

# Mathematics

*Senior 3 Part I*

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# **Introduction**

**Why this book?**

**Disclaimer**

**Acknowledgements**

# Contents

<b>Introduction</b>	<b>1</b>
<b>22 Function</b>	<b>4</b>
22.1 Definition of a Function . . . . .	4
22.1.1 Practice 1 . . . . .	4
22.1.2 Practice 2 . . . . .	6
22.2 Domain and Range . . . . .	6
22.3 Graphs of Functions and Their Transformations . . . . .	6
22.4 Composite Functions . . . . .	6
22.5 One to One Function, Onto Function and One to One Onto Function . . . . .	6
22.6 Inverse Functions . . . . .	6
<b>23 Exponents and Logarithms</b>	<b>7</b>
23.1 Exponents . . . . .	7
23.2 Logarithms . . . . .	7
23.3 Arithmetic Properties of Logarithms and Base Changing Formula . . . . .	7
23.4 Exponential Equations . . . . .	7
23.5 Logarithmic Equations . . . . .	7
23.6 Compound Interest and Annuity . . . . .	7
<b>24 Limits</b>	<b>8</b>
24.1 Concept of Limits . . . . .	8
24.2 Limits of Functions . . . . .	8
24.3 Arithmetic Properties of Limits of Functions . . . . .	8
<b>25 Differentiation</b>	<b>9</b>
25.1 Gradient of Tangent Line on a Curve . . . . .	9
25.2 Gradient of Tangent Line and Derivative . . . . .	9
25.3 Law of Differentiation . . . . .	9
25.4 Chain Rule - Differentiation of Composite Functions . . . . .	9

25.5 Higher Order Derivatives . . . . .	9
25.6 Implicit Differentiation . . . . .	9
25.7 Two Basic Limits . . . . .	9
25.8 Derivatives of Trigonometric Functions . . . . .	9
25.9 Derivatives of Exponential Functions . . . . .	9
25.10 Derivatives of Logarithmic Functions . . . . .	9

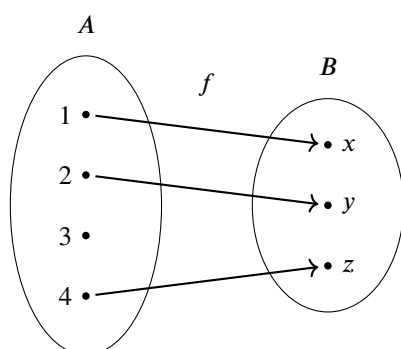
# Chapter 22

## Function

### 22.1 Definition of a Function

#### Mapping, Preimage and Image

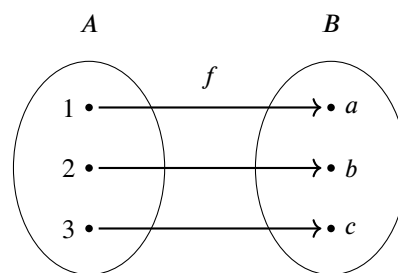
For two non-empty sets  $A$  and  $B$ , If an element  $a$  inside set  $A$  has a corresponding element  $b$  inside set  $B$ , denoted as  $a \rightarrow b$ , then we say that  $a$  is mapped to  $b$  or  $a$  and  $b$  are paired. The mapping between two sets is normally denoted as  $f, g, h$ , etc. The mapping shown in the diagram below can be denoted as  $f : 1 \rightarrow x, 2 \rightarrow y, 4 \rightarrow z$ .



Let  $f : A \rightarrow B$  is a mapping,  $a$  is an element in  $A$ . If  $a$  is mapped to  $b$  under the mapping  $f$ , then  $b$  is said to be the image of  $a$  under the mapping  $f$ , denoted as  $b = f(a)$ ;  $a$  is said to be the preimage of  $b$  under the mapping  $f$ . In the diagram above, under the mapping  $f$ , the image of 1, 2, and 4 are  $x, y$ , and  $z$  respectively, while the preimage of  $x, y$ , and  $z$  are 1, 2, and 4 respectively.

