Solution Book of Mathematic

Ssnior 2 Part I

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Contents

Chapter 17

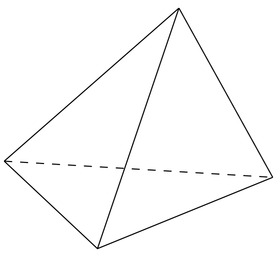
Solid Geometry, Longitude and Latitude

17.1 Solid Geometry

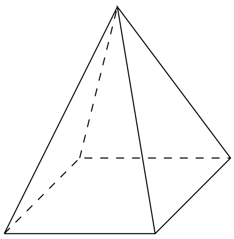
Polyhedron

A polyhedron is a solid bounded by a finite amount of flat polygon, and each side of the polygons must be the common edge of two polygons. Polyhedron can be classified into tetrahedron, pentahedron, hexahedron, etc. based on the number of flat surfaces, aka the *faces* of the polyhedron. The common side of two faces of a polyhedron is called an edge, and the common vertex of three edges is called an *apex*.

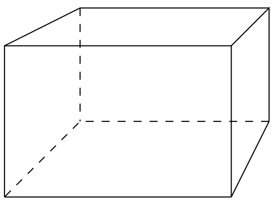
Besides, the angles formed by the faces intersecting at the same apex are called *polyhedral angles* or *solid angles*. The line segment connecting two apexes at different faces is called a *diagonal*.



Tetrahedron



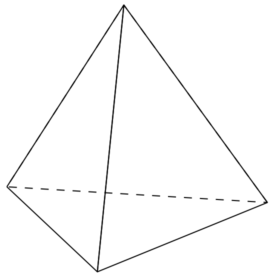
Pentahedron



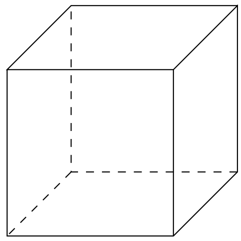
Hexahedron

Regular Polyhedron

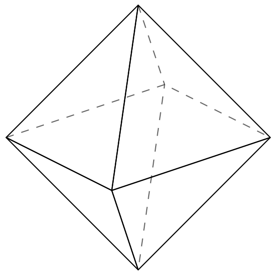
A *regular polyhedron* is a polyhedron with all faces being regular polygons, and all polyhedral angles being equal. The regular polyhedron can be classified into 5 types: *regular tetrahedron*, *regular octahedron*, *regular hexahedron*, *regular dodecahedron* and *regular icosahedron*.



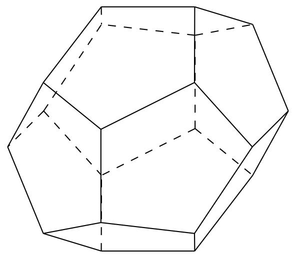
Regular Tetrahedron



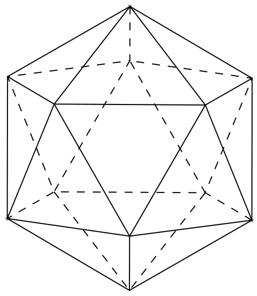
Regular Hexahedron



Regular Octahedron



Regular Dodecahedron

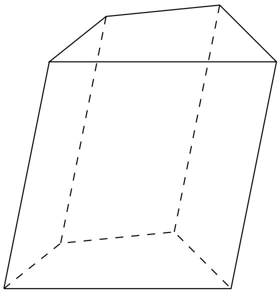


Regular Icosahedron

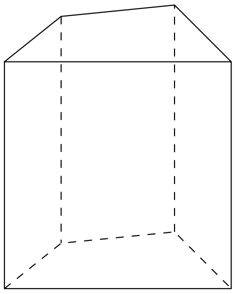
Prism

If two faces of a polyhedron are parallel, while the other faces intersect in sequence to form parallel lines, then the polyhedron is called a *prism*. The two faces which are parallel to each other are called the *bases of the prism*, and the other faces are called the *lateral faces of the prism*. The common sides that two adjacent lateral faces share is called the *lateral edges of the prism*. The distance between two bases is called the *height of the prism*.

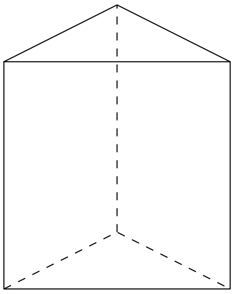
Prism with lateral edges that aren't parallel to each other are called *oblique prism*; prism with lateral edges that are parallel to each other are called *right prism*; regular prism with regular bases are called *regular prism*.



Oblique Prism

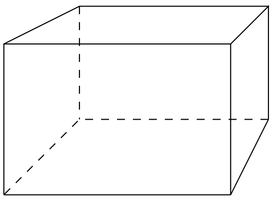


Right Prism

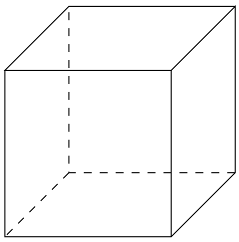


Regular Prism

Prism with bases of parallelogram are called *parallelepiped*. Parallelepiped with lateral edges that are parallel to each other are called *right parallelepiped*. Right parallelepiped with regular bases are called *cuboid*, and a cuboid with equal width, height, and depth is called a *cube*.



Cuboid

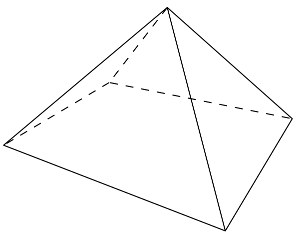


Cube

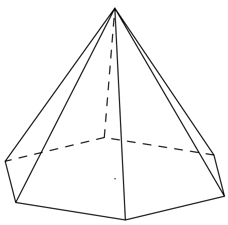
Pyramid

If a polyhedron has a polygonal base and all its lateral faces are triangles that shares a common apex, then the polyhedron is called a *pyramid*.

If the foot point of a pyramid is the centre of its base, then the pyramid is called a *right pyramid*. If the base of a right pyramid is a regular polygon, then the pyramid is called a *regular pyramid*.



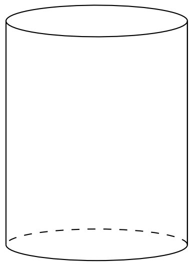
Right Pyramid



Regular Pyramid

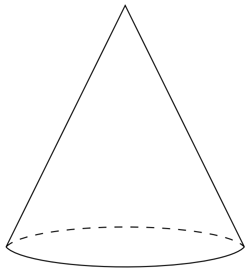
Right Circular Cylinder

A *right circular cylinder* is the solid of revolution generated by rotating a rectangle about one of its sides.



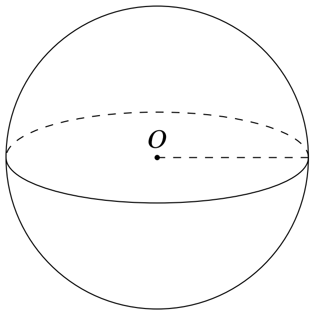
Right Circular Cone

A *right circular cone* is the solid of revolution generated by rotating a right-angled triangle about one of its sides.



Sphere

The surface of revolution generated by rotating a semicircle about its diameter is called a *spherical surface*, and the solid covered by it is called a *sphere*.

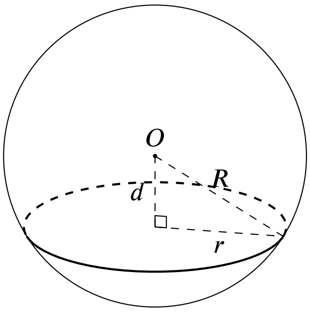


If the circle is cut with a plane, the plane has the following properties:

1. The line joining the centre of the sphere to the centre of the plane are perpendicular to the plane.

2. The distance of the plane from the centre of the sphere d, the radius of the sphere R and the radius of the plane r has the following relation:

$$r = \sqrt{R^2 - d^2}$$

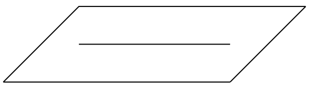


The circle cut by a plane passing through the centre of the sphere is called a *great circle*; the circle cut by a plane that does not pass through the centre of the sphere is called a *small circle*.

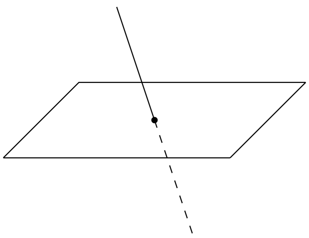
17.2 Angle Formed by Planes and Straight Lines

There are three types of positional relationship between a plane and a straight line:

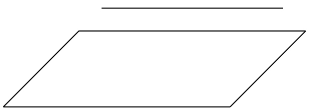
1. The line is on the plane



2. The line only intersects the plane at one point



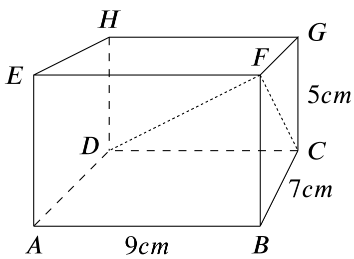
3. The line does not intersect the plane



The angle formed by a line and the orthoprojection of the line on the plane is called *the angle formed by the line and the plane*. This angle represents the inclination of the line with respect to the plane, thus it is called *the tilt angle of the line with respect to the plane*.

17.2.1 Practice 1

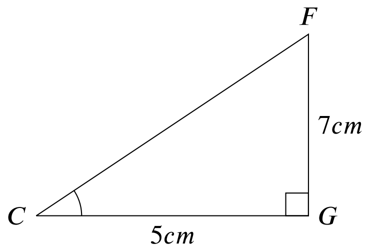
1. In the diagram below, $AB = 9cm$, $BC = 7cm$, $CG = 5cm$. Find:



(a) The angle formed by line CF and plane $GHDC$.

Sol.

The angle formed by line CF and plane $GHDC$ is $\angle FCG$.



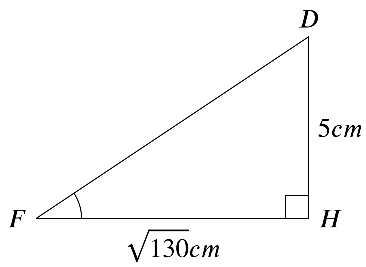
$$\begin{aligned} \tan \angle FCG &= \frac{FG}{CG} \\ &= \frac{7}{5} \\ \angle FCG &\approx 54.46^\circ \end{aligned}$$

(b) The angle formed by line DF and plane $EFGH$.

Sol.

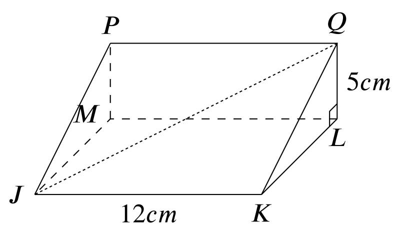
$$\begin{aligned} \text{In } EFGH,\ HF &= \sqrt{EF^2 + EH^2} \\ &= \sqrt{9^2 + 7^2} \\ &= \sqrt{130}cm \end{aligned}$$

The angle formed by line DF and plane $EFGH$ is $\angle DFH$.



$$\begin{aligned} \tan \angle DFH &= \frac{DH}{FH} \\ &= \frac{5}{\sqrt{130}} \\ \angle DFH &\approx 23.68^\circ \end{aligned}$$

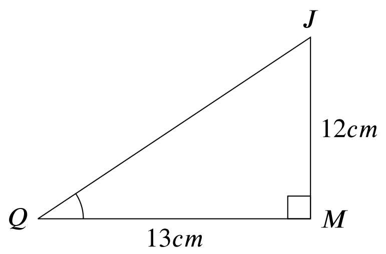
2. The diagram below shows a right prism, its base KQL is a right-angled triangle, $JKLM$ is a square. Given that $JK = 12cm$, $LQ = 5cm$, find the angle formed by line JQ and plane $PQLM$.



