1. $\sqrt{\sqrt{3}+\sqrt{\sqrt{3}+x}} = x$

**Sol.**

$\because \forall n \in \mathbb{R}, n >= 0,\ \sqrt{n} >= 0 \quad \therefore x > 0$

$$\begin{aligned}
\sqrt{\sqrt{3}+\sqrt{\sqrt{3}+x}} &= x \\
\sqrt{3}+\sqrt{\sqrt{3}+x} &= x^2 \\
\sqrt{\sqrt{3}+x} &= x^2 - \sqrt{3} \\
x+\sqrt{3} &= \left(x^2-\sqrt{3}\right)^2 \\
&= x^4 - 2\sqrt{3}x^2 + 3 \\
x^4 - 2\sqrt{3}x^2 + 3 - x - \sqrt{3} &= 0
\end{aligned}$$

Let $a = \sqrt{3}$,

$$\begin{aligned}
x^4 - 2ax^2 + a^2 - x - a &= 0 \\
a^2 - (2x^2+1)a + x^4 - x &= 0 \\
a^2 - (2x^2+1)a + x(x^3-1) &= 0 \\
a^2 - (2x^2+1)a + x(x-1)(x^2+x+1) &= 0 \\
a^2 - (2x^2+1)a + (x^2-x)(x^2+x+1) &= 0 \\
\left[a-(x^2-x)\right]\left[a-(x^2+x+1)\right] &= 0 \\
a = x^2 - x \text{ or } a &= x^2+x+1
\end{aligned}$$

When $a = x^2 - x$,

$$\begin{aligned}
x^2 - x &= \sqrt{3} \\
x^2 - x - \sqrt{3} &= 0 \\
x &= \frac{1 \pm \sqrt{1+4\sqrt{3}}}{2} \\
\because\ x &> 0 \\
\therefore x &= \frac{1 + \sqrt{1+4\sqrt{3}}}{2}
\end{aligned}$$

When $a = x^2 + x + 1$,

$$\begin{aligned}
x^2 + x + 1 &= \sqrt{3} \\
x^2 + x + 1 - \sqrt{3} &= 0 \\
x &= \frac{-1 \pm \sqrt{1-4(1-\sqrt{3})}}{2} \\
&= \frac{-1 \pm \sqrt{4\sqrt{3}-3}}{2} \\
\because\ x &> 0 \\
\therefore\ x &= \frac{-1 + \sqrt{4\sqrt{3}-3}}{2}
\end{aligned}$$

$\therefore\ x = \dfrac{1+\sqrt{1+4\sqrt{3}}}{2}$ or $x = \dfrac{-1+\sqrt{4\sqrt{3}-3}}{2}$

2. Given that $r$ is a rational number such that $0 < r < 1$ and

$$\sum_{n=1}^{\infty} n^2 r^n = 180,$$

find the value of $180r$.

**Sol.**

$$\begin{aligned}
\sum_{n=1}^{\infty} n^2 r^n &= \sum_{n=1}^{\infty} n \cdot n \cdot r \cdot r^{n-1} \\
&= \sum_{n=1}^{\infty} nr \cdot nr^{n-1} \\
&= \sum_{n=1}^{\infty} nr \cdot \frac{d}{dr}(r^n) \\
&= \sum_{n=1}^{\infty} nr \cdot \frac{d}{dr} \sum_{n=1}^{\infty} r^n \\
&= \sum_{n=1}^{\infty} nr \cdot \frac{d}{dr}\left(\frac{r}{1-r}\right) k
\end{aligned}$$

3. $\left(1+\dfrac{1}{x}\right)^{x+1} = \left(1+\dfrac{1}{2022}\right)^{2022}$, $x = ?$

**Sol.**

$$\begin{aligned}
\left(1+\frac{1}{x}\right)^{x+1} &= \left(1+\frac{1}{2022}\right)^{2022} \\
\left(\frac{x+1}{x}\right)^{x+1} &= \left(1+\frac{1}{2022}\right)^{2022} \\
\left(\frac{x}{x+1}\right)^{-(x-1)} &= \left(1+\frac{1}{2022}\right)^{2022} \\
\left(\frac{(x+1)-1}{x+1}\right)^{-(x-1)} &= \left(1+\frac{1}{2022}\right)^{2022} \\
\left(1+\frac{-1}{x+1}\right)^{-(x-1)} &= \left(1+\frac{1}{2022}\right)^{2022} \\
\left(1+\frac{1}{-(x+1)}\right)^{-(x-1)} &= \left(1+\frac{1}{2022}\right)^{2022} \\
-(x+1) &= 2022 \\
x+1 &= -2022 \\
x &= -2023
\end{aligned}$$

4. $(x^x)'$

**Sol.**

$$\begin{aligned}
(x^x)' &= (e^{\ln x^x})' \\
&= (e^{x \ln x})'
\end{aligned}$$