1. $\int_0^3 \int_0^{\sqrt{x}} xy \, dx \, dy$

$= \int_0^3 x \left(\int_0^{\sqrt{x}} y \, dy \right) dx$

$= \frac{1}{2} \int_0^3 x \left[y^2 \right]_0^{\sqrt{x}} dx$

$= \frac{1}{2} \int_0^3 x^2 \, dx$

$= \frac{1}{6} \left[x^3 \right]_0^3$

$= \frac{1}{6}(27)$

$= 4.5$

2. $\int_0^{y^2} \int_1^2 \frac{y^2}{\sqrt{x}} \, dx \, dy$

$= \int_2^1 y^2 \left(\int_0^{y^2} x^{-\frac{1}{2}} \, dx \right) dy$

$= 2 \int_1^2 y^2 \left[x^{\frac{1}{2}} \right]_0^{y^2} dy$

$= 2 \int_1^2 y^3 \, dy$

$= \frac{1}{2} \left[y^4 \right]_1^2$

$= \frac{1}{2}(16 - 1)$

$= 7.5$

3. $\int_1^4 \int_4^9 \frac{dx \, dy}{\sqrt{xy}}$

$= \int_1^4 y^{-\frac{1}{2}} \left(\int_4^9 x^{-\frac{1}{2}} \, dx \right) dy$

$= 2 \int_1^4 y^{-\frac{1}{2}} \left[x^{\frac{1}{2}} \right]_4^9 dy$

$= 2 \int_1^4 y^{-\frac{1}{2}} \, dy$

$= 4 \left[y^{\frac{1}{2}} \right]_1^4$

$= 4(2 - 1)$

$= 4$

4. $\int_0^5 \int_x^{2x} x^2 y \, dx \, dy$

$= \int_0^5 x^2 \left(\int_x^{2x} y \, dy \right) dx$

$= \frac{1}{2} \int_0^5 x^2 \left[y^2 \right]_x^{2x} dx$

$= \frac{3}{2} \int_0^5 x^4 \, dx$

$= \frac{3}{10} \left[x^5 \right]_0^5$

$= \frac{3}{10} \cdot 3125$

$= 937.5$

5. $\int_0^y \int_0^2 \int_0^x xy^2 z^3 \, dx \, dy \, dz$

$= \int_0^2 y^2 \int_0^y x \left(\int_0^x z^3 \, dz \right) dx \, dy$

$= \frac{1}{4} \int_0^2 y^2 \int_0^y x \left[z^4 \right]_0^x dx \, dy$

$= \frac{1}{4} \int_0^2 y^2 \left(\int_0^y x^5 \, dx \right) dy$

$= \frac{1}{24} \int_0^2 y^2 \left[x^6 \right]_0^y dy$

$= \frac{1}{24} \int_0^2 y^8 \, dx$

$= \frac{1}{216} \left[y^9 \right]_0^2$

$= \frac{64}{27}$

≈ 2.37

6. $\int_0^{\sqrt{y}} \int_0^{\sqrt{z}} \int_0^4 x^3 y \, dx \, dy \, dz$

$= \int_0^{\sqrt{z}} \int_0^4 y \left(\int_0^{\sqrt{y}} x^3 \, dx \right) dy \, dz$

$= \frac{1}{4} \int_0^4 \left(\int_0^{\sqrt{z}} y \left[x^4 \right]_0^{\sqrt{y}} dy \right) dz$

$= \frac{1}{4} \int_0^4 \left(\int_0^{\sqrt{z}} y^3 \, dy \right) dz$

$= \frac{1}{16} \int_0^4 \left[y^4 \right]_0^{\sqrt{z}} dz$

$= \frac{1}{16} \int_0^4 z^2 \, dz$

$= \frac{1}{48} \left[z^3 \right]_0^4$

$= \frac{4}{3}$

≈ 1.33