1. $\sqrt{\sqrt{3}+\sqrt{\sqrt{3}+x}}=x$

Sol.

$\because \forall n \in \mathbb{R}, n >= 0,\ \sqrt{n} >= 0 \quad \therefore x > 0$

$$\begin{aligned}
\sqrt{\sqrt{3}+\sqrt{\sqrt{3}+x}} &= x \\
\sqrt{3}+\sqrt{\sqrt{3}+x} &= x^2 \\
\sqrt{\sqrt{3}+x} &= x^2-\sqrt{3} \\
x+\sqrt{3} &= \left(x^2-\sqrt{3}\right)^2 \\
&= x^4-2\sqrt{3}x^2+3 \\
x^4-2\sqrt{3}x^2+3-x-\sqrt{3} &= 0
\end{aligned}$$

Let $a=\sqrt{3}$,

$$\begin{aligned}
x^4-2ax^2+a^2-x-a &= 0 \\
a^2-(2x^2+1)a+x^4-x &= 0 \\
a^2-(2x^2+1)a+x(x^3-1) &= 0 \\
a^2-(2x^2+1)a+x(x-1)(x^2+x+1) &= 0 \\
a^2-(2x^2+1)a+(x^2-x)(x^2+x+1) &= 0 \\
\left[a-(x^2-x)\right]\left[a-(x^2+x+1)\right] &= 0 \\
a=x^2-x \text{ or } a=x^2+x+1 &
\end{aligned}$$

When $a=x^2-x$,

$$\begin{aligned}
x^2-x &= \sqrt{3} \\
x^2-x-\sqrt{3} &= 0 \\
x &= \frac{1\pm\sqrt{1+4\sqrt{3}}}{2} \\
\because\ x &> 0 \\
\therefore x &= \frac{1+\sqrt{1+4\sqrt{3}}}{2}
\end{aligned}$$

When $a=x^2+x+1$,

$$\begin{aligned}
x^2+x+1 &= \sqrt{3} \\
x^2+x+1-\sqrt{3} &= 0 \\
x &= \frac{-1\pm\sqrt{1-4(1-\sqrt{3})}}{2} \\
&= \frac{-1\pm\sqrt{4\sqrt{3}-3}}{2} \\
\because\ x &> 0 \\
\therefore\ x &= \frac{-1+\sqrt{4\sqrt{3}-3}}{2}
\end{aligned}$$

$\therefore\ x=\dfrac{1+\sqrt{1+4\sqrt{3}}}{2}$ or $x=\dfrac{-1+\sqrt{4\sqrt{3}-3}}{2}$