Senior 2 Math Part I

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Chapter 1

Sequence and Series

1.1 Sequence and Series

1.1.1 Practice 1

1. Find the first 5 terms of the sequence $a_n = \frac{2^n}{n+1}$.

   Sol. $a_1 = \frac{2}{2} = 1, a_2 = \frac{4}{3}, a_3 = \frac{8}{4}, a_4 = \frac{16}{5}, a_5 = \frac{32}{6}$

2. Write the general term of the sequence 1, 8, 27, 64, ...

   Sol. $a_n = n^3$

1.1.2 Practice 2

1. Express the series $\sum_{n=1}^{10} n^2 + 1$ in the form of numbers.

   Sol. $$\sum_{n=1}^{10} n^2 + 1$$
   $$= (1^2 + 1) + (2^2 + 1) + (3^2 + 1) + (4^2 + 1)$$
   $$+ (5^2 + 1) + (6^2 + 1) + (7^2 + 1)$$
   $$+ (8^2 + 1) + (9^2 + 1) + (10^2 + 1)$$
   $$= 2 + 5 + 10 + 17 + 26 + 37 + 50 + 65$$
   $$+ 82 + 101$$

2. Write the first term, last term and the number of terms of the series $\sum_{n=1}^{10} (3^n - 2^n)$.

   Sol. $$First\ term = (3^1 - 2^1) = 1$$
   $$Last\ term = (3^{10} - 2^{10}) = 59049$$
   $$Number\ of\ terms = 10$$

3. Express the series $2\times5+3\times7+4\times9+\ldots+15\times31$ in the form of $\sum$.

   Sol.
   $$a_1 = 2 \times 5 = 10$$
   $$a_2 = 3 \times 7 = 21$$
   $$a_3 = 4 \times 9 = 36$$
   $$a_4 = 5 \times 11 = 55$$
   $$\vdots$$
   $$a_{15} = 15 \times 31 = 465$$
   $$\therefore 2 \times 5 + 3 \times 7 + 4 \times 9 + \ldots + 15 \times 31$$
   $$= \sum_{n=1}^{15} a_n$$

1.1.3 Exercise 12.1

1. Find the general term of the following sequences.

   (a) 5, 8, 11, 14, ...
   Sol. $a_n = 3n + 2$

   (b) 2, 4, 8, 16, ...
   Sol. $a_n = 2^n$

   (c) $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$
   Sol. $a_n = \frac{n+1}{n}$

   (d) $\frac{2}{5}, \frac{4}{7}, \frac{6}{9}, \frac{8}{11}, \ldots$
   Sol. $a_n = \frac{2n}{2n+1}$

2. Find the first 5 terms of the following sequences.

   (a) $a_n = 2n + 3$
   Sol. $a_1 = 2 \times 1 + 3 = 5, a_2 = 2 \times 2 + 3 = 7, a_3 = 2\times3+3 = 9, a_4 = 2\times4+3 = 11, a_5 = 2 \times 5 + 3 = 13$

   (b) $a_n = n(n - 2)$
   Sol. $a_1 = 1\times(-1) = -1, a_2 = 2\times0 = 0, a_3 = 3 \times 1 = 3, a_4 = 4 \times 2 = 8, a_5 = 5 \times 3 = 15$

   (c) $a_n = \frac{n}{2n+1}$
   Sol. $a_1 = \frac{1}{2\times1+1} = \frac{1}{3}, a_2 = \frac{2}{2\times2+1} = \frac{2}{5}, a_3 = \frac{3}{2\times3+1} = \frac{3}{7}, a_4 = \frac{4}{2\times4+1} = \frac{4}{9}, a_5 = \frac{5}{2\times5+1} = \frac{5}{11}$

   (d) $a_n = (-3)^n$
   Sol. $a_1 = (-3)^1 = -3, a_2 = (-3)^2 = 9, a_3 = (-3)^3 = -27, a_4 = (-3)^4 = 81, a_5 = (-3)^5 = -243$

3. Express the following series in the form of numbers.

(a) $\sum_{n=1}^{5} n(n+3)$

Sol. $\displaystyle\sum_{n=1}^{5} n(n+3)$
$= (1\times 4) + (2\times 5) + (3\times 6) + (4\times 7)$
$+ (5\times 8)$
$= 4 + 10 + 18 + 28 + 40$

