Solution Book of Mathematic

Ssnior 2 Part I

MELVIN CHIA

Written on 9 October 2022

Contents

Chapter 12

Marix and Determinant

12.1 Matrix

Definition of Matrix

A matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns. It is generally denoted as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

where m is the number of rows and n is the number of columns.

Each number in the matrix is called *an entry of the matrix*, the number in the i^{th} row and j^{th} column is denoted as a_{ij}. Thus, a matrix can also be denoted as $A = (a_{ij})$, or $A = (a_{ij})_{mn}$ where m is the number of rows and n is the number of columns.

A matrix with m rows and n columns is called an $m \times n$ matrix, where $m \times n$ is called the *order of the matrix*. For example, the following matrix is a **3×4 matrix**:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

When $m = n$, the matrix is called a *square matrix*. For example, the following matrix is a **third-order square matrix**:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

When $m = 1$, the matrix is called a *row matrix*. For example, the following matrix is a **row matrix**:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

When $n = 1$, the matrix is called a *column matrix*. For example, the following matrix is a **column matrix**:

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

Equal Matrices

Two matrices A and B are equal if they have the same order and the same entries. That is, $A = B$ if and only if $A_{ij} = B_{ij}$ for all i and j.

Zero Matrix

The matrix with all entries equal to zero is called the *zero matrix* and is denoted as O. Zero matrix can be in any order. For exmaple, the matrix below is a **2 × 2 zero matrix** or a **second-order square zero matrix**:

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Identity Matrix

The matrix with all entries equal to zero except the entries on the main diagonal, which are equal to one, is called the *identity matrix* and is denoted as I. Identity matrix can be in any order. The form of an identity matrix is:

$$I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Transpose Matrix

The transpose of a matrix A is denoted as A', A^t or A^T and is obtained by interchanging the rows and columns of A. For example, given the matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

The transpose of A is:

$$A' = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Thus, we know that the transpose matrix of $m \times n$ matrix is a $n \times m$ matrix.

12.1.1 Exercise 14.1

1. State the order of the following matrices.

   (a) $A = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

   Sol. A is a matrix with order $\mathbf{3 \times 1}$.

   (b) $B = \begin{bmatrix} 2 & 1 & 3 & 0 \\ 4 & 2 & 1 & 5 \end{bmatrix}$

   Sol. B is a matrix with order $\mathbf{2 \times 4}$.

   (c) $C = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 3 & 0 \\ 2 & 1 & -1 \end{bmatrix}$

   Sol. C is a matrix with order $\mathbf{3 \times 3}$.

2. Given $A = \begin{bmatrix} 1 & 5 & -2 & 4 \\ 2 & -4 & 3 & 1 \\ 0 & 6 & 4 & 7 \end{bmatrix}$, what is a_{23} and a_{34}?

   Sol. $a_{23} = 3$ and $a_{34} = 7$.

3. If $\begin{bmatrix} 2 & 0 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & x \end{bmatrix}$, what is x?

   Sol. $x = -4$.

12.2 Matrix Addition and Substraction

Given two matrices A and B of the same order, the sum of A and B is defined as the matrix $A + B$ whose (i, j)-th entry is the sum of the (i, j)-th entries of A and B. That is:

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{bmatrix}$$

The difference of A and B is defined as the matrix $A - B$ whose (i, j)-th entry is the difference of the (i, j)-th entries of A and B. That is:

$$A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \cdots & a_{2n} - b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} - b_{m1} & a_{m2} - b_{m2} & \cdots & a_{mn} - b_{mn} \end{bmatrix}$$

Note that the order of A and B must be the same. For example, the following metrices cannot be added or subtracted:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The addition of matrices has the following properties:

- Commutative: $A + B = B + A$.
- Associative: $(A + B) + C = A + (B + C)$.
- Identity: $A \pm O = A$.
- Inverse: $A + (-A) = O$.
- Transpose: $(A \pm B)' = A' \pm B'$.

where A, B, C are matrices of the same order and O is the zero matrix of the same order as A.

Given a matrix A, if $A = A'$, then A is called a *symmetric matrix*. If $A = -A'$, then A is called an *antisymmetric matrix*.

