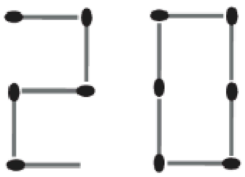
1. What is $\dfrac{20 \cdot 22}{(2+0) \cdot (2+2)}$?

   Sol.

   $$\begin{aligned}\frac{20 \cdot 22}{(2+0) \cdot (2+2)} &= \frac{20 \cdot 22}{2 \cdot 4} \\ &= \frac{440}{8} \\ &= 55 \ \textbf{(D)} \quad \square\end{aligned}$$

2. Karo has a box of matches with 30 matches. Using some of the matches she forms the number 2022. She has already formed the first two digits (see picture). How many matches will be left in the box when she has finished the number?



   Sol.

   There are 5 matches in the digit 2 and 6 matches in the digit 0. Therefore, the total number of matches required to form the number 2022 is $5 \cdot 3 + 6 = 21$.

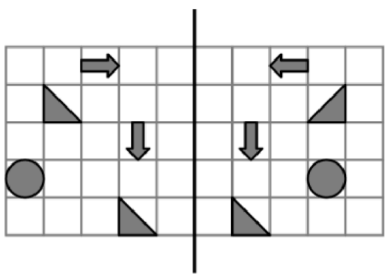
   Hence, the number of matches left in the box is $30 - 21 = 9$. **(B)** $\square$

3. An equilateral triangle with side length 12 has the same perimeter as a square with side length x. What is the value of x?

   Sol.

   The perimeter of an equilateral triangle is $3 \cdot 12 = 36$. Since the square has the same perimeter, the side length x of the square is $\dfrac{36}{4} = 9$. **(A)** $\square$

4. Various symbols are drawn on a piece of paper (see picture).



   The teacher folds the left side along the vertical line to the right.

   How many symbols of the left side are now congruent on top of a symbol on the right side?

   Sol.

   From top to bottom,

   The arrow on the left and the right side are 3 units away from the symmetry line, and they are flipped horizontally. Therefore, they are congruent when the the paper is fold.

   The triangle on the left and the right side are 4 units away from the symmetry line, and they are flipped horizontally. Therefore, they are congruent when the paper is fold.

   The arrow on the left and the right side are 2 units away from the symmetry line, and they are the same when being flipped horizontally. Therefore, they are congruent when the paper is fold.

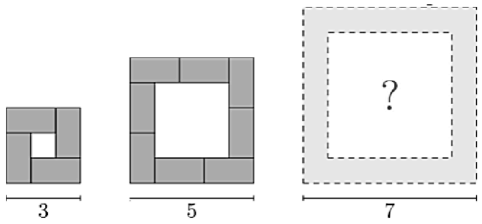
   The circle on the left side is 5 units away from the symmetry line, while the circle on the right side is 4 units away from the symmetry line. Therefore, they are not congruent when the paper is fold.

   The triangle on the left side and the right side are 2 units away from the symmetry line, but they are of the same rotation. Therefore, they are not congruent when the paper is fold.

   Hence, there are 3 congruent symbols when the paper is fold. **(C)** $\square$

5. Karin places a table of size 2×1 according to the number of participants in a meeting. The diagram shows the table arrangements from above for a small, medium and a large meeting.

   How many tables are used in a large meeting?



   Sol.

   In each arrangement, the table required is $\dfrac{n-1}{2} \cdot 4$, where n is the side length of the arrangement. Therefore, the number of tables required in the large meeting is $\dfrac{7-1}{2} \cdot 4 = 12$. **(C)** $\square$

6. I am smaller than my half and bigger than my double. The sum of me and my square is 0.

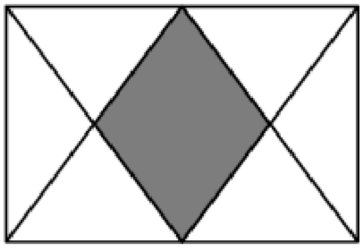
   Sol.

   Let x be the number. Then, we have the following statements:

   $$\begin{aligned}\text{I am smaller than my half} &\implies x < \frac{x}{2} \\ \text{I am bigger than my double} &\implies x > 2x \\ \text{The sum of me and my square is 0} &\implies x + x^2 = 0\end{aligned}$$

   $$\begin{aligned}\because 2x < x < \frac{x}{2} &\implies x < 0 \\ \because x + x^2 = 0 &\implies x = -1 \ \textbf{(B)} \quad \square\end{aligned}$$

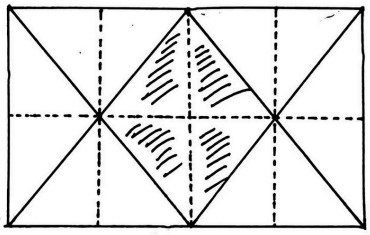
7. The midpoints of both longer sides of a rectangle are connected with the vertices (see diagram).



