Solution Book of Mathematic

Ssnior 2 Part I

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Written on 9 October 2022

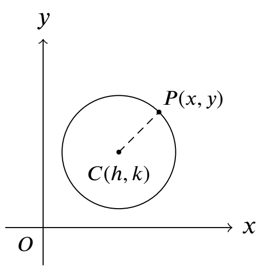
Contents

Chapter 15

Circle

15.1 Standard Equation of a Circle

The circle is a locus of points in a plane that are equidistant from a fixed point called the centre of the circle. The distance between the centre and the points on the circle is called the radius of the circle.



The standard equation of a circle is given by

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h, k) is the centre of the circle and r is the radius of the circle.

If the centre of the circle is at the origin, then the equation of the circle is

$$x^2 + y^2 = r^2 \quad (r > 0)$$

15.1.1 Practice 1

1. Find the equation of the circle with centre $(3, -1)$ and radius 2.

2. Find the equation of the circle with centre $(-2, 9)$ and passing through the point $(1, 5)$.

15.1.2 Exercise 16.1

1. Find the equation of the circle with centre at the origin and radius 7.

2. Find the equation of circle of each of the following description:

   (a) Passing through the points $(5, -3)$ and centre at $(2, 1)$.
   (b) Centre at $(3, 2)$ and radius 4.
   (c) Centre at (a, b) and radius $a + b$.

3. Given that the coordinates of two points on the end of the diameter of a circle are $(5, -3)$ and $(3, 1)$, find the equation of the circle.

4. Find the equation of the circle with a diameter connected by the points $(-3, 4)$ and $(9, 2)$.

5. Given two points $P(-2, 2)$ and $Q(4, 6)$, find the equation of the circle wuth line PQ as its diameter.

6. Turn the equation $x^2 + y^2 - 6x + 12y + 41 = 0$ into the standard form, and find the centre and radius of the circle.

15.2 General Equation of a Circle

Expand the standard equation of a circle, we get

$$x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$$

Let $g = -h$, $f = -k$, $c = h^2 + k^2 - r^2$, we get the general equation of a circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

From $c = h^2 + k^2 - r^2$, we have $r^2 = h^2 + k^2 - c$

$$\begin{aligned} r &= \sqrt{h^2 + k^2 - c} \\ &= \sqrt{(-g)^2 + (-f)^2 - c} \\ &= \sqrt{g^2 + f^2 - c} \end{aligned}$$

Thus,

1. When $g^2 + f^2 - c > 0$, the image is a real circle with centre (g, f) and radius $\sqrt{g^2 + f^2 - c}$.

2. When $g^2 + f^2 - c = 0$, the image is point (g, f).

3. When $g^2 + f^2 - c < 0$, the image does not exist.

15.2.1 Practice 2

1. Find the centre and radius of the circle with equation $x^2 + y^2 - 6x - 8y + 21 = 0$.

2. Find the equaton of the circle that passes through the following points:

   (a) $A(0, 0)$, $B(2, 0)$, $C(0, -3)$.
   (b) $K(0, 3)$, $L(1, 2)$, $M(2, -1)$.

3. Given that the vertices of ΔABC are $(1, 2)$, $(2, 5)$ and $(-1, 2)$, find the equation of the circumcircle of ΔABC.

15.2.2 Exercise 16.2

1. Find the centre and radius of the circle with the following equation:

   (a) $x^2 + y^2 - 64 = 0$

   (b) $x^2 + y^2 - 4x - 8y = 44$

   (c) $x^2 + y^2 - 8x = 0$

   (d) $9x^2 + 9y^2 + 2x - 6y - 6 = 0$

   (e) $9x^2 + 9y^2 + 2x - 6y - 6 = 0$

2. Find the equation of the circle that passes through the following points:

   (a) $A(1, 1)$, $B(1, -1)$, $C(-2, 1)$

   (b) $F(0, 0)$, $G(3, -3)$, $H(-1, 0)$

   (c) $P(1, 0)$, $Q(0, -3)$, $R(3, 4)$

3. A circle passes through point $A(2, 2)$ and $B(5, 3)$ while intersecting the line $x + y = 4$ at y-axis. Find the equation of the circle.