1. Solve the following system of linear equations:

$$x - 2y + 3z = 8 \ \cdots \ (1)$$
$$x + y - 3z = -10 \ \cdots \ (2)$$
$$2x + y - 2z = -12 \ \cdots \ (3)$$

Solution:

$$
\begin{aligned}
(1) \times 2 : 2x - 4y + 6z &= 16 \ \cdots \ (4) \\
(2) \times 2 : 2x + 2y - 6z &= -20 \ \cdots \ (5) \\
(4) - (5) : -6y + 12z &= 36 \\
-y + 2z &= 6 \ \cdots \ (6) \\
(3) - (5) : -y + 4z &= 8 \ \cdots \ (7) \\
(6) - (7) : -2z &= -2 \\
z &= 1
\end{aligned}
$$

Substituting $z = 1$ into equation (6),

$$
\begin{aligned}
-y + 2(1) &= 6 \\
-y + 2 &= 6 \\
-y &= 4 \\
y &= -4
\end{aligned}
$$

Substituting $y = -4$ and $z = 1$ into equation (1),

$$
\begin{aligned}
x - 2(-4) + 3(1) &= 8 \\
x + 8 + 3 &= 8 \\
x + 11 &= 8 \\
x &= -3
\end{aligned}
$$

Therefore, $x = -3$, $y = -4$ and $z = 1$.

2. The first term and the common ratio of a geometric progression is 3. The first term and the common difference of an arithmetic progression are also 3. A new sequence is formed by adding the corresponding terms of the two progressions. Find

   (a) the first four terms of the new sequence,
   Solution:
   The first four terms of the new sequence are

$$
\begin{aligned}
T_1 &= (1 \times 3) + (3^1) = 3 + 3 = 6, \\
T_2 &= (2 \times 3) + (3^2) = 6 + 9 = 15, \\
T_3 &= (3 \times 3) + (3^3) = 9 + 27 = 36, \\
T_4 &= (4 \times 3) + (3^4) = 12 + 81 = 93
\end{aligned}
$$

(b) the n^{th} term of the new sequence,

Solution:

The general formula of the arithmetic progression is

$$\begin{aligned} T_n &= a + (n-1)d, \\ &= 3 + (n-1)3, \\ &= 3 + 3n - 3, \\ &= 3n \end{aligned}$$

The general formula of the geometric progression is

$$\begin{aligned} T_n &= ar^{n-1}, \\ &= 3(3)^{n-1} \end{aligned}$$

The n^{th} term of the new sequence is

$$T_n = 3n + 3(3)^{n-1}$$

(c) the sum of the first 10 terms of the new sequence.

Solution:

$$\begin{aligned} S_{10} &= \frac{10}{2}\left[2(3) + 9(3)\right] + \frac{3(3^{10}-1)}{3-1} \\ &= 5(6+27) + \frac{3(59048)}{2} \\ &= 5(33) + 88572 \\ &= 165 + 88572 \\ &= 88737 \end{aligned}$$

3. (a) Solve the equation:

$$2^{x+3} - 2^{x+2} = \frac{1}{2}$$

Solution:

$$\begin{aligned} 2^{x+3} - 2^{x+2} &= \frac{1}{2} \\ 2^x \times 2^3 - 2^x \times 2^2 &= \frac{1}{2} \\ 8 \times 2^x - 4 \times 2^x &= \frac{1}{2} \\ 4 \times 2^x &= \frac{1}{2} \\ 2^x &= \frac{1}{8} = 2^{-3} \\ x &= -3 \end{aligned}$$

(b) It is given that $4^x = p$ and $3^y = p$. Express each of the following in terms of x and/or y.

i. $\log_4 4p$,

Solution:

$$\begin{aligned} 4^x = p &\implies x = \log_4 p \\ 3^y = p &\implies y = \log_3 p \end{aligned}$$

$$\begin{aligned} \log_4 4p &= \log_4 4 + \log_4 p \\ &= 1 + x \end{aligned}$$

ii. $\log_p 48$.

Solution:

$$\begin{aligned} \log_p 48 &= \frac{\log_4(4^2 \times 3)}{\log_4 p} \\ &= \frac{2\log_4 4 + \log_4 3}{x} \\ &= \frac{2 + \dfrac{\log_p 3}{\log_p 4}}{x} \\ &= \frac{2 + \dfrac{\log_4 p}{\log_3 p}}{x} \\ &= \frac{2 + \dfrac{x}{y}}{x} \\ &= \frac{2y + x}{xy} \end{aligned}$$

4. Given a triangle ABC such that $\overrightarrow{AB} = 3\vec{\imath} - 2\vec{\jmath}$ and $\overrightarrow{AC} = 6\vec{\imath} + 3\vec{\jmath}$. R lies on BC such that $BR = \dfrac{1}{2}BC$.

