Principle of conservation of Mechanical Energy

U78-7 Fig.U78-7 represents a frictionless track for trolley. If a trolley is set into motion from point X with an initial velocity u, and arrived at Z with a final velocity v, express u in terms of v, h_1 and h_2.

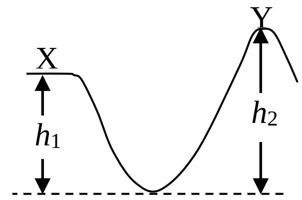


Fig.U78-7

A. $u = \sqrt{v^2 + 2g(h_2 - h_1)}$ B. $u = \sqrt{2g(h_2 - h_1)v}$

C. $u = v + 2g\sqrt{h_2 - h_1}$ D. $u = g(h_2 - h_1)v$

U78-P2a (i) A force F is exerted on a spring of force constant k to stretch it by x. Write down an equation in F, x and k.

(ii) What is the increase of the potential energy in (i)?

(b) A sphere of mass M is dropped from rest from a height of h above a spiral coil of force constant k and compresses it by x, as shown in Fig.U78-P2b. Neglecting the mass of the spring, find an equation involving x, M, h and k. If M = 10 kg, k = 600 N m^{-1}, h = 4 m and g = 10 m s^{-2}, calculate the value of x. [1.33 m]

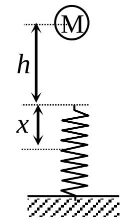


Fig.U78-P2

U86-6 As shown in Fig.U86-6, a body is to slide down freely from rest from point A. Its speed when reaching B would be ____________. (Neglecting all frictional forces)

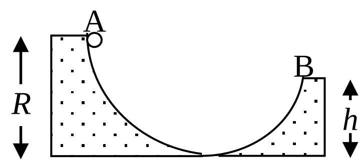


Fig.U86-6

A. $\sqrt{\dfrac{R-h}{2g}}$ B. $\sqrt{\dfrac{2gh}{R}}$ C. $\sqrt{2g(R+h)}$

D. $\sqrt{2g(R-h)}$ E. $\sqrt{2gRh}$

U86-P3 A block of wood A of mass 2 kg slides along a smooth horizontal surface with a velocity 10 m s^{-1} from left to right, as shown in Fig.U86-P3. Right in front of A, another wooden block B of mass 5 kg moves with 3 m s^{-1} in the same direction. Attached to the left side of B is a light spring S of force constant 1120 N m^{-1}. Assume that the spring obeys Hooke's Law when it is being bumped against by A and when it is being separated from A.

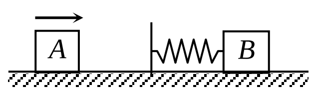


Fig.U86-P3

(a) What do you understand by *force constant*?
(b) In what condition would spring S be when A and B are in a collision state with S sandwiched in between and moving together with a common velocity? What is their common speed?
(c) Find the maximum compression of the spring when A and B are in collision state with the spring sandwiched in between.

[Hint: You may consider the fact that before and after the collision, A, B and S as a system has its total energy and momentum conserved.] [5 m s^{-1}; 0.25 m]

U90-3 Two bodies A and B, with a mass ratio of 1: 2, fall freely from different heights in the ratio 2: 5. If the air resistance is to be neglected, what would be the ratio of their kinetic energies when they strike the ground?

A. 1: 2 B. 1: 5 C. 2: 5 D. 4: 5 E. 5: 4

U91-P4 A man of mass 80 kg slides down a vertical rope with a constant acceleration of 7.8 ms^{-2}. He slides a distance of 6 m down before landing on the ground.
(a) What is the magnitude of frictional force exerted on the man by the rope?
(b) What is the tension in the rope?
(c) How much work is done on the man by the gravitational field?
(d) How much mechanical energy lost in traveling this distance? [160 N; 160 N; 4704 J; 960 J]

U97-8 An object of mass 2 kg is at rest on the top B of a slope AB, which is 1m in height and 4m in length, as shown in Fig.U97-8. The object slides from rest to A and gains a speed of 4 m s^{-1}. The potential energy of the object decreases by ___________ J.