For any given matrix A, $A + A'$ is symmetric, and $A - A'$ is antisymmetric.

12.2.1 Practice 1

Let $A = \begin{bmatrix} -4 & 2 & -7 \\ 5 & 4 & 0 \\ 3 & -2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & -5 \\ 4 & -1 & 1 \end{bmatrix}$. Find the following:

1. $A + B'$.

   Sol.

   $$B' = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 1 & -1 \\ 2 & -5 & 1 \end{bmatrix}$$

   $$A + B' = \begin{bmatrix} -3 & 5 & -3 \\ 8 & 5 & -1 \\ 5 & -7 & -2 \end{bmatrix}$$

2. (A-B)'

   Sol.

   $$A - B = \begin{bmatrix} -5 & -1 & -9 \\ 2 & 3 & 5 \\ -1 & -1 & -4 \end{bmatrix}$$

   $$(A - B)' = \begin{bmatrix} -5 & 2 & -1 \\ -1 & 3 & -1 \\ -9 & 5 & -4 \end{bmatrix}$$

12.2.2 Exercise 14.2

Let $P = \begin{bmatrix} -5 & 4 & 2 \\ 6 & -4 & 3 \\ -2 & 1 & 6 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix}$. Evaluate the following:

1. $(P + Q)'$

   Sol.

   $$P + Q = \begin{bmatrix} -4 & 2 & 2 \\ 9 & -2 & 4 \\ -2 & 1 & 10 \end{bmatrix}$$

   $$\therefore (P + Q)' = \begin{bmatrix} -4 & 9 & -2 \\ 2 & -2 & 1 \\ 2 & 4 & 10 \end{bmatrix}$$

2. $Q' - P'$

   Sol.

   $$Q - P = \begin{bmatrix} 6 & -6 & 2 \\ -3 & 6 & -2 \\ 2 & -1 & -2 \end{bmatrix}$$

   $$\therefore Q' - P' = (Q - P)' = \begin{bmatrix} 6 & -3 & 2 \\ -6 & 6 & -1 \\ 2 & -2 & -2 \end{bmatrix}$$

3. $(P' - Q)'$

Sol.

$$P' = \begin{bmatrix} -5 & 6 & -2 \\ 4 & -4 & 1 \\ 2 & 3 & 6 \end{bmatrix}$$

$$P' - Q = \begin{bmatrix} -6 & 8 & -2 \\ 1 & -6 & 0 \\ 2 & 3 & 2 \end{bmatrix}$$

$$\therefore (P' - Q)' = \begin{bmatrix} -6 & 1 & 2 \\ 8 & -6 & 3 \\ -2 & 0 & 2 \end{bmatrix}$$

4. $P' - (I - Q)'$

   Sol.

$$\begin{aligned} P' - (I - Q)' &= P' - I' + Q' \\ &= (P + Q)' - I' \\ &= (P + Q - I)' \end{aligned}$$

$$P + Q - I = \begin{bmatrix} -4 & 2 & 2 \\ 9 & -2 & 4 \\ -2 & 1 & 10 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 2 & 2 \\ 9 & -3 & 4 \\ -2 & 1 & 9 \end{bmatrix}$$

$$\begin{aligned} \therefore P' - (I - Q)' &= (P + Q - I)' \\ &= \begin{bmatrix} -5 & 9 & -2 \\ 2 & -3 & 1 \\ 2 & 4 & 9 \end{bmatrix} \end{aligned}$$

12.3 Scalar Product of Matrices

Let $A = (a_{ij})_{m\times n}$ be an $m \times n$ matrix, k be any real number, then $kA = (ka_{ij})_{m\times n}$. This is called scalar product of a matrix A and scalar k. For example:

$$k\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} k & 2k & 3k \\ 4k & 5k & 6k \end{bmatrix}$$

The scalar product of a matrix has the following properties:

- $r(A + B) = rA + sB$
- $(r + s)A = rA + sA$
- $(rs)A = r(sA)$

12.3.1 Practice 2

Let $A = \begin{bmatrix} 2 & 0 \\ -3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 2 & -4 \end{bmatrix}$. Evaluate the following:

1. $3A + B$

   Sol.

