Solution Book of Mathematic

Ssnior 2 Part I

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Contents

Chapter 15

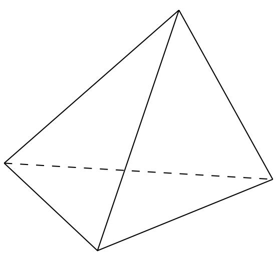
Solid Geometry, Longitude and Latitude

15.1 Solid Geometry

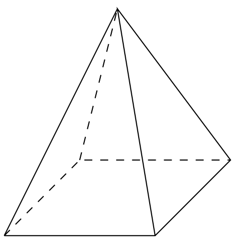
Polyhedron

A polyhedron is a solid bounded by a finite amount of flat polygon, and each side of the polygons must be the common edge of two polygons. Polyhedron can be classified into tetrahedron, pentahedron, hexahedron, etc. based on the number of flat surfaces, aka the *faces* of the polyhedron. The common side of two faces of a polyhedron is called an edge, and the common vertex of three edges is called an *apex*.

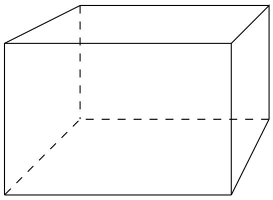
Besides, the angles formed by the faces intersecting at the same apex are called *polyhedral angles* or *solid angles*. The line segment connecting two apexes at different faces is called a *diagonal*.



Tetrahedron



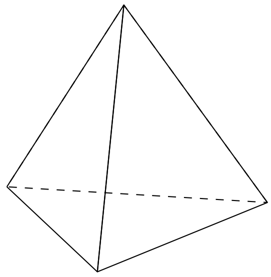
Pentahedron



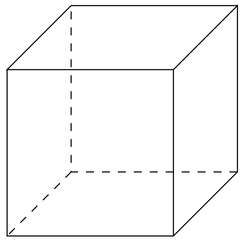
Hexahedron

Regular Polyhedron

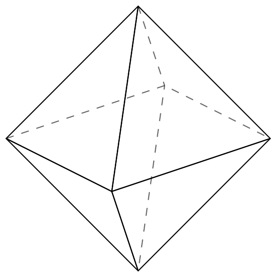
A *regular polyhedron* is a polyhedron with all faces being regular polygons, and all polyhedral angles being equal. The regular polyhedron can be classified into 5 types: *regular tetrahedron*, *regular octahedron*, *regular hexahedron*, *regular dodecahedron* and *regular icosahedron*.



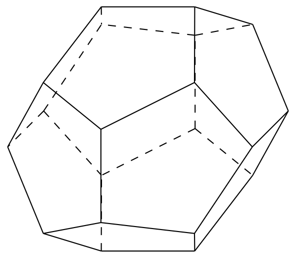
Regular Tetrahedron



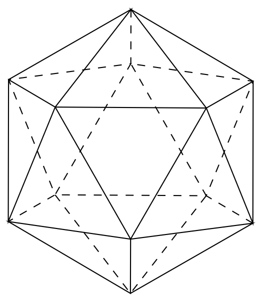
Regular Hexahedron



Regular Octahedron



Regular Dodecahedron

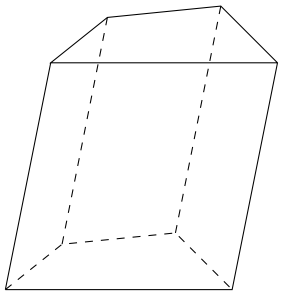


Regular Icosahedron

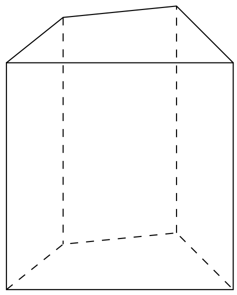
Prism

If two faces of a polyhedron are parallel, while the other faces intersect in sequence to form parallel lines, then the polyhedron is called a *prism*. The two faces which are parallel to each other are called the *bases of the prism*, and the other faces are called the *lateral faces of the prism*. The common sides that two adjacent lateral faces share is called the *lateral edges of the prism*. The distance between two bases is called the *height of the prism*.

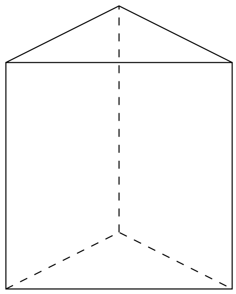
Prism with lateral edges that aren't parallel to each other are called *oblique prism*; prism with lateral edges that are parallel to each other are called *right prism*; regular prism with regular bases are called *regular prism*.



Oblique Prism

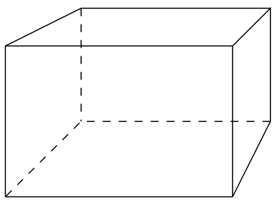


Right Prism

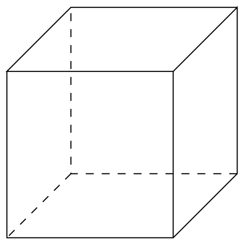


Regular Prism

Prism with bases of parallelogram are called *parallelepiped*. Parallelepiped with lateral edges that are parallel to each other are called *right parallelepiped*. Right parallelepiped with regular bases are called *cuboid*, and a cuboid with equal width, height, and depth is called a *cube*.



Cuboid

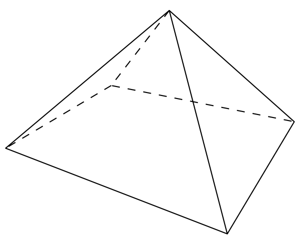


Cube

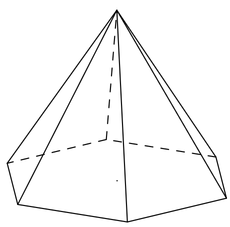
Pyramid

If a polyhedron has a polygonal base and all its lateral faces are triangles that shares a common apex, then the polyhedron is called a *pyramid*.

