

11.1

1. Given $A(0,1), B(-2,1)$

$$M_{AB} = \left(\frac{0-2}{2}, \frac{1+1}{2} \right)$$

$$= (-1, 1)$$

$$\therefore y_A = y_B$$

\therefore A and B lies on a horizontal line

\therefore any line $y=k$ has the same perpendicular distance with point A and B, being $|1-k|$.

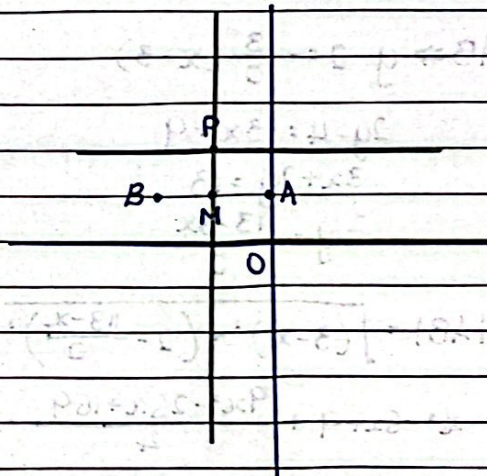
In order for the line to pass through $P(-1,2)$, $y=2$.

Also, any vertical line that passes through $M(A,B)$ is also equal in distance with point A and B, being $x=k$.

Notice that $P_x = M_x$

$$\therefore x = -1$$

\therefore There are two lines #

2. Let $A(a,b), B(-b,a), C(0,0), D(x,y)$

$$M_{AB} = \left(\frac{a-b}{2}, \frac{a+b}{2} \right)$$

$$M_{CD} = M_{AB}$$

$$\left(\frac{x+0}{2}, \frac{y+0}{2} \right) = \left(\frac{a-b}{2}, \frac{a+b}{2} \right)$$

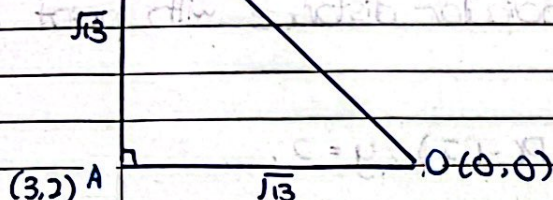
$$x = a-b, y = a+b$$

$$\therefore D(a-b, a+b) \#$$

11.2

$$1. |OA| = \sqrt{(3-0)^2 + (2-0)^2} \\ = \sqrt{9+4} \\ = \sqrt{13} = |AB|$$

B(x,y)



$$m_{OA} = \frac{2}{3}$$

$$m_{AB} = -\frac{3}{2} \quad (AO \perp AB)$$

$$AB \Rightarrow y - 2 = -\frac{3}{2}(x - 3)$$

$$2y - 4 = -3x + 9$$

$$3x + 2y = 13$$

$$y = \frac{13-3x}{2}$$

$$|AB| = \sqrt{(3-x)^2 + (2 - \frac{13-3x}{2})^2} = \sqrt{13}$$

$$x^2 - 6x + 9 + \frac{9x^2 - 78x + 169}{4} - 2(13-3x) + 4 = 13$$

$$4x^2 - 24x + 36 + 9x^2 - 78x + 169 - 8(13-3x) + 16 = 52$$

$$13x^2 - 102x + 205 - 104 + 24x - 36 = 0$$

$$13x^2 - 78x + 65 = 0$$

$$x^2 - 6x + 5 = 0$$

$$(x-5)(x-1) = 0$$

$$x = 1 \text{ or } x = 5$$

$$\text{When } x=1, y = \frac{13-3}{2} = 5$$

$$\text{When } x=5, y = \frac{13-15}{2} = -1$$

$$\therefore B(1,5) \text{ or } B(5,-1) \quad \#$$

11.3

1. Let $A(0,0)$, $B(x,0)$, $C(0,y)$

$$|y-x| = 3 \text{ --- (1)}$$

$$\Delta = \frac{1}{2} \begin{vmatrix} 0 & x & 0 & 0 \\ 0 & 0 & y & 0 \end{vmatrix} = 2$$

