# Mathematics

*Senior 3 Part I*

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# Introduction

## Why this book?

## Disclaimer

## Acknowledgements

# Contents

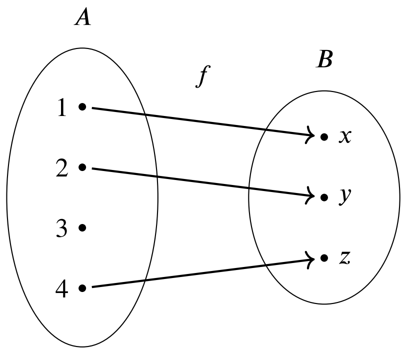
# Chapter 22

# Function

## 22.1 Definition of a Function

### Mapping, Preimage and Image

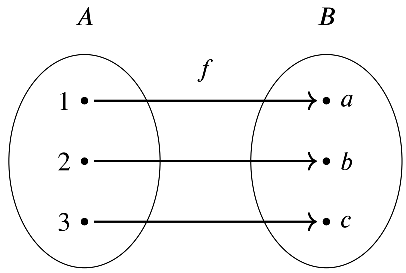
For two non-empty sets $A$ and $B$, If an element $a$ inside set $A$ has a corresponding element $b$ inside set $B$, denoted as $a \to b$, then we say that $a$ is mapped to $b$ or $a$ and $b$ are paired. The mapping between two sets is normally denoted as $f$, $g$, $h$, etc. The mapping shown in the diagram below can be denoted as $f : 1 \to x,\ 2 \to y,\ 4 \to z$.




Let $f : A \to B$ is a mapping, $a$ is an element in $A$. If $a$ is mapped to $b$ under the mapping $f$, then $b$ is said to be the image of $a$ under the mapping $f$, denoted as $b = f(a)$; $a$ is said to be the preimage of $b$ under the mapping $f$. In the diagram above, under the mapping $f$, the image of 1, 2, and 4 are $x$, $y$, and $z$ respectively, while the preimage of $x$, $y$, and $z$ are 1, 2, and 4 respectively.

| |
|---|
| Let $A$ and $B$ be two non-empty sets, $f$ is a mapping from $A$ to $B$ such that for all elements in $A$, there is a unique corresponding element in $B$, then $f$ is a function or a mapping from $A$ to $B$, denoted as $f : A \to B$. |

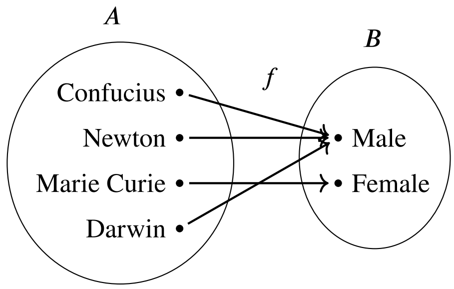
The mapping shown in the diagram below is a function.



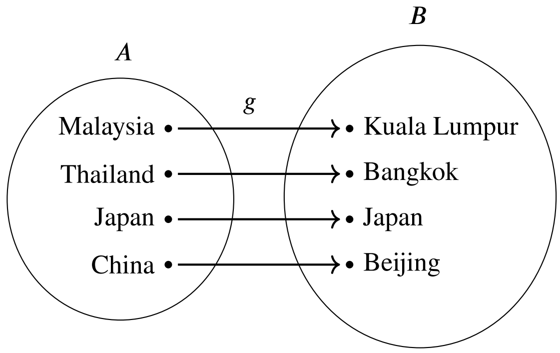

### 22.1.1 Practice 1

1. For the following mappings, list the image of each element in $A$ and the preimage of each element in $B$, and determine whether the mapping is a function or not:

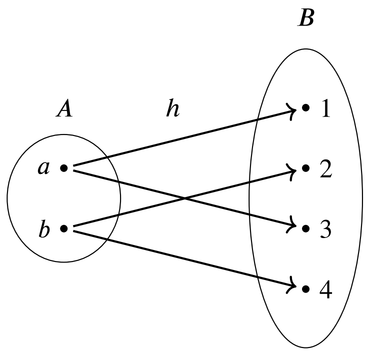
   (a)



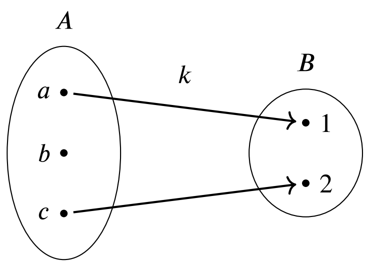

   (b)




   (c)



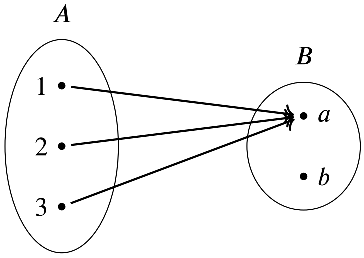

   (d)



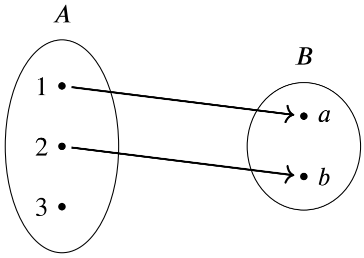

2. Given a mapping $g : x \to x + 3$, $x \in \{$-2, -1, 0, 1, 2, 3$\}$, find the image of each $x$.

3. Determine whether the following mappings are functions.

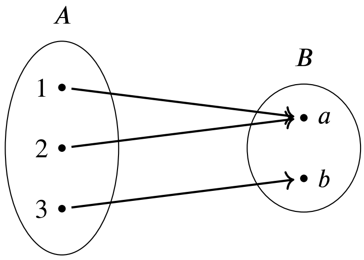
(a)



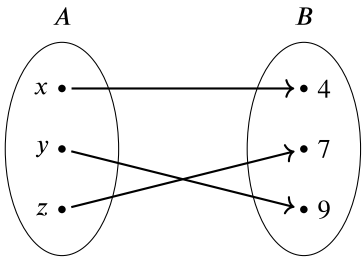

(b)



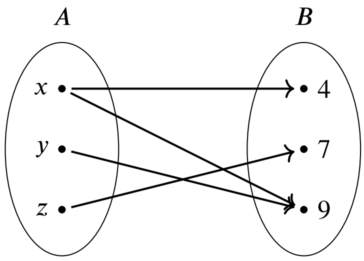

(c)



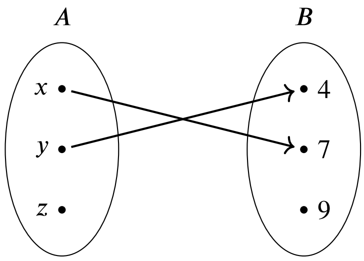

(d)




(e)




(f)




The function $f : A \to B$ can be written as $y = f(x)$, $x$ is the element of $A$ and $y$ is the element of $B$. When $x$ changes, $y$ changes as well. $x$ is called independent variable, while $y$ is called dependent variable.

Keep in mind that $f(x)$ is NOT the product of $f$ and $x$.

## Representation of Functions

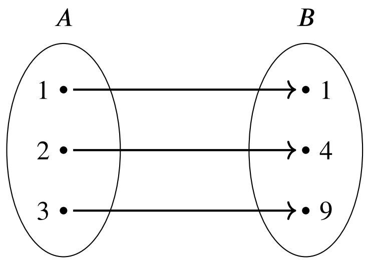
Generally speaking, there are a few ways to represent a function:

1. **Narrative Form**: express the function of two sets in words. For example, Let $A = \{1,2,3\}$ and $B = \{1,4,9\}$, $f$ is a function from $A$ to $B$, its definition is that for any element $x$ in $A$, its corresponding element is $x^2$ in $B$.

2. **Arrow Method**: draw an arrow to connect the preimage and image of a function such that the preimage is corresponding to the image. To express the example above, we express it as $f : 1 \to 1,\ 2 \to 4,\ 3 \to 9$.

3. **Analytical Method**: express the function in the form of mathematical expression to represent the relationship between the independent variable and the dependent variable. For example, $f(x) = x^2, x \in \mathbb{A}$.

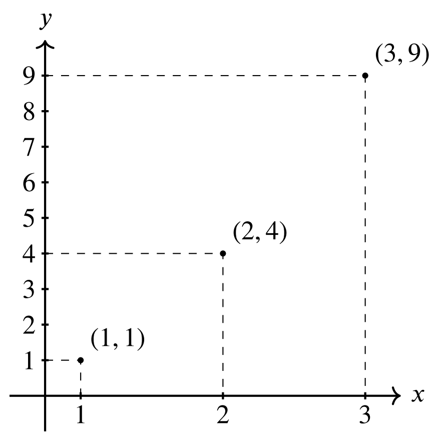
4. **Venn Diagram**: draw arrows between the venn diagram of two sets to represent the function, as shown below:




5. **Table Method**: express the function in the form of table, showing the relationship of the chosen value between independent variable $x$ and the value of its corresponding dependent variable $y$, as shown below:

| | | | |
|---|---|---|---|
| $x$ | 1 | 2 | 3 |
| $y$ | 1 | 4 | 9 |

6. **Graphical Method**: draw a graph to represent the function of the two variables, as shown below:



### 22.1.2 Practice 2

Express the following functions using analytical method, venn diagram, table method and graphical method.

(a) $f$ mapping each integers from $-3$ to 3 to its squares plus 4.

(b) $g$ mapping each natural numbers from 1 to 4 to its cubes.

## 22.2 Domain and Range

## 22.3 Graphs of Functions and Their Transformations

## 22.4 Composite Functions

## 22.5 One to One Function, Onto Function and One to One Onto Function

## 22.6 Inverse Functions

# Chapter 23

# Exponents and Logarithms

# Chapter 24

# Limits

# Chapter 25

# Differentiation