(b) $\sum_{n=2}^{6} \frac{1}{3^n}$

Sol. $\displaystyle\sum_{n=2}^{6} \frac{1}{3^n}$
$= \dfrac{1}{3^2} + \dfrac{1}{3^3} + \dfrac{1}{3^4} + \dfrac{1}{3^5} + \dfrac{1}{3^6}$
$= \dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81} + \dfrac{1}{243} + \dfrac{1}{729}$

(c) $\sum_{n=1}^{6} \frac{1}{n(2n+1)}$

Sol. $\displaystyle\sum_{n=1}^{6} \frac{1}{n(2n+1)}$
$= \dfrac{1}{1(2\times 1+1)} + \dfrac{1}{2(2\times 2+1)}$
$+ \dfrac{1}{3(2\times 3+1)} + \dfrac{1}{4(2\times 4+1)}$
$+ \dfrac{1}{5(2\times 5+1)} + \dfrac{1}{6(2\times 6+1)}$
$= \dfrac{1}{3} + \dfrac{1}{10} + \dfrac{1}{21} + \dfrac{1}{36} + \dfrac{1}{55} + \dfrac{1}{78}$

(d) $\sum_{n=2}^{5} \frac{1}{n^2+2}$

Sol. $\displaystyle\sum_{n=2}^{5} \frac{1}{n^2+2}$
$= \dfrac{1}{4+2} + \dfrac{1}{9+2} + \dfrac{1}{16+2} + \dfrac{1}{25+2}$
$= \dfrac{1}{6} + \dfrac{1}{11} + \dfrac{1}{18} + \dfrac{1}{27}$

4. Find the first term, last term and the number of terms of the following series.

(a) $\sum_{n=3}^{10} 2^2$
Sol. $a_3 = 2^2 = 4, a_{10} = 2^2 = 4, n = 10 - 3 + 1 = 8$

(b) $\sum_{n=1}^{8} \frac{n+2}{n}$
Sol. $a_1 = \frac{1+2}{1} = \frac{3}{1} = 3, a_8 = \frac{8+2}{8} = \frac{10}{8} = \frac{5}{4}, n = 8 - 1 + 1 = 8$

(c) $\sum_{n=1}^{10} 3n^2 - n$
Sol. $a_1 = 3\times 1^2 - 1 = 2, a_{10} = 3\times 10^2 - 10 = 290, n = 10 - 1 + 1 = 10$

(d) $\sum_{n=9}^{14} n^2(n-7)$
Sol. $a_9 = 9^2(9-7) = 9^2 \times 2 = 162, a_{14} = 14^2(14-7) = 14^2\times 7 = 2744, n = 14-9+1 = 6$

5. Express the following series in the form of $\sum$.

(a) $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{30}$

Sol.

$$\begin{aligned}
a_1 &= 1\\
a_2 &= \frac{1}{2}\\
a_3 &= \frac{1}{3}\\
&\vdots\\
a_{30} &= \frac{1}{30}\\
\therefore 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{30} &= \sum_{n=1}^{30} \frac{1}{n}
\end{aligned}$$

(b) $1^3 + 2^3 + 3^3 + \ldots + 50^3$

Sol.

$$\begin{aligned}
a_1 &= 1^3\\
a_2 &= 2^3\\
a_3 &= 3^3\\
&\vdots\\
a_{50} &= 50^3\\
\therefore 1^3 + 2^3 + 3^3 + \ldots + 50^3 &= \sum_{n=1}^{50} n^3
\end{aligned}$$

(c) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16}$

Sol.

$$\begin{aligned}
a_1 &= (-\frac{1}{2})^{1-1} \\
a_2 &= (-\frac{1}{2})^{2-1} \\
a_3 &= (-\frac{1}{2})^{3-1} \\
a_4 &= (-\frac{1}{2})^{4-1} \\
a_5 &= (-\frac{1}{2})^{5-1} \\
\therefore 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} \\
&= \sum_{n=1}^{5} (-\frac{1}{2})^{n-1}
\end{aligned}$$

(d) $2 \times 4 + 4 \times 7 + 6 \times 10 + 8 \times 13 + 10 \times 16$

Sol.

