

$$\begin{aligned}
\frac{d^2x}{dt^2} + \frac{ga^2}{x^2} &= 0 \\
\frac{d^2x}{dt^2} &= -\frac{ga^2}{x^2} \\
2\frac{dx}{dt} \frac{d^2x}{dt^2} &= -2\frac{dx}{dt} \frac{ga^2}{x^2} \\
\left(\frac{dx}{dt}\right)^2 &= -2ga^2 \int \frac{dx}{x^2} \\
&= 2ga^2 \left(\frac{1}{x} + A\right) \\
&= \frac{2ga^2}{x} + A
\end{aligned}$$

When $x = h$, $\frac{dx}{dt} = 0$,

$$\begin{aligned}
0 &= \frac{2ga^2}{h} + A \\
A &= -\frac{2ga^2}{h}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{dx}{dt}\right)^2 &= \frac{2ga^2}{x} - \frac{2ga^2}{h} \\
&= 2ga^2 \left(\frac{1}{x} - \frac{1}{h}\right) \\
&= 2ga^2 \left(\frac{h-x}{xh}\right) \\
\frac{dx}{dt} &= \pm a\sqrt{2g} \cdot \frac{\sqrt{h-x}}{\sqrt{xh}} \\
\frac{\sqrt{xh}}{\sqrt{h-x}} dx &= \pm a\sqrt{2g} dt \\
\sqrt{h} \int \frac{\sqrt{x}}{\sqrt{h-x}} dx &= \pm a\sqrt{2g} t + B
\end{aligned}$$

Let $x = u^2$, $dx = 2u du$,

$$\begin{aligned}
\sqrt{h} \int \frac{2u^2}{\sqrt{h-u^2}} du &= \pm a\sqrt{2g} t + B \\
2\sqrt{h} \int \frac{u^2}{\sqrt{h-u^2}} du &= \pm a\sqrt{2g} t + B
\end{aligned}$$

Let $u = \sqrt{h} \sin \theta$, $du = \sqrt{h} \cos \theta d\theta$,

$$2\sqrt{h} \int \frac{h \sin^2 \theta}{\sqrt{h - h \sin^2 \theta}} \sqrt{h} \cos \theta d\theta = \pm a \sqrt{2gt} + B$$

$$2\sqrt{h} \int \frac{h \sin^2 \theta}{\sqrt{h(1 - \sin^2 \theta)}} \sqrt{h} \cos \theta d\theta = \pm a \sqrt{2gt} + B$$

$$2\sqrt{h} \int \frac{h \sin^2 \theta}{\sqrt{h \cos^2 \theta}} \sqrt{h} \cos \theta d\theta = \pm a \sqrt{2gt} + B$$

$$2\sqrt{h} \int \frac{h \sin^2 \theta}{\sqrt{h} \cos \theta} \sqrt{h} \cos \theta d\theta = \pm a \sqrt{2gt} + B$$

$$2\sqrt{h} \int h \sin^2 \theta d\theta = \pm a \sqrt{2gt} + B$$

$$2h^{\frac{3}{2}} \int \sin^2 \theta d\theta = \pm a \sqrt{2gt} + B$$

$$2h^{\frac{3}{2}} \int \frac{1 - \cos 2\theta}{2} d\theta = \pm a \sqrt{2gt} + B$$

$$h^{\frac{3}{2}} \int (1 - \cos 2\theta) d\theta = \pm a \sqrt{2gt} + B$$

$$h^{\frac{3}{2}} \left(\theta - \frac{\sin 2\theta}{2} \right) = \pm a \sqrt{2gt} + B$$

$$h^{\frac{3}{2}} (\theta - \sin \theta \cos \theta) = \pm a \sqrt{2gt} + B$$

$$h^{\frac{3}{2}} \left(\arcsin \frac{u}{\sqrt{h}} - \frac{u}{\sqrt{h}} \sqrt{\frac{h - u^2}{h}} \right) = \pm a \sqrt{2gt} + B$$

$$h^{\frac{3}{2}} \left(\arcsin \frac{u}{\sqrt{h}} - \frac{u \sqrt{h - u^2}}{h} \right) = \pm a \sqrt{2gt} + B$$

$$h^{\frac{3}{2}} \left(\arcsin \frac{\sqrt{x}}{\sqrt{h}} - \frac{\sqrt{x} \sqrt{h - x}}{h} \right) = \pm a \sqrt{2gt} + B$$

$$h^{\frac{3}{2}} \left(\arcsin \sqrt{\frac{x}{h}} - \frac{\sqrt{(h - x)x}}{h} \right) = \pm a \sqrt{2gt} + B$$

$$h^{\frac{3}{2}} \arcsin \sqrt{\frac{x}{h}} - \sqrt{h} \sqrt{hx - x^2} = \pm a \sqrt{2gt} + B$$

When $x = h$, $t = 0$,

$$\begin{aligned}
 h^{\frac{3}{2}} \arcsin \sqrt{\frac{h}{h}} - \sqrt{h} \sqrt{h^2 - h^2} &= \pm a \sqrt{2g} 0 + B \\
 h^{\frac{3}{2}} \arcsin 1 - \sqrt{h} \sqrt{0} &= B \\
 h^{\frac{3}{2}} \frac{\pi}{2} - 0 &= B \\
 B &= \frac{\pi}{2} h^{\frac{3}{2}}
 \end{aligned}$$

$$h^{\frac{3}{2}} \arcsin \sqrt{\frac{x}{h}} - \sqrt{h} \sqrt{hx - x^2} = \pm a \sqrt{2gt} + \frac{\pi}{2} h^{\frac{3}{2}}$$