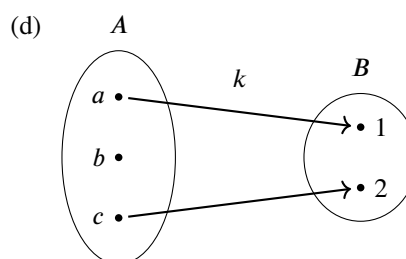
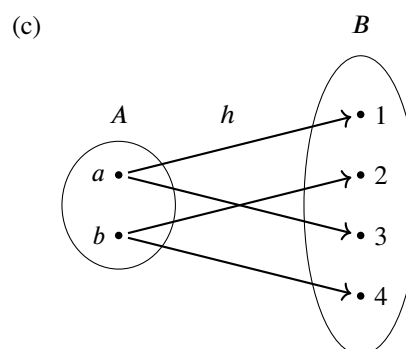
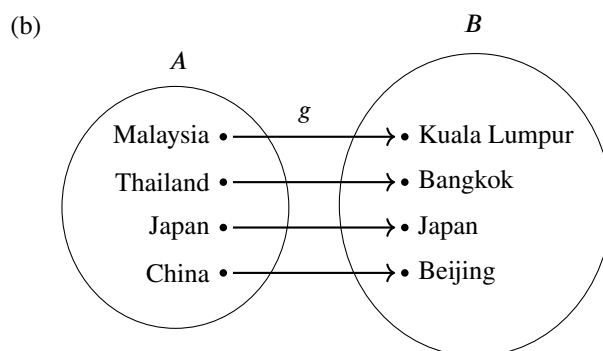
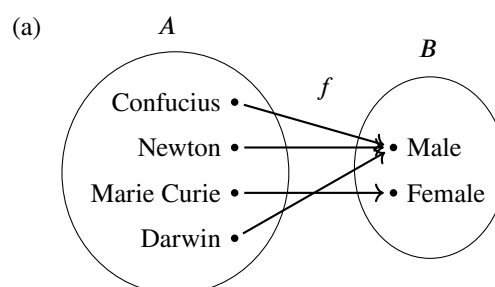
Let  $A$  and  $B$  be two non-empty sets,  $f$  is a mapping from  $A$  to  $B$  such that for all elements in  $A$ , there is a unique corresponding element in  $B$ , then  $f$  is a function or a mapping from  $A$  to  $B$ , denoted as  $f : A \rightarrow B$ .

The mapping shown in the diagram below is a function.



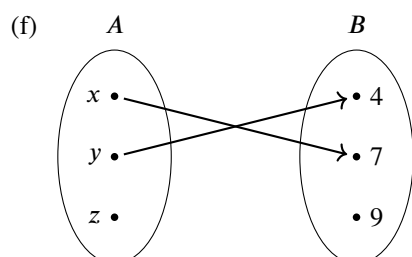
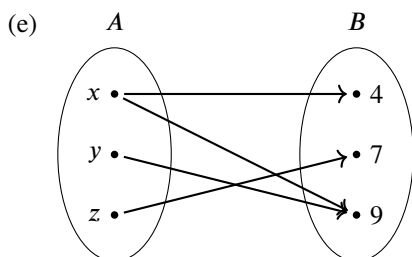
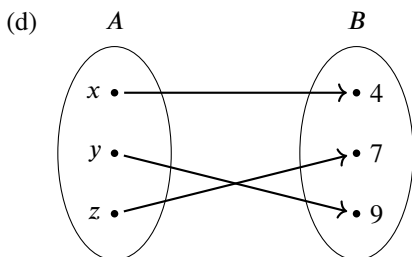
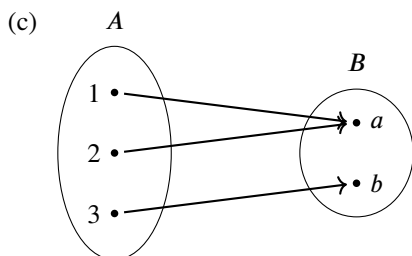
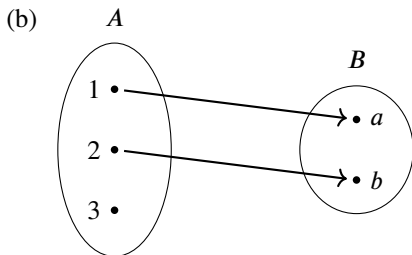
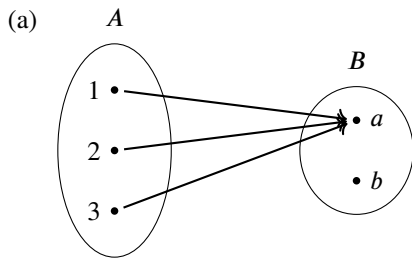
#### 22.1.1 Practice 1

- For the following mappings, list the image of each element in  $A$  and the preimage of each element in  $B$ , and determine whether the mapping is a function or not:



- Given a mapping  $g : x \rightarrow x + 3, x \in \{-2, -1, 0, 1, 2, 3\}$ , find the image of each  $x$ .