Sol.

$$\text{In } PQLM,\ QM = \sqrt{JK^2 + KL^2}$$
$$= \sqrt{12^2 + 5^2}$$
$$= 13cm$$

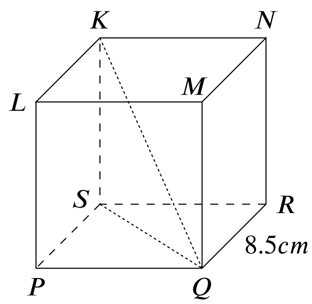
The angle formed by line JQ and plane $PQLM$ is $\angle JQM$.



$$\tan\angle JQM = \frac{JM}{QM}$$
$$= \frac{12}{13}$$
$$\angle JQM \approx 42.71°$$

17.2.2 Exercise 17.2

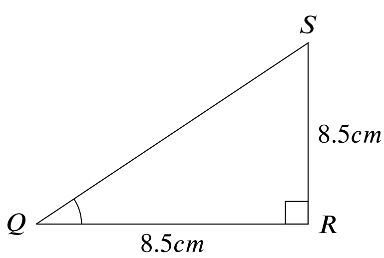
1. The diagram below shows a cube with side length of $8.5cm$. Find:



(a) The angle formed by line QS and plane $MNRQ$.

Sol.

The angle formed by line QS and plane $MNRQ$ is $\angle SQR$.



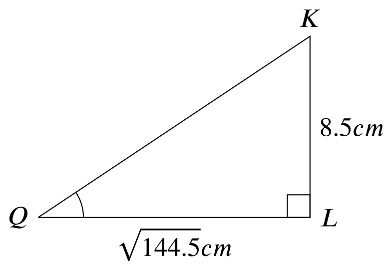
$$\tan\angle SQR = \frac{SR}{QR}$$
$$= \frac{8.5}{8.5}$$
$$= 1$$
$$\angle SQR = 45°$$

(b) The angle formed by line KQ and plane $PQML$.

Sol.

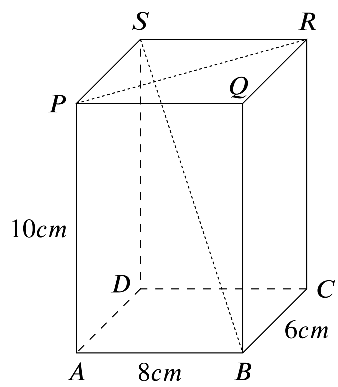
$$\text{In } KLMN,\ KM = \sqrt{8.5^2 + 8.5^2}$$
$$= \sqrt{144.5}\text{cm}$$

The angle formed by line KQ and plane $PQML$ is $\angle KQL$.



$$\tan\angle KQL = \frac{KL}{QL}$$
$$= \frac{8.5}{\sqrt{144.5}}$$
$$\angle KQL \approx 35.26°$$

2. THe diagram below shows a cuboid, $AB = 8cm$, $BC = 6cm$, $AP = 10cm$. Find:



(a) The length of PR.

Sol.

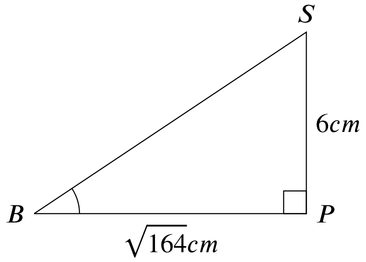
$$PR = \sqrt{PQ^2 + QR^2}$$
$$= \sqrt{8^2 + 6^2}$$
$$= 10cm$$

(b) The angle formed by line SB and plane $APQB$.

Sol.

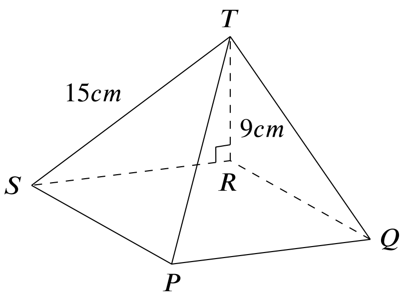
$$\text{In } APQB,\ PB = \sqrt{PA^2 + AB^2}$$
$$= \sqrt{10^2 + 8^2}$$
$$= \sqrt{164}cm$$

The angle formed by line SB and plane $APQB$ is $\angle SBP$.



$$\tan \angle SBP = \frac{SP}{BP}$$
$$= \frac{6}{\sqrt{164}}$$
$$\angle SBP \approx 25.10°$$

3. The diagram below shows a pyramid. Given that its base $PQRS$ is a square, TR is perpendicular to the base, $TS = 15cm$, $TR = 9cm$. Find:



(a) The length of RS.

Sol.

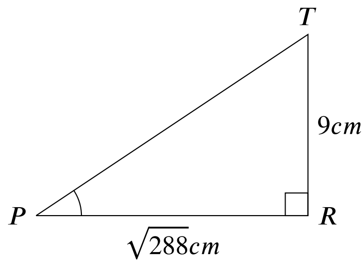
$$RS = \sqrt{ST^2 - TR^2}$$
$$= \sqrt{15^2 - 9^2}$$
$$= 12cm$$

(b) The angle formed by line PT and plane $PQRS$.

Sol.

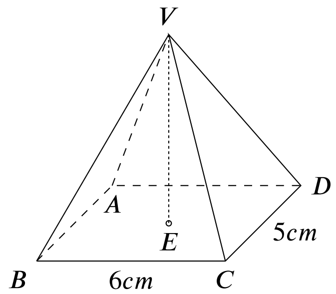
$$\text{In } PQRS,\ PR = \sqrt{PQ^2 + RQ^2}$$
$$= \sqrt{12^2 + 12^2}$$
$$= \sqrt{288}cm$$

The angle formed by line PT and plane $PQRS$ is $\angle TPR$.



$$\tan \angle TPR = \frac{TR}{PR}$$
$$= \frac{9}{\sqrt{288}}$$
$$\angle TPR \approx 27.94°$$

4. The diagram below shows a right pyramid with height of $8cm$, its base is a rectangle, E is the foot point from V to the base. Given that $CD = 5cm$, $BC = 6cm$. Find:

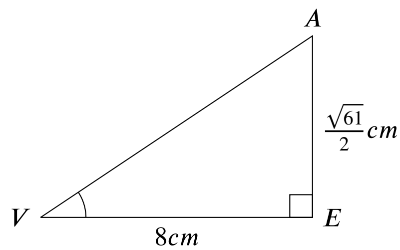


(a) The angle formed by line VA and line VE.

Sol.

$$\text{In } ABCD,\ AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{5^2 + 6^2}$$
$$= \sqrt{61}cm$$
$$AE = \frac{AC}{2}$$
$$= \frac{\sqrt{61}}{2}cm$$

The angle formed by line VA and line VE is $\angle AVE$.



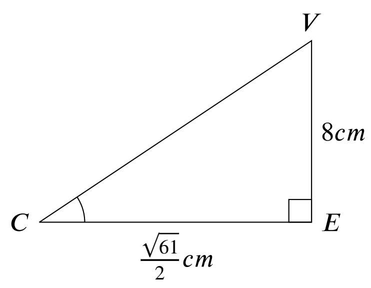
$$\tan \angle AVE = \frac{AE}{VE}$$
$$= \frac{\frac{\sqrt{61}}{2}}{8}$$
$$\angle AVE \approx 26.02°$$

(b) The angle formed by line VC and plane $ABCD$.

Sol.

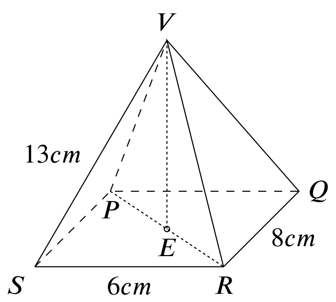
$$\text{In } ABCD,\ EC = \frac{AC}{2} = \frac{\sqrt{61}}{2}cm$$

The angle formed by line VC and plane $ABCD$ is $\angle VCE$.



$$\tan \angle VCE = \frac{VE}{CE} = \frac{8}{\frac{\sqrt{61}}{2}}$$
$$\angle VCE \approx 63.98°$$

5. The diagram below shows a right pyramid, its base $PQRS$ is a regtangle. Given that $SR = 6cm$, $QR = 8cm$, $VS = 13cm$. Find:



(a) The length of PR.

Sol.

$$PR = \sqrt{SR^2 + SP^2} \quad (1)$$
$$= \sqrt{6^2 + 8^2} \quad (2)$$
$$= 10cm \quad (3)$$

(b) The height of the pyramid.

Sol.

Let the foot point of the pyramid be E.

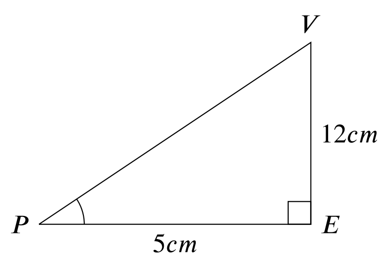
Sol.

$$\text{In } PQRS,\ PE = \frac{PR}{2} = \frac{10}{2} = 5cm$$

$$VE = \sqrt{VP^2 - PE^2} = \sqrt{13^2 - 5^2} = 12cm$$

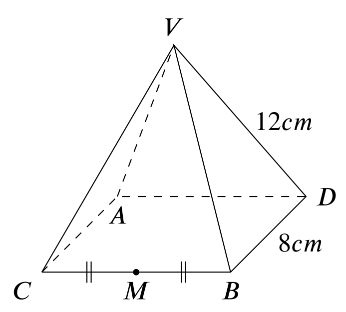
(c) The angle of the line VP and plane $PQRS$.

Sol. The angle of the line VP and plane $PQRS$ is $\angle VPE$.



$$\tan \angle VPE = \frac{VE}{PE} = \frac{12}{5}$$
$$\angle VPE \approx 67.38°$$

6. The diagram below shows a regular pyramid, the length of its lateral edge is $12cm$, its base $ABCD$ is a square with side length of $8cm$, M is the midpoint of BC. Find:



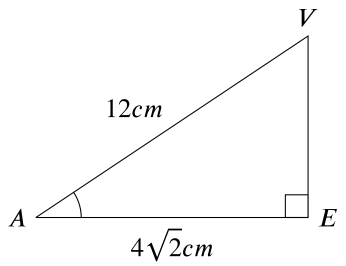
(a) The angle formed by the lateral edge and the base of the pyramid.

Sol.

Let the foot point of the pyramid be E.

$$\begin{aligned} \text{In } ABCD,\ AB &= \sqrt{AD^2 + BD^2} \\ &= \sqrt{8^2 + 8^2} \\ &= \sqrt{128}cm \\ &= 8\sqrt{2} \\ AE &= \frac{AB}{2} \\ &= 4\sqrt{2}cm \end{aligned}$$

The angle formed by the lateral edge and the base of the pyramid is $\angle VAE$.



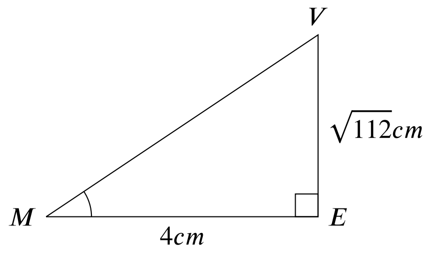
$$\begin{aligned} \cos\angle VAE &= \frac{AE}{AV} \\ &= \frac{4\sqrt{2}}{12} \\ &= \frac{\sqrt{2}}{3} \\ \angle VAE &= 61.87^\circ \end{aligned}$$

(b) The angle formed by line VM and the base of the pyramid.