If the foot point of a pyramid is the centre of its base, then the pyramid is called a *right pyramid*. If the base of a right pyramid is a regular polygon, then the pyramid is called a *regular pyramid*.



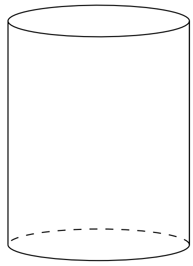
Right Pyramid



Regular Pyramid

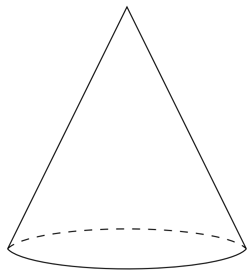
Right Circular Cylinder

A *right circular cylinder* is the solid of revolution generated by rotating a rectangle about one of its sides.



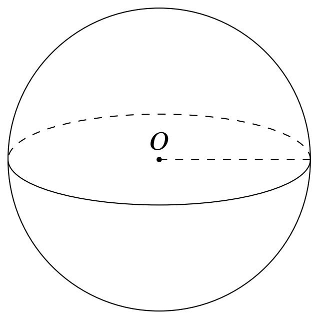
Right Circular Cone

A *right circular cone* is the solid of revolution generated by rotating a right-angled triangle about one of its sides.



Sphere

The surface of revolution generated by rotating a semicircle about its diameter is called a *spherical surface*, and the solid covered by it is called a *sphere*.

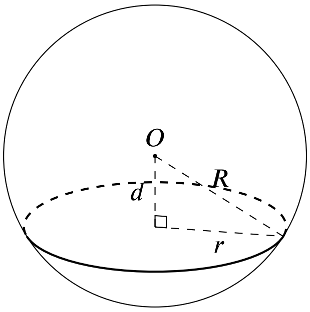


If the circle is cut with a plane, the plane has the following properties:

1. The line joining the centre of the sphere to the centre of the plane are perpendicular to the plane.

2. The distance of the plane from the centre of the sphere d, the radius of the sphere R and the radius of the plane r has the following relation:

$$r = \sqrt{R^2 - d^2}$$

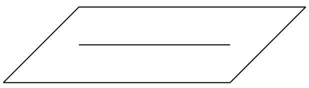


The circle cut by a plane passing through the centre of the sphere is called a *great circle*; the circle cut by a plane that does not pass through the centre of the sphere is called a *small circle*.

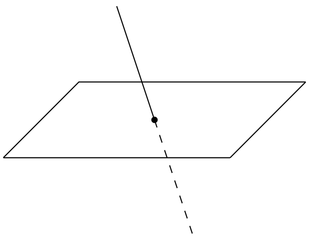
15.2 Angle Formed by Planes and Straight Lines

There are three types of positional relationship between a plane and a straight line:

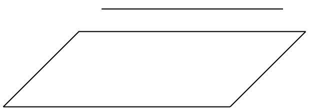
1. The line is on the plane



2. The line only intersects the plane at one point



3. The line does not intersect the plane



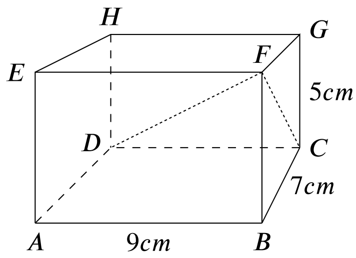
The angle formed by a line and the orthoprojection of the line on the plane is called *the angle formed by the line and the plane*. This angle represents the inclination of the line with respect to the plane, thus it is called *the tilt angle of the line with respect to the plane*.

15.2.1 Practice 1

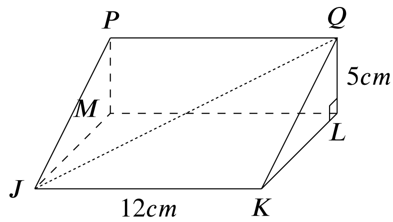
1. In the diagram below, $AB = 9cm$, $BC = 7cm$, $CG = 5cm$. Find:

   (a) The angle formed by line CF and plane $GHDC$.

   (b) The angle formed by line DF and plane $EFGH$.



2. The diagram below shows a right prism, its base KQL is a right-angled triangle, $JKLM$ is a square. Given that $JK = 12cm$, $LQ = 5cm$, find the angle formed by line JQ and plane $PQLM$.

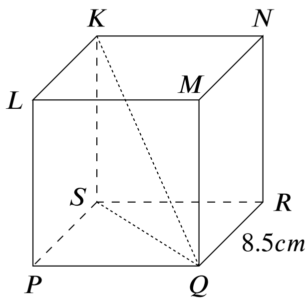


15.2.2 Exercise 17.2

1. The diagram below shows a cube with side length of $8.5cm$. Find:

   (a) The angle formed by line QS and plane $MNRQ$.

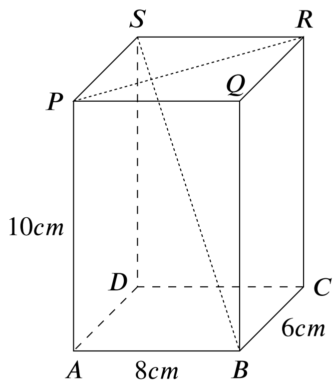
   (b) The angle formed by line KQ and plane $PQML$.



2. THe diagram below shows a cuboid, $AB = 8cm$, $BC = 6cm$, $AP = 10cm$. Find:

   (a) The length of PR.

