Solution Book of Mathematic

Ssnior 2 Part I

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Chapter 15

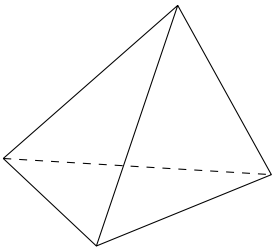
Solid Geometry, Longitude and Latitude

15.1 Solid Geometry

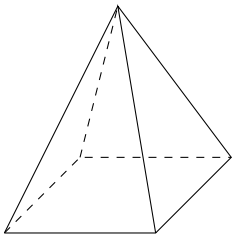
Polyhedron

A polyhedron is a solid bounded by a finite amount of flat polygon, and each side of the polygons must be the common edge of two polygons. Polyhedron can be classified into tetrahedron, pentahedron, hexahedron, etc. based on the number of flat surfaces, aka the *faces* of the polyhedron. The common side of two faces of a polyhedron is called an edge, and the common vertex of three edges is called an *apex*.

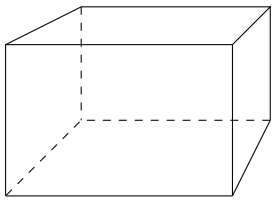
Besides, the angles formed by the faces intersecting at the same apex are called *polyhedral angles* or *solid angles*. The line segment connecting two apexes at different faces is called a *diagonal*.



Tetrahedron



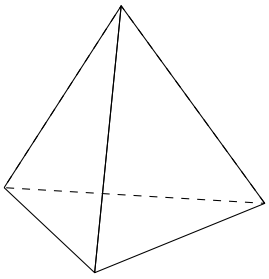
Pentahedron



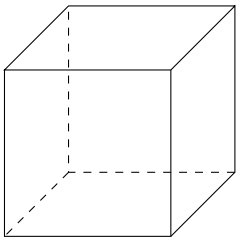
Hexahedron

Regular Polyhedron

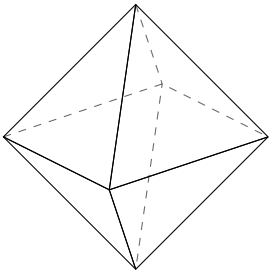
A regular polyhedron is a polyhedron with all faces being regular polygons, and all polyhedral angles being equal. The regular polyhedron can be classified into 5 types: regular tetrahedron, regular octahedron, regular hexahedron, regular dodecahedron and regular icosahedron.



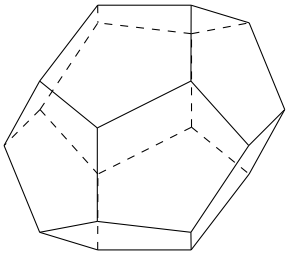
Regular Tetrahedron



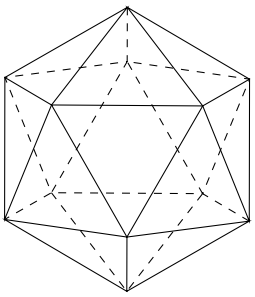
Regular Hexahedron



Regular Octahedron



Regular Dodecahedron

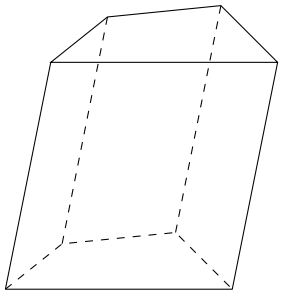


Regular Icosahedron

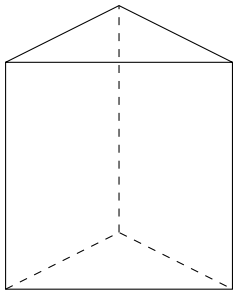
Prism

If two faces of a polyhedron are parallel, while the other faces intersect in sequence to form parallel lines, then the polyhedron is called a prism. The two faces which are parallel to each other are called the bases of the prism, and the other faces are called the lateral faces of the prism. The common sides that two adjacent lateral faces share is called the lateral edges of the prism. The distance between two bases is called the height of the prism.

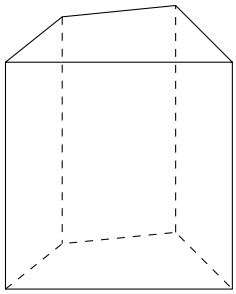
Prism with lateral edges that aren't parallel to each other are called oblique prism; prism with lateral edges that are parallel to each other are called regular prism; regular prism with regular bases are called regular prism.