Sol.

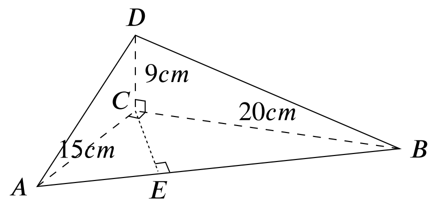
$$\begin{aligned} EM &= \frac{BD}{2} \\ &= \frac{8}{2} \\ &= 4cm \\ VE &= \sqrt{AV^2 + AE^2} \\ &= \sqrt{12^2 - (4\sqrt{2})^2} \\ &= \sqrt{144 - 32} \\ &= \sqrt{112} \end{aligned}$$

The angle formed by line VM and the base of the pyramid is $\angle VME$.



$$\begin{aligned} \cos\angle VME &= \frac{VE}{VM} \\ &= \frac{\sqrt{112}}{4} \\ \angle VME &= 69.30^\circ \end{aligned}$$

7. In the pyramid shown below, ΔABC is a right-angled triangle, CD is perpendicular to plane ABC, CE is perpendicular to AB. Given that $AC = 15cm$, $BC = 20cm$ and $CD = 9cm$. Find:

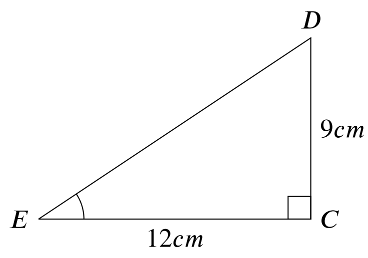


(a) The length of CE.

Sol.

$$\begin{aligned} \text{In } \Delta ABC,\ \tan\angle CBA &= \frac{AC}{AB} \\ &= \frac{15}{20} \\ &= \frac{3}{4} \\ \angle CBA &= 36.87^\circ \\ \text{In } \Delta CBE,\ \sin\angle CBE &= \frac{CE}{CB} \\ \sin 36.87^\circ &= \frac{CE}{20} \\ CE &= 20\sin 36.87^\circ \\ &= 12cm \end{aligned}$$

(b) $\angle DEC$.

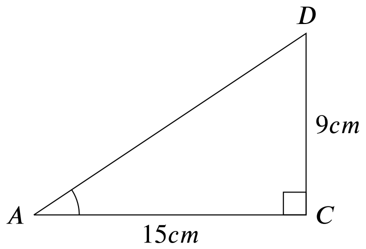


$$\begin{aligned}\cos\angle DEC &= \frac{DC}{EC}\\ &= \frac{9}{12}\\ &= \frac{3}{4}\\ \angle DEC &= 36.87^\circ\end{aligned}$$

(c) The angle formed by line AD and plane ABC.

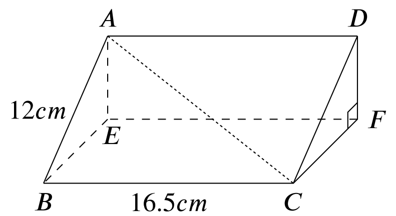
Sol.

The angle formed by line AD and plane ABC is $\angle DAC$.



$$\begin{aligned}\cos\angle DAC &= \frac{DC}{AC}\\ &= \frac{9}{15}\\ &= \frac{3}{5}\\ \angle DAC &= 30.96^\circ\end{aligned}$$

8. The diagram below shows a right prism, its base CDF is a right-angled triangle. Given that $BC = 16.5cm$ and $AB = 12cm$. Assume that $CF = 2DF$, find:

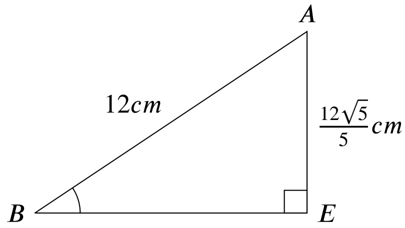


(a) The angle formed by line AB and plane $BCFE$.

Sol.

$$\begin{aligned}CF &= 2DF\\ DF^2 + (2DF)^2 &= 12^2\\ DF^2 + 4DF^2 &= 144\\ 5DF^2 &= 144\\ DF^2 &= \frac{144}{5}\\ DF &= \frac{12}{\sqrt{5}}\\ &= \frac{12\sqrt{5}}{5}cm\end{aligned}$$

The angle formed by line AB and plane $BCFE$ is $\angle ABE$.



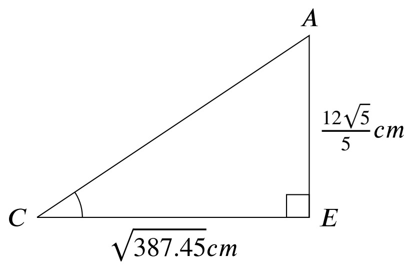
$$\begin{aligned}\sin\angle ABE &= \frac{AE}{AB}\\ &= \frac{\frac{12\sqrt{5}}{5}}{12}\\ &= \frac{\sqrt{5}}{5}\\ \angle ABE &= 26.57^\circ\end{aligned}$$

(b) The angle formed by line AC and plan $BCFE$.

Sol.

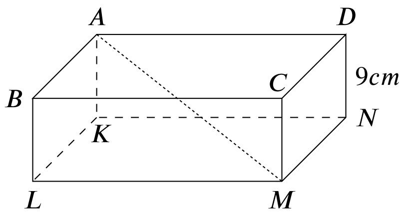
$$\begin{aligned}CF &= 2DF\\ &= \frac{24}{\sqrt{5}}\\ \text{In } BCEF,\ EC &= \sqrt{BC^2 + BE^2}\\ &= \sqrt{16.5^2 + \frac{24^2}{5}}\\ &= \sqrt{387.45}\end{aligned}$$

The angle formed by line AC and plane $BCFE$ is $\angle ACE$.



$$\begin{aligned}\sin\angle ACE &= \frac{AE}{EC}\\ &= \frac{\frac{12\sqrt{5}}{5}}{\sqrt{387.45}}\\ \angle ACE &= 15.25^\circ\end{aligned}$$

9. The diagram below shows a cuboid with volume of $300cm^3$. Given that $AD = 2DC$ and $DN = 9cm$. Find the angle formed by line AM and plane $KLMN$.

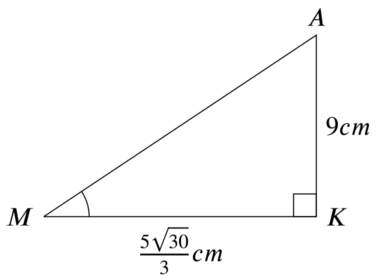


Sol.

$$
\begin{aligned}
AD &= 2DC \\
AD \times DC \times DN &= 300 \\
2DC \times DC \times 9 &= 300 \\
2DC^2 \times 9 &= 300 \\
2DC^2 &= \frac{100}{3} \\
DC^2 &= \frac{50}{3} \\
DC &= \frac{5\sqrt{2}}{\sqrt{3}} \\
&= \frac{5\sqrt{6}}{3} \\
AD &= 2DC \\
&= \frac{10\sqrt{6}}{3}
\end{aligned}
$$

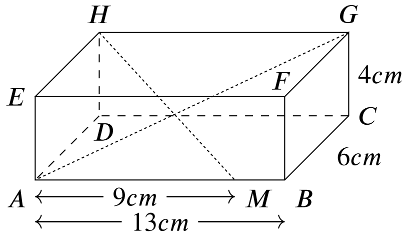
$$
\begin{aligned}
\text{In } KLMN,\ KM &= \sqrt{MN^2 + KN^2} \\
&= \sqrt{\left(\frac{5\sqrt{6}}{3}\right)^2 + \left(\frac{10\sqrt{6}}{3}\right)^2} \\
&= \sqrt{\frac{50}{3} + \frac{200}{3}} \\
&= \frac{5\sqrt{30}}{3}
\end{aligned}
$$

The angle formed by line AM and plane $KLMN$ is $\angle AMK$.



$$
\begin{aligned}
\tan\angle AMK &= \frac{AK}{MK} \\
&= \frac{9}{\frac{5\sqrt{30}}{3}} \\
\angle AMK &\approx 44.59^\circ
\end{aligned}
$$

10. The diagram below shows a cuboid. Given that $AB = 13cm$, $BC = 6cm$, $CG = 4cm$. M is a point on AB, $AM = 9cm$. Find:

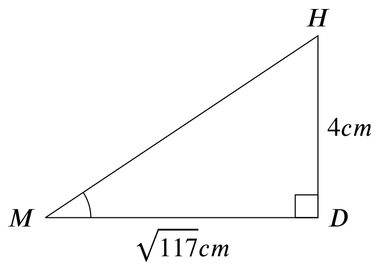


(a) The angle formed by line HM and plane $ABCD$.

Sol.

$$
\begin{aligned}
\text{In } ABCD,\ DM &= \sqrt{AM^2 + AD^2} \\
&= \sqrt{9^2 + 6^2} \\
&= \sqrt{117}cm
\end{aligned}
$$

The angle formed by line HM and plane $ABCD$ is $\angle HMD$.



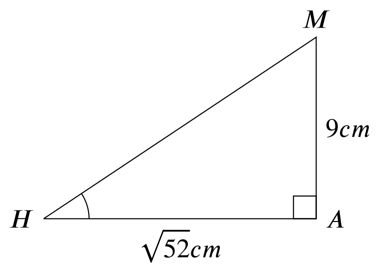
$$
\begin{aligned}
\tan\angle HMD &= \frac{HD}{MD} \\
&= \frac{4}{\sqrt{117}} \\
\angle HMD &\approx 20.29^\circ
\end{aligned}
$$

(b) The angle formed by line HM and plane $HDAE$.

Sol.

$$
\begin{aligned}
\text{In } HDAE,\ HA &= \sqrt{AD^2 + HD^2} \\
&= \sqrt{6^2 + 4^2} \\
&= \sqrt{52}cm
\end{aligned}
$$

The angle formed by line HM and plane $HDAE$ is $\angle MHA$.

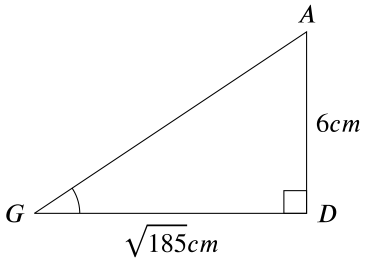


$$\tan \angle MHA = \frac{MA}{HA} = \frac{9}{\sqrt{52}}$$
$$\angle MHA \approx 51.30°$$

(c) The angle formed by line AG and plane $CDHG$.
Sol.

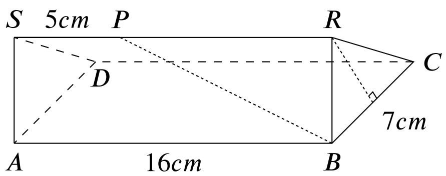
$$\text{In } CDHG,\ DG = \sqrt{DC^2 + GC^2} = \sqrt{13^2 + 4^2} = \sqrt{185}cm$$

The angle formed by line AG and plane $CDHG$ is $\angle AGD$.



$$\tan \angle AGD = \frac{AD}{GD} = \frac{6}{\sqrt{185}}$$
$$\angle AGD \approx 23.80°$$

11. The diagram below shows a regular prism, its bases ADS and BCR are equiliteral triangles. Given that $AB = 16cm$, $BC = 7cm$, $SP = 5cm$. Find:



(a) The length of BP.
Sol.