$$|0+xy+0-0-0-0| = 4$$

$$|xy| = 4$$

$\therefore \Delta$ is in 1st quadrant

$$\therefore x > 0, y > 0$$

$$\therefore xy > 0$$

$$\therefore xy = 4$$

$$y = \frac{4}{x}$$

Sub into (1),

$$|y-x| = 3$$

$$|\frac{4}{x} - x| = 3$$

$$\frac{4}{x} - x = \pm 3$$

$$\frac{4-x^2}{x} = \pm 3$$

$$4-x^2 = \pm 3x$$

$$x^2 \pm 3x - 4 = 0$$

When $x^2 + 3x - 4 = 0$

$$(x+4)(x-1) = 0$$

$$x = -4 \text{ or } x = 1 \text{ (rejected)}$$

When $x^2 - 3x - 4 = 0$

$$(x-4)(x+1) = 0$$

$$x = 4 \text{ or } x = -1 \text{ (rejected)}$$

When $x = 1$, $y = 4 \Rightarrow B(1,0), C(0,4)$

$$\frac{y-0}{x-1} = \frac{4-0}{0-1}$$

$$\frac{y}{x-1} = \frac{4}{-1}$$

$$-y = 4x - 4$$

$$4x + y - 4 = 0 \#$$

When $x = 4$, $y = 1 \Rightarrow B(4,0), C(0,1)$

$$\frac{y-0}{x-4} = \frac{1-0}{0-4}$$

$$\frac{y}{x-4} = \frac{1}{-4}$$

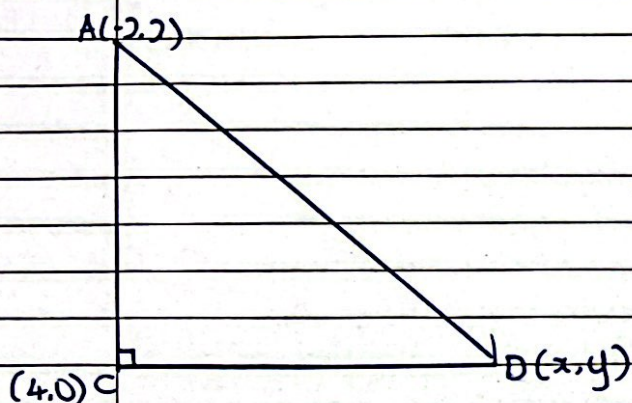
$$-4y = x - 4$$

$$x + 4y - 4 = 0 \#$$

$$2. A_{\triangle ABC} = \frac{1}{2} \begin{vmatrix} -2 & 3 & 4 & -2 \\ 5 & 7 & 0 & 2 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} -14 + 0 + 8 - 6 - 28 - 0 \end{vmatrix}$$

$$= 20$$



$$m_{AC} = \frac{0-2}{4-(-2)} = -\frac{1}{3}$$

$$m_{CD} = 3 \quad (AC \perp CD)$$

$$y-0 = 3(x-4)$$

$$CD \Rightarrow y = 3x - 12$$

$$A_{\triangle ACD} = \frac{1}{2} \begin{vmatrix} -2 & 4 & x & -2 \\ 2 & 0 & y & 2 \end{vmatrix} = 20$$