3. Determine whether the following mappings are functions.



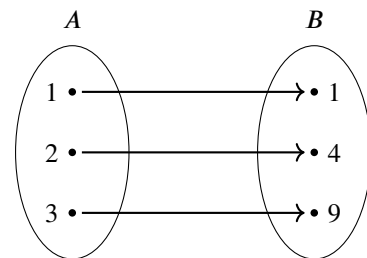
The function  $f : A \rightarrow B$  can be written as  $y = f(x)$ ,  $x$  is the element of  $A$  and  $y$  is the element of  $B$ . When  $x$  changes,  $y$  changes as well.  $x$  is called independent variable, while  $y$  is called dependent variable.

Keep in mind that  $f(x)$  is NOT the product of  $f$  and  $x$ .

## Representation of Functions

Generally speaking, there are a few ways to represent a function:

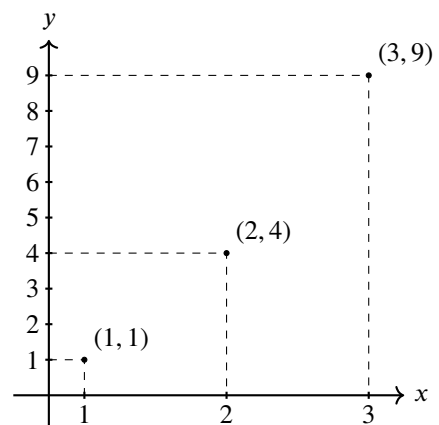
1. **Narrative Form:** express the function of two sets in words. For example, Let  $A = \{1, 2, 3\}$  and  $B = \{1, 4, 9\}$ ,  $f$  is a function from  $A$  to  $B$ , its definition is that for any element  $x$  in  $A$ , its corresponding element is  $x^2$  in  $B$ .
2. **Arrow Method:** draw an arrow to connect the preimage and image of a function such that the preimage is corresponding to the image. To express the example above, we express it as  $f : 1 \rightarrow 1, 2 \rightarrow 4, 3 \rightarrow 9$ .
3. **Analytical Method:** express the function in the form of mathematical expression to represent the relationship between the independent variable and the dependent variable. For example,  $f(x) = x^2, x \in \mathbb{A}$ .
4. **Venn Diagram:** draw arrows between the venn diagram of two sets to represent the function, as shown below:



5. **Table Method:** express the function in the form of table, showing the relationship of the chosen value between independent variable  $x$  and the value of its corresponding dependent variable  $y$ , as shown below:

$x$	1	2	3
$y$	1	4	9

6. **Graphical Method:** draw a graph to represent the function of the two variables, as shown below:



### **22.1.2 Practice 2**

Express the following functions using analytical method, venn diagram, table method and graphical method.

- (a)  $f$  mapping each integers from  $-3$  to  $3$  to its squares plus  $4$ .
- (b)  $g$  mapping each natural numbers from  $1$  to  $4$  to its cubes.

## **22.2 Domain and Range**

## **22.3 Graphs of Functions and Their Transformations**

## **22.4 Composite Functions**

## **22.5 One to One Function, Onto Function and One to One Onto Function**

## **22.6 Inverse Functions**

## **Chapter 23**

# **Exponents and Logarithms**

### **23.1 Exponents**

### **23.2 Logarithms**

### **23.3 Arithmetic Properties of Logarithms and Base Changing Formula**

### **23.4 Exponential Equations**

### **23.5 Logarithmic Equations**

### **23.6 Compound Interest and Annuity**



## **Chapter 24**

# **Limits**

### **24.1 Concept of Limits**

### **24.2 Limits of Functions**

### **24.3 Arithmetic Properties of Limits of Functions**

## **Chapter 25**

# **Differentiation**

- 25.1 Gradient of Tangent Line on a Curve**
- 25.2 Gradient of Tangent Line and Derivative**
- 25.3 Law of Differentiation**
- 25.4 Chain Rule - Differentiation of Composite Functions**
- 25.5 Higher Order Derivatives**
- 25.6 Implicit Differentiation**
- 25.7 Two Basic Limits**
- 25.8 Derivatives of Trigonometric Functions**
- 25.9 Derivatives of Exponential Functions**
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