$$\begin{aligned} 3A + B &= \begin{bmatrix} 6 & 0 \\ -9 & 15 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 7 & -1 \\ -7 & 11 \end{bmatrix} \end{aligned}$$

2. $2A - 3B$

   Sol.

$$\begin{aligned} 2A - 3B &= \begin{bmatrix} 4 & 0 \\ -6 & 10 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ 6 & -12 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 3 \\ -12 & 22 \end{bmatrix} \end{aligned}$$

3. $4B - 2A$

   Sol.

$$\begin{aligned} 4B - 2A &= 2(2B - A) \\ &= 2\left(\begin{bmatrix} 2 & -2 \\ 4 & -8 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ -3 & 5 \end{bmatrix}\right) \\ &= 2\begin{bmatrix} 0 & -2 \\ 7 & -13 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -4 \\ 14 & -26 \end{bmatrix} \end{aligned}$$

4. $A' - 2B'$

Sol.

$$
\begin{aligned}
A' - 2B' &= (A - 2B)' \\
&= \left(\begin{bmatrix} 2 & 0 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ 4 & -8 \end{bmatrix} \right)' \\
&= \left(\begin{bmatrix} 0 & 2 \\ -7 & 13 \end{bmatrix} \right)' \\
&= \begin{bmatrix} 0 & -7 \\ 2 & 13 \end{bmatrix}
\end{aligned}
$$

12.3.2 Exercise 14.3

1. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$, Calculate the following:

   (a) $2A - 3B + 4C$

   Sol.

$$
\begin{aligned}
&2A - 3B + 4C \\
&= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 18 & 3 \\ 9 & 6 \end{bmatrix} + \begin{bmatrix} 12 & 4 \\ 4 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 16 & 10 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} 18 & 3 \\ 9 & 6 \end{bmatrix} \\
&= \begin{bmatrix} -2 & 7 \\ -3 & -6 \end{bmatrix}
\end{aligned}
$$

   (b) $4A' - (C + B)'$

   Sol.

$$
\begin{aligned}
&4A' - (C + B') \\
&= 4 \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} - \left(\begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 1 & 2 \end{bmatrix} \right) \\
&= \begin{bmatrix} 8 & 4 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} 9 & 4 \\ 2 & 2 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 0 \\ 10 & -2 \end{bmatrix}
\end{aligned}
$$

2. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 4 & -1 \\ 3 & 1 \\ 2 & -3 \end{bmatrix}$, evaluate the following:

   (a) $3A + B - 2C$

   Sol.

$$
\begin{aligned}
3A + B - 2C &= \begin{bmatrix} 3 & 6 \\ 0 & 3 \\ 9 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 8 & -2 \\ 6 & 2 \\ 4 & -6 \end{bmatrix} \\
&= \begin{bmatrix} 5 & 7 \\ 4 & 6 \\ 10 & 3 \end{bmatrix} - \begin{bmatrix} 8 & -2 \\ 6 & 2 \\ 4 & -6 \end{bmatrix} \\
&= \begin{bmatrix} -3 & 9 \\ -2 & 4 \\ 6 & -3 \end{bmatrix}
\end{aligned}
$$

   (b) $3(A + C)' - B'$

   Sol.

$$
\begin{aligned}
&3(A + C)' - B' \\
&= 3 \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -1 \\ 3 & 1 \\ 2 & -3 \end{bmatrix} \right)' - \begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{bmatrix}' \\
&= 3 \left(\begin{bmatrix} 5 & 1 \\ 3 & 2 \\ 5 & -2 \end{bmatrix} \right)' - \begin{bmatrix} 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 15 & 9 & 15 \\ 3 & 6 & -6 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 13 & 5 & 14 \\ 2 & 3 & -6 \end{bmatrix}
\end{aligned}
$$

3. Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$, Find the matrix X in the following expression:

   (a) $X + 4A = 3(X + B) - A$

   Sol.