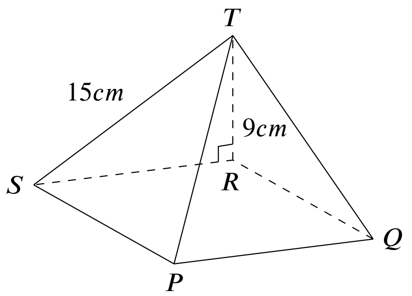
   (b) The angle formed by line SB and plane $AQQB$.



3. The diagram below shows a pyramid. Given that its base $PQRS$ is a square, TR is perpendicular to the base, $TS = 15cm$, $TR = 9cm$. Find:

   (a) The length of RS.

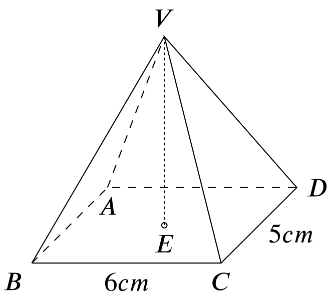
   (b) The angle formed by line PT and plane $PQRS$.



4. The diagram below shows a right pyramid with height of $8cm$, its base is a rectangle, E is the foot point from V to the base. Given that $CD = 5cm$, $BC = 6cm$. Find:

   (a) The angle formed by line VA and line VE.

   (b) The angle formed by line VC and plane $ABCD$.

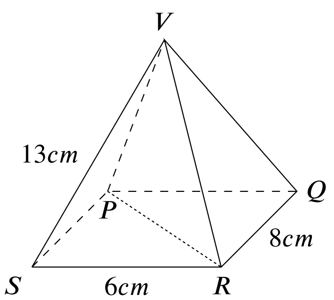


5. The diagram below shows a right pyramid, its base $PQRS$ is a regtangle. Given that $SR = 6cm$, $QR = 8cm$, $VS = 13cm$. Find:

   (a) The length of PR.

   (b) The height of the pyramid.

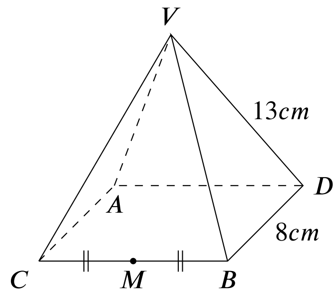
   (c) The angle of the line VP and plane $PQRS$.



6. The diagram below shows a regular pyramid, the length of its lateral edge is $12cm$, its base $ABCD$ is a square with side length of $8cm$, M is the midpoint of BC. Find:

   (a) The angle formed by the lateral edge and the base of the pyramid.

   (b) The angle formed by line VM and the base of the pyramid.

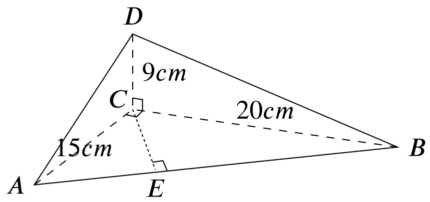


7. In the pyramid shown below, ΔABC is a right-angled triangle, CD is perpendicular to plane ABC, CE is perpendicular to AB. Given that $AC = 15cm$, $BC = 20cm$ and $CD = 9cm$. Find:

   (a) The length of CE.

   (b) $\angle CDE$.

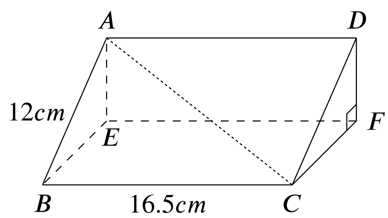
   (c) The angle formed by line AD and plane ABC.



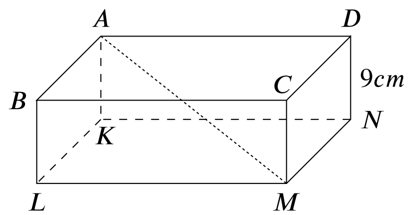
8. The diagram below shows a right prism, its base CDF is a right-angled triangle. Given that $BC = 16.5cm$ and $AB = 12cm$. Assume that $CF = 2DF$, find:

   (a) The angle formed by line AB and plane $BCFE$.

   (b) The angle formed by line AC and plan $BCFE$.



9. The diagram below shows a cuboid with volume of $300cm^3$. Given that $AD = 2DC$ and $DN = 9cm$. Find the angle formed by line AM and plane $KLMN$.

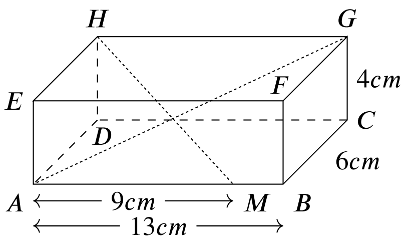


10. The diagram below shows a cuboid. Given that $AB = 13cm$, $BC = 6cm$, $CG = 4cm$. M is a point on AB, $AM = 9cm$. Find:

    (a) The angle formed by line HM and plane $ABCG$.

    (b) The angle formed by line HM and plane $HDAE$.

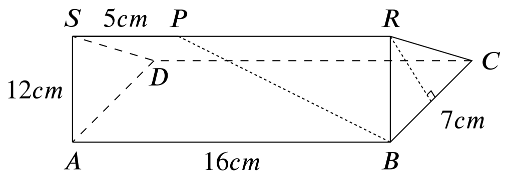
    (c) The angle formed by line AG and plane $CDHG$.



11. The diagram below shows a regular prism, its bases ADS and BCR are equiliteral triangles. Given that $AB = 16cm$, $BC = 7cm$, $SP = 5cm$. Find:

    (a) The length of BP.

    (b) The angle formed by line BP and plane $ABCD$.

