Given a polynomial $(x + b)^n + (x + 4)^{2m+1}$ where $m, n \in \mathbb{Z}^+$. Prove that $x + 5$ is a factor of this polynomial.

**Solution**

$$
\begin{aligned}
f(x) &= (x + b)^n + (x + 4)^{2m+1} \\
f(-5) &= (-5 + b)^n + (-1)^{2m+1} \\
&= -(b - 5)^n && \left[\forall m \in \mathbb{Z}^+, \ (-1)^{2m+1} = -1\right] \\
&= -\sum_{k=0}^{n} \binom{n}{k} b^{n-k} 5^k \\
&= -b^n + \binom{n}{1} b^{n-1} 5 + \binom{n}{2} b^{n-2} 5^2 + \ldots + \binom{n}{n} 5^n
\end{aligned}
$$