$$
\begin{aligned}
X + 4A &= 3(X + B) - A \\
&= 3X + 3B - A \\
2X &= 5A - 3B \\
2X &= 5 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} - 3 \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 5 & 10 & 15 \\ 0 & 5 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 3 & 9 \\ 3 & 6 & 0 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 7 & 6 \\ -3 & -1 & 5 \end{bmatrix} \\
x &= \begin{bmatrix} -\frac{1}{2} & \frac{7}{2} & 3 \\ -\frac{3}{2} & -\frac{1}{2} & \frac{5}{2} \end{bmatrix}
\end{aligned}
$$

   (b) $2A - B + X' = B$

Sol.

$$
\begin{aligned}
2A - B + X' &= B \\
X' &= -2A + 2B \\
&= -2(A - B) \\
&= -2\left(\begin{bmatrix}1 & 2 & 3\\0 & 1 & 1\end{bmatrix}\right. \\
&\left. - \begin{bmatrix}2 & 1 & 3\\1 & 2 & 0\end{bmatrix}\right) \\
&= -2\begin{bmatrix}-1 & 1 & 0\\-1 & -1 & 1\end{bmatrix} \\
&= \begin{bmatrix}2 & -2 & 0\\2 & 2 & -2\end{bmatrix} \\
X &= \begin{bmatrix}2 & 2\\-2 & 2\\0 & -2\end{bmatrix}
\end{aligned}
$$

12.4 Multiplication of Matrices

Let A and B be matrices of order $m \times n$ and $n \times p$ respectively. Then the product of A and B is defined as the matrix AB of order $m \times p$ such that the $(i, j)^{th}$ element of AB is given by

$$(AB)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

The multiplication of matrices has the following properties:

- Associative: $A(BC) = (AB)C$
- Distributive: $A(B + C) = AB + AC$ and $(B + C)A = BA + CA$
- $k(AB) \neq (kA)B$ for $k \neq 0$

12.4.1 Practice 3

Evaluate the following:

1. $\begin{bmatrix}-1 & -1\\2 & 3\end{bmatrix}\begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix}$

Sol.

$$
\begin{aligned}
&\begin{bmatrix}-1 & -1\\2 & 3\end{bmatrix}\begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix} \\
&= \begin{bmatrix}-1(-1) + (-1)(2) & -1(2) + (-1)(1)\\2(2) + 3(-1) & 2(1) + 3(2)\end{bmatrix} \\
&= \begin{bmatrix}1 & -3\\1 & 8\end{bmatrix}
\end{aligned}
$$

2. $\begin{bmatrix}3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 2\\3 & -3 & 2\end{bmatrix}$

Sol.

$$
\begin{aligned}
&\begin{bmatrix}3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 2\\3 & -3 & 2\end{bmatrix} \\
&= \begin{bmatrix}3(0) + 4(3) & 3(1) + 4(-3) & 3(2) + 4(2)\\-1(0) + 1(3) & -1(1) + 1(-3) & -1(2) + 1(2)\end{bmatrix} \\
&= \begin{bmatrix}12 & -9 & 14\\3 & -4 & 0\end{bmatrix}
\end{aligned}
$$

3. $\begin{bmatrix}1 & 0\\2 & 4\\3 & -5\end{bmatrix}\begin{bmatrix}6 & 1 & 5\\-2 & 3 & 2\end{bmatrix}$

Sol.

$$
\begin{aligned}
&\begin{bmatrix}1 & 0\\2 & 4\\3 & -5\end{bmatrix}\begin{bmatrix}6 & 1 & 5\\-2 & 3 & 2\end{bmatrix} \\
&= \begin{bmatrix}1(6) + 0(-2) & 1(1) + 0(3) & 1(5) + 0(2)\\2(6) + 4(-2) & 2(1) + 4(3) & 2(5) + 4(2)\\3(6) + (-5)(-2) & 3(1) + (-5)(3) & 3(5) + (-5)(2)\end{bmatrix} \\
&= \begin{bmatrix}6 & 1 & 5\\4 & 14 & 18\\28 & -12 & 5\end{bmatrix}
\end{aligned}
$$

4. $\begin{bmatrix}1 & 3 & 2\\0 & 1 & 5\end{bmatrix}\begin{bmatrix}1 & 3\\2 & 1\\-1 & 3\end{bmatrix}$

Sol.