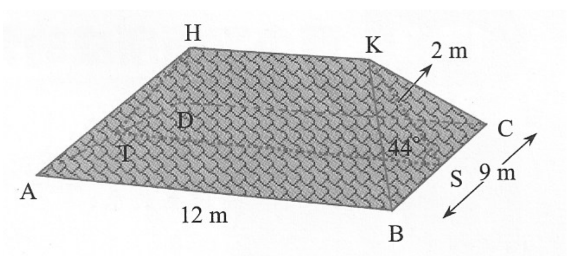


12. The diagram below shows a roof, HK is the ridge of the roof, its edges HA, HD, KB, KC are euqal in length. Both of the planes HAD and KBC form a 44^o angle with plane $ABCD$. Given that S and T are the midpoints of BC and AD respectively. Find:

    (a) The distance from line HK to plane $ABCD$.

    (b) The length of HK.

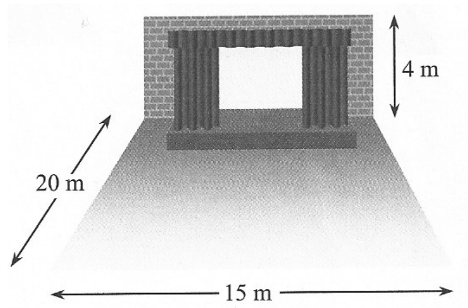
    (c) The angle formed by line HA and plane $ABCD$.



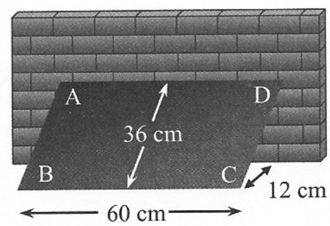
13. The length, width and height of a hall are $20m$, $15m$, and $4m$ respectively. Find:

    (a) The length of the diagonal of the hall.

    (b) The angle formed by the diagonal and the floor of the hall.



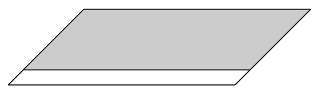
14. In the diagram below, $ABCD$ represents a rectangular plank with length and width of $60cm$ and $36cm$ respectively, its base BC is on the ground and the top of it lies on the wall. Assume that the distance between BC and the corner of the wall is $12cm$, find the angle formed by the diagonal BD of the plank and the ground.



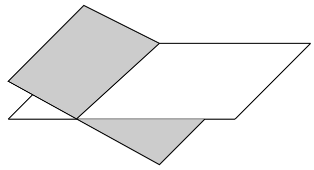
15.3 Angle Formed by Two Planes

There are three types positional relationship between two planes:

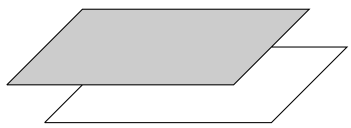
1. Two planes coincide with each other.



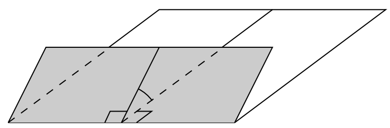
2. Two planes intersect with each other at a line.



3. Two planes are parallel to each other and do not intersect with each other.



Two non-parallel planes intersect with each other at a line, the line is called the *common edge*. At any point on the common edge, draw a line perpendicular to the common edge on each plane, the acute angles formed by these two perpendicular lines are called *the angle formed by the two planes*.

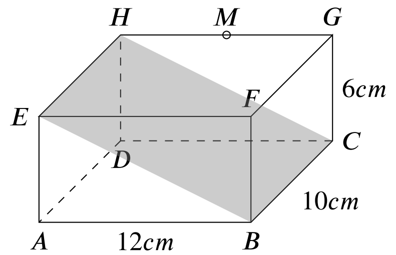


15.3.1 Practice 2

1. The diagram below shows a cuboid with length of $12cm$, width of $10cm$ and height of $6cm$.

   (a) Find the angle formed by plane $EBCH$ and plane $ABCD$.

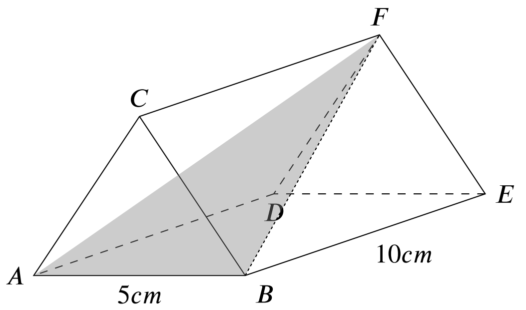
   (b) Assume that M is a point on HG, find the angle formed by plane MAB and plane $ABCD$.



2. The diagram below shows a regular prism, its bases ABC and DEF are equilateral triangles with side length of $5cm$. Given that the height of the prism is $10cm$, find:

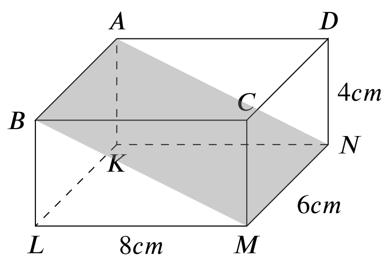
   (a) The length of BF.

   (b) The angle formed by plane ABF and plane ABC.

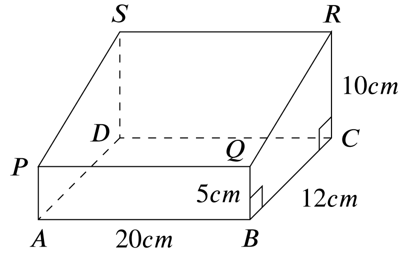


15.3.2 Exercise 17.3

1. The diagram below shows a cuboid with length of $8cm$, width of $6cm$ and height of $4cm$. Find the angle formed by plane $ABMN$ and $KLMN$.



