

# Mathematics

*Senior 3 Part I*

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# **Introduction**

**Why this book?**

**Disclaimer**

**Acknowledgements**

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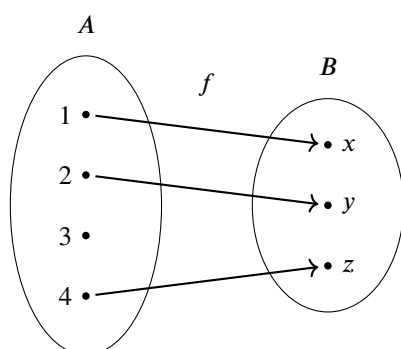
# Chapter 22

## Function

### 22.1 Definition of a Function

#### Mapping, Preimage and Image

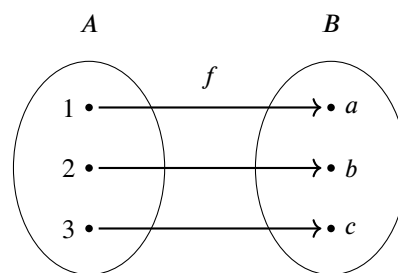
For two non-empty sets  $A$  and  $B$ , If an element  $a$  inside set  $A$  has a corresponding element  $b$  inside set  $B$ , denoted as  $a \rightarrow b$ , then we say that  $a$  is mapped to  $b$  or  $a$  and  $b$  are paired. The mapping between two sets is normally denoted as  $f, g, h$ , etc. The mapping shown in the diagram below can be denoted as  $f : 1 \rightarrow x, 2 \rightarrow y, 4 \rightarrow z$ .



Let  $f : A \rightarrow B$  is a mapping,  $a$  is an element in  $A$ . If  $a$  is mapped to  $b$  under the mapping  $f$ , then  $b$  is said to be the image of  $a$  under the mapping  $f$ , denoted as  $b = f(a)$ ;  $a$  is said to be the preimage of  $b$  under the mapping  $f$ . In the diagram above, under the mapping  $f$ , the image of 1, 2, and 4 are  $x, y$ , and  $z$  respectively, while the preimage of  $x, y$ , and  $z$  are 1, 2, and 4 respectively.

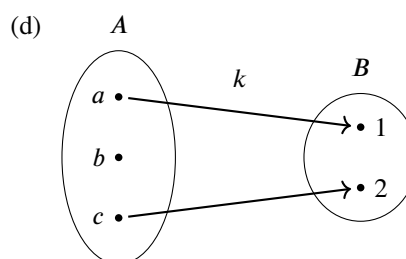
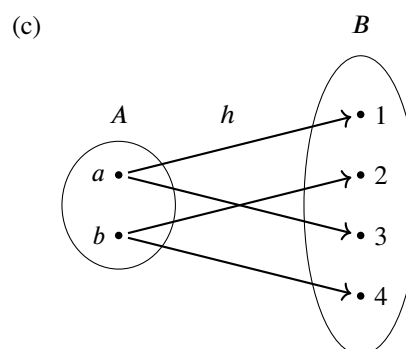
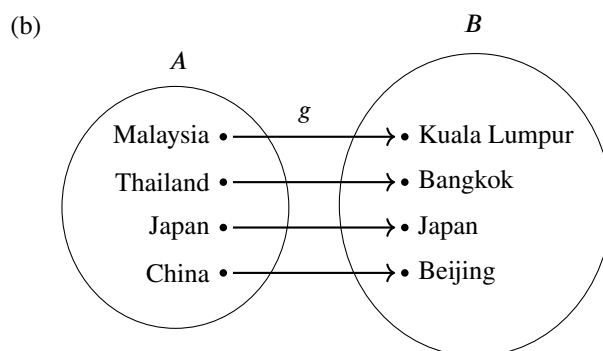
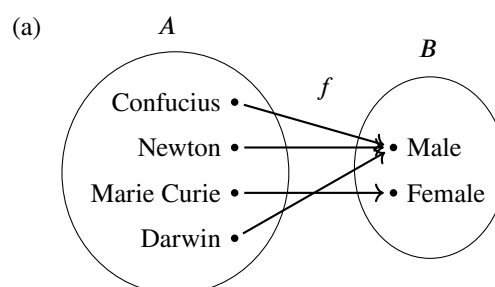
Let  $A$  and  $B$  be two non-empty sets,  $f$  is a mapping from  $A$  to  $B$  such that for all elements in  $A$ , there is a unique corresponding element in  $B$ , then  $f$  is a function or a mapping from  $A$  to  $B$ , denoted as  $f : A \rightarrow B$ .