$$
\begin{aligned}
&\begin{bmatrix}1 & 3 & 2\\0 & 1 & 5\end{bmatrix}\begin{bmatrix}1 & 3\\2 & 1\\-1 & 3\end{bmatrix} \\
&= \begin{bmatrix}1(1) + 3(2) + 2(-1) & 1(3) + 3(1) + 2(3)\\0(1) + 1(2) + 5(-1) & 0(3) + 1(1) + 5(3)\end{bmatrix} \\
&= \begin{bmatrix}5 & 12\\-3 & 16\end{bmatrix}
\end{aligned}
$$

12.4.2 Exercise 14.4

Calculate the following products (Question 1 to 8):

1. $\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}$

Sol.

$$
\begin{aligned}
&\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix} \\
&= \begin{bmatrix}1(1) + 2(2) + 3(3)\end{bmatrix} \\
&= \begin{bmatrix}14\end{bmatrix}
\end{aligned}
$$

2. $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix}$

Sol.

$$\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix}$$
$$=\begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\3 & 6 & 9\end{bmatrix}$$

3. $\begin{bmatrix}2 & -3\\1 & 5\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Sol.

$$\begin{bmatrix}2 & -3\\1 & 5\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}2(1)+(-3)(0) & 2(0)+(-3)(1)\\1(1)+5(0) & 1(0)+5(1)\end{bmatrix}$$
$$=\begin{bmatrix}2 & -3\\1 & 5\end{bmatrix}$$

4. $\begin{bmatrix}-6 & -4 & 2\\7 & 8 & -5\end{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix}$

Sol.

$$\begin{bmatrix}-6 & -4 & 2\\7 & 8 & -5\end{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix}$$
$$=\begin{bmatrix}-6(1)+(-4)(2)+2(3)\\7(1)+8(2)+(-5)(3)\end{bmatrix}$$
$$=\begin{bmatrix}-8\\8\end{bmatrix}$$

5. $\begin{bmatrix}2 & 3 & 4\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}2 & 0\\3 & 1\\4 & 2\end{bmatrix}$

Sol.

$$\begin{bmatrix}2 & 3 & 4\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}2 & 0\\3 & 1\\4 & 2\end{bmatrix}$$
$$=\begin{bmatrix}2(2)+3(3)+4(4) & 2(0)+3(1)+4(2)\\0(2)+1(3)+2(4) & 0(0)+1(1)+2(2)\end{bmatrix}$$
$$=\begin{bmatrix}27 & 11\\11 & 5\end{bmatrix}$$

6. $\begin{bmatrix}6 & 4 & 2\\5 & -2 & 0\\0 & 3 & 1\end{bmatrix}\begin{bmatrix}5\\2\\3\end{bmatrix}$

Sol.

$$\begin{bmatrix}6 & 4 & 2\\5 & -2 & 0\\0 & 3 & 1\end{bmatrix}\begin{bmatrix}5\\2\\3\end{bmatrix}$$
$$=\begin{bmatrix}6(5)+4(2)+2(3)\\5(5)+(-2)(2)+0(3)\\0(5)+3(2)+1(3)\end{bmatrix}$$
$$=\begin{bmatrix}44\\21\\9\end{bmatrix}$$

7. $\begin{bmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{bmatrix}$

Sol.

$$\begin{bmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}0(0)+1(1)+0(0) & 0(1)+1(0)+0(0) & 0(0)+1(0)+0(1)\\1(0)+0(1)+0(0) & 1(1)+0(0)+0(0) & 1(0)+0(0)+0(1)\\0(0)+0(1)+1(0) & 0(1)+0(0)+1(0) & 0(0)+0(0)+1(1)\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

8. $\begin{bmatrix}1 & -1 & 1\\-3 & 2 & -1\\-2 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\1 & 2 & 3\end{bmatrix}$

Sol.

$$\begin{bmatrix}1 & -1 & 1\\-3 & 2 & -1\\-2 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\1 & 2 & 3\end{bmatrix}$$
$$=\begin{bmatrix}1+(-2)+1 & 2+(-4)+2 & 3+(-6)+3\\(-3)+4+(-1) & (-6)+8+(-2) & (-9)+12+(-3)\\(-2)+2+0 & (-4)+4+0 & (-6)+6+0\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$