2. In the right prism shown below, $ABCD$ is a rectangle with length of $20cm$ and width of $12cm$, $BCRQ$ is a trapezoid, $\angle QBC$ and $\angle RCB$ are both right angles, $BQ = 5cm$, $CR = 10cm$. Find the angle formed by plane $PQRS$ and plane $ABCD$.

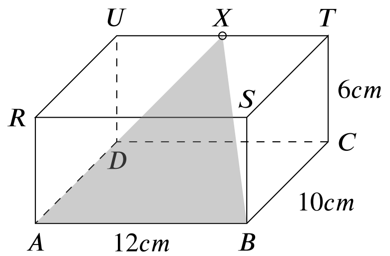


3. The diagram below shows a cuboid, $AB = 8cm$, $BC = 6cm$, $CT = 5cm$, X is the midpoint of TU. Find:

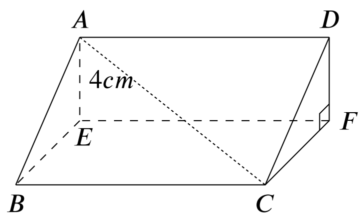
   (a) The angle formed by plane XAB and plane $ABCD$.

   (b) The angle formed by plane $BCUR$ and plane $ADUR$.

   (c) The angle formed by plane $ABTU$ and plane $ABCD$.



4. The diagram below shows a right pyramid, its bases ABE and DCF are right-angled triangles. Given that $AE = 4cm$, $BE = \frac{2}{3}EF$, $EF = 4DF$, find the angle formed by plane $ABCD$ and plane $BCFE$.

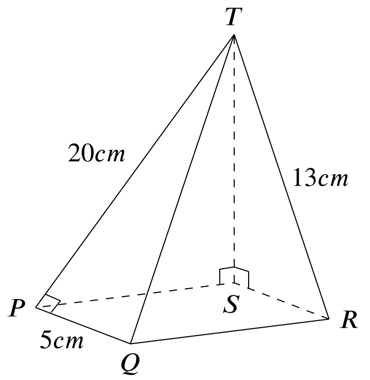


5. In the pyramid shown below, PQT, SPT, and SRT are all right-angled triangles, $PQRS$ is a triangle. Given that $PQ = 5cm$, $RT = 13cm$, $PT = 20cm$. Find:

   (a) The height of the prism.

   (b) The angle formed by line TQ and plane QST.

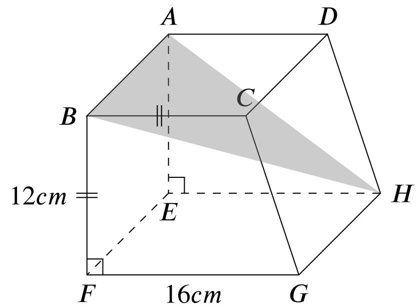
   (c) The angle formed by plane RST and PQT.



6. The diagram below shows a right prism, its base $BCGF$ is a trapezoid, $BC = BF = 12cm$, $FG = 16cm$. The lateral face $EFGH$ is a square, and is perependicular to another lateral face $ABFE$. Find:

   (a) The angle formed by plane $CDHG$ and plane $EFGH$.

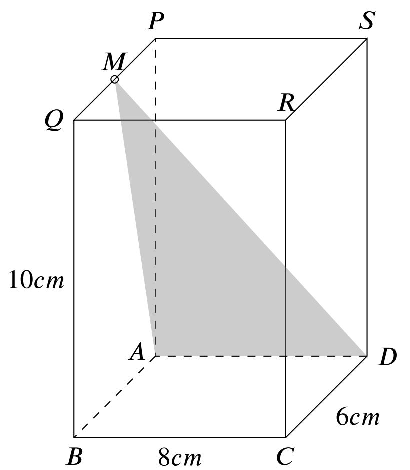
   (b) The angle formed by plane ABH and plane $ABFE$.



7. In the rectangle shown below, $BC = 8cm$, $CD = 6cm$, $BQ = 10cm$. Given that M is the midpoint of PQ. Find:

   (a) The angle formed by line MD and plane $PQBA$.

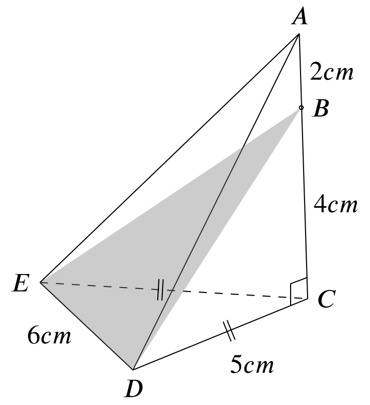
   (b) The angle formed by plane AMD and plane $ABCD$.



8. The diagram below shows a pyramid with an isoceles triangle base. Given that $CD = CE = 5cm$, $ED = 6cm$, ACD is a right-angled triangle, B is a point on AC, $AD = 2cm$, $BC = 4cm$. Find:

   (a) The angle formed by plane BDE and plane CDE.

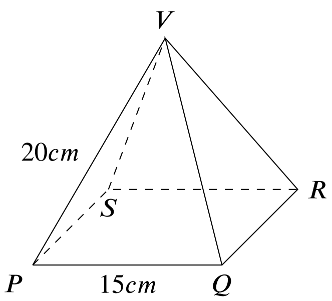
   (b) The angle formed by the plane ADE and CDE.



9. The diagram below shows a regular pyramid with a square base. Given that $PQ = 15cm$, $PV = 20cm$. Find:

   (a) The angle formed by linee PV and plane $PQRS$.

