

# **Solution Book of Mathematic**

*Senior 2 Part I*

MELVIN CHIA

Written on 9 October 2022

# Contents

<b>15 Solid Geometry, Longitude and Latitude</b>	<b>2</b>
15.1 Solid Geometry . . . . .	2
15.2 Angles Formed by Planes and Straight Lines . . . . .	4
15.2.1 Practice 1 . . . . .	4
15.2.2 Exercise 17.2 . . . . .	4
15.3 Angle Formed by Two Planes . . . . .	6
15.3.1 Practice 2 . . . . .	6
15.3.2 Exercise 17.3 . . . . .	7

## Chapter 15

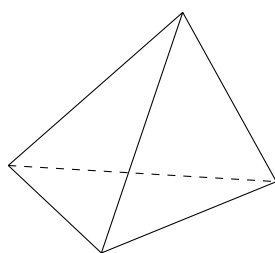
# Solid Geometry, Longitude and Latitude

### 15.1 Solid Geometry

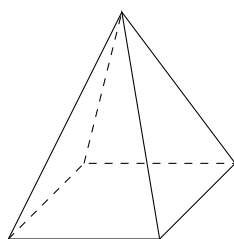
#### Polyhedron

A polyhedron is a solid bounded by a finite amount of flat polygon, and each side of the polygons must be the common edge of two polygons. Polyhedron can be classified into tetrahedron, pentahedron, hexahedron, etc. based on the number of flat surfaces, aka the *faces* of the polyhedron. The common side of two faces of a polyhedron is called an edge, and the common vertex of three edges is called an *apex*.

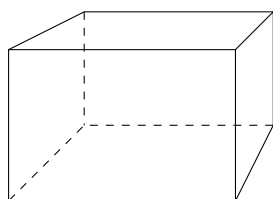
Besides, the angles formed by the faces intersecting at the same apex are called *polyhedral angles* or *solid angles*. The line segment connecting two apexes at different faces is called a *diagonal*.



Tetrahedron



Pentahedron

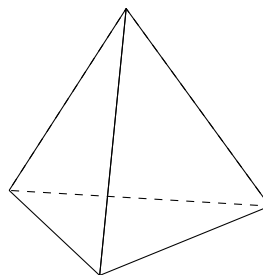


Hexahedron

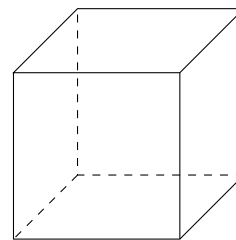
#### Regular Polyhedron

A *regular polyhedron* is a polyhedron with all faces being regular polygons, and all polyhedral angles being equal. The regular polyhedron can be classified into 5 types: *regular*

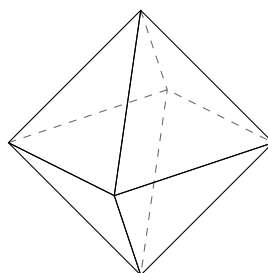
*tetrahedron, regular octahedron, regular hexahedron, regular dodecahedron and regular icosahedron.*



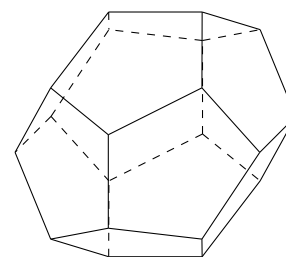
Regular Tetrahedron



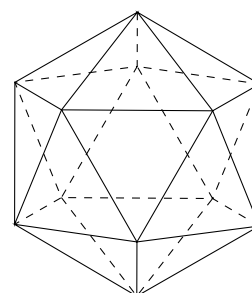
Regular Hexahedron



Regular Octahedron



Regular Dodecahedron

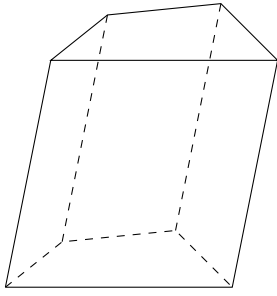


Regular Icosahedron

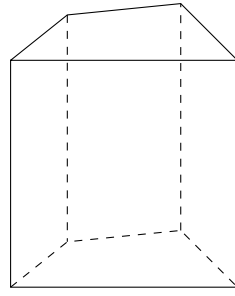
#### Prism

If two faces of a polyhedron are parallel, while the other faces intersect in sequence to form parallel lines, then the polyhedron is called a *prism*. The two faces which are parallel to each other are called the *bases of the prism*, and the other faces are called the *lateral faces of the prism*. The common sides that two adjacent lateral faces share is called the *lateral edges of the prism*. The distance between two bases is called the *height of the prism*.

