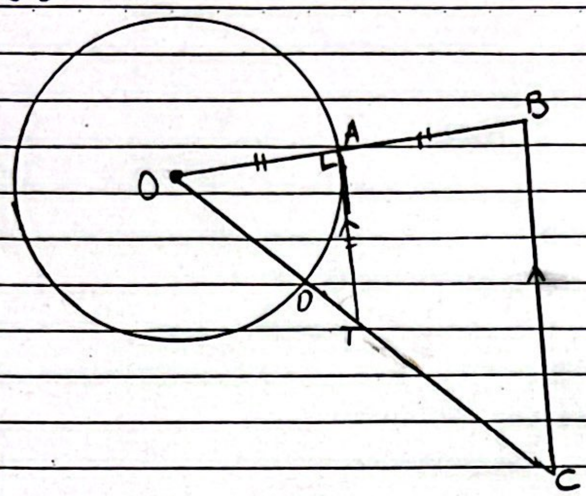
NO: DATE: 02 APR 2024

6.2



1. Given $OC = 4\sqrt{2}$

$\because \triangle OAT \sim \triangle OBC$

$\therefore \frac{OA}{AB} = \frac{OT}{TC}$

$\because OA = OB$

$\frac{OA}{OB} = 1$

$\therefore \frac{OT}{TC} = 1$

$OT = TC$

$OC = OT + TC = 4\sqrt{2}$

$2OT = 4\sqrt{2}$

$OT = 2\sqrt{2}$

$OA^2 + AT^2 = OT^2$

$\because OA = AT$

$\therefore 2OA^2 = OT^2$

$2OA^2 = 8$

$OA^2 = 4$

$OA = 2 \ (OA > 0)$

$\because OA = AT$

$\therefore \triangle OAT$ is an isoceles $\triangle$.

$\therefore \angle AOT = \angle ATO = (180° - 90°) \div 2 = 45° = \frac{\pi}{4}$

$\therefore$ Area of sector AOD $= \frac{1}{2} \times \cancel{(2)^2} \times \frac{\pi}{\cancel{4}} = \frac{\pi}{2}$ #

2. Area of sector DAB = $15cm^2$

Length of arc AB = 6

$\frac{1}{2}r^2\theta = 15$

$r^2\theta = 30$

$\theta = \frac{30}{r^2}$ —— ①

$\theta r = 6$

$\theta = \frac{6}{r}$ —— ②

① = ②:

$\frac{6}{r} = \frac{30}{r^2}$

$6r^2 = 30r$

$r^2 = 5r$

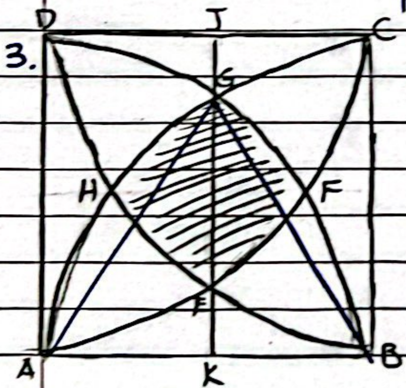
$r(r-5) = 0$

$r = 5$ $(r \neq 0)$

Sub into ②,

$\theta = \frac{6}{5} = 1.2$ rad #

3.



$\triangle AGB$ is equilateral $\triangle$

$\because \angle GAB = 60°$

$\angle DAG = 90° - 60° = 30° = \frac{\pi}{6}$

Area of sector DAG $= \frac{1}{2}(1)^2\left(\frac{\pi}{6}\right) = \frac{\pi}{12}$

Area of DJG = Area of AKJD − Area of sector DAG − Area of $\triangle AGK$

$= \frac{1}{2}(1) - \frac{\pi}{12} - \frac{1}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2} - \frac{\sqrt{3}}{8} - \frac{\pi}{12}$

Area of DBA: $1 - \frac{1}{2} \times \frac{\pi}{2} = 1 - \frac{\pi}{4}$

Area of DB: $1 - 2\left(1 - \frac{\pi}{4}\right)$

$= \frac{\pi}{2} - 1$

Area of DHG = Area of DBA − 4(Area of DJG)

$= 1 - \frac{\pi}{4} - 4\left(\frac{1}{2} - \frac{\sqrt{3}}{8} - \frac{\pi}{12}\right)$