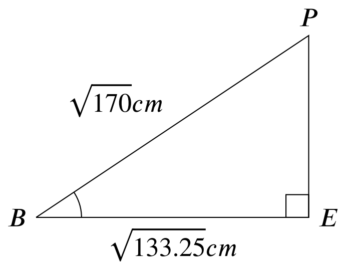
$$PR = SR - SP = 16 - 5 = 11cm$$
$$BP = \sqrt{BR^2 = PR^2} = \sqrt{7^2 + 11^2} = \sqrt{170} \approx 13.04cm$$

(b) The angle formed by line BP and plane $ABCD$.
Sol.

Let the foot point of P be E.

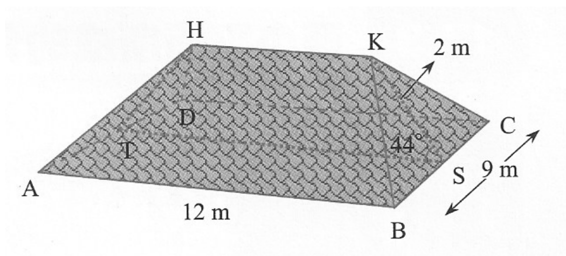
$$EB = \sqrt{3.5^2 + 11^2} = \sqrt{133.25cm}$$

The angle formed by line BP and plane $ABCD$ is $\angle PBE$.



$$\cos \angle PBE = \frac{BE}{BP} = \frac{\sqrt{133.25}}{\sqrt{170}}$$
$$\angle PBE \approx 27.71°$$

12. The diagram below shows a roof, HK is the ridge of the roof, its edges HA, HD, KB, KC are euqal in length. Both of the planes HAD and KBC form a 44^o angle with plane $ABCD$. Given that S and T are the midpoints of BC and AD respectively. Find:



(a) The distance from line HK to plane $ABCD$.

Sol.

Let the foot point of K on plane $ABCD$ be P.

$$\text{In } \Delta KPS, \sin \angle KSP = \frac{KP}{KS}$$
$$\sin 44° = \frac{KP}{2}$$
$$KP = 2\sin 44° \approx 1.39m$$

(b) The length of HK.

Sol.

$$\cos \angle KSP = \frac{PS}{KS}$$
$$\cos 44° = \frac{PS}{2}$$
$$PS = 2\cos 44°$$
$$\approx 1.44m$$
$$HK \approx 12 - 2PS$$
$$\approx 12 - 2.88$$
$$\approx 9.12m$$

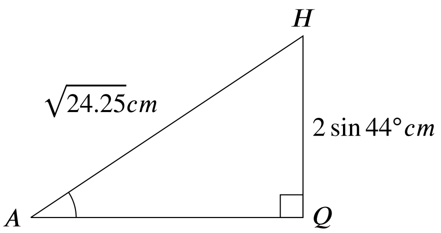
(c) The angle formed by line HA and plane $ABCD$.

Sol.

Let the foot point of H on plane $ABCD$ be Q.

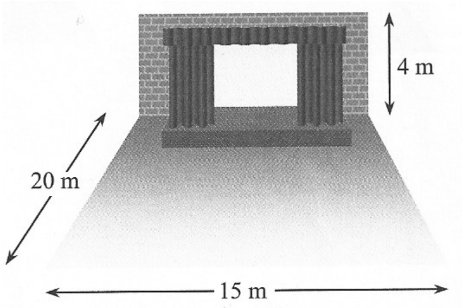
$$HA = \sqrt{HT^2 + AT^2}$$
$$= \sqrt{2^2 + 4.5^2}$$
$$= \sqrt{24.25}cm$$

The angle formed by line HA and plane $ABCD$ is $\angle HAQ$.



$$\sin \angle HAQ = \frac{HQ}{HA}$$
$$\sin \angle HAQ = \frac{2\sin 44°}{\sqrt{24.25}}$$
$$\angle HAQ \approx 16.38°$$

13. The length, width and height of a hall are $20m$, $15m$, and $4m$ respectively. Find:



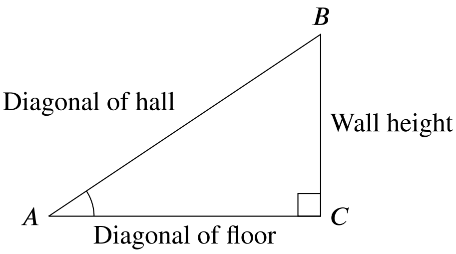
(a) The length of the diagonal of the hall.

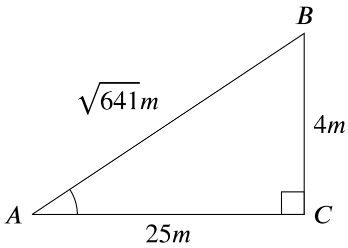
Sol.

$$\text{Diagonal of floor} = \sqrt{20^2 + 15^2}$$
$$= \sqrt{625}m$$
$$= 25m$$
$$\text{Diagonal of hall} = \sqrt{4^2 + 25^2}$$
$$= \sqrt{641}m$$
$$= 25.32m$$

(b) The angle formed by the diagonal and the floor of the hall.

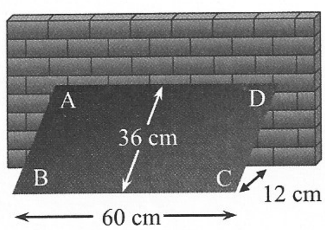
Sol.





$$\tan \angle BAC = \frac{4}{25}$$
$$\angle BAC \approx 9.09°$$

14. In the diagram below, $ABCD$ represents a rectangular plank with length and width of $60cm$ and $36cm$ respectively, its base BC is on the ground and the top of it lies on the wall. Assume that the distance between BC and the corner of the wall is $12cm$, find the angle formed by the diagonal BD of the plank and the ground.

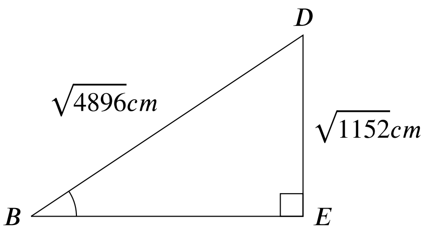


Sol.

Let the footpoint of D on the ground be E.

$$
\begin{aligned}
BD &= \sqrt{BC^2 + CD^2} \\
&= \sqrt{60^2 + 36^2} \\
&= \sqrt{4896}cm \\
DE &= \sqrt{DC^2 - CE^2} \\
&= \sqrt{36^2 - 12^2} \\
&= \sqrt{1152}cm
\end{aligned}
$$

The angle formed by the diagonal BD and the ground is $\angle DBE$.



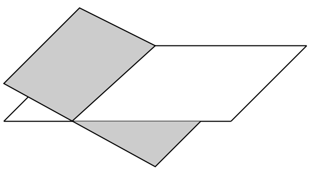
$$
\begin{aligned}
\sin \angle DBE &= \frac{\sqrt{1152}}{\sqrt{4896}} \\
\angle DBE &\approx 29.02°
\end{aligned}
$$

17.3 Angle Formed by Two Planes

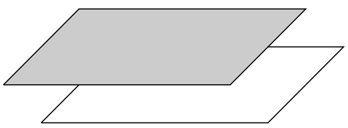
There are three types positional relationship between two planes:

1. Two planes coincide with each other.

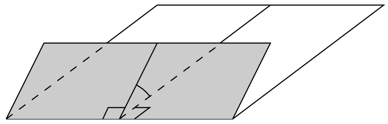
2. Two planes intersect with each other at a line.



3. Two planes are parallel to each other and do not intersect with each other.

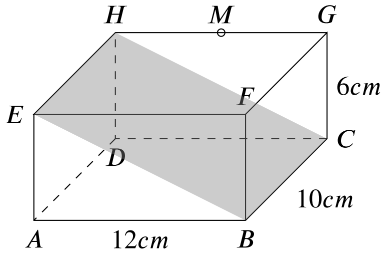


Two non-parallel planes intersect with each other at a line, the line is called the *common edge*. At any point on the common edge, draw a line perpendicular to the common edge on each plane, the acute angles formed by these two perpendicular lines are called *the angle formed by the two planes*.



17.3.1 Practice 2

1. The diagram below shows a cuboid with length of $12cm$, width of $10cm$ and height of $6cm$.

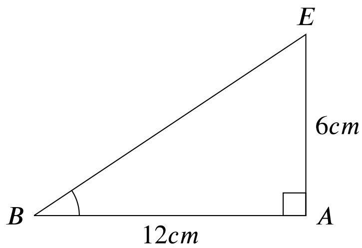


(a) Find the angle formed by plane $EBCH$ and plane $ABCD$.

Sol.

$\because$ BC is the common edge of plane $EBCH$ and plane $ABCD$, $AB \perp BC$ and $EB \perp BC$.

$\therefore$ The angle formed by plane $EBCH$ and plane $ABCD$ is $\angle EBA$.



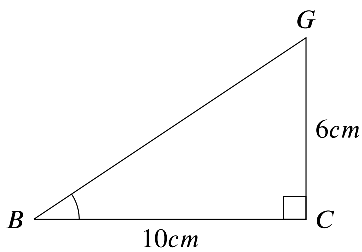
$$
\begin{aligned}
\tan \angle EAB &= \frac{6}{12} \\
&= \frac{1}{2} \\
\angle EAB &\approx 26.57°
\end{aligned}
$$

(b) Assume that M is a point on HG, find the angle formed by plane MAB and plane $ABCD$.

Sol.

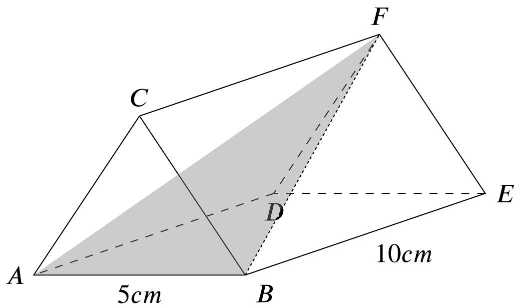
$\because$ AB is the common edge of plane MAB and plane $ABCD$, M is on HG, $HG \perp AB$, $BC \perp AB$.

$\therefore$ The angle formed by plane MAB and plane $ABCD$ is $\angle GBC$.



$$\tan\angle GBC = \frac{6}{10} = \frac{3}{5}$$
$$\angle GBC \approx 30.96°$$

2. The diagram below shows a regular prism, its bases ABC and DEF are equilateral triangles with side length of $5cm$. Given that the height of the prism is $10cm$, find:



(a) The length of BF.

Sol.

$$\begin{aligned} BF &= \sqrt{EF^2 + BE^2} \\ &= \sqrt{10^2 + 5^2} \\ &= \sqrt{125} \\ &\approx 11.18cm \end{aligned}$$

(b) The angle formed by plane ABF and plane ABC.

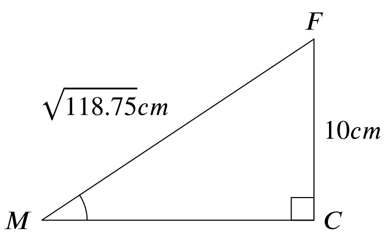
Sol.

Let the midpoint of AB be M.

$$\begin{aligned} MF &= \sqrt{FB^2 - BM^2} \\ &= \sqrt{125 - 2.5^2} \\ &= \sqrt{118.75}cm \end{aligned}$$

$\because$ AB is the common edge of plane ABF and plane ABC, $MF \perp AB$, $CF \perp AB$.