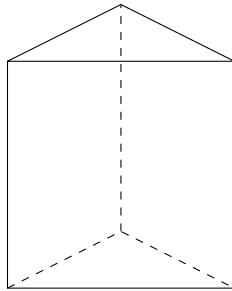
Prism with lateral edges that aren't parallel to each other are called *oblique prism*; prism with lateral edges that are parallel to each other are called *right prism*; regular prism with regular bases are called *regular prism*.



Oblique Prism

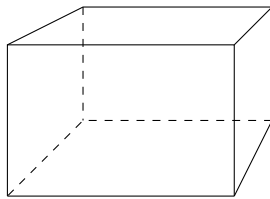


Right Prism

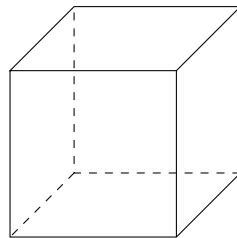


Regular Prism

Prism with bases of parallelogram are called *parallelepiped*. Parallelepiped with lateral edges that are parallel to each other are called *right parallelepiped*. Right parallelepiped with regular bases are called *cuboid*, and a cuboid with equal width, height, and depth is called a *cube*.



Cuboid

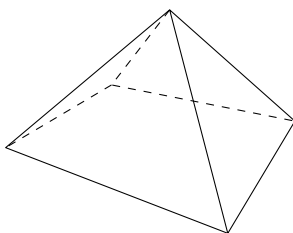


Cube

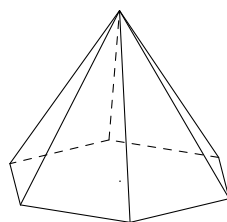
## Pyramid

If a polyhedron has a polygonal base and all its lateral faces are triangles that share a common apex, then the polyhedron is called a *pyramid*.

If the foot point of a pyramid is the centre of its base, then the pyramid is called a *right pyramid*. If the base of a right pyramid is a regular polygon, then the pyramid is called a *regular pyramid*.



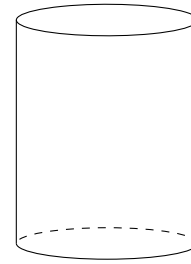
Right Pyramid



Regular Pyramid

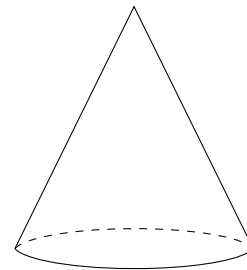
## Right Circular Cylinder

A *right circular cylinder* is the solid of revolution generated by rotating a rectangle about one of its sides.



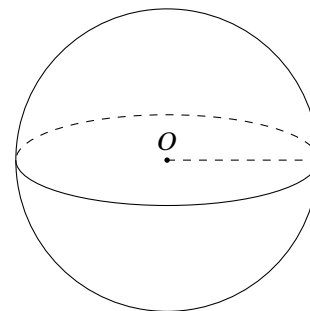
## Right Circular Cone

A *right circular cone* is the solid of revolution generated by rotating a right-angled triangle about one of its sides.



## Sphere

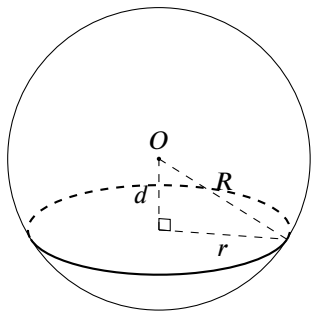
The surface of revolution generated by rotating a semicircle about its diameter is called a *spherical surface*, and the solid covered by it is called a *sphere*.



If the circle is cut with a plane, the plane has the following properties:

1. The line joining the centre of the sphere to the centre of the plane are perpendicular to the plane.
2. The distance of the plane from the centre of the sphere  $d$ , the radius of the sphere  $R$  and the radius of the plane  $r$  has the following relation:

$$r = \sqrt{R^2 - d^2}$$

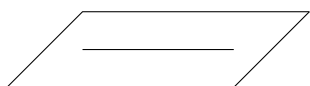


The circle cut by a plane passing through the centre of the sphere is called a *great circle*; the circle cut by a plane that does not pass through the centre of the sphere is called a *small circle*.

