Mathematics

Senior 3 Part I

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Actual time spent: XX days

Introduction

Why this book?

Disclaimer

Acknowledgements

Contents

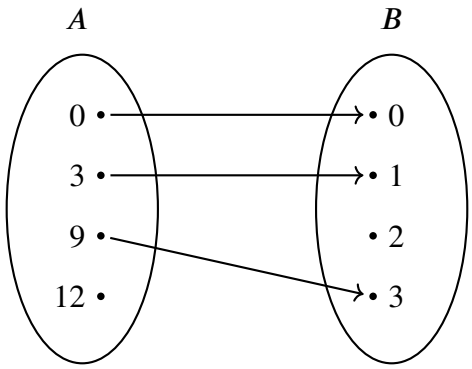
Exercise 22.1

1. Express the mapping from set A to set B using venn diagram, and determine which of the following mappings are functions.

	Set A	Set B	Mapping
(a)	{0, 3, 9, 12}	{0, 1, 2, 3}	Divide by 3
(b)	{-2, -1, 0, 1, 2}	{0, 1, 4, 9, 16}	Power of 4
(c)	{-2, -1, 0, 1, 2}	{0, 1, 4}	Square
(d)	{30°, 45°, 60°}	$\left\{\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}\right\}$	Sine
(e)	{-1, 0, 1, 2}	{-1, 0, 1}	Cube

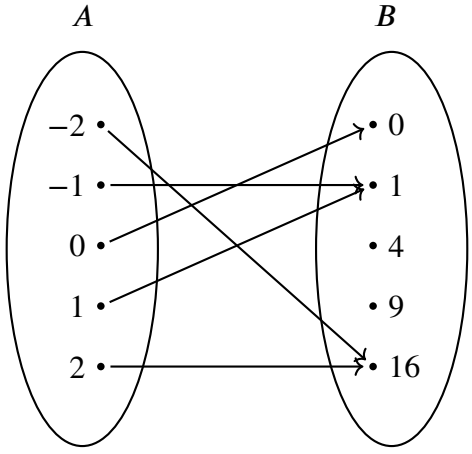
Sol.

(a)



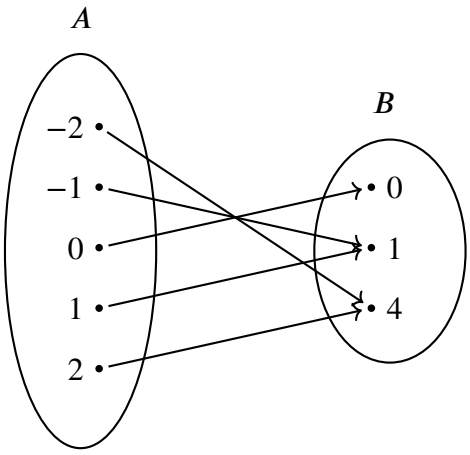
Since $12 \in A$ has no image in B, this mapping is not a function.

(b)



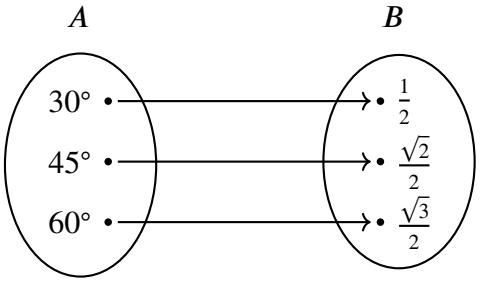
Since each element in A has an image in B, this mapping is a function.

(c)



Since each element in A has an image in B, this mapping is a function.

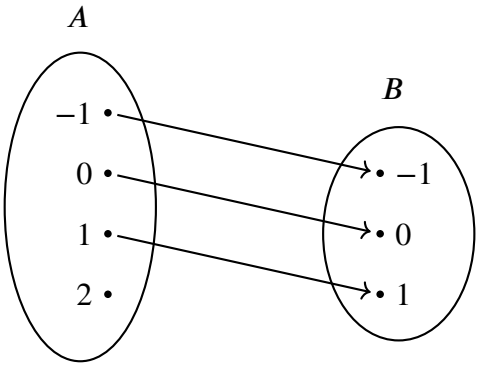
(d)



Since each element in A has an image in

B, this mapping is a function.

(e)



Since $2 \in A$ does not have an image in B, this mapping is not a function.