$= 1 - \frac{\pi}{4} - 2 + \frac{\sqrt{3}}{2} + \frac{\pi}{3}$

$= \frac{\sqrt{3}}{2} + \frac{\pi}{12} - 1$

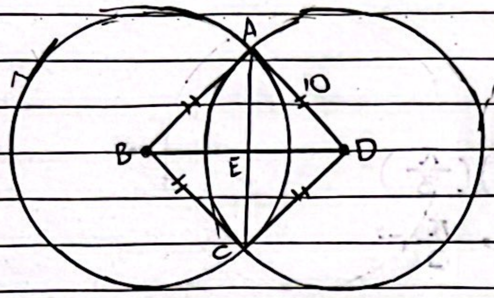
Area of HGFE = Area of DB − 2(Area of DHG)

$= \frac{\pi}{2} - 1 - 2\left(\frac{\sqrt{3}}{2} + \frac{\pi}{12} - 1\right)$

$= \frac{\pi}{2} - 1 - \sqrt{3} - \frac{\pi}{6} + 2$

$= \frac{\pi}{3} + 1 - \sqrt{3}$ #

1.



BD = 12cm = AC

BC = CD = DA = AB = 10cm

BE = ED = 6cm

$\cos\angle ABE = \frac{BE}{AB} = \frac{6}{10} = \frac{3}{5}$

$\angle ABE = 0.9273$

$\angle ABC = 2\angle ABE = 1.8546$

Area of sector ABC $= \frac{1}{2}(10)^2 \times 1.8546$

$= 92.73$

$AE = \sqrt{AB^2 - BE^2}$

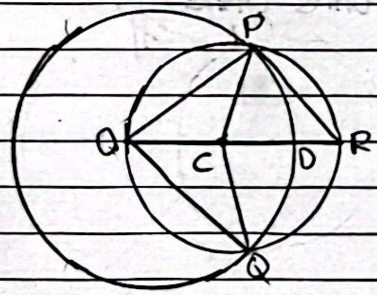
$= \sqrt{100 - 36}$

$= 8$

Area of $\Delta ABE = \frac{1}{2}(8)(6) = 24$

Area of $\Delta ABC = 2$(Area of ΔABE) $= 48$

Area of leaf AC $= 2(92.73 - 48) = 89.46\text{cm}^2$ #



(i) $\angle QCP = 4\theta$

$\angle QOP = 2\theta$

$\overset{\frown}{QRP} = \frac{2}{\sqrt{3}}\overset{\frown}{QDP}$

$OC = CP = r$

$\angle POC = \angle COQ = \theta$

$\frac{OP}{\sin\angle OCP} = \frac{PC}{\sin\angle POC}$

$\frac{OP}{\sin(\pi - 2\theta)} = \frac{r}{\sin\theta}$

$\frac{OP}{2\cancel{\sin\theta}\cos\theta} = \frac{r}{\cancel{\sin\theta}}$

$\cancel{2\theta(2r\cos\theta)} = \cancel{\frac{2}{\sqrt{3}}(4\theta r)}$

$\cancel{4\theta r} = \frac{2}{\sqrt{3}}(\cancel{2\theta}\cdot\cancel{2r}\cos\theta)$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}$ #

(ii) Area of $\Delta PCO = \frac{1}{2} \times r \times r \times \sin\left(\frac{2\pi}{3}\right)$

$= \frac{\sqrt{3}}{4} r^2$

Area of sector $POQ = \frac{1}{2}\left(4r^2\cos^2\frac{\pi}{6}\right)\left(\frac{\pi}{3}\right)$

$= \frac{1}{2}(3r^2)\left(\frac{\pi}{3}\right) = \frac{\pi}{2} r^2$

Area of circle $OPRQ = \pi r^2$

~~∴ Area of shaded area~~ $\cancel{: \pi r^2 - 2\left(\frac{\sqrt{3}}{4}r^2\right) - \frac{\pi}{2}r^2}$

$\cancel{= \frac{\pi}{2}r^2 - \frac{\sqrt{3}}{2}r^2}$

$\cancel{= \frac{1}{2}r^2(\pi - \sqrt{3})}$

Area of sector $OCP = \frac{1}{2} \times r^2 \times \frac{2\pi}{3} = \frac{\pi}{3} r^2$

Area of half leaf $OP = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) r^2$