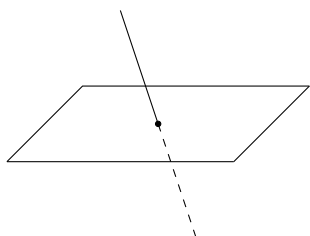
## 15.2 Angles Formed by Planes and Straight Lines

There are three types of positional relationship between a plane and a straight line:

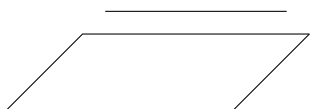
1. The line is on the plane



2. The line only intersects the plane at one point



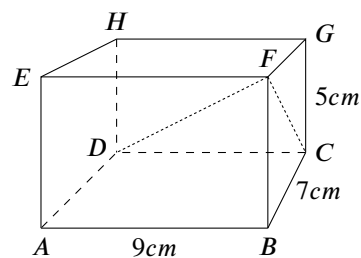
3. The line does not intersect the plane



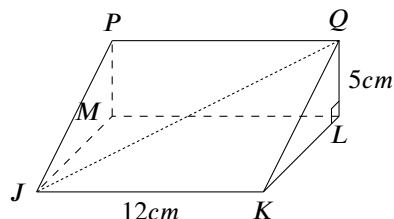
The angle formed by a line and the orthoprojection of the line on the plane is called *the angle formed by the line and the plane*. This angle represents the inclination of the line with respect to the plane, thus it is called *the tilt angle of the line with respect to the plane*.

### 15.2.1 Practice 1

1. In the diagram below,  $AB = 9\text{cm}$ ,  $BC = 7\text{cm}$ ,  $CG = 5\text{cm}$ . Find:
  - (a) The angle formed by line  $CF$  and plane  $GHDC$ .
  - (b) The angle formed by line  $DF$  and plane  $EFGH$ .



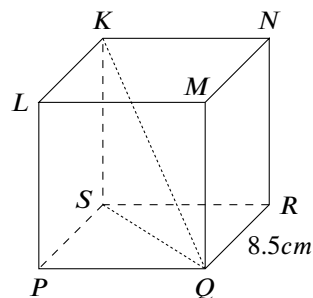
2. The diagram below shows a right prism, its base  $KQL$  is a right-angled triangle,  $JKLM$  is a square. Given that  $JK = 12\text{cm}$ ,  $LQ = 5\text{cm}$ , find the angle formed by line  $JQ$  and plane  $PQML$ .



### 15.2.2 Exercise 17.2

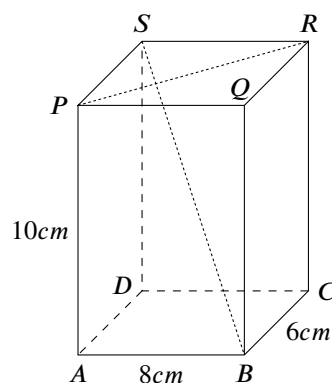
1. The diagram below shows a cube with side length of  $8.5\text{cm}$ . Find:

- (a) The angle formed by line  $QS$  and plane  $MNRQ$ .
- (b) The angle formed by line  $KQ$  and plane  $PQML$ .



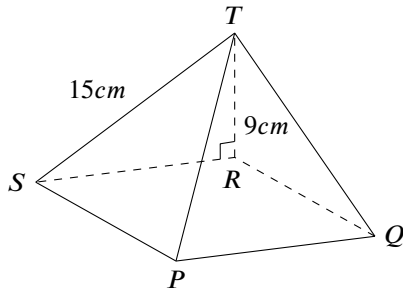
2. The diagram below shows a cuboid,  $AB = 8\text{cm}$ ,  $BC = 6\text{cm}$ ,  $AP = 10\text{cm}$ . Find:

- (a) The length of  $PR$ .
- (b) The angle formed by line  $SB$  and plane  $AQQB$ .