Which fraction of the rectangle is shaded?

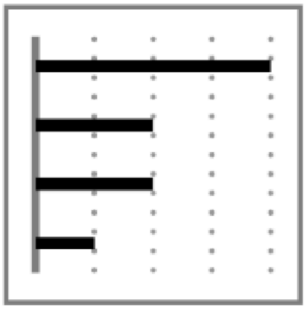
Sol.

Splitting the rectangle into eight equal parts, each part is split into two equal triangles, as shown below.



There are 16 triangles in total, while the shaded area consists of 4 triangles. Hence, the fraction of the rectangle that is shaded is $\frac{4}{16} = \frac{1}{4}$. **(B)** □

8. Sonja's smartphone displays the diagram on the right. It shows how long he has worked for four different apps in the previous week. This week he has spent only half the amount of time using two of the apps and the same amount of time as last weeks using the other two apps. Which of the following pictures could be the diagram for the current week?



Sol.

From the picture, from top to bottom, the time spent on the apps are 4, 2, 2, 1 respectively. In the options:

(A) 2 (half), 1(half), 1(half), 0.5 (half)

(B) 4, 2, 1 (half), 1

(C) 2 (half), 2, 1 (half), 1

(D) 3 (three quarter), 1 (half), 1 (half), 0.5 (half)

(E) 2 (half), 1 (half), 1 (half), 1

Hence, the correct answer is **(C)**. □

9. In the multiplication grid displayed, each white cell should show the product of the numbers on the grey cells that are in the same row and column respectively. One number is already entered. The integer x is bigger than the positive integer y. What is the value of y?

Sol.

·	x	x+1
y		
y+1		77

If $y = 6$, $y + 1 = 7$, $x + 1 = 77 \div 7 = 11$, $x = 10$.

If $y = 7$, $y+1 = 8$, $x+1 = 77 \div 8 = 9.625$, $x = 8.625$.

If $y = 8$, $y+1 = 9$, $x+1 = 77 \div 9 = 8.555$, $x = 7.555$.

If $y = 10$, $y + 1 = 11$, $x + 1 = 77 \div 11 = 7$, $x = 6$.

If $y = 11$, $y + 1 = 12$, $x + 1 = 77 \div 12 = 6.416$, $x = 5.416$.

Since $x, y \in \mathbb{Z}^+$, $x > y$, $y = 6$ is the only possible answer. **(A)** □

10. There are 5 people to choose from on a ballot paper. After counting 90% of the votes in the intermediate results looks as shown in the table. How many of the 5 people cannot win the election anymore?

Alex	Bella	Clint	Diana	Eddy
14	11	10	8	2

The number of votes counted is $14+11+10+8+2 = 45$. The number of votes haven't been counted yet is $45 \div \frac{100}{90} \cdot \frac{10}{100} = 5$.

After all the votes are counted, if all the previously uncounted votes are for the same person, then

For Alex, he will get $14 + 5 = 19$ votes, which is enough to win the election.

For Bella, she will get $11 + 5 = 16$ votes, which is enough to win the election.

For Clint, he will get $10 + 5 = 15$ votes, which is enough to win the election.

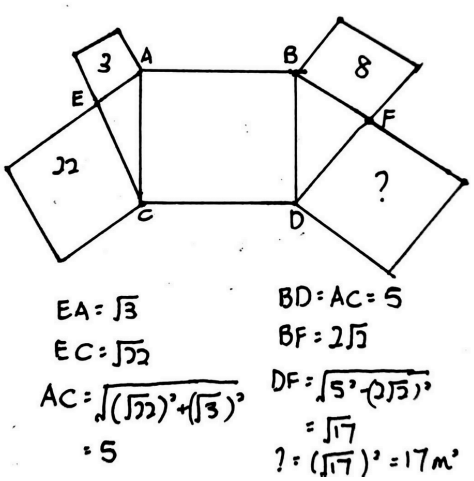
For Diana, she will get $8 + 5 = 13$ votes, which is not enough to win the election.

For Eddy, he will get $2 + 5 = 7$ votes, which is not enough to win the election.

Hence, the number of people who cannot win the election is 2. **(B)** □

11. Five squares and two right-angled triangles are placed as shown in the diagram. The number 3, 8, 22 in the squares state the size of the area in m^2. How big is the area (in m^2) of the square with the question mark?

Sol.



The area of the square with the question mark is $17m^2$. **(D)** □

12. 2022 tiles are placed in one long row. Adam removes every sixth tile. Then Beate removes every fifth of the remaining tiles.

