8. Find the height from vertex B to side AC in $\triangle ABC$, where the vertices are $A(3, 6)$, $B(2, -3)$, and $C(-1, 4)$.

Sol.

$$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4 - 6}{-1 - 3}$$
$$= \frac{-2}{-4}$$
$$= \frac{1}{2}$$
$$y - y_A = m_{AC}(x - x_A)$$
$$y - 6 = \frac{1}{2}(x - 3)$$
$$2y - 12 = x - 3$$
$$x - 2y + 9 = 0$$
$$d = \left| \frac{Ax_B + By_B + C}{\sqrt{A^2 + B^2}} \right|$$
$$= \left| \frac{2 + 6 + 9}{\sqrt{1^2 + (-2)^2}} \right|$$
$$= \left| \frac{17}{\sqrt{5}} \right|$$
$$= \frac{17\sqrt{5}}{5}$$

9. Find the vertices of triangle whose mid point of sides are (-2, -5), (-1, 1) and (4, -1).

Sol.

Let the vertices be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$

$$\frac{x_1 + x_2}{2} = -2 \tag{1}$$

$$\frac{x_2 + x_3}{2} = -1 \tag{2}$$

$$\frac{x_3 + x_1}{2} = 4 \tag{3}$$

$$(1) \Rightarrow x_1 + x_2 = -4 \tag{4}$$
$$(2) \Rightarrow x_2 + x_3 = -2 \tag{5}$$
$$(3) \Rightarrow x_3 + x_1 = 8 \tag{6}$$

$$(4) + (5) + (6) \Rightarrow 2x_1 + 2x_2 + 2x_3 = 2$$
$$x_1 + x_2 + x_3 = 1 \tag{7}$$

Sub (4) in (7) $\Rightarrow -4 + x_3 = 1$
$$x_3 = 5$$
Sub $x_3 = 5$ in (3) $\Rightarrow x_1 = 3$
Sub $x_3 = 5$ in (2) $\Rightarrow x_2 = -7$

$$\frac{y_1 + y_2}{2} = -5 \tag{8}$$

$$\frac{y_2 + y_3}{2} = 1 \tag{9}$$

$$\frac{y_3 + y_1}{2} = -1 \tag{10}$$

$$(8) \Rightarrow y_1 + y_2 = -10 \tag{11}$$
$$(9) \Rightarrow y_2 + y_3 = 2 \tag{12}$$
$$(10) \Rightarrow y_3 + y_1 = -2 \tag{13}$$

$$(11) + (12) + (13) \Rightarrow 2y_1 + 2y_2 + 2y_3 = -10$$
$$y_1 + y_2 + y_3 = -5 \tag{14}$$

Sub (11) in (14) $\Rightarrow -10 + y_3 = -5 \tag{15}$
$$y_3 = 5 \tag{16}$$
Sub $y_3 = 5$ in (13) $\Rightarrow y_1 = -7 \tag{17}$
Sub $y_3 = 5$ in (12) $\Rightarrow y_2 = -3 \tag{18}$

$\therefore A(-7, -3)$, $B(3, -7)$ and $C(5, 5)$

10. If (-1, -2), (2, t), and (3, 6) are on the same line, find the value of t.

Sol.

$$\frac{1}{2} |(-t + 12 - 6) - (-4 + 3t - 6)| = 0$$

$$-t + 6 + 10 - 3t =$$

$$-4t = -16$$

$$t = 4$$