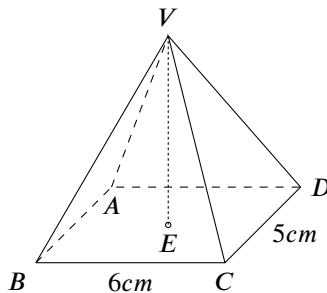
3. The diagram below shows a pyramid. Given that its base  $PQRS$  is a square,  $TR$  is perpendicular to the base,  $TS = 15\text{cm}$ ,  $TR = 9\text{cm}$ . Find:

- The length of  $RS$ .
- The angle formed by line  $PT$  and plane  $PQRS$ .



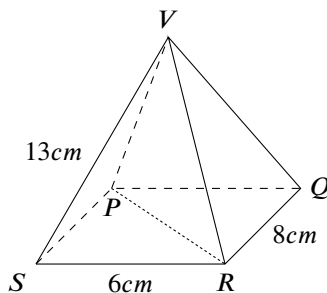
4. The diagram below shows a right pyramid with height of  $8\text{cm}$ , its base is a rectangle,  $E$  is the foot point from  $V$  to the base. Given that  $CD = 5\text{cm}$ ,  $BC = 6\text{cm}$ . Find:

- The angle formed by line  $VA$  and line  $VE$ .
- The angle formed by line  $VC$  and plane  $ABCD$ .



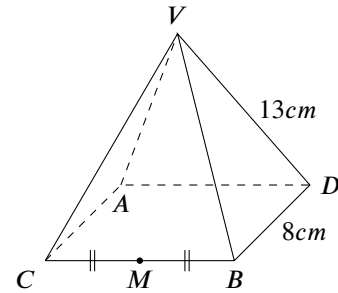
5. The diagram below shows a right pyramid, its base  $PQRS$  is a rectangle. Given that  $SR = 6\text{cm}$ ,  $QR = 8\text{cm}$ ,  $VS = 13\text{cm}$ . Find:

- The length of  $PR$ .
- The height of the pyramid.
- The angle of the line  $VP$  and plane  $PQRS$ .



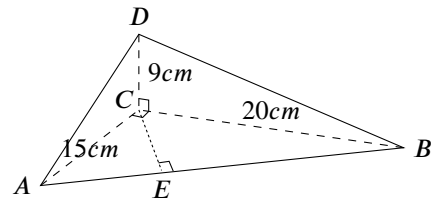
6. The diagram below shows a regular pyramid, the length of its lateral edge is  $12\text{cm}$ , its base  $ABCD$  is a square with side length of  $8\text{cm}$ ,  $M$  is the midpoint of  $BC$ . Find:

- The angle formed by the lateral edge and the base of the pyramid.
- The angle formed by line  $VM$  and the base of the pyramid.



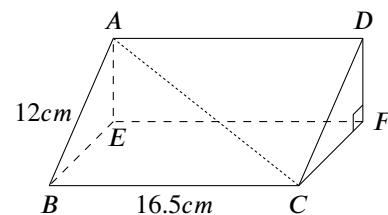
7. In the pyramid shown below,  $\triangle ABC$  is a right-angled triangle,  $CD$  is perpendicular to plane  $ABC$ ,  $CE$  is perpendicular to  $AB$ . Given that  $AC = 15\text{cm}$ ,  $BC = 20\text{cm}$  and  $CD = 9\text{cm}$ . Find:

- The length of  $CE$ .
- $\angle CDE$ .
- The angle formed by line  $AD$  and plane  $ABC$ .

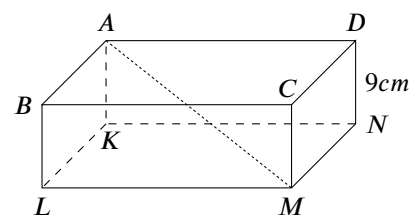


8. The diagram below shows a right prism, its base  $CDF$  is a right-angled triangle. Given that  $BC = 16.5\text{cm}$  and  $AB = 12\text{cm}$ . Assume that  $CF = 2DF$ , find:

- The angle formed by line  $AB$  and plane  $BCFE$ .
- The angle formed by line  $AC$  and plane  $BCFE$ .