$$\begin{aligned}
a_1 &= 2 \times 1 \times (3 \times 1 + 1) \\
a_2 &= 2 \times 2 \times (3 \times 2 + 1) \\
a_3 &= 2 \times 3 \times (3 \times 3 + 1) \\
a_4 &= 2 \times 4 \times (3 \times 4 + 1) \\
a_5 &= 2 \times 5 \times (3 \times 5 + 1) \\
\therefore 2 \times 4 + 4 \times 7 + 6 \times 10 + 8 \times 13 \\
+ 10 \times 16 &= \sum_{n=1}^{5} 2n(3n+1)
\end{aligned}$$

1.2 Arithmetic Progression

General term of an Arithmetic Progression (AP) is given by

$$a_n = a_1 + (n-1)d$$

where a_1 is the first term, d is the common difference and n is the number of terms.

1.2.1 Practice 3

1. Find the number of terms of the AP $-4 - 2\frac{3}{4} - 1\frac{1}{2} - \frac{1}{4} + \ldots + 16$.

$$\begin{aligned}
a_1 &= -4 \\
a_n &= 16 \\
d &= -2\frac{3}{4} - (-4) \\
&= -2\frac{3}{4} + 4 \\
&= \frac{5}{4} \\
16 &= -4 + (n-1)\frac{5}{4} \\
20 &= \frac{5}{4}(n-1) \\
80 &= 5(n-1) \\
n - 1 &= 16 \\
n &= 17
\end{aligned}$$

2. Given that $a_2 = 4$ and $a_6 = -8$, find the 10th term of the AP.

Sol.

$$\begin{aligned}
a_2 &= 4 \\
a + (2-1)d &= 4 \\
a_6 &= -8 \\
a + (6-1)d &= -8
\end{aligned}$$

$$\begin{cases} a + d &= 4 & (1.1) \\ a + 5d &= \text{-8} & (1.2) \end{cases}$$

$$\begin{aligned}
(2) - (1) : 4d &= -12 \\
d &= -3 \\
a + (-3) &= 4 \\
a &= 7 \\
\therefore a_{10} &= 7 + (10-1)(-3) \\
&= 7 - 27 \\
&= -20
\end{aligned}$$

3. How many multiples of 7 are there between 50 and 500?

Sol.

$$\begin{aligned}
a_1 &= 56 \\
a_n &= 497 \\
d &= 7 \\
497 &= 56 + (n-1)7 \\
441 &= 7(n-1) \\
n - 1 &= 63 \\
n &= 64
\end{aligned}$$

4. Find 5 numbers between 30 and 54 such that these numbers form an AP.

   Sol.

$$\begin{aligned} a_1 &= 30 \\ a_7 &= 54 \\ 54 &= 30 + (7-1)d \\ 24 &= 6d \\ d &= 4 \end{aligned}$$

$$\therefore \textit{These } 5 \textit{ numbers are } 34,\ 38,\ 42,\ 46, \textit{ and } 50.$$

Arithmetic mean

If A is in between x and y, and x, A, y are in AP, then

$$A = \frac{x+y}{2}$$

1.2.2 Practice 4

1. If 9, x, 17 are in AP, find x.

   Sol.

$$\begin{aligned} x &= \frac{9+17}{2} \\ &= \frac{26}{2} \\ &= 13 \end{aligned}$$

2. Find the arithmetic mean of 26 and -11.

   Sol.

$$\begin{aligned} A &= \frac{26-11}{2} \\ &= \frac{15}{2} \end{aligned}$$

3. Find x and y when 3, x, 12, y, 21 are in AP.

   Sol.

$$\begin{aligned} x &= \frac{3+12}{2} \\ &= \frac{15}{2} \\ y &= \frac{12+21}{2} \\ &= \frac{33}{2} \end{aligned}$$

Summation of Arithmetic Progression

The summation formula for AP is given by

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

or

$$S_n = \frac{n}{2}(a_1 + a_n)$$

1.2.3 Practice 5

1. Find the sum of the first 16 terms of the AP $22 + 18 + 14 + 10 + \ldots$

   Sol.