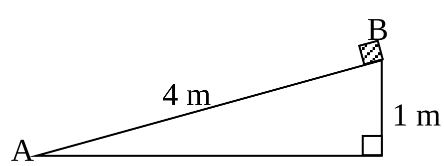


Fig.U97-8

A. 80 B. 20 C. 16 D. 4

U93-9 Fig.U93-9 shows two small identical balls resting at the same height on top of two smooth planes inclined to horizontal floor at different angles. If they slide down from rest, what physical quantities possessed by these balls are equal at the time when they reach the floor?

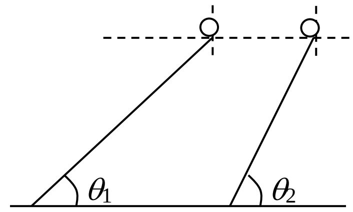


Fig.U93-9

I. potential energy II. kinetic energy
III. power IV. speed
A. I, III B. II, III C. I, II, IV D. I, III, IV

U2k-P5c A wooden block A of mass 2 kg is connected to one end of a spring of force constant k = 980 N m^{-1} and placed on a smooth horizontal plane, and the other end of the spring is fixed to an upholder, as shown in Fig.U2k-P5c. A small ball B of mass 0.5 kg strikes right on block A with a velocity of 10 m s^{-1}. Find the **maximum** compressed length of the spring. [0.18 m]

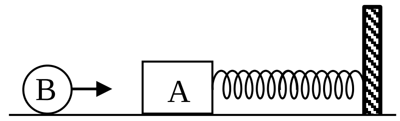


Fig.U2k-P5c

U2k03-P5a Explain whether the mechanical energy of each of the following objects is reserved during its motion:
(i) A small ball that is moving in uniform circular motion horizontally;
(ii) A wooden block that is sliding down a slope with uniform velocity.

(b) In Fig.U2k03-P5b, two small balls A and B of masses m_A = 0.5 kg and m_B = 0.3 kg respectively are hung from the same point O. A is fixed at the lower end of an inelastic light string of length 1 m while B is fixed at the other end of a light spring of length 0.8 m and force constant 40 Nm^{-1}. The spring is kept straight horizontally to be equal to its original length OB. After B is released from rest horizontally, it makes a head-on strike with A at its lowest point, and its velocity becomes zero instantaneously. Find
(i) the velocity of B before it strikes on A;
(ii) the **greatest** height that A can achieve after being struck by B. [3.78 m s^{-1}; 0.26 m]

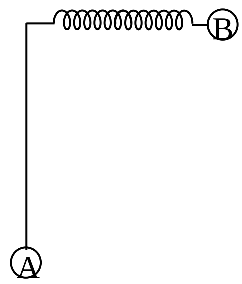


Fig.U2k03-P5b

U2k05-P2 (c) A bullet of mass 20 g is fired horizontally into a block of wood of mass 4.98 kg which is hung vertically by two thin threads, as shown in Fig.U2k05-P2c. After the firing, the bullet become imbedded in the block which swings upwards, raising a height h = 10 cm. Find the speed v of the bullet immediately before it strikes the block. [350 m s^{-1}] [4%]

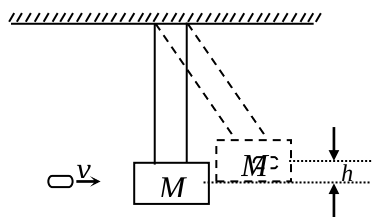
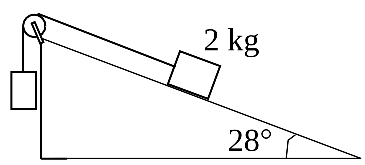


Fig.U2k05-P2c

U2k06-13 As shown in Fig.U2k06-13, both the pulley and the inclined plane are frictionless and the mass of the pulley is negligible. If the two objects are released from rest with the connecting cord taut, what is their total kinetic energy when the object of mass 3 kg has fallen 35 cm?
A. 7.1 J B. 6.7 J C. 5.3 J D. 4.8 J



U2k06-13

U2k06-P3 A body A of mass 1.0 kg is placed at a point 4 m above the ground. When released, it slides down a frictionless smooth track and collides with another body B of mass 1.0 kg, and the two bodies move forward together as shown in Fig.U2k06-P3.