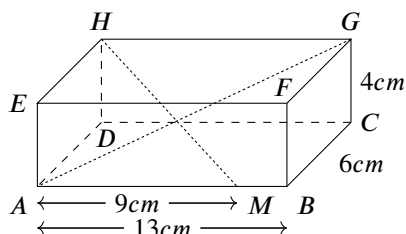


9. The diagram below shows a cuboid with volume of  $300\text{cm}^3$ . Given that  $AD = 2DC$  and  $DN = 9\text{cm}$ . Find the angle formed by line  $AM$  and plane  $KLMN$ .



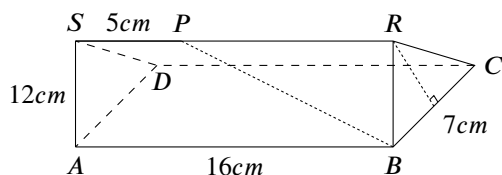
10. The diagram below shows a cuboid. Given that  $AB = 13\text{cm}$ ,  $BC = 6\text{cm}$ ,  $CG = 4\text{cm}$ .  $M$  is a point on  $AB$ ,  $AM = 9\text{cm}$ . Find:

- The angle formed by line  $HM$  and plane  $ABCG$ .
- The angle formed by line  $HM$  and plane  $HDAE$ .
- The angle formed by line  $AG$  and plane  $CDHG$ .



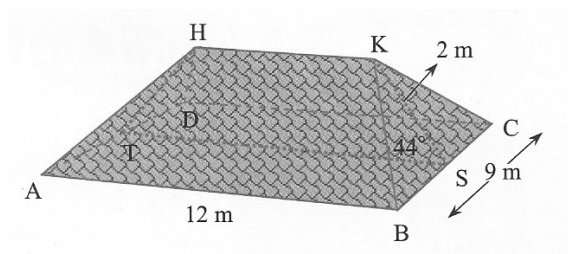
11. The diagram below shows a regular prism, its bases  $ADS$  and  $BCR$  are equilateral triangles. Given that  $AB = 16\text{cm}$ ,  $BC = 7\text{cm}$ ,  $SP = 5\text{cm}$ . Find:

- The length of  $BP$ .
- The angle formed by line  $BP$  and plane  $ABCD$ .



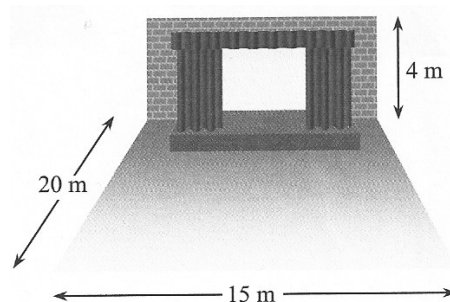
12. The diagram below shows a roof,  $HK$  is the ridge of the roof, its edges  $HA$ ,  $HD$ ,  $KB$ ,  $KC$  are equal in length. Both of the planes  $HAD$  and  $KBC$  form a  $44^\circ$  angle with plane  $ABCD$ . Given that  $S$  and  $T$  are the midpoints of  $BC$  and  $AD$  respectively. Find:

- The distance from line  $HK$  to plane  $ABCD$ .
- The length of  $HK$ .
- The angle formed by line  $HA$  and plane  $ABCD$ .

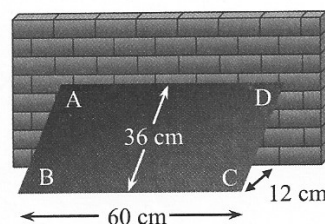


13. The length, width and height of a hall are  $20\text{m}$ ,  $15\text{m}$ , and  $4\text{m}$  respectively. Find:

- The length of the diagonal of the hall.
- The angle formed by the diagonal and the floor of the hall.



14. In the diagram below,  $ABCD$  represents a rectangular plank with length and width of  $60\text{cm}$  and  $36\text{cm}$  respectively, its base  $BC$  is on the ground and the top of it lies on the wall. Assume that the distance between  $BC$  and the corner of the wall is  $12\text{cm}$ , find the angle formed by the diagonal  $BD$  of the plank and the ground.



## 15.3 Angle Formed by Two Planes

There are three types positional relationship between two planes:

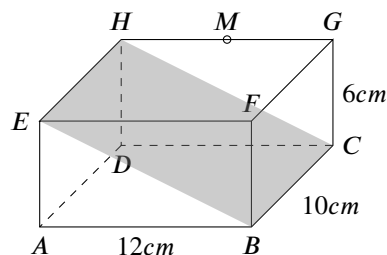
- Two planes coincide with each other.
- Two planes intersect with each other at a line.
- Two planes are parallel to each other and do not intersect with each other.