$$\begin{aligned} a_1 &= 22 \\ n &= 16 \\ d &= -4 \\ S_n &= \frac{16}{2}(2 \times 22 + (-4)(16-1)) \\ &= \frac{16}{2}(44 + (-4)(15)) \\ &= \frac{16}{2}(44 - 60) \\ &= \frac{16}{2}(-16) \\ &= -128 \end{aligned}$$

2. If the sum of AP $23 + 19 + 15 + \ldots$ is 72, find the number of terms.

   Sol.

$$\begin{aligned} a_1 &= 23 \\ S_n &= 72 \\ d &= -4 \\ 72 &= \frac{n}{2}(2 \times 23 + (-4)(n-1)) \\ 72 &= \frac{n}{2}(46 + (-4)(n-1)) \\ 144 &= n(46 + (-4)(n-1)) \\ 144 &= n(46 - 4n + 4) \\ 144 &= n(50 - 4n) \\ 144 &= 50n - 4n^2 \\ 72 &= 25n - 2n^2 \\ 2n^2 - 25n + 72 &= 0 \\ (n-8)(2n-9) &= 0 \\ n &= 8 \end{aligned}$$

3. Given that $S_n = 2n + 3n^2$, find the first term and the common difference of the AP.

Sol.

$$\begin{aligned} S_n &= 2n + 3n^2 \\ 2n + 3n^2 &= \frac{n}{2}(2a + (n-1)d) \\ 4n + 6n^2 &= n(2a + (n-1)d) \\ 4n + 6n^2 &= 2na + (n-1)nd \\ 4n + 6n^2 &= 2na + n^2d - nd \\ 4n + 6n^2 &= (2a - d)n + dn^2 \end{aligned}$$

$$\begin{aligned} &\textit{Comparing both sides}, \\ &2a - d = 4 \\ &a = 6 \\ &d = 2 \end{aligned}$$

1.2.4 Exercise 12.2

1. Find the 10th terms of the AP $5, 13, 21, \ldots$

Sol.

$$\begin{aligned} a_1 &= 5 \\ n &= 10 \\ d &= 8 \\ a_{10} &= 5 + (10-1) \times 8 \\ &= 5 + 72 \\ &= 77 \end{aligned}$$

2. Find the 8th term of the AP $5, 4\frac{1}{4}, 3\frac{1}{2}, 2\frac{3}{4}, \ldots$

Sol.

$$\begin{aligned} a_1 &= 5 \\ n &= 8 \\ d &= -\frac{3}{4} \\ a_8 &= 5 + (8-1) \times -\frac{3}{4} \\ &= 5 - \frac{3}{4} \times 7 \\ &= 5 - \frac{21}{4} \\ &= -\frac{1}{4} \end{aligned}$$

3. Find the number of terms of the following AP.

(a) $4, 9, \ldots, 64$

Sol.

$$\begin{aligned} a_1 &= 4 \\ a_n &= 64 \\ d &= 5 \\ 64 &= 4 + (n-1) \times 5 \\ 60 &= 5(n-1) \\ 12 &= n - 1 \\ n &= 13 \end{aligned}$$

(b) $4\frac{1}{3}, 3\frac{2}{3}, 3, \ldots, -10\frac{1}{3}$

Sol.

$$\begin{aligned} a_1 &= 4\frac{1}{3} \\ a_n &= -10\frac{1}{3} \\ d &= -\frac{2}{3} \\ -10\frac{1}{3} &= 4\frac{1}{3} + (n-1) \times -\frac{2}{3} \\ -\frac{31}{3} &= \frac{13}{3} - \frac{1}{3}(n-1) \\ -31 &= 13 - 2n + 2 \\ -46 &= 2n \\ n &= 23 \end{aligned}$$

4. The 6th term of an AP is 43, and its 10th term is 75. Find the first term and common difference of this AP.

Sol.

$$\begin{aligned} a_6 &= 43 \\ a_{10} &= 75 \\ 43 &= a + (6-1)d \\ 75 &= a + (10-1)d \\ 32 &= 4d \\ d &= 8 \\ 43 &= a + 5 \times 8 \\ 43 &= a + 40 \\ 3 &= a \\ a &= 3 \\ \therefore a_1 &= 3,\ d = 8 \end{aligned}$$

5. The 7th term of an AP is -10, and the 12th term -25, find the 15th term of this AP.

Sol.