The mapping shown in the diagram below is a function.



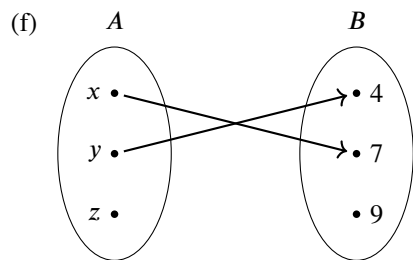
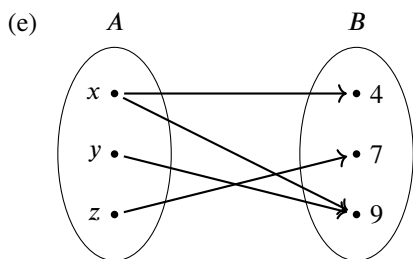
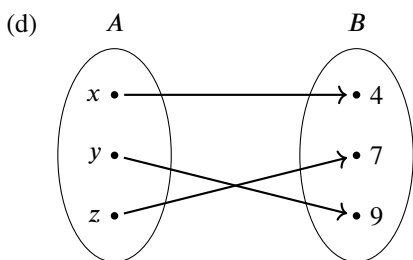
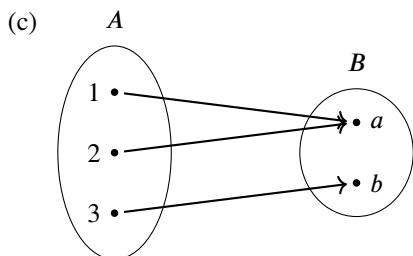
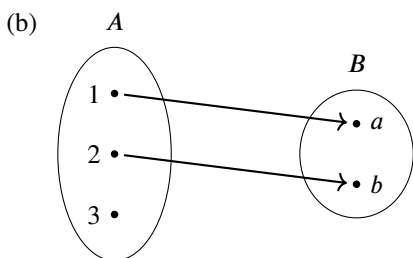
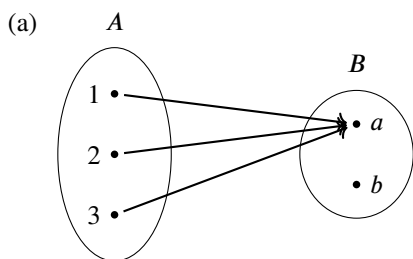
#### 22.1.1 Practice 1

- For the following mappings, list the image of each element in  $A$  and the preimage of each element in  $B$ , and determine whether the mapping is a function or not:



- Given a mapping  $g : x \rightarrow x + 3, x \in \{-2, -1, 0, 1, 2, 3\}$ , find the image of each  $x$ .

3. Determine whether the following mappings are functions.



The function  $f : A \rightarrow B$  can be written as  $y = f(x)$ ,  $x$  is the element of  $A$  and  $y$  is the element of  $B$ . When  $x$  changes,  $y$  changes as well.  $x$  is called independent variable, while  $y$  is called dependent variable.

## 22.2 Domain and Range

## 22.3 Graphs of Functions and Their Transformations

## 22.4 Composite Functions

## 22.5 One to One Function, Onto Function and One to One Onto Function

## 22.6 Inverse Functions

## **Chapter 23**

# **Exponents and Logarithms**

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