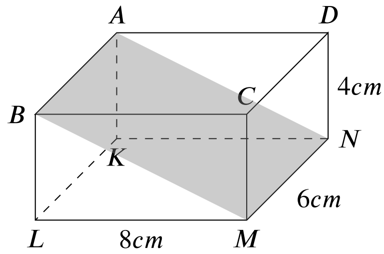
$\therefore$ The angle formed by plane ABF and plane ABC is $\angle FMC$.



$$\begin{aligned} \sin\angle FMC &= \frac{FC}{MF} \\ &= \frac{10}{\sqrt{118.75}} \end{aligned}$$
$$\angle FMC \approx 66.59°$$

17.3.2 Exercise 17.3

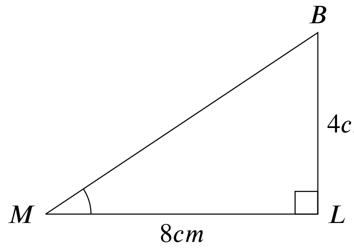
1. The diagram below shows a cuboid with length of $8cm$, width of $6cm$ and height of $4cm$. Find the angle formed by plane $ABMN$ and $KLMN$.



Sol.

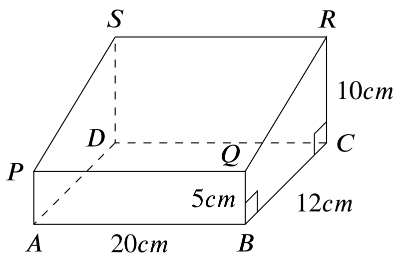
$\because$ MN is the common edge of $ABMN$ and $KLMN$, $LM \perp MN$ and $BM \perp MN$.

$\therefore$ The angle formed by plane $ABMN$ and $KLMN$ is $\angle BML$.



$$\begin{aligned} \tan\angle BML &= \frac{BL}{LM} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$
$$\angle BML \approx 26.57°$$

2. In the right prism shown below, $ABCD$ is a rectangle with length of $20cm$ and width of $12cm$, $BCRQ$ is a trapezoid, $\angle QBC$ and $\angle RCB$ are both right angles, $BQ = 5cm$, $CR = 10cm$. Find the angle formed by plane $PQRS$ and plane $ABCD$.

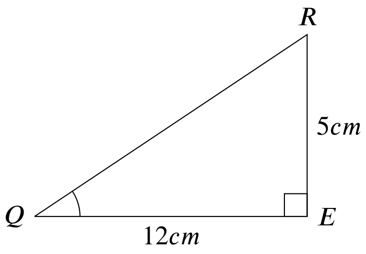


Sol.

Let the midpoint of RC and SD be E and F respectively.

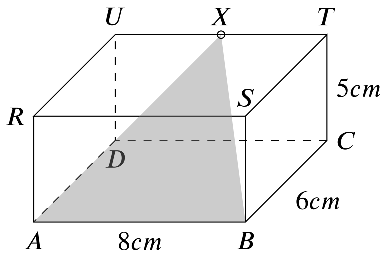
$\because$ $PQEF \parallel ABCD$, PQ is the common edge of $PQRS$ and $PQER$, $PQ \perp QE$, and $PQ \perp QR$.

$\therefore$ The angle formed by plane $PQRS$ and $ABCD$ is $\angle RQE$.



$$\tan\angle RQE = \frac{RE}{QE} = \frac{5}{12}$$
$$\angle RQE \approx 22.62^\circ$$

3. The diagram below shows a cuboid, $AB = 8cm$, $BC = 6cm$, $CT = 5cm$, X is the midpoint of TU. Find:



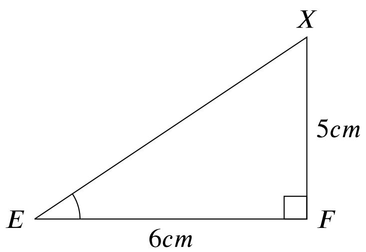
(a) The angle formed by plane XAB and plane $ABCD$.

Sol.

Let the midpoint of AB and CD be E and F respectively.

$\because$ AB is the common edge of $ABCD$ and XAB, $AB \perp XE$, and $AB \perp EF$.

$\therefore$ The angle formed by plane $ABCD$ and XAB is $\angle XEF$.



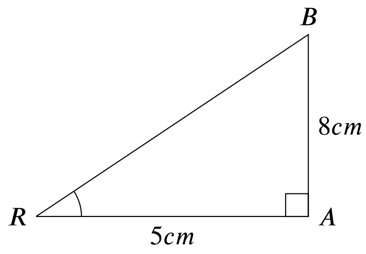
$$\tan\angle XEF = \frac{XF}{EF} = \frac{5}{6}$$
$$\angle XEF \approx 39.81^\circ$$

(b) The angle formed by plane $BCUR$ and plane $ADUR$.

Sol.

$\because$ UR is the common edge of $BCUR$ and $ADUR$, $UR \perp RB$, and $UR \perp AR$.

$\therefore$ The angle formed by plane $BCUR$ and $ADUR$ is $\angle BRA$.



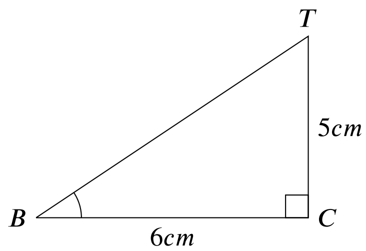
$$\tan\angle BRA = \frac{BA}{RA} = \frac{8}{5}$$
$$\angle BRA \approx 57.99^\circ$$

(c) The angle formed by plane $ABTU$ and plane $ABCD$.

Sol.

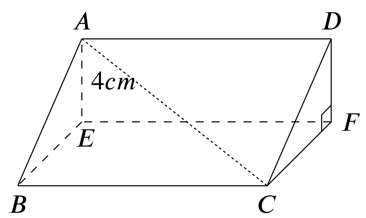
$\because$ AB is the common edge of $ABTU$ and $ABCD$, $AB \perp TB$, and $AB \perp BC$.

$\therefore$ The angle formed by plane $ABTU$ and $ABCD$ is $\angle TBC$.



$$\tan\angle TBC = \frac{TC}{BC} = \frac{5}{6}$$
$$\angle TBC \approx 39.81^\circ$$

4. The diagram below shows a right pyramid, its bases ABE and DCF are right-angled triangles. Given that $AE = 4cm$, $BE = \frac{2}{3}EF$, $EF = 4DF$, find the angle formed by plane $ABCD$ and plane $BCFE$.

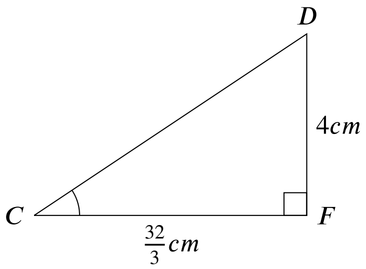


Sol.

$$\begin{aligned} EF &= 4DF \\ &= 4 \times 4 \\ &= 16cm \\ BE &= \frac{2}{3}EF \\ &= \frac{32}{3}cm \end{aligned}$$

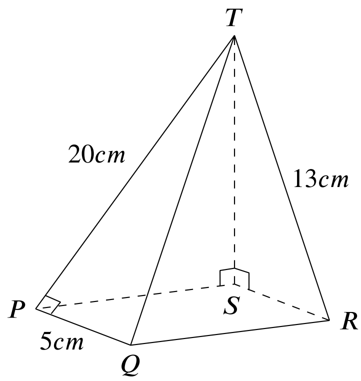
$\because$ BC is the common edge of $ABCD$ and $BCFE$, $BC \perp CD$, and $BC \perp CF$.

$\therefore$ The angle formed by plane $ABCD$ and $BCFE$ is $\angle DCF$.



$$\begin{aligned} \tan \angle DCF &= \frac{DF}{CF} \\ &= \frac{4}{\frac{32}{3}} \\ \angle DCF &\approx 20.56° \end{aligned}$$

5. In the pyramid shown below, PQT, SPT, and SRT are all right-angled triangles, $PQRS$ is a triangle. Given that $PQ = 5cm$, $RT = 13cm$, $PT = 20cm$. Find:



(a) The height of the prism.

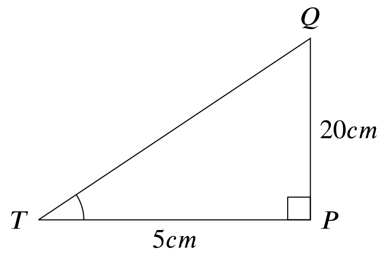
Sol.

$$\begin{aligned} \text{Height of the prism} &= TS \\ &= \sqrt{TR^2 - RS^2} \\ &= \sqrt{13^2 - 5^2} \\ &= 12cm \end{aligned}$$

(b) The angle formed by line TQ and plane PST.

Sol.

The angle formed by line TQ and plane PST is $\angle QTP$.



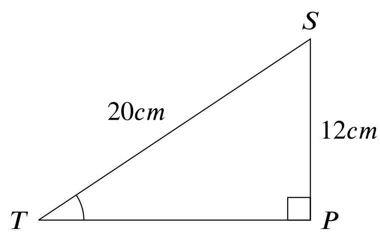
$$\begin{aligned} \tan \angle QTP &= \frac{PQ}{PT} \\ &= \frac{5}{20} \\ &= \frac{1}{4} \\ \angle QTP &\approx 14.04° \end{aligned}$$

(c) The angle formed by plane RST and PQT.

Sol.

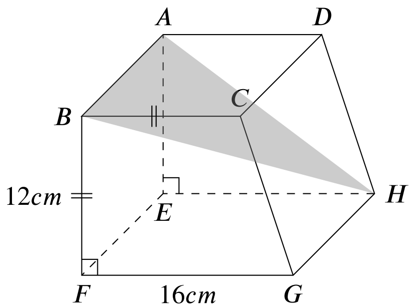
The angle formed by plane RST and PQT is $\angle STP$.

$$\begin{aligned} \text{In } \Delta TRS,\ TS &= \sqrt{TR^2 - SR^2} \\ &= \sqrt{13^2 - 5^2} \\ &= 12cm \end{aligned}$$



$$\begin{aligned} \cos \angle STP &= \frac{TS}{TP} \\ &= \frac{12}{20} \\ &= \frac{3}{5} \\ \angle STP &\approx 53.13° \end{aligned}$$

6. The diagram below shows a right prism, its base $BCGF$ is a trapezoid, $BC = BF = 12cm$, $FG = 16cm$. The lateral face $EFGH$ is a square, and is perependicular to another lateral face $ABFE$. Find:



(a) The angle formed by plane $CDHG$ and plane $EFGH$.

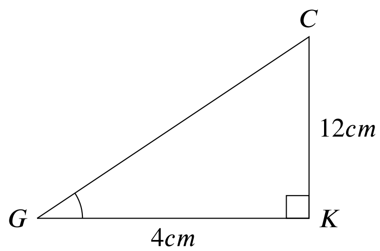
Sol.

Let the foot point of C be K.

$\because$ GH is the common edge of the plane $CDHG$ and plane $EFGH$, $CG \perp GH$, and $KG \perp GH$.

$\therefore$ The angle formed by plane $CDHG$ and plane $EFGH$ is $\angle CGK$.

$$\begin{aligned} KG &= FG - FK \\ &= 16 - 12 \\ &= 4cm \end{aligned}$$



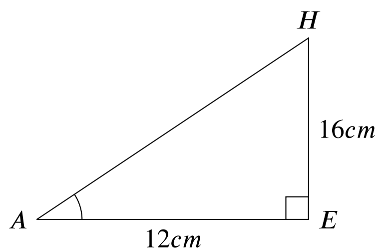
$$\begin{aligned} \tan \angle CGK &= \frac{CK}{KG} \\ &= \frac{12}{4} \\ &= 3 \\ \angle CGK &\approx 71.57^\circ \end{aligned}$$

(b) The angle formed by plane ABH and plane $ABFE$.