2. Let function $f(x) = 3x^2 + 1$.

(a) Find the image of the following elements:

i. -3

Sol.

$$f(-3) = 3(-3)^2 + 1$$
$$= 28$$

ii. -2

Sol.

$$f(-2) = 3(-2)^2 + 1$$
$$= 13$$

iii. 0

Sol.

$$f(0) = 3(0)^2 + 1$$
$$= 1$$

iv. 2

Sol.

$$f(2) = 3(2)^2 + 1$$
$$= 13$$

v. 5

Sol.

$$f(5) = 3(5)^2 + 1$$
$$= 76$$

(b) Find the preimage of the following elements:

i. 13

Sol.

$$13 = 3x^2 + 1$$
$$12 = 3x^2$$
$$4 = x^2$$
$$x = \pm 2$$

ii. 28

Sol.

$$28 = 3x^2 + 1$$
$$27 = 3x^2$$
$$9 = x^2$$
$$x = \pm 3$$

iii. 1

Sol.

$1 = 3x^2 + 1$

$0 = 3x^2$

$0 = x^2$

$x = 0$

iv. 0

Sol.

$0 = 3x^2 + 1$

$-1 = 3x^2$

$-\frac{1}{3} = x^2$

x is not a real no.

v. 4

Sol.

$4 = 3x^2 + 1$

$3 = 3x^2$

$1 = x^2$

$x = \pm 1$

3. Let function $g(x) = 5x - 2$. Find:

(a) $g(-2)$

Sol.

$g(-2) = 5(-2) - 2$

$= -12$

(b) $g(-1)$

Sol.

$g(-1) = 5(-1) - 2$

$= -7$

(c) $g(0)$

Sol.

$g(0) = 5(0) - 2$

$= -2$

4. Let function $f(x) = \begin{cases} 2x, & x \leq -1 \\ x-1, & -1 \leq x < 3 \\ 4x+2, & x \geq 3 \end{cases}$, find

(a) $f(-5)$

Sol.

$$f(-5) = 2(-5)$$
$$= -10$$

(b) $f(-2)$

Sol.

$$f(-2) = 2(-2)$$
$$= -4$$

(c) $f(0)$

Sol.

$$f(0) = 0 - 1$$
$$= -1$$

(d) $f(2)$

Sol.

$$f(2) = 2 - 1$$
$$= 1$$

(e) $f(10)$

Sol.

$$f(10) = 4(10) + 2$$
$$= 42$$

5. Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^4$. Find the image of -1, 0, 1, and 2 under f.

Sol.

$$f(-1) = (-1)^4 = 1$$
$$f(0) = (0)^4 = 0$$
$$f(1) = (1)^4 = 1$$
$$f(2) = (2)^4 = 16$$

6. Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$. Find the preimage of 0, 1, and 4 under f.

   In $\mathbb{R}$, which element does not have a preimage?

   Sol.

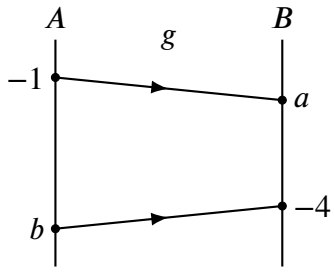
   $0 = x^4$ $\qquad$ $1 = x^4$ $\qquad$ $4 = x^4$

   $x = 0$ $\qquad$ $x = \pm 1$ $\qquad$ $x = \pm 2$

   $\because \forall x \in \mathbb{R}, f(x) \geq 0$

   $\therefore x \in \mathbb{R}^-$ does not have a preimage.

7. In the diagram below, given that function $g : A \to B$ is defined as $g : x \to 2x - 8$. Find the value of a and b.



   Sol.

   $a = 2(-1) - 8$ $\qquad$ $-4 = 2b - 8$

   $= -10$ $\qquad$ $2b = 4$

   $b = 2$

8. Using narrative form, arrow method, venn diagram, table method and graphical method, express the function $f(x) = 2x$, $x \in \{-2, -1, 0, 1, 2\}$.

   Sol.

   Narrative form:

   Let $A = \{-2, -1, 0, 1, 2\}$ and $B = \{-4, -2, 0, 2, 4\}$, f is a function from A to B, its definition is that for any element x in A, its corresponding element is $2x$ in B.

Arrow method:

$f : -2 \to -4, -1 \to -2, 0 \to 0, 1 \to 2, 2 \to 4$

Venn diagram:

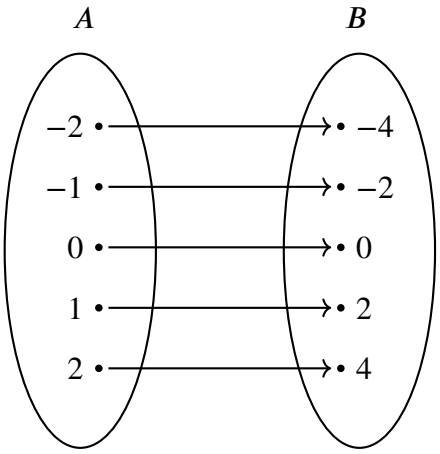


Table method:

x	-2	-1	0	1	2
$f(x)$	-4	-2	0	2	4

Graphical method: