Exercise 11f

Find the following indefinite integrals:

1. $\displaystyle\int \sin 2x \cos x \, dx$

   Sol.

   $$\begin{aligned} a \int \sin 2x \cos x \, dx &= \frac{1}{2} \int (\sin 3x + \sin x) \, dx \\ &= -\frac{1}{6} \cos 3x - \frac{1}{2} \cos x + C \end{aligned}$$

2. $\displaystyle\int 2 \sin 3x \cos x \, dx$

   Sol.

   $$\begin{aligned} \int 2 \sin 3x \cos x \, dx &= \int (\sin 4x + \sin 2x) \, dx \\ &= -\frac{1}{4} \cos 4x - \frac{1}{2} \cos 2x + C \end{aligned}$$

3. $\displaystyle\int 2 \cos 3x \cos x \, dx$

   Sol.

   $$\begin{aligned} \int 2 \cos 3x \cos x \, dx &= \int (\cos 4x + \cos 2x) \, dx \\ &= \frac{1}{4} \sin 4x + \frac{1}{2} \sin 2x + C \end{aligned}$$

4. $\displaystyle\int 2 \sin 5x \sin 3x \, dx$

   Sol.

   $$\begin{aligned} \int 2 \sin 5x \sin 3x \, dx &= \int (\cos 2x - \cos 8x) \, dx \\ &= \frac{1}{2} \sin 2x - \frac{1}{8} \sin 8x + C \end{aligned}$$

5. $\displaystyle\int \frac{\sin x}{\sec 3x} \, dx$

   Sol.

   $$\begin{aligned} \int \frac{\sin x}{\sec 3x} \, dx &= \int \sin x \cos 3x \, dx \\ &= \frac{1}{2} \int (\sin 4x - \sin 2x) \, dx \\ &= -\frac{1}{8} \cos 4x + \frac{1}{4} \cos 2x + C \end{aligned}$$

6. $\displaystyle\int \frac{\cos 5x}{\operatorname{cosec} 2x} \, dx$

   Sol.

   $$\begin{aligned} \int \frac{\cos 5x}{\operatorname{cosec} 2x} \, dx &= \int \sin 2x \cos 5x \, dx \\ &= \frac{1}{2} \int (\sin 7x - \sin 3x) \, dx \\ &= -\frac{1}{14} \cos 7x + \frac{1}{6} \cos 3x + C \end{aligned}$$

7. $\displaystyle\int 4\cos\frac{x}{2}\cos\frac{x}{3}\sin\frac{x}{2}\sin\frac{x}{3}\,dx$

Sol.

$$\begin{aligned}
\int 4\cos\frac{x}{2}\cos\frac{x}{3}\sin\frac{x}{2}\sin\frac{x}{3}\,dx &= \int(\sin x+\sin 0)(\sin\frac{2x}{3}+\sin 0)\,dx\\
&= \int \sin x\sin\frac{2x}{3}\,dx\\
&= \frac{1}{2}\int(\cos\frac{x}{3}-\cos\frac{5x}{3})\,dx\\
&= \frac{3}{2}\sin\frac{x}{3}-\frac{3}{10}\sin\frac{5x}{3}+C
\end{aligned}$$

8. $\displaystyle\int \sin(2x-1)\cos x\,dx$

Sol.

$$\begin{aligned}
\int \sin(2x-1)\cos x\,dx &= \frac{1}{2}\int\left[\sin(3x-1)-\sin(x-1)\right]\,dx\\
&= -\frac{1}{6}\cos(3x-1)-\frac{1}{2}\cos(x-1)+C
\end{aligned}$$

9. $\displaystyle\int \cos mx\cos nx\,dx$

Sol.

$$\begin{aligned}
\int \cos mx\cos nx\,dx &= \frac{1}{2}\int\left[\cos(m+n)x+\cos(m-n)x\right]\,dx\\
&= \frac{1}{2(m+n)}\sin(m+n)x+\frac{1}{2(m-n)}\sin(m-n)x+C
\end{aligned}$$

10. $\displaystyle\int \sin mx\cos nx\,dx$

Sol.

$$\begin{aligned}
\int \sin mx\cos nx\,dx &= \frac{1}{2}\int\left[\sin(m+n)x+\sin(m-n)x\right]\,dx\\
&= -\frac{1}{2(m+n)}\cos(m+n)x-\frac{1}{2(m-n)}\cos(m-n)x+C
\end{aligned}$$