Sol.

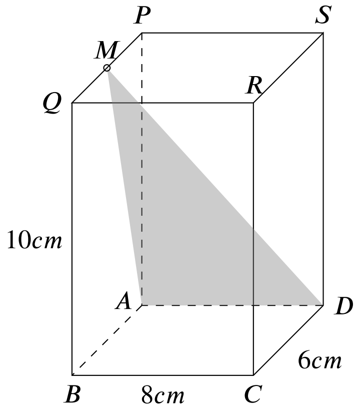
$\because$ AB is the common edge of the plane ABH and plane $ABFE$, $AB \perp AH$ and $AB \perp AE$.

$\therefore$ The angle formed by plane ABH and plane $ABFE$ is $\angle HAE$.



$$\begin{aligned} \tan \angle HAE &= \frac{HE}{AE} \\ &= \frac{16}{12} \\ &= \frac{4}{3} \\ \angle HAE &\approx 53.13^\circ \end{aligned}$$

7. In the cuboid shown below, $BC = 8cm$, $CD = 6cm$, $BQ = 10cm$. Given that M is the midpoint of PQ. Find:

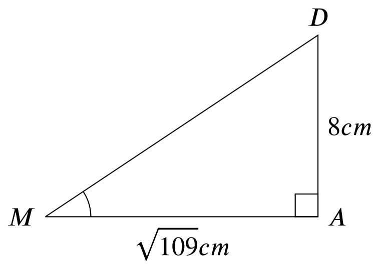


(a) The angle formed by line MD and plane $PQBA$.

Sol.

The angle formed by line MD and plane $PQBA$ is $\angle DMA$.

$$\begin{aligned} \text{In } \Delta MPA,\ MA &= \sqrt{PA^2 + MP^2} \\ &= \sqrt{10^2 + 3^2} \\ &= \sqrt{109}cm \end{aligned}$$



$$\begin{aligned} \tan \angle DMA &= \frac{DA}{MA} \\ &= \frac{8}{\sqrt{109}} \\ \angle DMA &\approx 37.46^\circ \end{aligned}$$

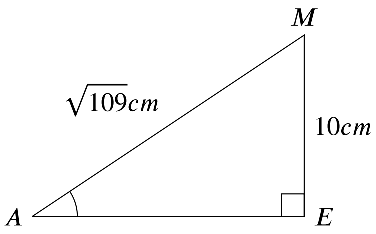
(b) The angle formed by plane AMD and plane $ABCD$.

Sol.

Let the midpoint of AB be E.

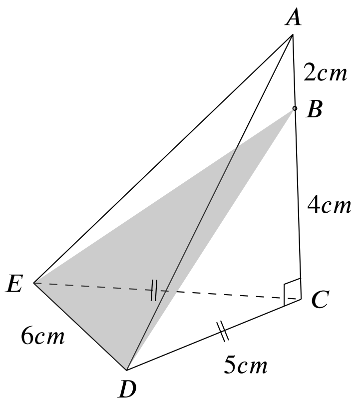
$\because$ AD is the common edge of plane AMD and plane $ABCD$, $AM \perp AD$, and $EA \perp AD$.

$\therefore$ The angle formed by plane AMD and plane $ABCD$ is $\angle MAE$.



$$\tan\angle MAE = \frac{ME}{MA} = \frac{10}{\sqrt{109}}$$
$$\angle MAE \approx 73.30^\circ$$

8. The diagram below shows a pyramid with an isosceles triangle base. Given that $CD = CE = 5cm$, $ED = 6cm$, ACD is a right-angled triangle, B is a point on AC, $AD = 2cm$, $BC = 4cm$. Find:



(a) The angle formed by plane BDE and plane CDE.

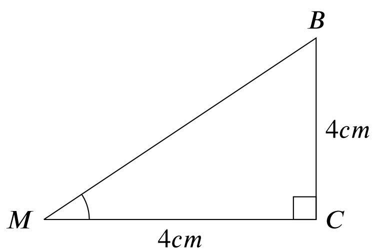
Sol.

Let the midpoint of ED be M.

$\because$ DE is the common edge of plane BDE and plane CDE, $BM \perp DE$, and $CM \perp DE$.

$\therefore$ The angle formed by plane BDE and plane CDE is $\angle BMC$.

$$MC = \sqrt{DC^2 - DM^2} = \sqrt{5^2 - 3^2} = 4cm$$



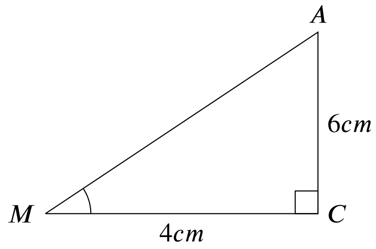
$$\tan\angle BMC = \frac{BC}{CM} = \frac{4}{4} = 1$$
$$\angle BMC = 45^\circ$$

(b) The angle formed by the plane ADE and CDE.

Sol.

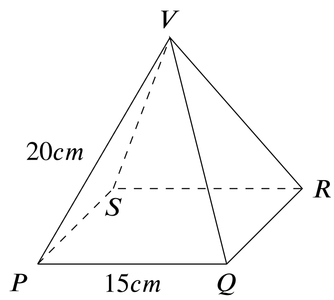
$\because$ DE is the common edge of plane ADE and plane CDE, $CM \perp DE$, and $AM \perp DE$.

$\therefore$ The angle formed by plane ADE and plane CDE is $\angle AMC$.



$$\tan\angle AMC = \frac{AC}{CM} = \frac{6}{4} = \frac{3}{2}$$
$$\angle AMC = 56.31^\circ$$

9. The diagram below shows a regular pyramid with a square base. Given that $PQ = 15cm$, $PV = 20cm$. Find:



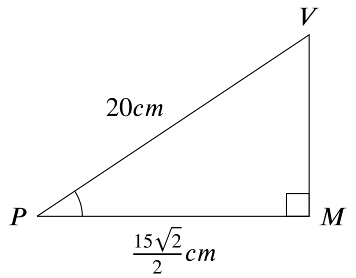
(a) The angle formed by line PV and plane $PQRS$.

Sol.

Let the footpoint of V be M.

The angle formed by line PV and plane $PQRS$ is $\angle VPM$.

$$PR = \sqrt{PQ^2 + QR^2} = \sqrt{15^2 + 15^2} = 15\sqrt{2}cm$$
$$PM = \frac{PR}{2} = \frac{15\sqrt{2}}{2}$$



$$\begin{aligned}\cos\angle VPM &= \frac{PM}{PV}\\ &= \frac{\frac{15\sqrt{2}}{2}}{20}\\ &= \frac{3\sqrt{2}}{8}\\ \angle VPM &= 57.97^\circ\end{aligned}$$

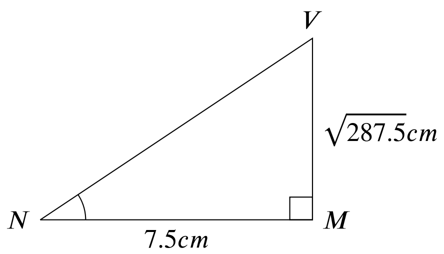
(b) The angle formed by the lateral faces and the base of the pyramid.

Sol.

$$\begin{aligned}VM &= \sqrt{VP^2 - PM^2}\\ &= \sqrt{20^2 - \left(\frac{15\sqrt{2}}{2}\right)^2}\\ &= \sqrt{287.5}cm\end{aligned}$$

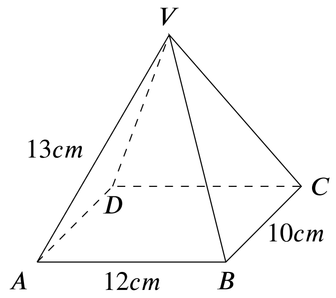
Let the midpoint of PQ be N.

The angle formed by the lateral faces and the base of the pyramid is $\angle VNM$.



$$\begin{aligned}\tan\angle VNM &= \frac{VM}{NM}\\ &= \frac{\sqrt{287.5}}{7.5}\\ \angle VNM &= 66.14^\circ\end{aligned}$$

10. The diagram below shows a right pyramid with lateral edges of $13cm$. Its base $ABCD$ is a rectangle with length of $12cm$ and width of $10cm$. Find:



(a) The angle formed by plane VBC and plane $ABCD$.

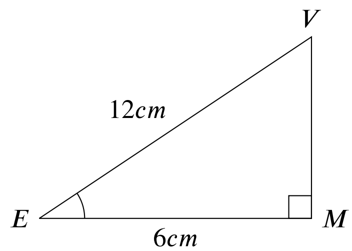
Sol.

Let the midpoint of BC be E, and the footpoint of V be M.

$\because$ BC is the common edge of plane VBC and plane $ABCD$, $VE \perp BC$, and $ME \perp BC$.

$\therefore$ The angle formed by plane VBC and plane $ABCD$ is $\angle VEM$.

$$\begin{aligned}VE &= \sqrt{VB^2 - BE^2}\\ &= \sqrt{13^2 - 5^2}\\ &= 12cm\end{aligned}$$



$$\begin{aligned}\cos\angle VEM &= \frac{ME}{EV}\\ &= \frac{1}{2}\\ \angle VEM &= 60^\circ\end{aligned}$$

(b) The angle formed by plane VCD and plane $ABCD$.

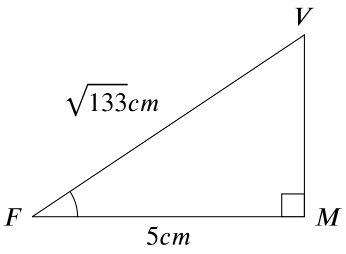
Sol.

Let the midpoint of CD be F

$\because$ CD is the common edge of plane VCD and plane $ABCD$, $VF \perp CD$, and $MF \perp CD$.

$\therefore$ The angle formed by plane VCD and plane $ABCD$ is $\angle VFM$.

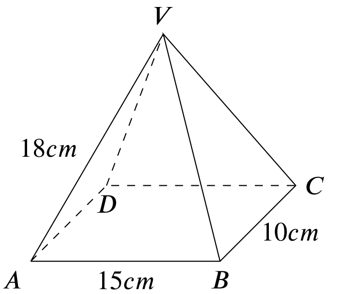
$$\begin{aligned}VF &= \sqrt{VD^2 - DF^2}\\ &= \sqrt{13^2 - 6^2}\\ &= \sqrt{133}cm\end{aligned}$$



$$\cos \angle VEM = \frac{MF}{VF}$$
$$= \frac{5}{\sqrt{133}}$$
$$\angle VEM = 64.31°$$

RIGHT HERE LMAO

11. The diagram below shows a right pyramid with lateral edges of $18cm$, its base $ABCD$ is a rectangle with length of $15cm$ and width of $10cm$. Find:



(a) The height of the pyramid.

Sol.

Let the footpoint of V on $ABCD$ be M.

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{15^2 + 10^2}$$
$$= 5\sqrt{13}cm$$
$$AM = \frac{AC}{2}$$
$$= \frac{5\sqrt{13}}{2}$$
$$\text{Height of the pyramid} = VM$$
$$= \sqrt{AV^2 - AM^2}$$
$$= \sqrt{18^2 - \left(\frac{5\sqrt{13}}{2}\right)^2}$$
$$= \sqrt{242.75}$$
$$\approx 15.58cm$$

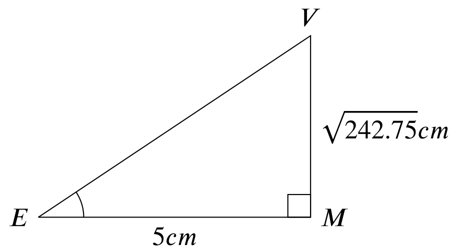
(b) The angle formed by plane VAB and plane $ABCD$.