$$\begin{aligned}
a_7 &= -10\\
a_{12} &= -25\\
-10 &= a + (7-1)d\\
-25 &= a + (12-1)d\\
-15 &= 5d\\
d &= -3\\
-10 &= a + 6 \times -3\\
-10 &= a - 18\\
a &= 8\\
a_{15} &= 8 + (15-1) \times -3\\
&= 8 - 42\\
&= -34
\end{aligned}$$

6. How many multiples of 7 are there between 100 and 200?

   Sol.

$$\begin{aligned}
a &= 105\\
d &= 7\\
a_n &= 196\\
196 &= 105 + (n-1) \times 7\\
91 &= 7(n-1)\\
13 &= n-1\\
n &= 14
\end{aligned}$$

7. Find the arithmetic mean o fthe following number pairs.

   (a) $(8, 20)$

   Sol.

$$\frac{8+20}{2} = 14$$

   (b) $(-9, 17)$

   Sol.

$$\frac{-9+17}{2} = 4$$

8. Find 5 numbers between 22 and 58 such that these 7 numbers are in AP.

   Sol.

$$\begin{aligned}
a_1 &= 22\\
a_7 &= 58\\
58 &= 22 + (7-1)d\\
36 &= 6d\\
d &= 6\\
&\therefore \mathit{These}\ 5\ \mathit{numbers\ are}\ 22, 28, 34, 40, 46
\end{aligned}$$

9. Find the sum of first 20 terms of AP $12 + 15 + 18 + \ldots$

   Sol.

$$\begin{aligned}
a_1 &= 12\\
n &= 20\\
d &= 3\\
S_{20} &= \frac{20}{2}(2 \times 12 + (20-1) \times 3)\\
&= 10(24 + 57)\\
&= 10(81)\\
&= 810
\end{aligned}$$

10. Find the sum of first 12 terms of the AP $18 + 10 + 2 - 6 - \ldots$

    Sol.

$$\begin{aligned}
a_1 &= 18\\
n &= 12\\
d &= -8\\
S_{12} &= \frac{12}{2}(2 \times 18 + (12-1) \times -8)\\
&= 6(36 - 88)\\
&= 6(-52)\\
&= -312
\end{aligned}$$

11. Find the sum of first 14 terms of the AP $\frac{1}{6} + \frac{4}{3} + \frac{5}{2} + \ldots$

Sol.

$$\begin{aligned}
a_1 &= \frac{1}{6}\\
n &= 14\\
d &= \frac{7}{6}\\
S_{14} &= \frac{14}{2}(2 \times \frac{1}{6} + (14-1) \times \frac{7}{6})\\
&= 7(\frac{1}{3} + \frac{91}{6})\\
&= 7 \times \frac{93}{6}\\
&= 7 \times \frac{31}{2}\\
&= \frac{217}{2}
\end{aligned}$$

12. Find the sum of all the multiples of 13 in between 200 and 800.

Sol.

$$\begin{aligned}
a_1 &= 208\\
a_n &= 793\\
d &= 13\\
793 &= 208 + (n-1) \times 13\\
585 &= 13(n-1)\\
45 &= n-1\\
n &= 46
\end{aligned}$$

$$\begin{aligned}
S_{46} &= \frac{46}{2}(2 \times 208 + (46-1) \times 13)\\
&= 23(416 + 585)\\
&= 23(1001)\\
&= 23023
\end{aligned}$$

13. If the sum of first n terms of the AP $-3, -7, -11, \ldots$ is -903, find the value of n.

Sol.

$$\begin{aligned}
a_1 &= -3\\
d &= -4\\
-903 &= \frac{n}{2}(2 \times (-3) - 4(n-1))\\
-1806 &= -2n - 4n^2\\
4n^2 + 2n - 1806 &= 0\\
2n^2 + n - 903 &= 0\\
(n-21)(2n+43) &= 0\\
n &= 21, -43(invalid)\\
\therefore\ n &= 21
\end{aligned}$$

14. Given that the first 3 terms of an AP are $x,\ 3x-4,\ 2x+7$, find:

(a) The value of x

Sol.

$$\begin{aligned}
3x - 4 &= \frac{x + 2x + 7}{2}\\
6x - 8 &= 3x + 7\\
3x &= 15\\
x &= 5
\end{aligned}$$

(b) The common difference

Sol.

$$\begin{aligned}
a_1 &= x = 5\\
a_2 &= 3x - 4 = 3 \times 5 - 4 = 11\\
d &= 11 - 5\\
&= 6
\end{aligned}$$

(c) The sum of first 10 terms.

Sol.

$$\begin{aligned}
a_1 &= x = 5\\
n &= 10\\
d &= 6\\
S_{10} &= \frac{10}{2}(2 \times 5 + (10-1) \times 6)\\
&= 5(10 + 54)\\
&= 5(64)\\
&= 320
\end{aligned}$$

15. Let the sum of the first n terms of an AP to be $S_n = \frac{n(n+1)}{4}$, find:

(a) The first term

Sol.

$$\begin{aligned}\frac{n(n+1)}{4} &= \frac{n}{2}(2a+(n-1)d)\\ n(n+1) &= 2n(2a+dn-d)\\ n^2+n &= 4na+2dn^2-2nd\\ n^2+n &= 2dn^2+(4a-2d)n\end{aligned}$$

$$\begin{aligned}&Comparing\ both\ sides,\\ 2d &= 1\\ d &= \frac{1}{2}\\ 4a-2d &= 1\\ 4a-1 &= 1\\ 4a &= 2\\ a &= \frac{1}{2}\end{aligned}$$

(b) The common difference

Sol.

$$d = \frac{1}{2}$$

gg

(c) The 6th terms

Sol.

$$\begin{aligned}a_1 &= \frac{1}{2}\\ n &= 6\\ d &= \frac{1}{2}\\ a_6 &= \frac{1}{2}+(6-1)\times\frac{1}{2}\\ &= \frac{1}{2}+\frac{5}{2}\\ &= 3\end{aligned}$$

(d) The sum from 6th term to 10th term

Sol.

$$\begin{aligned}a &= \frac{1}{2}\\ d &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}S_{10} &= \frac{10}{2}(2\times\frac{1}{2}+(10-1)\times\frac{1}{2})\\ &= \frac{10}{2}(1+\frac{9}{2})\\ &= 5\times\frac{11}{2}\\ &= \frac{55}{2}\end{aligned}$$

$$\begin{aligned}S_5 &= \frac{5}{2}(2\times\frac{1}{2}+(5-1)\times\frac{1}{2})\\ &= \frac{5}{2}(1+2)\\ &= \frac{15}{2}\end{aligned}$$

$$\begin{aligned}S_{10}-S_6 &= \frac{55}{2}-\frac{15}{2}\\ &= \frac{40}{2}\\ &= 20\end{aligned}$$

16. Given three numbers in an AP, the sum of these three numbers is 30, and the sum of square of these numbers is 318, find these three numbers.

Sol.

$$\begin{aligned}
a_1 + a_2 + a_3 &= 30 \\
a_1^2 + a_2^2 + a_3^2 &= 318 \\
a_2 - a_1 &= a_3 - a_2 \\
a_1 - 2a_2 + a_3 &= 0 \\
3a_2 &= 30 \\
a_2 &= 10 \\
a_1 - 20 + a_3 &= 0 \\
a_1 + a_3 &= 20 \\
a_3 &= 20 - a_1 \\
a_1^2 + 100 + (20 - a_1)^2 &= 318 \\
a_1^2 + 100 + 400 + a_1^2 - 40a_1 &= 318 \\
2a_1^2 - 40a_1 + 182 &= 0 \\
a_1^2 - 20a_1 + 91 &= 0 \\
(a_1 - 7)(a_1 - 13) &= 0 \\
a_1 &= 7 or a_1 = 13
\end{aligned}$$

$$\therefore\ These\ three\ numbers\ are\ 7,\ 10,\ and\ 13$$

17. Find the sum of all the numbers between 100 and 200 that are both the multiples of 2 and 3.

Sol.

$$\begin{aligned}
a_1 &= 102 \\
d &= 6 \\
a_n &= 198 \\
198 &= 102 + (n-1) \times 6 \\
96 &= 6(n-1) \\
6n - 6 &= 96 \\
6n &= 102 \\
n &= 17
\end{aligned}$$

$$\begin{aligned}
S_{17} &= \frac{17}{2}(2 \times 102 + (17-1) \times 6) \\
&= \frac{17}{2}(204 + 96) \\
&= \frac{17}{2}(300) \\
&= 150 \times 17 \\
&= 2550
\end{aligned}$$

18. Given an AP $-100 - 96 - 92 - \ldots$:

(a) Find the term where the number become positive.

