$$\begin{aligned}
s &= \int v dt \\
&= \int (5t - t^2 + 14) dt \\
&= \frac{5}{2}t^2 - \frac{1}{3}t^3 + 14t + C
\end{aligned}$$

When $t = 0$, $s = 0$.

$$\begin{aligned}
0 &= \frac{5}{2}(0)^2 - \frac{1}{3}(0)^3 + 14(0) + C \\
C &= 0
\end{aligned}$$

Therefore, $s = \frac{5}{2}t^2 - \frac{1}{3}t^3 + 14t$.

$$\begin{aligned}
s(3) &= \frac{5}{2}(3)^2 - \frac{1}{3}(3)^3 + 14(3) \\
&= \frac{45}{2} - 9 + 42 \\
&= 55.5 \text{ m}
\end{aligned}$$

ii. the particle is instantaneously at rest.

Solution:

$$\begin{aligned}
v &= 0 \\
5t - t^2 + 14 &= 0 \\
t^2 - 5t - 14 &= 0 \\
(t - 7)(t + 2) &= 0 \\
t &= 7 \text{ or } t = -2 \text{ (rejected)}
\end{aligned}$$

When $t = 7$,

$$\begin{aligned}
s &= \frac{5}{2}(7)^2 - \frac{1}{3}(7)^3 + 14(7) \\
&\approx 106.17 \text{ m}
\end{aligned}$$

- (b) Then, find the distance, in m , travelled by the particle at the 8th second.

Solution:

$$\begin{aligned}
 s &= |s(8) - s(7)| \\
 &= \left| \left(\frac{5}{2}(8)^2 - \frac{1}{3}(8)^3 + 14(8) \right) - \left(\frac{5}{2}(7)^2 - \frac{1}{3}(7)^3 + 14(7) \right) \right| \\
 &= \left| -4\frac{5}{6} \right| \\
 &= 4\frac{5}{6} \text{ m}
 \end{aligned}$$

13. Use graph paper to answer this question.

A bookshop buys x English storybooks and y Malay storybooks. The prices of each English storybook and Malay storybook are RM40 and RM30 respectively. The purchase of the storybooks is based on the following constraints:

- I: The total amount allocated is RM2 000.
 - II: The number of Malay storybooks is not more than four times the number of English storybooks.
 - III: The number of English storybooks is at most three times the number of Malay storybooks.
- (a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$ that satisfy all the given constraints.

Solution:

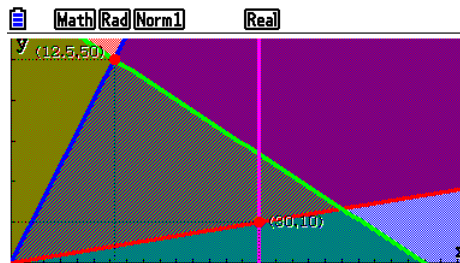
I: $40x + 30y \leq 2000$

II: $y \leq 4x$

III: $x \leq 3y$

- (b) Using a scale of 2 cm to 10 storybooks on both axes, construct and shade the region R which satisfies all the given constraints.

Solution:



- (c) Using the graph constructed in 13(b), find
- i. the minimum number of Malay storybooks bought if the number of English storybooks bought is 30,

Solution:

According to the graph, the minimum number of Malay storybooks bought is 10.

- ii. the maximum total number of storybooks that can be bought.

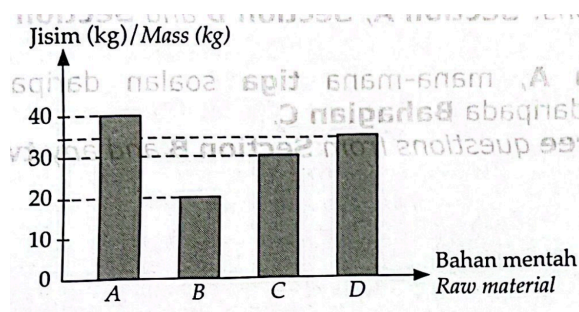
Solution:

According to the graph, the maximum total number of storybooks that can be bought is 62.

14. Table 2 shows the price indices and changes in price indices of four raw materials, A, B, C and D , used in the production of a type of snack in a factory.

Raw material	Price index in the year 2020 based on the year 2019	Change in price index from the year 2020 to the year 2022
A	130	15% increase
B	110	5% increase
C	150	No change
D	140	10% decrease

Diagram 5 shows a bar chart which represents the mass of the raw materials used to make the snacks in the year 2019.



- (a) The price of raw material A in the year 2020 was RM110. Find the corresponding price in the year 2019.