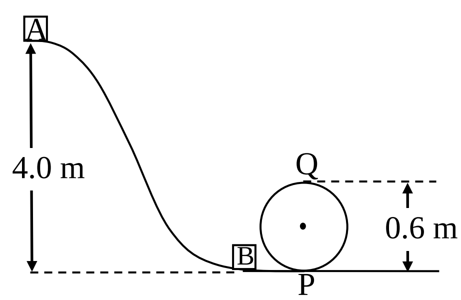


Fig.U2k06-P3

(a) Find the velocity of A immediately before and after its collision with B. [8.85 m s^{-1}; 4.42 m s^{-1}] [3%]
(b) When coming to point P, both A and B embark on a circular motion of diameter 0.60 m.
Draw and label the forces on A at point P as well as at point Q, which is the highest point of the circular track. (Note: Two diagram sto be drawn) [2%]
(c) Calculate the normal reaction on A at both points P and Q. (Note: Two answers to be calculated) [74.92 N; 16.15 N] [5%]

U2k07-P3b Fig.U2k07-P3b shows a pendulum of mass "*m*" is hung by a massless cord of length "*r*" at a fixed point. It is set into motion and the maximum angle the cord makes with the vertical is θ. ($0 \leq \theta < 90°$)

(i) What is the total energy "*E*" of the system in term sof "*g*", "θ", "m" and "*r*"?

(ii) Once you have established "*E*", find the expressions for the Potential Energy, Kinetic Energy and velocity in term sof "*m*", "*r*", "θ", "ϕ" and "*g*". When the cord makes an angle ϕ with the vertical, where $-\theta < \phi < \theta$.

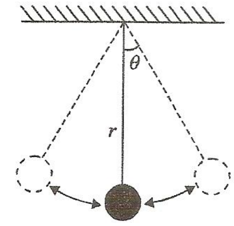


Fig.U2k07-P

U2k08-P5 (b) Fig.U2k08-P5b, shows a 250 kg car which is at rest at point A on a smooth roller coaster track. The car carries a 75 kg passenger and is 20 meters above the ground at point A.

(i) Calculate the total gravitational potential energy of the car and the passenger at point A relative to the ground.

(ii) Calculate the speed of the car and passenger at point B.

(iii) What happen to the total mechanical energy of the roller coaster car and passenger at points A, B and C?

[6.64×10^4 J; 20 m s^{-1}; remains the same]

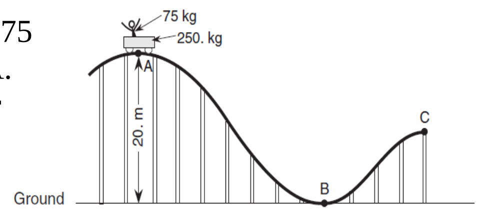


Fig.U2k08-P5b

U2k10-P5a State the condition for conservation of momentum when two objects collide.

(b) In Fig.U2k10-P5b(i), a ring R of mass 0.8 kg is put into a smooth horizontal bar, and a ball B of mass 1 kg is hanging with a string from the ring. The system is in equilibrium. The mass of the string and the frictional force between the ring and the bar are neglected. A bullet of mass 50 g, moving with velocity 100 m s^{-1}, is fired horizontally into the ball and is embedded in the ball. The ball then swings to a height of 0.6 m and moves together with the ring with velocity *v* as shown in Fig.U2k10-P5b(ii).