Subsequently Cora removes every fourth of the remaining tiles.

How many tiles are left?

Sol.

For Adam, he removes $2022 \div 6 = 337$ tiles.

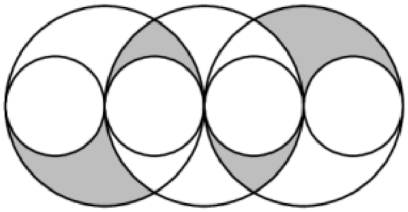
For Beate, she removes $(2022 - 337) \div 5 = 337$ tiles.

For Cora, she removes $(2022 - 337 - 337) \div 4 = 337$ tiles.

Hence, the number of tiles left is $2022 - 3 \times 337 = 1011$. **(A)** □

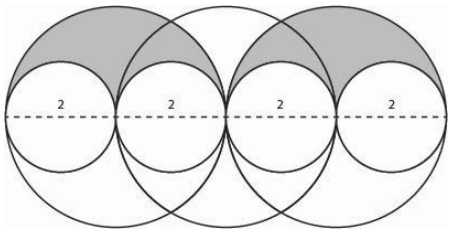
13. The diagram shows three big circles of equal size and four small circles. Each small circle touches two big circles and has radius 1.

How big is the shaded area?



Sol.

Move the bottom two shaded area to the top, and draw a line separating shape into top and bottom parts, we get the following:



Having inspected the diagram, we can see that there are two shaded region, the area of each of them is the area of a big semicircle minus 2 times the area of a small semicircle:

$$\text{Area of the small semicircle} = \frac{1}{2}\pi \times 1^2 = \frac{\pi}{2}$$

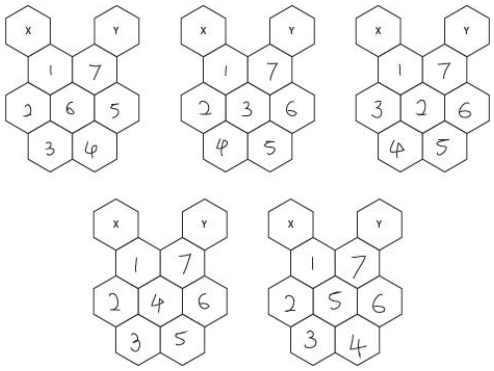
$$\text{Area of the big semicircle} = \frac{1}{2}\pi \times 2^2 = 2\pi$$

$$\text{Area of the shaded region} = 2\pi - 2 \times \frac{\pi}{2} = \pi$$

Since there are two shaded region, the total area of the shaded region is $2 \times \pi = 2\pi$. **(B)** □

14. A bee called Maja wants to hike from honeycomb X to honeycomb Y. Sje can only move from one honeycomb to the neighbouring honeycomb if they share an edge. How many, different ways are there for Maja to go from X to Y if she has to step onto every one of the seven honeycombs exactly once?

Sol.



After some manual calculation, there are 5 different ways for Maja to achieve her goal. **(D)** □

15. The sum of two positive integers is three times as big as their difference.

The product of the two numbers is four times as big as their sum.

How big is the sum of the two numbers?

Sol.

Let the two numbers be x and y, $x > y > 0$.

$$x + y = 3(x - y) \quad \cdots (1)$$
$$xy = 4(x + y) \quad \cdots (2)$$

$$(1) \Rightarrow x + y = 3x - 3y$$
$$2x - 4y = 0$$
$$x = 2y$$

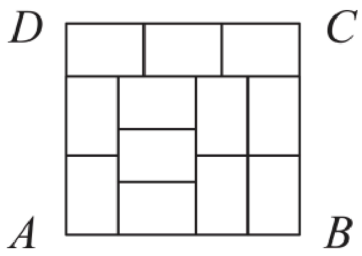
$$(2) \Rightarrow xy = 4x + 4y \quad \cdots (4)$$

Substituting $x = 2y$ into (4), we get

$$\begin{aligned}(2y)(y) &= 4(2y) + 4y \\ 2y^2 &= 8y + 4y \\ 2y^2 &= 12y \\ y^2 - 6y &= 0 \\ y(y-6) &= 0 \\ y &= 6 \ (y > 0) \\ x &= 2 \times 6 = 12 \\ x + y &= 12 + 6 = 18 \end{aligned}$$ **(E)** □

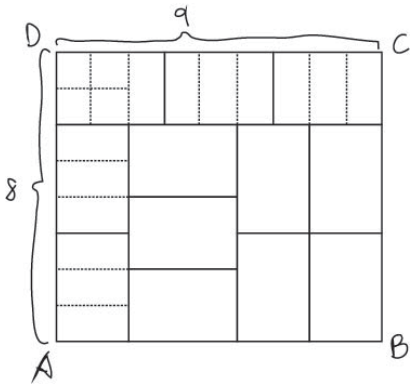
16. The rectangle $ABCD$ is made up of 12 congruent rectangles (see diagram).