Oblique Prism

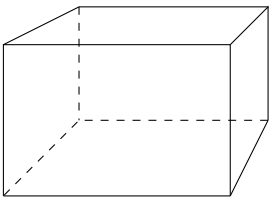


Regular Prism

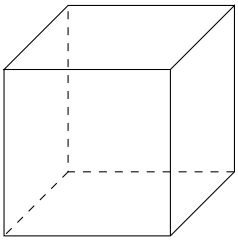


Right Prism

Prism with bases of parallelogram are called parallelepiped. Parallelepiped with lateral edges that are parallel to each other are called right parallelepiped. Right parallelepiped with regular bases are called cuboid, and a cuboid with equal width, height, and depth is called a cube.



Cuboid

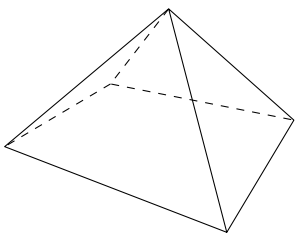


Cube

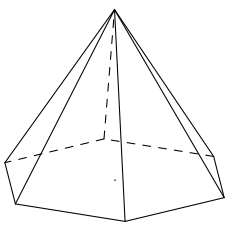
Pyramid

If a polyhedron has a base and all its lateral faces are triangles that shares a common apex, then the polyhedron is called a pyramid.

If the foot point of a pyramid is the centre of its base, then the pyramid is called a right pyramid. If the base of a right pyramid is a regular polygon, then the pyramid is called a regular pyramid.



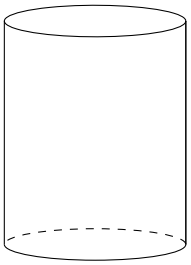
Right Pyramid



Regular Pyramid

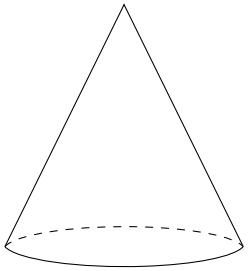
Right Circular Cylinder

A right circular cylinder is the solid of revolution generated by rotating a rectangle about one of its sides.



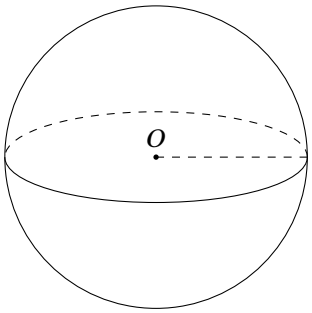
Right Circular Cone

A right circular cone is the solid of revolution generated by rotating a right-angled triangle about one of its sides.



Sphere

The surface of revolution generated by rotating a semicircle about its diameter is called a spherical surface, and the solid covered by it is called a sphere.

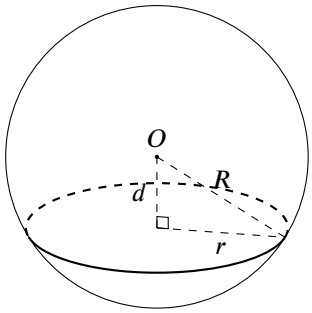


If the circle is cut with a plane, the plane has the following properties:

1. The line joining the centre of the sphere to the centre of the plane are perpendicular to the plane.

2. The distance of the plane from the centre of the sphere d, the radius of the sphere R and the radius of the plane r has the following relation:

$$r = \sqrt{R^2 - d^2}$$

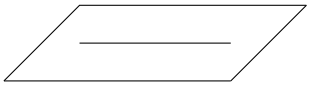


The circle cut by a plane passing through the centre of the sphere is called a great circle; the circle cut by a plane that does not pass through the centre of the sphere is called a small circle.

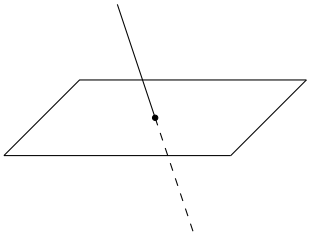
15.2 Angles Formed by Planes and Straight Lines

There are three types of angles formed by planes and straight lines:

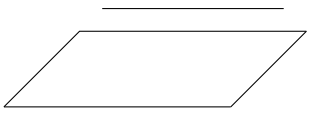
1. The line is on the plane



2. The line only intersects the plane at one point



3. The line does not intersect the plane



The angle formed by a line and the orthoprojection of the line on the plane is called the angle formed by the line and the plane. This angle represents the inclination of the line with respect to the plane, thus it is called the tilt angle of the line with respect to the plane.