(a) $\overrightarrow{BC}$

Solution:

$$\begin{aligned} \overrightarrow{BC} &= \overrightarrow{AC} - \overrightarrow{AB} \\ &= (6\vec{\imath} + 3\vec{\jmath}) - (3\vec{\imath} - 2\vec{\jmath}) \\ &= 6\vec{\imath} + 3\vec{\jmath} - 3\vec{\imath} + 2\vec{\jmath} \\ &= 3\vec{\imath} + 5\vec{\jmath} \end{aligned}$$

(b) the unit vector in the direction of $\overrightarrow{BC}$,

Solution:

$$\begin{aligned}\overrightarrow{BC} &= 3\vec{\imath} + 5\vec{\jmath} \\ \text{Let } \vec{u} &= \frac{\overrightarrow{BC}}{|\overrightarrow{BC}|} \\ &= \frac{3\vec{\imath} + 5\vec{\jmath}}{\sqrt{3^2+5^2}} \\ &= \frac{3\vec{\imath} + 5\vec{\jmath}}{\sqrt{34}}\end{aligned}$$

(c) $\overrightarrow{AR}$.

Solution:

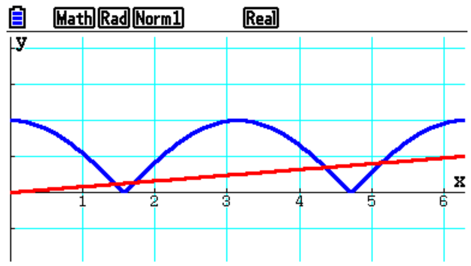
$$\begin{aligned}\overrightarrow{AR} &= \overrightarrow{AB} + \overrightarrow{BR} \\ &= 3\vec{\imath} - 2\vec{\jmath} + \frac{1}{2}\overrightarrow{BC} \\ &= 3\vec{\imath} - 2\vec{\jmath} + \frac{1}{2}(3\vec{\imath} + 5\vec{\jmath}) \\ &= 3\vec{\imath} - 2\vec{\jmath} + \frac{3}{2}\vec{\imath} + \frac{5}{2}\vec{\jmath} \\ &= \frac{9}{2}\vec{\imath} + \frac{1}{2}\vec{\jmath}\end{aligned}$$

5. (a) Prove that $\sin 2x = \tan x + \tan x \cos 2x$.

Proof:

$$\begin{aligned}\text{R.H.S.} &= \tan x + \tan x \cos 2x \\ &= \tan x(1+\cos 2x) \\ &= \frac{\sin x}{\cos x}(1 + 2\cos^2 x - 1) \\ &= \frac{\sin x}{\cos x}(2\cos^2 x) \\ &= 2\sin x \cos x \\ &= \sin 2x \\ &= \text{L.H.S.}\end{aligned}$$

(b) i. Sketch the graph of $y = 2|\cos x|$ for $0 \leqslant x \leqslant 2\pi$.
Solution:

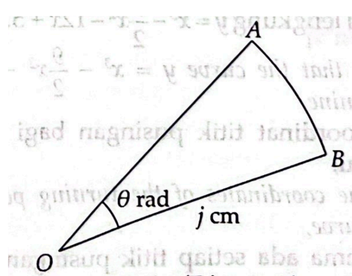


ii. Hence, using the same axes, sketch a suitable straight line to determine the number of solutions for the equation $4\pi|\cos x| - x = 0$ for $0 \leqslant x \leqslant 2\pi$. State the number of solutions.
Solution:

$$\begin{aligned} 4\pi|\cos x| &= x \\ 2|\cos x| &= \frac{x}{2\pi} \end{aligned}$$

From the graph, the straight line intersects the curve at 4 points. Therefore, the number of solutions is 4.

6. Diagram 1 shows the sector AOB of a circle with centre O such that $\angle AOB$ is an acute angle. Given the perimeter and the area of the sector AOB are 12 cm and 5 cm^2 respectively.



(a) Form two equations that relate j and θ based on the above information.
Solution:

$$\begin{aligned} \text{Perimeter} &= 2j + \theta j = 12 \;\cdots\; (1) \\ \text{Area} &= \frac{1}{2}\theta j^2 = 5 \\ \theta j^2 &= 10 \\ \theta &= \frac{10}{j^2} \;\cdots\; (2) \end{aligned}$$

(b) Hence, find the value of j and of θ.

Solution: Substituting equation (2) into equation (1),

$$\begin{aligned}
2j + \frac{10}{j^2} \cdot j &= 12 \\
2j + \frac{10}{j} &= 12 \\
2j^2 + 10 &= 12j \\
2j^2 - 12j + 10 &= 0 \\
j^2 - 6j + 5 &= 0 \\
(j-5)(j-1) &= 0 \\
j &= 5 \text{ or } j = 1 \text{ (rejected)} \\
\theta &= \frac{10}{5^2} = \frac{10}{25} \\
&= \frac{2}{5} \\
&= 0.4 \text{ rad}
\end{aligned}$$

Therefore, $j = 5$ and $\theta = 0.4$ rad.

7. (a) Determine the number of ways to form 4-digit odd numbers from the digits 4, 5, 7 and 9 if the number must be less than 7000.