How big is the ratio $\dfrac{AD}{DC}$?



Sol.

Each rectangle can be divided into three parts horizontally and two parts vertically:



Therefore, the ratio $\dfrac{AD}{DC}$ is $\dfrac{8}{9}$. **(A)**

17. A rabbit and a hedgehog enter a race against each other. The circular racecourse is $550m$ long. The starting line and the finish line are the same. The speed of the rabbit is a constant $10m/s$, the speed of the hedgehog is a constant $1m/s$. They start at the same time, but the hedgehog tries to cheat by going in the opposite direction. When the two meets, the hedgehog turns around immediately and follows the rabbit.

How many seconds after the rabbit does the hedgehog reach the finish line?

Sol. Let the distance travelled by the hedgehog before meeting the rabbit be x.

$$\begin{aligned}x + 10x &= 550 \\ 11x &= 550 \\ x &= 50m\end{aligned}$$

After the hedgehog turns around, it needs to travel $\dfrac{50m}{1m/s} = 50s$ more to reach the finish line, while the rabbit only needs to travel $\dfrac{50m}{10m/s} = 5s$ more. Hence, the hedgehog reaches the finish line $50 - 5 = 45s$ seconds after the rabbit. **(A)** □

18. The grandchildren ask their grandma how old she is. The grandma invites them to guess the age.

The first child says 75, the second says 78 and the third says 81. It turns out that one child is wrong by 1 year, one by 2 years and one by 4 years. How many possibilities are there for the age of the grandma? are there for the age of the grandma?

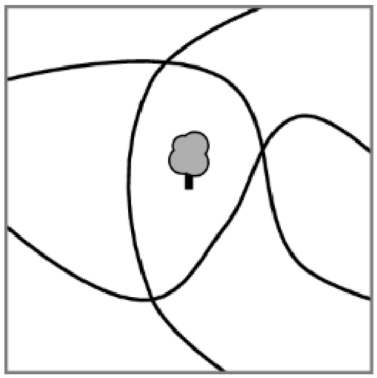
Sol.

If the first child says wrong by 2 years, the actual age of the grandma is 77. Which means, the second child is wrong by 1 year, while the third child is wrong by 4 years, which is valid.

If the first child says wrong by 4 years, the actual age of the grandma is 79. Which means, the second child is wrong by 4 years, while the third child is wrong by 2 years, which is valid.

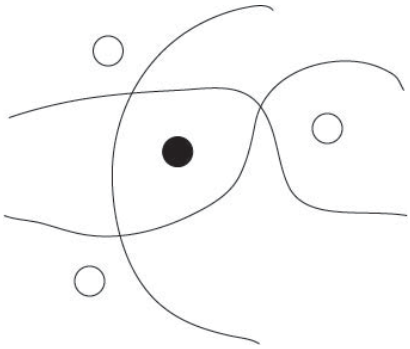
The other possibilities are not valid. Hence, there are 2 possibilities for the actual age of grandma. **(C)** □

19. There are three paths running through our park in the city (see diagram).



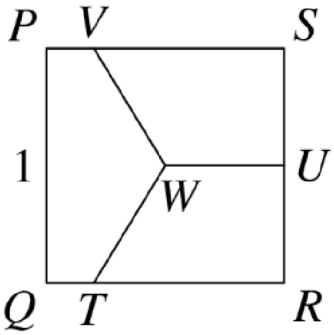
What is the minimum number of trees that have to be planted additionally so that there are the same number of trees on either side of each path?

Sol.



With some manual calculation, we can see that the minimum number of trees that have to be planted is 3. **(C)** ∎

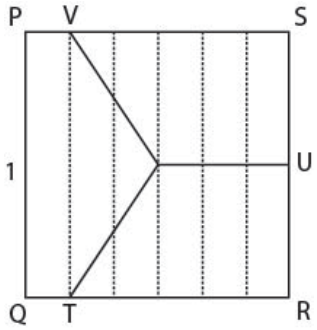
20. The diagram shows a square $PQRS$ with side length 1. The point U is the midpoint of the side RS and the point W is the midpoint of the square.



The three line segments, TW, UW, and VW split the square into three equal big areas.

How long is the line segment SV?

Sol.



The square can be divided horizontally into 6 equal parts. Those, the length of the line segment SV is $\frac{5}{6}$. **(E)** ∎