Two non-parallel planes intersect with each other at a line, the line is called the *common edge*. At any point on the common edge, draw a line perpendicular to the common edge on each plane, the acute angles formed by these two perpendicular lines are called *the angle formed by the two planes*.

### 15.3.1 Practice 2

1. The diagram below shows a cuboid with length of  $12\text{cm}$ , width of  $10\text{cm}$  and height of  $6\text{cm}$ .

- Find the angle formed by plane  $EBCH$  and plane  $ABCD$ .
- Assume that  $M$  is a point on  $HG$ , find the angle formed by plane  $MAB$  and plane  $ABCD$ .

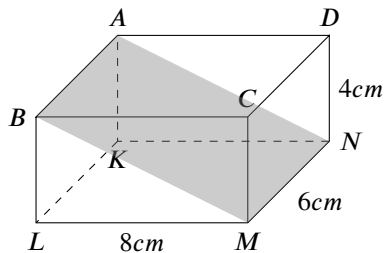


2. The diagram below shows a regular prism, its bases  $ABC$  and  $DEF$  are equilateral triangles with side length of  $5\text{cm}$ . Given that the height of the prism is  $10\text{cm}$ , find:

- The length of  $BF$ .
- The angle formed by plane  $ABF$  and plane  $ABC$ .

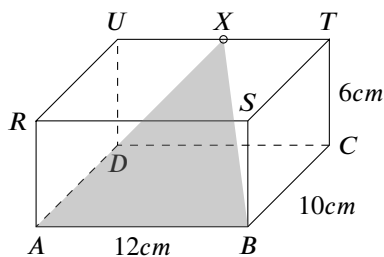
### 15.3.2 Exercise 17.3

1. The diagram below shows a cuboid with length of  $8\text{cm}$ , width of  $6\text{cm}$  and height of  $4\text{cm}$ . Find the angle formed by plane  $ABMN$  and  $KLMN$ .



2. In the right prism shown below,  $ABCD$  is a rectangle with length of  $20\text{cm}$  and width of  $12\text{cm}$ ,  $BCRQ$  is a trapezoid,  $\angle QBC$  and  $\angle RCB$  are both right angles,  $BQ = 5\text{cm}$ ,  $CR = 10\text{cm}$ . Find the angle formed by plane  $PQRS$  and plane  $ABCD$ .
3. The diagram below shows a cuboid,  $AB = 8\text{cm}$ ,  $BC = 6\text{cm}$ ,  $CT = 5\text{cm}$ ,  $X$  is the midpoint of  $TU$ . Find:

- The angle formed by plane  $XAB$  and plane  $ABCD$ .
- The angle formed by plane  $BCUR$  and plane  $ADUR$ .
- The angle formed by plane  $ABTU$  and plane  $ABCD$ .



4. The diagram below shows a right pyramid, its bases  $ABE$  and  $DCF$  are right-angled triangles. Given that  $AE = 4\text{cm}$ ,  $BE = \frac{2}{3}EF$ ,  $EF = 4DF$ , find the angle formed by plane  $ABCD$  and plane  $BCFE$ .

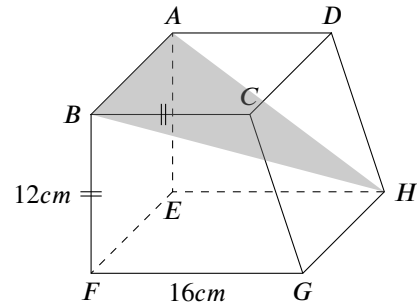
5. In the pyramid shown below,  $PQT$ ,  $SPT$ , and  $SRT$  are all right-angled triangles,  $PQRS$  is a triangle. Given that  $PQ = 5\text{cm}$ ,  $RT = 13\text{cm}$ ,  $PT = 20\text{cm}$ . Find:

- The height of the prism.
- The angle formed by line  $TQ$  and plane  $QST$ .

- The angle formed by plane  $RST$  and  $PQT$ .