Sol.

$$\begin{aligned}
a_1 &= -100 \\
d &= 4 \\
a_n &= -100 + (n-1) \times 4 > 0 \\
-100 + 4n - 4 &> 0 \\
4n &> 104 \\
n &> 26
\end{aligned}$$

$$\therefore\ n = 27$$

(b) Find the term where the sum of this AP becomes positive.

Sol.

$$\begin{aligned}
S_n &= \frac{n}{2}(2(-100) + (n-1) \times (4)) > 0 \\
\frac{n}{2}(-200 + 4n - 4) &> 0 \\
\frac{n}{2}(-204 + 4n) &> 0 \\
n(2n - 102) &> 0 \\
n(n - 51) &> 0 \\
n &> 51
\end{aligned}$$

$$\therefore\ n = 52$$

19. Find the first negative term of the AP $20, 19\frac{1}{5}, 18\frac{2}{5}, \ldots$

Sol.

$$\begin{aligned}
a_1 &= 20 \\
d &= -\frac{4}{5} \\
a_n &= 20 + (n-1) \times (-\frac{4}{5}) < 0 \\
100 - 4n + 4 &< 0 \\
4n &> 104 \\
n &> 26
\end{aligned}$$

$$\therefore\ n = 27$$

20. Given an AP $10 + 9\frac{1}{5} + 8\frac{2}{5} + \ldots$, what is the first negative term? When will the sum of the terms become negative, and what's the value of it?

Sol.

$$
\begin{aligned}
a_1 &= 10 \\
d &= -\frac{4}{5} \\
a_n = 10 + (n-1) \times (-\frac{4}{5}) &< 0 \\
10 - \frac{4}{5}(n-1) &< 0 \\
50 - 4n + 4 &< 0 \\
-4n &< -54 \\
n &> 13\frac{1}{2}
\end{aligned}
$$

$$\therefore\ n = 14$$

$$
\begin{aligned}
S_n = \frac{n}{2}(2 \times 10 + (n-1) \times (-\frac{4}{5})) &< 0 \\
\frac{n}{2}(20 - \frac{4}{5}(n-1)) &< 0 \\
20n - \frac{4}{5}(n^2 - n) &< 0 \\
100n - 4n^2 + 4n &< 0 \\
25n - n^2 + n &< 0 \\
26n - n^2 &< 0 \\
n(n-26) &> 0 \\
n &> 26
\end{aligned}
$$

$$\therefore\ n = 27$$

$$
\begin{aligned}
S_{27} &= \frac{27}{2}(2 \times 10 + (27-1) \times (-\frac{4}{5})) \\
&= \frac{27}{2}(20 - \frac{4}{5}(27-1)) \\
&= \frac{27}{2}(20 - \frac{4}{5}(26)) \\
&= \frac{27}{2} \times (-\frac{4}{5}) \\
&= -\frac{54}{5}
\end{aligned}
$$

$\therefore$ *The first negative term is the* $14th$ *term*

$\therefore$ *The first term where the sum of the terms becomes negative is the* $27th$ *term*

$\therefore$ *The value of the sum of the terms when it becomes negative is* $-\frac{54}{5}$

21. Given a polygon which all their internal angles are in AP. The common difference of this AP is $6°$, the largest angle is $135°$. How many sides does this polygon have?

    Sol.

$$
\begin{aligned}
a_1 &= 135 \\
d &= -6 \\
\frac{n}{2}(2 \times 135 + (n-1) \times (-6)) &= 180(n-2) \\
n(270 - 6(n-1)) &= 360(n-2) \\
n(276 - 6n) &= 360n - 720 \\
276n - 6n^2 &= 360n - 720 \\
46n - n^2 &= 60n - 120 \\
n^2 + 14n - 120 &= 0 \\
(n+20)(n-6) &= 0 \\
n &= -20\ (invalid) \\
n &= 6
\end{aligned}
$$

$\therefore$ *The number of sides is* 6