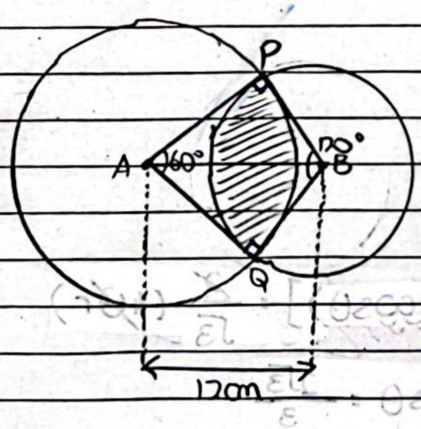
∴ Area of shaded region $= \pi r^2 - 2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) r^2 - \frac{\pi}{2} r^2$

$= \frac{\pi}{2} r^2 - \frac{2\pi}{3} r^2 + \frac{\sqrt{3}}{2} r^2$

$= \frac{\sqrt{3}}{2} r^2 - \frac{\pi}{6} r^2$

$= \frac{1}{2} r^2 \left(\sqrt{3} - \frac{\pi}{3}\right)$ sq. units (~~shown~~) #

3.



(i) $\angle PAB = \angle BAQ = 30°$

$\angle PBA = \angle ABQ = 60°$

$\angle APB = \angle AQB = 180° - 90° = 90°$

$\cos\angle QAB = \frac{QA}{AB}$ $\qquad$ $\cos\angle QBA = \frac{QB}{BA}$

$QA = 12\cos 30° = 6\sqrt{3}$ cm # $\qquad$ $QB = 12\cos 60° = 6$ cm #

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(ii) Area of $\Delta AQP = \frac{1}{2} \times (6\sqrt{3})^2 \times \sin 60° = 27\sqrt{3}$

Area of sector $AQP = \frac{1}{2} \times (6\sqrt{3})^2 \times \frac{\pi}{3} = 18\pi$

~~Area of left half leaf PQ: $27\pi - 27\sqrt{3}$~~

Area of $\Delta QBP = \frac{1}{2}(6)^2 \times \sin 120° = 9\sqrt{3}$

Area of sector $QBP = \frac{1}{2}(6)^2 \times \frac{2\pi}{3} = 12\pi$

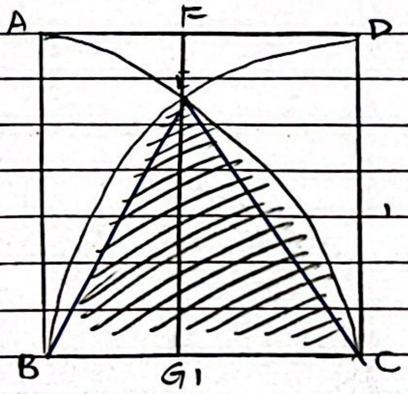
Area of left half leaf PQ: $12\pi - 9\sqrt{3}$

Area of right half leaf PQ: $18\pi - 27\sqrt{3}$

$\therefore$ Area of shaded region: $12\pi - 9\sqrt{3} + 18\pi - 27\sqrt{3}$

$= 30\pi - 36\sqrt{3}$ (得证) #

4.



$\because BE = BC =$ radius of circle $= 1 = EC$

$\therefore \Delta EBC$ is and equilateral Δ.

$\therefore \angle EBC = \angle BCE = \angle BEC = 60°$

$\angle ABE = 90° - 60° = 30°$

Area of sector $ABE = \frac{1}{2}(1)^2\left(\frac{\pi}{6}\right) = \frac{\pi}{12}$

Area of $\Delta BGE = \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{8}$

Area of $AEF = \frac{1}{2} - \frac{\pi}{12} - \frac{\sqrt{3}}{8}$

Area of sector $ABC = \frac{1}{2}(1)^2\left(\frac{\pi}{2}\right) = \frac{\pi}{4} =$ Area of sector BCD

Area of wierd shape $ABE = 1 - \frac{\pi}{4} - 2\left(\frac{1}{2} - \frac{\pi}{12} - \frac{\sqrt{3}}{8}\right)$

$= \cancel{1} - \frac{\pi}{4} - \cancel{1} + \frac{\pi}{6} + \frac{\sqrt{3}}{4}$

$= \frac{\sqrt{3}}{4} - \frac{\pi}{12}$

Area of leaf $BE = \frac{\pi}{12} - \frac{\sqrt{3}}{4} + \frac{\pi}{12}$

$= \frac{\pi}{6} - \frac{\sqrt{3}}{4}$

Area of $\Delta BEC = \frac{1}{2} \times 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$\therefore$ Area of shaded region: $\frac{\sqrt{3}}{4} + 2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

$= \frac{\pi}{3} - \frac{\sqrt{3}}{4}$ unit² #