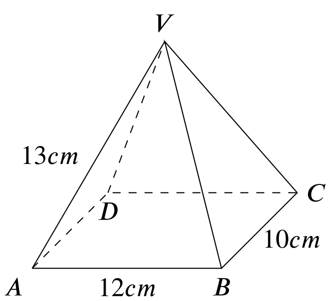
   (b) The angle formed by the lateral faces and the base of the pyramid.



10. The diagram below shows a right pyramid with lateral edges of $13cm$. Its base $ABCD$ is a rectangle with length of $12cm$ and width of $10cm$. Find:

    (a) The angle formed by plane VBC and plane $ABCD$.

    (b) The angle formed by plane VCD and plane $ABCD$.

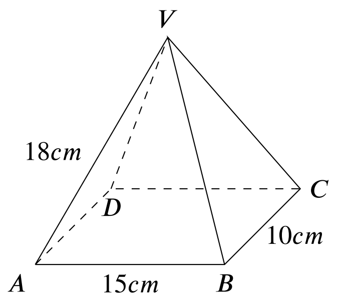


11. The diagram below shows a right pyramid with lateral edges of $18cm$, its base $ABCD$ is a rectangle with length of $15cm$ and width of $10cm$. Find:

(a) The height of the pyramid.

(b) The angle formed by plane VAB and plane $ABCD$.

(c) The angle formed by plane VBC and plane VAD.

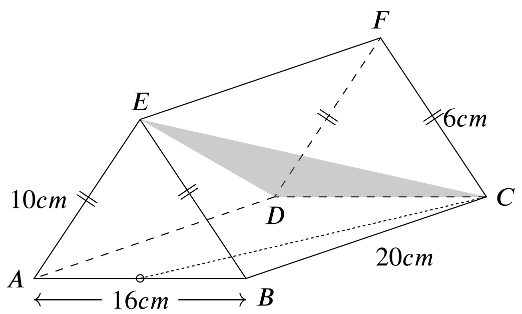


12. The diagram below shows a right prism with isoceles triangle bases. The side length and base length of the triangle base are $10cm$ and $16cm$ respectively, the height of the prism is $20cm$. Given that P is the midpoint of AB. Find:

    (a) The length of PC.

    (b) The angle formed by line EC and plane $ABCD$.

    (c) The angle formed by plane DCE and plane $ABCD$.



15.4 Longitude and Latitude

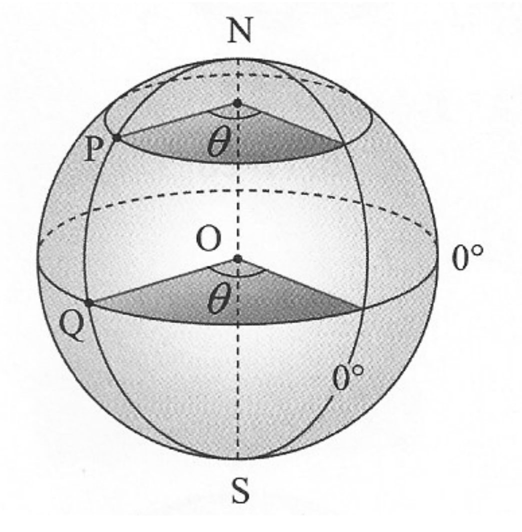
The earth is approximately spherical in shape, its radius is about $6,370km$, and its axis is a line that passes through the north (N) and south (S) poles. The earth rotating around its axis once is called a day, and the earth rotating around the sun once is called a year.

Any point on the earth's surface can be identified by two angles, the first is the angle between the point and the equator, called the *latitude* of the point, and the second is the angle between the point and the prime meridian, called the *longitude* of the point.

Longitude and Lines of Longitude

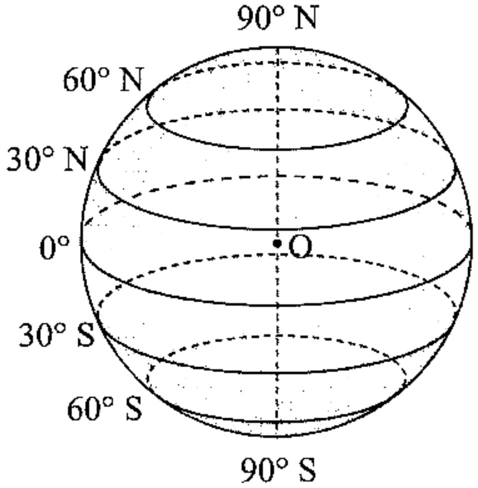
The two semicircles that are formed by the intersection of the earth's surface with the plane that passes through the north and south poles are called the *lines of longitude*, also called *meridians*. The lines of longitude that passes through the *Greenwich Observatory* in England are considered as 0° longitude, called the *Greenwich Meridian* or *prime meridian*.

The angle between the Greenwich Meridian and the line of longitude that passes through the point P is called the *longitude of P*. There are 360 degrees of longitude (+180° eastward and −180° westward.). The prime meridian divides the world into the Eastern Hemisphere and the Western Hemisphere. $180°E$ and $180°W$ coincide with each other at the same line of longitude, called the 180^{th} *Meridian* or *Antimeridian*.

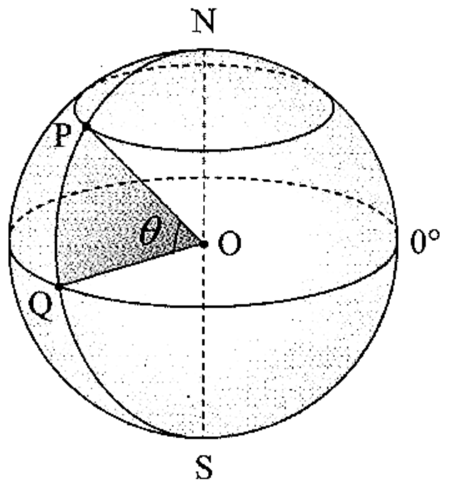


Latitude and Parallels of Latitude

The lines of latitude are the circles that are perpendicular to the plane that passes through the north and south poles. The *equator* is the one and only great circle among the parallels of latitude.