15.2.1 Practice 1

1. In the diagram below, $AB = 9cm$, $BC = 7cm$, $CG = 5cm$. Find:
   (a) The angle formed by the line CF and the plane $GHDC$.
   (b) The angle formed by the line DF and the plane $EFGH$.
2. The diagram below shows a right prism, its base KQL is a right-angled triangle, $JKLM$ is a square. Given that $JK = 12cm$, $LQ = 5cm$, find the angle formed by the line JQ and the plane $PQLM$.

15.2.2 Exercise 17.2

1. The diagram below shows a cube with side length of $8.5cm$. Find:
   (a) The angle formed by the line QS and the plane $MNRQ$.
   (b) The angle formed by the line KQ and the plane $PQML$.
2. THe diagram below shows a cuboid, $AB = 8cm$, $BC = 6cm$, $AP = 10cm$. Find:
   (a) The length of PR.
   (b) The angle formed by the line SB and the plane $AQQB$.
3. The diagram below shows a pyramid. Given that its base $PQRS$ is a square, TR is perpendicular to the base, $TS = 15cm$, $TR = 9cm$. Find:
   (a) The length of RS.
   (b) The angle formed by the line PT and the plane $PQRS$.
4. The diagram below shows a right pyramid with height of $8cm$, its base is a rectangle, E is the foot point from V to the base. Given that $CD = 5cm$, $BC = 6cm$. Find:
   (a) The angle formed by the line VA and VE.
   (b) The angle formed by the line VC and the planew $ABCD$.
5. The diagram below shows a right pyramid, its base $PQRS$ is a regtangle. Given that $SR = 6cm$, $QR = 8cm$, $VS = 13cm$. Find:
   (a) The length of PR.
   (b) The height of the pyramid.
   (c) The angle of the line VP and the plane $PQRS$.
6. The diagram below shows a regular pyramid, the length of its lateral edge is $12cm$, its base $ABCD$ is a square with side length of $8cm$, M is the midpoint of BC. Find:
   (a) The angle formed by the lateral edge and the base of the pyramid.
   (b) The angle formed by the line VM and the base of the pyramid.
7. In the pyramid shown below, ΔABC is a right-angled triangle, CD is perpendicular to plane ABC, CE is perpendicular to AB. Given that $AC = 15cm$, $BC = 20cm$ and $CD = 9cm$. Find:
   (a) The length of CE.
   (b) $\angle CDE$.

(c) The angle formed by the line AD and the plane ABC.

8. The diagram below shows a right prism, its base CDF is a right-angled triangle. Given that $BC = 16.5cm$ and $AB = 12cm$. Assume that $CF = 2DF$, find:

   (a) The angle formed by the line AB and the plane $BCFE$.

   (b) The angle formed by the line AC and the plan $BCFE$.

9. The diagram below shows a cuboid with volume of $300cm^3$. Given that $AD = 2DC$ and $DN = 9cm$. Find the angle formed by the line AM and the plane $KLMN$.

10. The diagram below shows a cuboid. Given that $AB = 13cm$, $BC = 6cm$, $CG = 4cm$. M is a point on AB, $AM = 9cm$. Find:

    (a) The angle formed by the line HM and the plane $ABCG$.

    (b) The angle formed by the line HM and the plane $HDAE$.

    (c) The angle formed by the line AG and the plane $CDHG$.

11. The diagram below shows a regular prism, its base ADS and BCR is an equiliteral triangle. Given that $AB = 16cm$, $BC = 7cm$, $SP = 5cm$. Find:

    (a) The length of BP.

    (b) The angle formed by the line BP and the plane $ABCD$.

12. The diagram below shows a roof, HK is the ridge of the roof, its edges HA, HD, KB, KC are euqal in length. Both of the planes HAD and KBC form a 44^o angle with the plane $ABCD$. Given that S and T are the midpoints of BC and AD respectively. Find:

    (a) The distance from the line HK to the plane $ABCD$.

    (b) The length of HK.

    (c) The angle formed by the line HA and the plane $ABCD$.

13. The length, width and height of a hall are $20m$, $15m$, and $4m$ respectively. Find:

14. (a) The length of the diagonal of the hall.

    (b) The angle formed by the diagonal and the floor of the hall.

15. In the diagram below, $ABCD$ represents a rectangular plank with length and width of $60cm$ and $36cm$ respectively, its base BC is on the ground and the top of it lies on the wall. Assume that the distance between BC and the corner of the wall is $12cm$, find the angle formed by the diagonal BD of the plank and the ground.