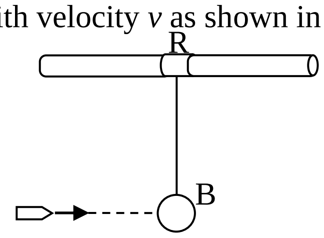


Fig.U2k10-P5b(i)

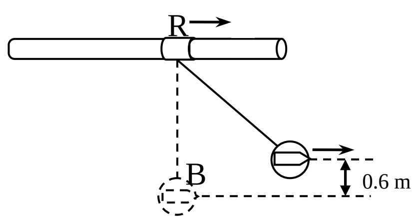


Fig.U2k10-P5b(ii)

Find

(i) the velocity of the system, *v* after the collision;

(ii) the loss in kinetic energy of the system during the collision.

[2.7 m s^{-1}; 237.09 J]

U2k12-4 As shown in Fig.U2k12-P4, a bullet A of mass 100 g travels horizontally with initial velocity *u* towards a wooden ball B of mass 500 g, which is suspended from point O by a light string of length $L = 0.5$ m. After the collision, the bullet A remains embedded in ball B and move upwards in a vertical circle. If the objects can just reach the highest point C and complete the circular motion, determine

(a) the speed of the bodies at C;

(b) the speed of the bodies immediately after the collision;

(c) the initial velocity *u* of the bullet;

(d) the energy lost by the system due to the collision.

[2.21 m s^{-1}; 4.95 m s^{-1}; 29.71 m s^{-1}; -36.75 J]

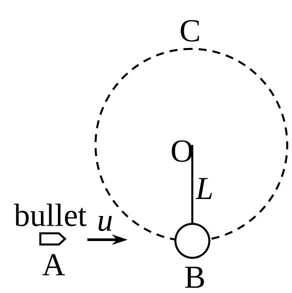


Fig.U2k12-P3

U2k13-10 Fig.U2k13-10 shows the path of a body M sliding from P along a smooth curved path to Q. Given that its velocity at P is 2 m s^{-1}, what would be its velocity when it reaches Q? (Take $g = 10$ m s^{-2})

A. 4 m s^{-1} B. 6 m s^{-1}

C. 8 m s^{-1} D. 10 m s^{-1}

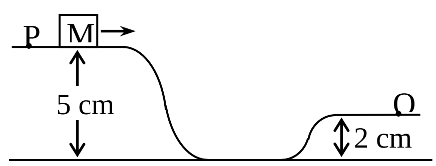
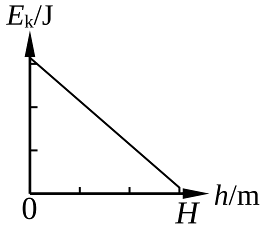
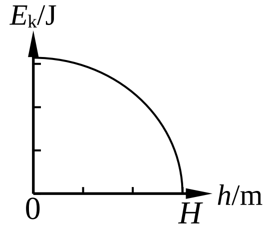
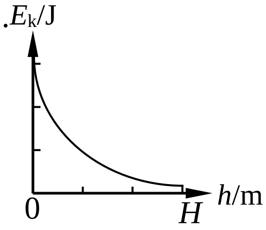


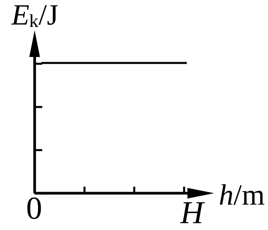
Fig.U2k13-10

U2k11-6 A ball is thrown vertically upwards from the ground and rises to a height H. Neglecting air resistance, which of the following graphs best describes the variation of the kinetic energy E_k of the ball with the height h of the ball.

A. 

B. 

C. 

D. 

U2k14-9 An object of mass 10 kg falls vertically from rest to the ground from a height of 10 m with acceleration of 2.5 m s^{-2}. Which of the following statements is **correct** in the whole process of falling?

A. The kinetic energy of the object increased by 250 J.
B. The mechanical energy of the object increased by 750 J.
C. The gravitational potential energy of the object decreased by 750 J.
D. The mechanical energy of the object remains unchanged at 1000 J.