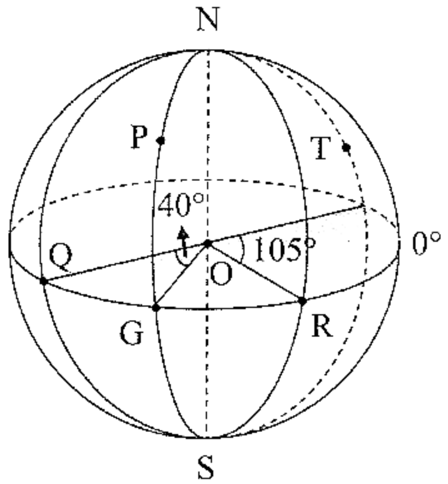


The angle between the equator and the line of latitude that passes through the point P is called the *latitude of P*. There are 180 degrees of latitude (+90° northward and −90° southward). The equator divides the world into the Northern Hemisphere and the Southern Hemisphere.

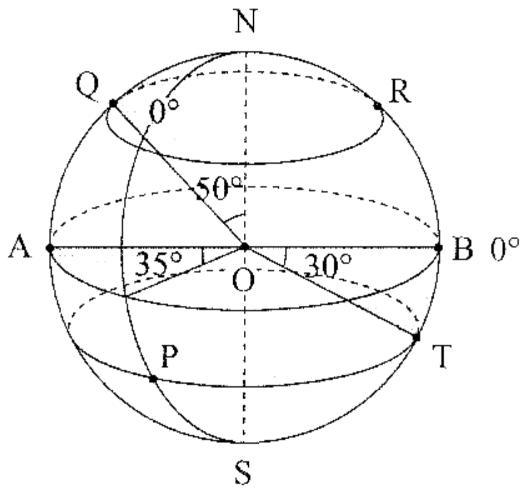


15.4.1 Practice 3

1. In the diagram below, NGS is the prime meridian, O is the centre of the earth. Find the longitude of locations P, Q, R and T.

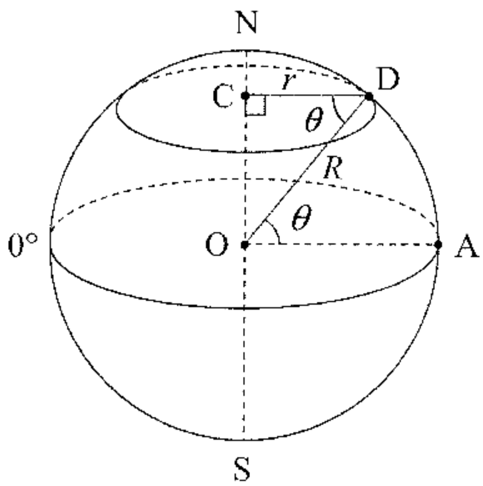


2. In the diagram below, O is the centre of the earth, location A and B are on the equator. Find the location of P, Q, R and T.



Radius of the Parallel of Latitude

Let R be the radius of the earth, r be the radius of latitude θ, then $r = R\cos\theta$.



Nautical Miles

The arc length corresponding to $1' (= \frac{1}{60}^\circ)$ of the great circle on earth is called a *nautical mile* $(1NM)$, that is, $1NM = \frac{1}{60 \times 360} \times 2\pi \times 6370km = 1.853km$.

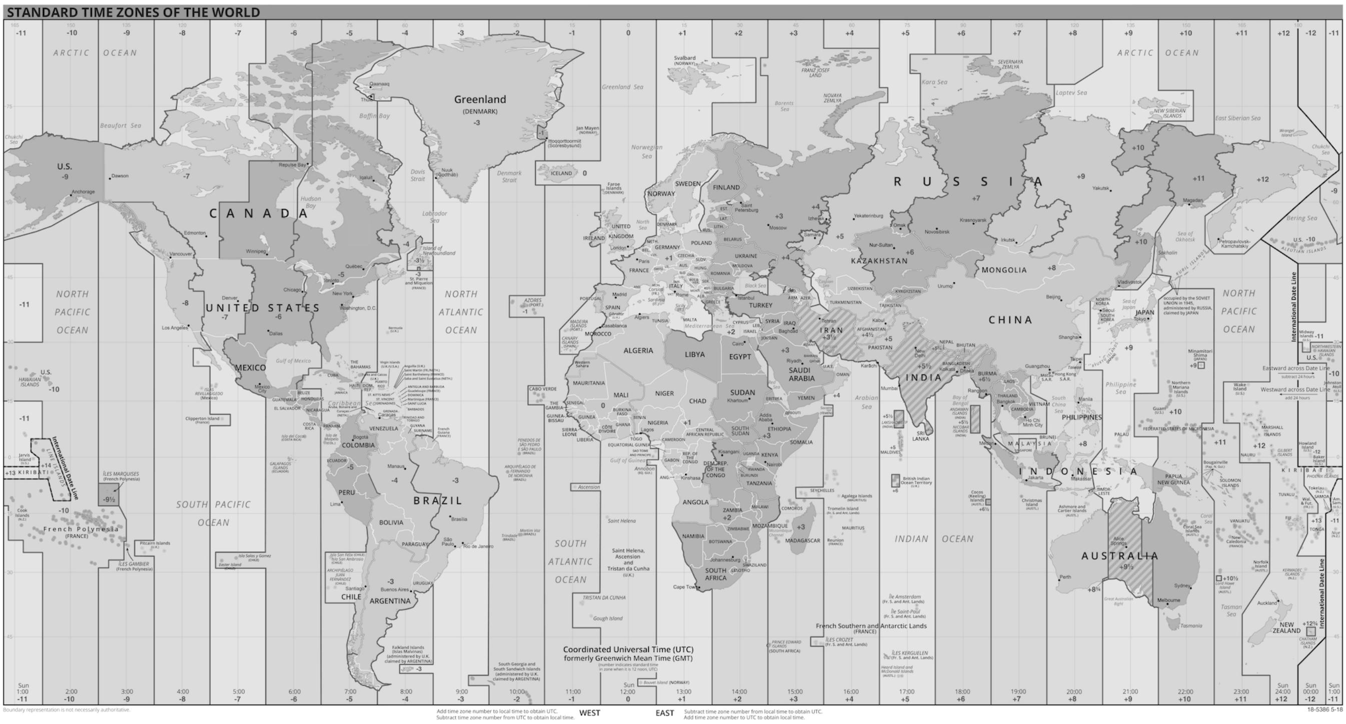
Time Difference and Longitude

The time is calculated by the rotation of the earth around its axis. The earth rotates around its axis from west to east once in $24h$. That is, the earth rotates $15°$ in $1h$. Thus, the time difference between two locations on the earth is equal to the difference of their longitudes. Thus, the time difference is $1hr$ per $15°$ of longitude difference.

1. **Local Time**
   The local time is the time at a location on the earth. The local time for any location on the same line of longitude is the same.

2. **Standard Time**
   Back in the year 1844, International Meridian Conference was held in Washington DC. The conference decided to divide the world into 24 time zones base on the Greenwich Meridian, called the *Greenwich Meridian Time (GMT)*. There is zero time offset $7.5°$ eastward and $7.5°$ westward of the Greenwich Meridian. The time offset is $1hr$ per $15°$ of longitude difference. All places in the same time zone share the same local time with the location located on the line of longitude that passes through the centre of the time zone, called the *standard time* or *zone time*.

   When entering a new time zone from the east, the local time is advanced by $1hr$ per $15°$ of longitude difference. When entering a new time zone from the west, the local time is delayed by $1hr$ per $15°$ of longitude difference.



15.5 Distance of Two Location on the Same Line of Longitude

The distance of two location on the same line of longitude is the arc length corresponding to the difference of their latitudes. Given two location P and Q on the same line of longitude, according to the definition of nautical mile, the distance between P and Q can be acquired by the arc length of PQ. That is, $\overset{\frown}{PQ} = \theta \times 60NM$, where θ is the difference of their latitudes.

15.5.1 Practice 4

1. Given that location A and B are on the same line of longitude. Base on the following longitude, find the distance between A and B (Express your answer in nautical miles):
   (a) $A(50°N)$, $B(75°N)$
   (b) $A(0°)$, $B(42°S)$
   (c) $A(43°N)$, $B(38°S)$
2. Given that location P and Q are on the same line of longitude. The distane between two locations is $1000NM$, P is located at $7°30'$ north of the equator. Base on the following criteria, find the latitude of Q:
   (a) Q is located at the north of P
   (b) Q is located at the south of P

15.5.2 Exercise 17.5

1. Given that A and B are on the same line of longitude. Base on the following difference of latitude of two lo-

cations, find the distance between A and B (Express your answer in nautical miles):

(a) $\theta = 39°$

(b) $\theta = 80°30'$

(c) $\theta = 64°20'$

2. Given that A and B are on the same line of longitude. Base on the following distance between two locations, find the difference of latitude of A and B (Round your answer to the nearest minute):

   (a) $700NM$

   (b) $318NM$

   (c) $3450NM$

3. Find the distance between two locations along the same line of longitude:

   (a) $A(21°S, 110°E)$, $B(33°S, 110°E)$

   (b) $X(38°N, 40°W)$, $Y(19°N, 40°W)$

   (c) $E(34°45'S, 80°E)$, $F(0°, 80°E)$

   (d) $P(18°15'N, 90°W)$, $Q(43°30'N, 90°W)$

   (e) $T(15°30'N, 120°E)$, $M(24°30'N, 120°E)$

4. Location X and Y are on the same line of longitude, the distane between them is $400NM$. Find the difference of latitude of X and Y.

5. Location P and Q are on the same line of longitude, and their distance along the line of longitude is $600NM$, find the difference between their latitude.

6. X city and Y city are on the same line of longitude, the latitude of X city is $2°15'$ north of the equator, the latitude of Y city is $6°$ north of the equator. Find the distance between X city and Y city (Express your answer in kilometers).

7. A plane is flying $1000km$ due north from airport $A(15°N, 115°E)$ to airport B. Find the longitude and latitude of airport B.

8. A plane is flying $1500km$ due south from airport $A(5°N, 100°E)$ to airport B. Find the longitude and latitude of airport B.

9. Find the distance from $A(18°30'S)$ to the north pole along the same line of longitude.

10. The distance between location C and D is $700NM$, C is located at $5°30'$ north of the equator. Find the latitude of D.

11. A plane takes off from $P(60°N, 60°E)$ and flies pass north pole along the great circle route to $Q(50°N, 120°W)$. Find the flying distance.

12. A ship sails from $P(50°S, 160°E)$ due north to another port $Q(30°N, 160°E)$. The sailing time is 10 days. Find the average speed of the ship. (Express your answer in NM/hr)

13. Given that PQ is the diameter of the parallel of latitude $35°S$. A plane takes off from location P, flies pass the south pole along the line of longitude, and lands at location Q after $13hr40mins$. Find the average speed of the plane for the whole flight duration. (Express your answer in NM/hr)