Sol.

Let the midpoint of AB be E.

$\because$ AB is the common edge of plane VAB and plane $ABCD$, $ME \perp AB$, and $VE \perp AB$.

$\therefore$ The angle formed by plane VAB and plane $ABCD$ is $\angle VEM$.



$$\tan \angle VEM = \frac{VM}{ME}$$
$$= \frac{\sqrt{242.75}}{5}$$
$$\angle VEM \approx 72.21°$$

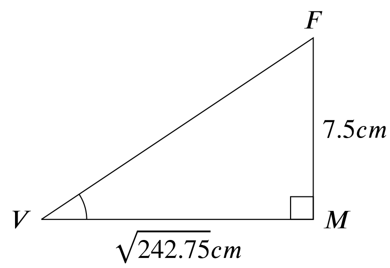
(c) The angle formed by plane VBC and plane VAD.

Sol.

Let the midpoint of AD and BC be F and G respectively.

The angle formed by plane VBC and plane VAD is $\angle FVG$.

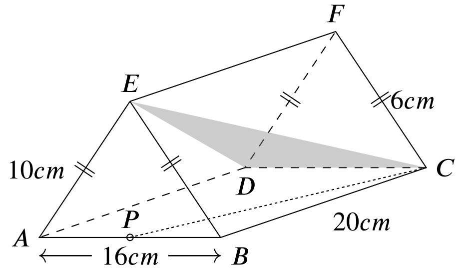
$$\angle FVG = \angle FVM + \angle MVG$$
$$= 2\angle FVM$$



$$\tan \angle FVM = \frac{FM}{VM}$$
$$= \frac{7.5}{\sqrt{242.75}}$$
$$\angle FVM \approx 25.705°$$

$$FVG = 2\angle FVM$$
$$\approx 2 \times 25.705°$$
$$\approx 51.41°$$

12. The diagram below shows a right prism with isoceles triangle bases. The side length and base length of the triangle base are $10cm$ and $16cm$ respectively, the height of the prism is $20cm$. Given that P is the midpoint of AB. Find:



(a) The length of PC.

Sol.

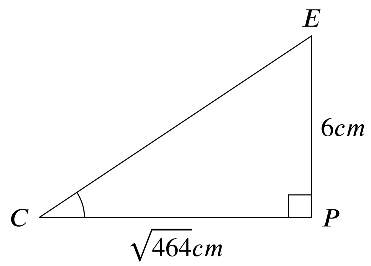
$$\begin{aligned} PC &= \sqrt{BC^2 - PB^2} \\ &= \sqrt{20^2 + 8^2} \\ &= \sqrt{464} \\ &\approx 21.54cm \end{aligned}$$

(b) The angle formed by line EC and plane $ABCD$.

Sol.

The angle formed by line EC and plane $ABCD$ is $\angle ECP$.

$$\begin{aligned} EP &= \sqrt{AE^2 - AP^2} \\ &= \sqrt{10^2 - 8^2} \\ &= 6cm \end{aligned}$$



$$\begin{aligned} \tan\angle ECP &= \frac{EP}{CP} \\ &= \frac{6}{\sqrt{464}} \\ \angle ECP &\approx 15.56° \end{aligned}$$

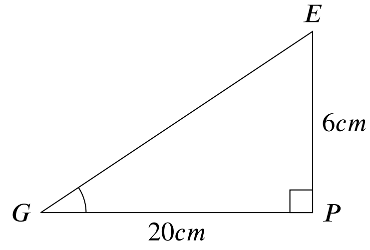
(c) The angle formed by plane DCE and plane $ABCD$.

Sol.

Let the midpoint of CD be G.

$\because$ CD is the common edge of plane DCE and plane $ABCD$, $PG \perp CD$, and $EG \perp CD$.

$\therefore$ The angle formed by plane DCE and plane $ABCD$ is $\angle EGP$.



$$\begin{aligned} \tan\angle EGP &= \frac{EP}{GP} \\ &= \frac{6}{20} \\ \angle EGP &= 16.70° \end{aligned}$$

17.4 Longitude and Latitude

The earth is approximately spherical in shape, its radius is about $6,370km$, and its axis is a line that passes through the north (N) and south (S) poles. The earth rotating around its axis once is called a day, and the earth rotating around the sun once is called a year.

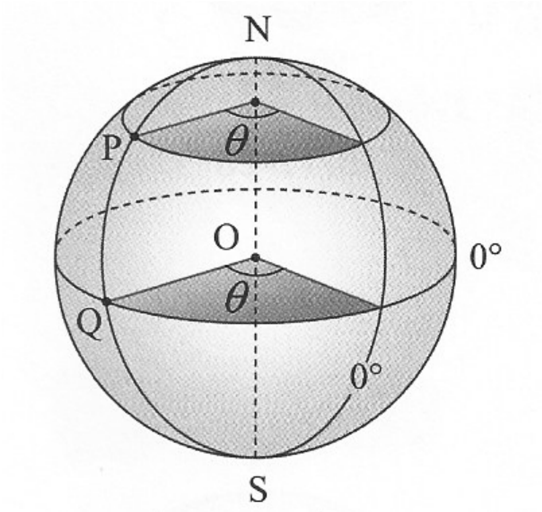
Any point on the earth's surface can be identified by two angles, the first is the angle between the point and the equator, called the *latitude* of the point, and the second is the angle between the point and the prime meridian, called the *longitude* of the point.

Longitude and Lines of Longitude

The two semicircles that are formed by the intersection of the earth's surface with the plane that passes through the north and south poles are called the *lines of longitude*, also called *meridians*. The lines of longitude that passes through the *Greenwich Observatory* in England are considered as 0° longitude, called the *Greenwich Meridian* or *prime meridian*.

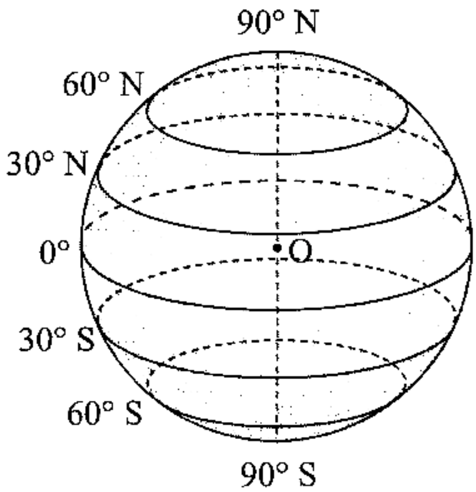
Prime meridian

The angle between the Greenwich Meridian and the line of longitude that passes through the point P is called the *longitude of P*. There are 360 degrees of longitude (+180° eastward and −180° westward.). The prime meridian divides the world into the Eastern Hemisphere and the Western Hemisphere. $180°E$ and $180°W$ coincide with each other at the same line of longitude, called the 180^{th} *Meridian* or *Antimeridian*.

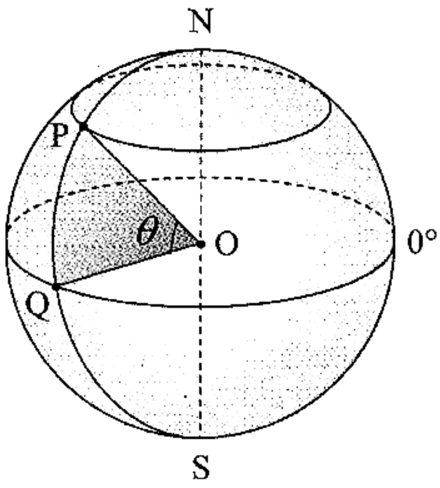


Latitude and Parallels of Latitude

The lines of latitude are the circles that are perpendicular to the plane that passes through the north and south poles. The *equator* is the one and only great circle among the parallels of latitude.

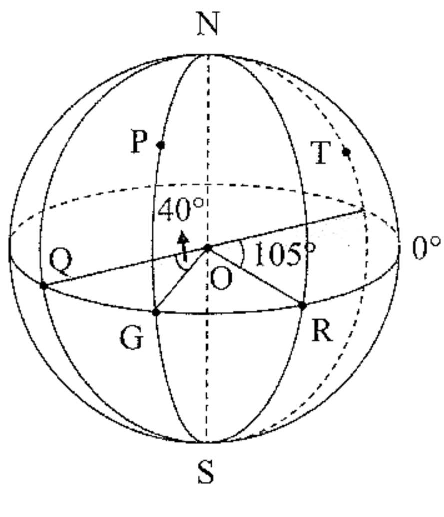


The angle between the equator and the line of latitude that passes through the point P is called the *latitude of P*. There are 180 degrees of latitude (+90° northward and −90° southward). The equator divides the world into the Northern Hemisphere and the Southern Hemisphere.

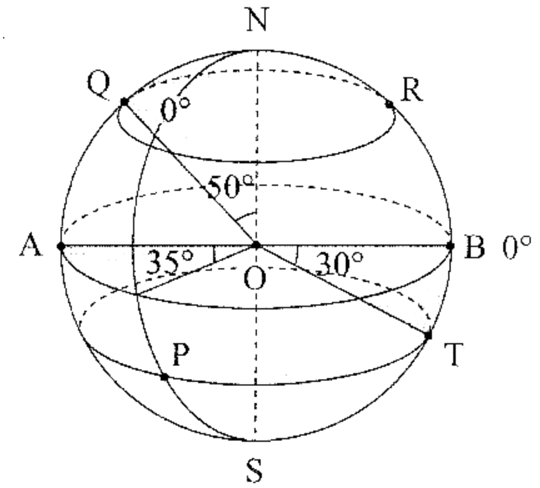


17.4.1 Practice 3

1. In the diagram below, NGS is the prime meridian, O is the centre of the earth. Find the longitude of locations P, Q, R and T.

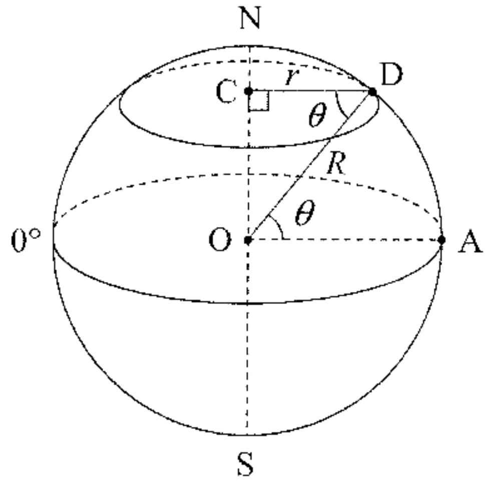


2. In the diagram below, O is the centre of the earth, location A and B are on the equator. Find the location of P, Q, R and T.



Radius of the Parallel of Latitude

Let R be the radius of the earth, r be the radius of latitude θ, then $r = R\cos\theta$.



Nautical Miles

The arc length corresponding to $1'$ ($= \frac{1}{60}^{\circ}$) of the great circle on earth is called a *nautical mile* ($1NM$), that is, $1NM = \frac{1}{60\times360} \times 2\pi \times 6370km = 1.853km$.