U2k14-P2a Fig.U2k14-P2a shows a uniform wooden plank AB of length 4 m and weighs 200 N. Its end B is placed on a smooth horizontal platform, while its end A is attached to a light string which passes over a fixed smooth pulley to a hanging object C of weight 550 N.

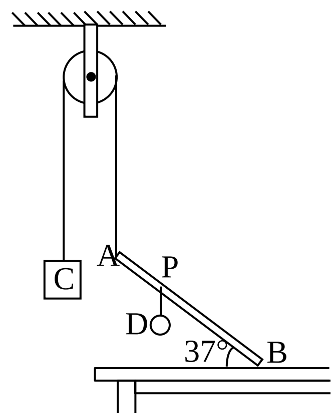


Fig.U2k14-P2

(a) When an object D of weight 500 N is hung with a light string at point P on the plank, the system is in static equilibrium. The plank is inclined at an angle of 37° with the platform. Given that g = 10 m s^{-2}. Find
  (i) the length of PB.
  (ii) the reaction force at B. [3.6 m; 150 N]
(b) When the object D is removed from the plank by cutting its string, the plank is being pulled upwards. When the end B of the plank is 1 m above the platform, find
  (i) the change of height of object C and the center of the mass of the plank respectively.
  (ii) the velocity of the object C at this instant.
  [- 2.6 m, 1.8 m; 5.34 m s^{-1}]

U2k15-P4 In Fig.U2k15-P4, a uniform iron rod AB of length 1 m and mass 2 kg is hinged to the wall at the end A and is pivoted horizontally at point C on a platform. The distance of C from A is 40 cm. A small ball R of mass 0.4 kg is hung at the end B of the rod by using a light string of length 20 cm. The system is in static equilibrium. [Given g = 10 m s^{-2}]

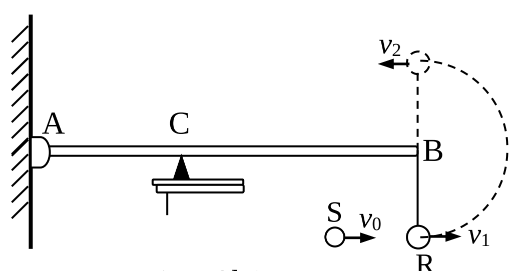


Fig.U2k15-P4

(a) Find the reaction force on the rod at pivot C. [2%]
(b) A small ball S of mass 0.1 kg moving horizontally with velocity v_0 collides head-on with the ball R and rebounds with velocity 0.6 v_0. This causes the ball R to swing along a semi-circular path to the highest point.
  (i) If the ball R swings with an initial velocity v_1, and reaches the highest point with final velocity v_2, derive a relationship between v_1 and v_2. [2%]
  (ii) At the moment the ball R reaches the highest point, the reaction force at point C on the rod is zero. Find the instantaneous tension T in the string and the velocities v_1 and v_2 of the ball R. [2%+2%]
  (iii) Find the initial velocity v_0 of the ball S. [2%]
  [35 N, upwards; $v_1 = \sqrt{v_2^2 + 8}\ \mathrm{m\,s^{-1}}$; 10 N upwards, 2.65 m s^{-1}, 3.88 m s^{-1}; 9.70 m s^{-1}]

U2k16-08 Fig.U2k16-08 shows a block of 2 kg being held against a light spring which has been compressed by 0.2 m at position *i*. The mass of the spring is negligible and its force constant is 2500 N m^{-1}. When the spring projects the block up the inclined slope at 30°, the block comes to rest momentarily at position *f*. If the frictional force is negligible, determine the distance *S* between position *i* and *f*.

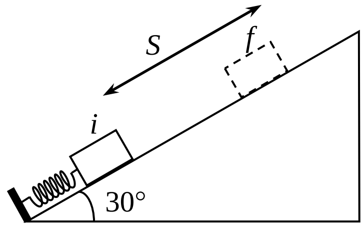


Fig.U2k16-08

A. 2.6 m B. 3.2 m C. 4.7 m D. 5.1 m

U2k16-P4 Fig.U2k16-P4(i) shows a pendulum hanging from point O by a string of length 0.5 m. The mass of the bob A is 1 kg. A block B of mass 2 kg rest on a horizontal plane and is in contact with bob A. The coefficient of sliding friction between the block and the plane is 0.2.