Solution:

First, choose the thousands place. Since the number must be less than 7000, the thousands place can only be 4 or 5.

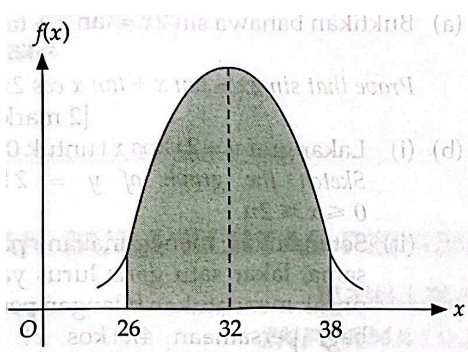
If the thousands place is 4, then the units place can be filled in 3 ways.

If the thousands place is 5, then the units place can be filled in 2 ways.

The tens and hundreds place can be filled in ${}_2P_2 = 2$ ways.

The total number of ways to form the 4-digit odd numbers is $(3 + 2)2 = 10$.

(b) Diagram 2 shows a normal distribution graph which is symmetrical at $X = 32$.



i. State the mean, μ.

Solution:

The mean, $\mu = 32$.

ii. Express the shaded region in probability notation.
Solution:
The shaded region is $P(26 < X < 38)$.

iii. If the probability of the shaded region is 0.68 , find $P(X < 26)$.
Solution:

$$\begin{aligned}P(X < 26) &= 0.5 - \frac{0.68}{2} \\ &= 0.5 - 0.34 \\ &= 0.16\end{aligned}$$

8. (a) Find the equation of the tangent and normal to the curve $f(x) = x^3 - 3x^2 + 6$ at point $A(3, 6)$.
Solution:

$$\begin{aligned}f'(x) &= 3x^2 - 6x \\ f'(3) &= 3(3)^2 - 6(3) \\ &= 27 - 18 \\ &= 9\end{aligned}$$

The gradient of the tangent is 9. The equation of the tangent is

$$\begin{aligned}y - 6 &= 9(x - 3) \\ y &= 9x - 27 + 6 \\ y &= 9x - 21\end{aligned}$$

The gradient of the normal is $-\frac{1}{9}$. The equation of the normal is

$$\begin{aligned}y - 6 &= -\frac{1}{9}(x - 3) \\ y &= -\frac{1}{9}x + \frac{1}{3} + 6 \\ y &= -\frac{1}{9}x + \frac{19}{3}\end{aligned}$$

(b) Given that the curve $y = x^3 - \frac{9}{2}x^2 - 12x + 5$. Determine

i. the coordinates of the turning point of the curve,
Solution:

$$\begin{aligned}y' = 3x^2 - 9x - 12 &= 0 \\ x^2 - 3x - 4 &= 0 \\ (x - 4)(x + 1) &= 0 \\ x = 4 \text{ or } x &= -1\end{aligned}$$

When $x = 4$, $y = 4^3 - \frac{9}{2}(4)^2 - 12(4) + 5 = -51$.
When $x = -1$, $y = (-1)^3 - \frac{9}{2}(-1)^2 - 12(-1) + 5 = \frac{23}{2}$.
The coordinates of the turning points are $(4, -51)$ and $\left(-1, \frac{23}{2}\right)$.

ii. whether each turning point is a maximum or minimum point.
Solution:

$$\begin{aligned} y''(x) &= 6x - 9 \\ y''(4) &= 6(4) - 9 = 15 \\ y''(-1) &= 6(-1) - 9 = -15 \end{aligned}$$

The turning point $(4, -51)$ is a minimum point and the turning point $\left(-1, \dfrac{23}{2}\right)$ is a maximum point.

9. Use graph paper to answer this question.

   Table 1 shows the values of two variables, x and y, obtained from an experiment. The variables x and y are related by the equation $py = x^2 + qx$, where p and q are constants.

x	1	2	3	4	5	6
y	2	5	9	14	20	27

   (a) Plot $\dfrac{y}{x}$ against x by using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 0.5 unit on the $\dfrac{y}{x}$-axis. Hence, draw the line of best fit.

   (b) Using the graph in $9(a)$, find the value of

   i. p,

   ii. q.

10. Diagram 3 shows a straight line AB which intersects a straight line BC at point B. The point C lies on the y-axis.

   (a) Find

   i. the equation of the straight line AB,

   ii. the coordinates of point B.

   (b) The straight line AB is extended to point D such that $AB : BD = 1 : 3$. Find the coordinates of point D.

   (c) Point P moves such that its distance from point A is always 5 units. Find the equation of the locus of point P.

   (d) Determine whether point $(2, 6)$ lies in the locus of point P.

11. (a) Find the range of values of x if $(x - 2)^2 > 16 - 3x$.

   (b) Diagram 4 shows the curve of a quadratic function $f(x) = x^2 + mx + 8$. The curve has a minimum point $B(3, h)$ and intersects the $f(x)$-axis at point A.

   i. Find the coordinates of point A.

   ii. By using the method of completing the square, find the value of m and of h.

   iii. Determine the range of values of x if $f(x) < 8$.