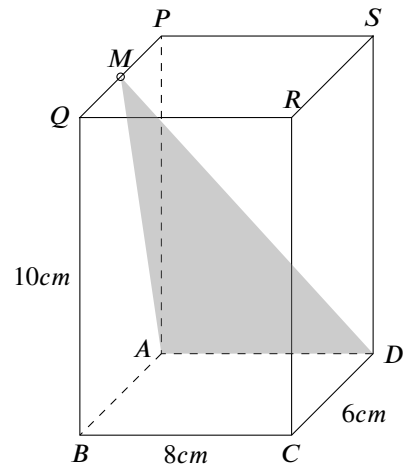
6. The diagram below shows a right prism, its base  $BCGF$  is a trapezoid,  $BC = BF = 12\text{cm}$ ,  $FG = 16\text{cm}$ . The lateral face  $EFGH$  is a square, and is perpendicular to another lateral face  $ABFE$ . Find:

- The angle formed by plane  $CDHG$  and plane  $EFGH$ .
- The angle formed by plane  $ABH$  and plane  $ABFE$ .



7. In the rectangle shown below,  $BC = 8\text{cm}$ ,  $CD = 6\text{cm}$ ,  $BQ = 10\text{cm}$ . Given that  $M$  is the midpoint of  $PQ$ . Find:

- The angle formed by line  $MD$  and plane  $PQBA$ .
- The angle formed by plane  $AMD$  and plane  $ABCD$ .



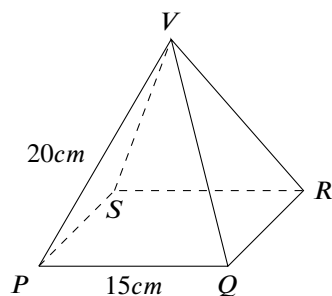
8. The diagram below shows a pyramid with an isosceles triangle base. Given that  $CD = CE = 5\text{cm}$ ,  $ED = 6\text{cm}$ ,  $ACD$  is a right-angled triangle,  $B$  is a point on  $AC$ ,  $AD = 2\text{cm}$ ,  $BC = 4\text{cm}$ . Find:

- The angle formed by plane  $BDE$  and plane  $CDE$ .
- The angle formed by the plane  $ADE$  and  $CDE$ .

9. The diagram below shows a regular pyramid with a square base. Given that  $PQ = 15\text{cm}$ ,  $PV = 20\text{cm}$ . Find:

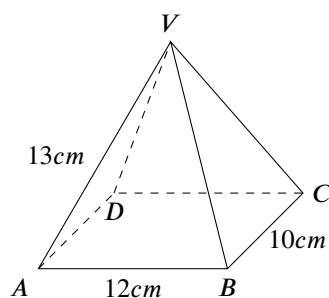
- The angle formed by line  $PV$  and plane  $PQRS$ .
- The angle formed by the lateral faces and the base of the pyramid.





10. The diagram below shows a right pyramid with lateral edges of  $13\text{cm}$ . Its base  $ABCD$  is a rectangle with length of  $12\text{cm}$  and width of  $10\text{cm}$ . Find:

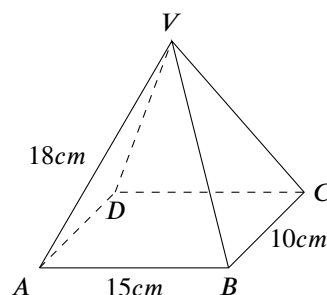
- The angle formed by plane  $VBC$  and plane  $ABCD$ .
- The angle formed by plane  $VCD$  and plane  $ABCD$ .



11. The diagram below shows a right pyramid with lateral edges of  $18\text{cm}$ , its base  $ABCD$  is a rectangle with

length of  $15\text{cm}$  and width of  $10\text{cm}$ . Find:

- The height of the pyramid.
- The angle formed by plane  $VAB$  and plane  $ABCD$ .
- The angle formed by plane  $VBC$  and plane  $VAD$ .



12. The diagram below shows a right prism with isosceles triangle bases. The side length and base length of the triangle base are  $10\text{cm}$  and  $16\text{cm}$  respectively, the height of the prism is  $20\text{cm}$ . Given that  $P$  is the mid-point of  $AB$ . Find:

- The length of  $PC$ .
- The angle formed by line  $EC$  and plane  $ABCD$ .
- The angle formed by plane  $DCE$  and plane  $ABCD$ .