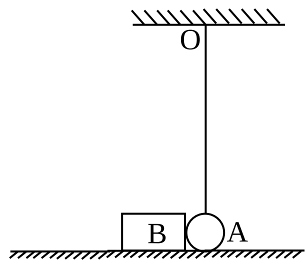


Fig.U2k16-P4(i)

The bob A is displaced an angle of θ with the vertical and it is then released from rest, as shown in Fig.U2k16-P4(ii). At its lowest point, bob A strikes block B with velocity u_A. After the collision, bob A rebounds with a speed of $\frac{1}{3}u_A$, while block B slides on the plane along a straight line, and stops after traveling a distance of 1 m. Taking g = 10 m s^{-2}, find

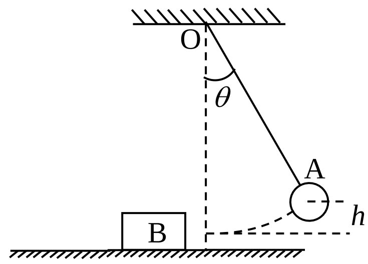


Fig.U2k16-P4(ii)

(a) the frictional force acting on block B when it is sliding on the plane;
(b) the velocity u_B of block B after the collision;
(c) the velocity u_A of bob A just before collision with block B;
(d) the tension in the string when the bob A swings to its lowest point;
(e) the height h_A that the bob A drops from rest;
(f) the value of θ.
[4 N; 2 m s^{-1}; 3 m s^{-1}; 28 N; 0.45 m; 84.26°]

U2k19-P8Tarzan who had a mass of 70 kg planned to swing himself to the opposite river bank using a rattan rope from the top of a tree. The rope was tied on a branch of the tree at a height of 20 m and at a horizontal distance of 5 m from the river. Holding the rope tightly so that the centre of mass was 8 m from the knot on the tree, he jumped off from the branch which was 20 m above the ground and he let go at the position when the rope was at 45° with the vertical, as shown in Fig.U2k19-P8.

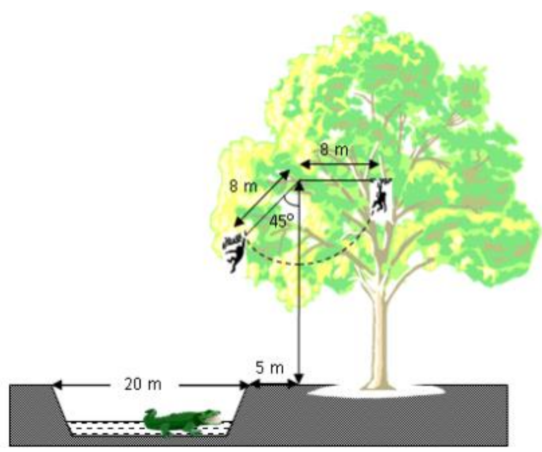


Fig.U2k19-P8

(a) ~~Explain why Tarzan chose to let go himself when the rope was making an angle of 45° with the vertical?~~ [1%]
(b) What was his speed *v* when he let go of the rattan rope? [2%]
(c) If the width of the river is 20 m, will he be able to reach the opposite river bank? Answer with the proof of calculation. [4%]
(d) If the maximum tensile force which the rattan rope can withstand is 1800 N, determine whether he can complete his mission. Answer with the proof of calculation. [3%]
[10.53 m s^{-1}; 25.25 m, yes; 2057.57 N, break]

U2k19-10 A person throws a small ball from a platform with initial speed v_0. If air resistance is negligible, which of the following statements about the speed of the ball at the instant it hits the ground is correct?
A. It is maximum if it is thrown vertically downwards.
B. It is minimum if it is thrown vertically upwards.
C. It is maximum if it is thrown upwards at 45° with horizontal.
D. It is the same regardless of which direction it is thrown.