Solution:

$$\begin{aligned}
 \text{Price index} &= \frac{P_{2020}}{P_{2019}} \times 100 \\
 130 &= \frac{110}{P_{2019}} \times 100 \\
 P_{2019} &= \frac{110}{130} \times 100 \\
 &= \text{RM } 84.62
 \end{aligned}$$

- (b) Find the price index of each material in the year 2022 based on the year 2019. **Solution:**

$$\begin{aligned}
 \text{Price Index of } A_{2022/2019} &= 130 \times 115\% = 149.5 \\
 \text{Price Index of } B_{2022/2019} &= 110 \times 105\% = 115.5 \\
 \text{Price Index of } C_{2022/2019} &= 150 \times 100\% = 150 \\
 \text{Price Index of } D_{2022/2019} &= 140 \times 90\% = 126
 \end{aligned}$$

- (c) i. Calculate the composite index for the cost of producing the snacks in the year 2022 based on the year 2019.

Solution:

$$\begin{aligned}\text{Composite index} &= \frac{149.5 \times 40 + 115.5 \times 20 + 150 \times 30 + 126 \times 35}{40 + 20 + 30 + 35} \\ &= \frac{5980 + 2310 + 4500 + 4410}{125} \\ &= \frac{17200}{125} \\ &= 137.6\end{aligned}$$

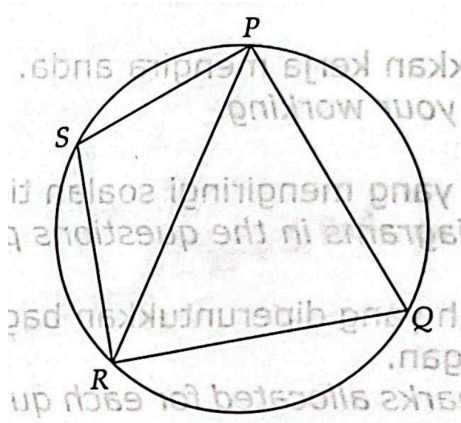
- ii. Hence, find the cost of producing the snacks in the year 2019 if the corresponding cost in the year 2022 is RM325.40.

Solution:

$$\begin{aligned}\frac{P_{2022}}{P_{2019}} \times 100 &= 137.6 \\ \frac{325.40}{P_{2019}} \times 100 &= 137.6 \\ P_{2019} &= \frac{325.40}{137.6} \times 100 \\ &= \text{RM } 236.48\end{aligned}$$

15. Diagram 6 shows a quadrilateral $PQRS$ inscribed in a circle.

It is given that $PQ = 12$ cm, $QR = 8$ cm, $RS = 6$ cm and $\angle PQR = 70^\circ$.



- (a) Calculate

- i. the length, in cm, of PR ,

Solution:

$$\begin{aligned}PR &= \sqrt{12^2 + 8^2 - 2(12)(8) \cos 70^\circ} \\ &\approx 11.93 \text{ cm}\end{aligned}$$

ii. $\angle PRS$.

Solution:

$$\begin{aligned}
 \angle PSR &= 180^\circ - 70^\circ \\
 &= 110^\circ \\
 \frac{PR}{\sin \angle PSR} &= \frac{RS}{\sin \angle SPR} \\
 \frac{11.93}{\sin 110^\circ} &= \frac{6}{\sin \angle SPR} \\
 \sin \angle SPR &= \frac{6 \sin 110^\circ}{11.93} \\
 &\approx 0.47 \\
 \angle SPR &\approx 28.20^\circ \\
 \angle PRS &= 180^\circ - 110^\circ - 28.20^\circ \\
 &= 41.80^\circ
 \end{aligned}$$

(b) Find the area, in cm^2 , of triangle PQR .

Solution:

$$\begin{aligned}
 \text{Area of triangle } PQR &= \frac{1}{2} \times 12 \times 8 \times \sin 70^\circ \\
 &\approx 45.11 \text{ cm}^2
 \end{aligned}$$

(c) Find the shortest distance, in cm, from point Q to the straight line PR .

Solution:

The shortest distance from point Q to the straight line PR is the perpendicular distance from point Q to the straight line PR .

Let the shortest distance be h .

$$\begin{aligned}
 \frac{QR}{\sin \angle RPQ} &= \frac{PR}{\sin \angle PQR} \\
 \frac{8}{\sin \angle RPQ} &= \frac{11.93}{\sin 70^\circ} \\
 \sin \angle RPQ &= \frac{8 \sin 70^\circ}{11.93} \\
 &\approx 0.63 \\
 \angle RPQ &\approx 39.06^\circ \\
 \frac{h}{PQ} &= \sin 39.06^\circ \\
 h &= 12 \sin 39.06^\circ \\
 &\approx 7.56 \text{ cm}
 \end{aligned}$$