$$|0 + 4y + 2x - 8 - 0 + 2y| = 40$$

$$|2x + 6y - 8| = 40$$

$$2x + 6y - 8 = \pm 40$$

$$x + 3y - 4 = \pm 20$$

$$x + 3y = 4 \pm 20$$

$$x + 3y = 24 \text{ or } x + 3y = -16$$

$$\text{When } x + 3y = 24,$$

$$x + 3(3x - 12) = 24$$

$$x + 9x - 36 = 24$$

$$10x = 60$$

$$x = 6$$

$$y = 3(6) - 12$$

$$= 6$$

$$\therefore D(6, 6) \#$$

$$\text{When } x + 3y = -16$$

$$x + 3(3x - 12) = -16$$

$$x + 9x - 36 = -16$$

$$10x = 20$$

$$x = 2$$

$$y = 3(2) - 12$$

$$= -6$$

$$\therefore D(2, -6) \#$$

11.4

$$1. A_{OABC} = \frac{1}{2} \begin{vmatrix} 0 & 1 & 5 & 4 & 0 \\ 0 & 6 & 4 & 3 & 0 \end{vmatrix}$$

$$= \frac{1}{2} |0 + 4 + 15 + 0 - 0 - 30 - 16 - 0|$$

$$= \frac{1}{2} (-27)$$

$$= 13.5$$

~~11.5~~
~~1. $3(3) + 1$~~
~~4~~

11.5

$$1. \quad \begin{array}{ccc} & 2 & 1 \\ A(1, -2) & B(3, m) & P(n, 1) \end{array}$$

$$AP : BP = 3 : 1$$

$$(AB + BP) : BP = 3 : 1$$

$$AB : BP = 2 : 1$$

$$\left(\frac{2n+1}{3}, \frac{2(1)+(-2)}{3} \right) = (3, m)$$

$$\frac{2n+1}{3} = 3$$

$$2n+1 = 9$$

$$2n = 8$$

$$n = 4$$

$$\frac{2-2}{3} = m$$

$$m = 0$$

$$\therefore m = 0, n = 4 \quad \#$$

$$1. (i) A_{\triangle ABC} = \frac{1}{2} \begin{vmatrix} 1 & 2 & 3 \\ 8 & 5 & 6 \\ 1 & 8 & 6 \end{vmatrix} = \frac{1}{2} (1 \cdot 5 \cdot 6 + 2 \cdot 6 \cdot 1 + 3 \cdot 1 \cdot 8 - 1 \cdot 8 \cdot 6 - 2 \cdot 1 \cdot 6 - 3 \cdot 8 \cdot 1)$$

$$= \frac{1}{2} (30 + 12 + 24 - 48 - 12 - 24)$$

$$= \frac{1}{2} (4)$$

$$= 2 \text{ units}^2 \#$$

$$(ii) |AB| = \sqrt{(2-1)^2 + (5-8)^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

$$|BC| = \sqrt{(6-5)^2 + (3-2)^2}$$

$$= \sqrt{2}$$

$$|CA| = \sqrt{(3-1)^2 + (6-8)^2}$$

$$= \sqrt{8}$$

$$\cos \angle B = \frac{BC^2 + AB^2 - CA^2}{2 \cdot BC \cdot AB}$$

$$= \frac{2 + 10 - 8}{2 \cdot \sqrt{2} \cdot \sqrt{10}}$$

$$= \frac{4}{2 \cdot \sqrt{2} \cdot \sqrt{10}}$$

$$= \frac{4}{4\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}$$

$$\angle B = 63.43^\circ \#$$

$$(iii) \begin{array}{ccc} & 2 & 1 \\ B(2,5) & & A(1,8) & & D(x,y) \end{array}$$

$$BD = 3AD$$

$$BA + AD = 3AD$$

$$BA = 2AD$$

$$\frac{BA}{AD} = 2$$

$$BA : AD = 2 : 1$$

$$\left(\frac{2x+2}{3}, \frac{2y+5}{3} \right) = (1, 8)$$

$$\frac{2x+2}{3} = 1$$

$$\frac{2y+5}{3} = 8$$

$$2x+2 = 3$$

$$2y+5 = 24$$

$$x = \frac{1}{2}$$

$$y = \frac{19}{2} = 9\frac{1}{2}$$

$$\therefore D\left(\frac{1}{2}, 9\frac{1}{2}\right) \#$$