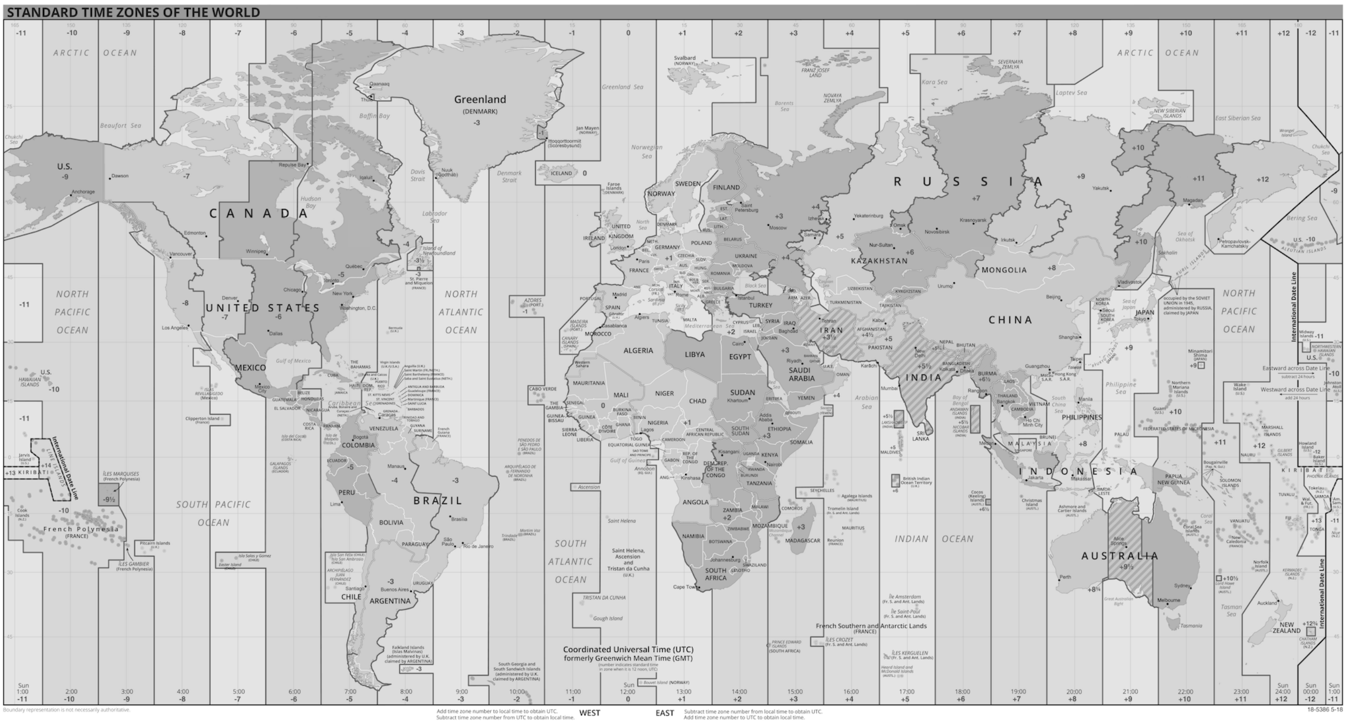
Time Difference and Longitude

The time is calculated by the rotation of the earth around its axis. The earth rotates around its axis from west to east once in $24h$. That is, the earth rotates 15° in $1h$. Thus, the time difference between two locations on the earth is equal to the difference of their longitudes. Thus, the time difference is $1hr$ per 15° of longitude difference.

1. **Local Time**
   The local time is the time at a location on the earth. The local time for any location on the same line of longitude is the same.

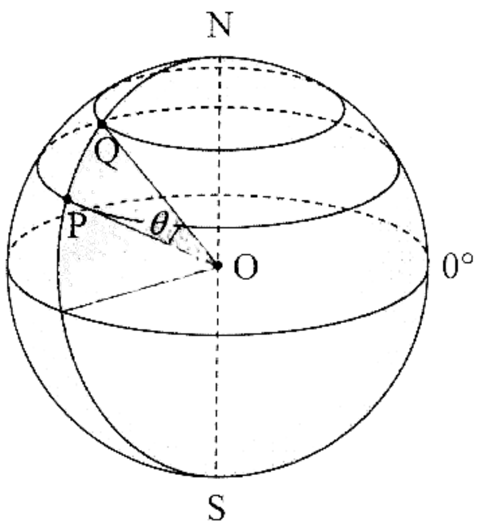
2. **Standard Time**
   Back in the year 1844, International Meridian Conference was held in Washington DC. The conference decided to divide the world into 24 time zones based on the Greenwich Meridian, called the *Greenwich Meridian Time (GMT)*. There is zero time offset 7.5° eastward and 7.5° westward of the Greenwich Meridian. The time offset is $1hr$ per 15° of longitude difference. All places in the same time zone share the same local time with the location located on the line of longitude that passes through the centre of the time zone, called the *standard time* or *zone time*.

   When entering a new time zone from the east, the local time is advanced by $1hr$ per 15° of longitude difference. When entering a new time zone from the west, the local time is delayed by $1hr$ per 15° of longitude difference.



17.5 Distance of Two Locations on the Same Line of Longitude

The distance of two location on the same line of longitude is the arc length corresponding to the difference of their latitudes. Given two location P and Q on the same line of longitude, according to the definition of nautical mile, the distance between P and Q can be acquired by the arc length of PQ. That is, $\overset{\frown}{PQ} = \theta \times 60NM$, where θ is the difference of their latitudes.



17.5.1 Practice 4

1. Given that location A and B are on the same line of longitude. Based on the following longitude, find the distance between A and B (Express your answer in nautical miles):
   - (a) $A(50°N)$, $B(75°N)$
   - (b) $A(0°)$, $B(42°S)$
   - (c) $A(43°N)$, $B(38°S)$
2. Given that location P and Q are on the same line of longitude. The distane between two locations is $1000NM$, P is located at $7°30'$ north of the equator. Based on the following criteria, find the latitude of Q:
   - (a) Q is located at the north of P
   - (b) Q is located at the south of P

17.5.2 Exercise 17.5

1. Given that A and B are on the same line of longitude. Based on the following difference of latitude of two locations, find the distance between A and B (Express your answer in nautical miles):
   - (a) $\theta = 39°$
   - (b) $\theta = 80°30'$
   - (c) $\theta = 64°20'$
2. Given that A and B are on the same line of longitude. Based on the following distance between two locations, find the difference of latitude of A and B (Round your answer to the nearest minute):
   - (a) $700NM$
   - (b) $318NM$
   - (c) $3450NM$
3. Find the distance between two locations along the same line of longitude:
   - (a) $A(21°S, 110°E)$, $B(33°S, 110°E)$
   - (b) $X(38°N, 40°W)$, $Y(19°N, 40°W)$
   - (c) $E(34°45'S, 80°E)$, $F(0°, 80°E)$

(d) $P(18°15'N, 90°W)$, $Q(43°30'N, 90°W)$

(e) $T(15°30'N, 120°E)$, $M(24°30'N, 120°E)$

4. Location X and Y are on the same line of longitude, the distane between them is $400NM$. Find the difference of latitude of X and Y.

5. Location P and Q are on the same line of longitude, and their distance along the line of longitude is $600NM$, find the difference between their latitude.

6. X city and Y city are on the same line of longitude, the latitude of X city is $2°15'$ north of the equator, the latitude of Y city is $6°$ north of the equator. Find the distance between X city and Y city (Express your answer in kilometers).

7. A plane is flying $1000km$ due north from airport $A(15°N, 115°E)$ to airport B. Find the longitude and latitude of airport B.

8. A plane is flying $1500km$ due south from airport $A(5°N, 100°E)$ to airport B. Find the longitude and latitude of airport B.

9. Find the distance from $A(18°30'S)$ to the north pole along the same line of longitude.

10. The distance between location C and D is $700NM$, C is located at $5°30'$ north of the equator. Find the latitude of D.

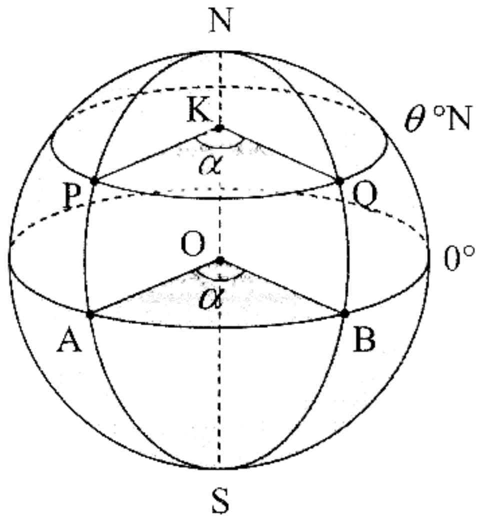
11. A plane takes off from $P(60°N, 60°E)$ and flies pass north pole along the great circle route to $Q(50°N, 120°W)$. Find the flying distance.

12. A ship sails from $P(50°S, 160°E)$ due north to another port $Q(30°N, 160°E)$. The sailing time is 10 days. Find the average speed of the ship. (Express your answer in NM/hr)

13. Given that PQ is the diameter of the parallel of latitude $35°S$. A plane takes off from location P, flies pass the south pole along the line of longitude, and lands at location Q after $13hrs40mins$. Find the average speed of the plane for the whole flight duration. (Express your answer in NM/hr)

17.6 Distance of Two Locations on the Same Parallel of Latitude

The distance between two locations on the same parallel of latitude is the arc length on the parallel of latitude corresponding to the difference of their longitudes.



In the diagram above, P and Q are on the same parallel of latitude θ, their difference of latitude is α. A and B are locations on the equator.

Given that $\angle PKQ = \angle AOB = \alpha$. Let R be the radius of the earth, r be the radius of the parallel of latitude.

$$\frac{\overset{\frown}{PQ}}{\overset{\frown}{AB}} = \frac{\frac{\alpha}{360°} \times 2\pi r}{\frac{\alpha}{360°} \times 2\pi R} = \frac{r}{R}$$

From the radius of the the parallel of latitude $r = R\cos R$, we have $\frac{r}{R} = \cos\theta$.

$$\therefore \frac{\overset{\frown}{PQ}}{\overset{\frown}{AB}} = \cos\theta$$

$$\begin{aligned}\overset{\frown}{PQ} &= \overset{\frown}{AB} \times \cos\theta \\ &= \alpha \times 60 \times \cos\theta NM \text{ or} \\ &= \alpha \times 60 \times \cos\theta \times 1.853km\end{aligned}$$

17.6.1 Practice 5

1. Fidn the distance of the following pairs of location on the same parallel of latitude (Express your answer in nautical miles):

   (a) $P(80°N, 105°W), Q(80°N, 48°W)$

   (b) $M(50°S, 48°E), N(50°S, 100°E)$

   (c) $X(40°N, 28°15'E), Y(40°N, 42°45'W)$

   (d) $K(20°S, 160°E), L(20°S, 140°W)$

2. Given that A is located at the west of $B(46°N, 72°W)$ with a distance of $2350NM$. Find the longitude and latitude of A.

17.6.2 Exercise 17.6

1. Find the distance of the following pairs of location on the same parallel of latitude (Express your answer in

nautical miles):

(a) $P(45°S, 20°E), Q(45°S, 100°E)$

(b) $M(36°N, 45°W), N(36°N, 105°W)$

(c) $A(80°S, 130°E), B(80°S, 165°E)$

(d) $K(70°N, 40°E), L(70°N, 20°W)$

(e) $T(0°, 128°W), M(0°, 120°E)$

2. Based on the following distances of location P and Q and the longitude and latitude of P, find the longitude and latitude of Q:

(a) $PQ = 800NM$, Q is located at the west of $P(50°S, 100°W)$

(b) $PQ = 3400NM$, Q is located at the east of $P(35°N, 68°E)$

(c) $PQ = 1450NM$, Q is located at the east of $P(42°N, 150°W)$

3. Given that two places are on the parallel of latitude 60° north to the equator, and their difference of longitude is 160°. Find the distance of the two places. (Express your answer in kilometers)

4. City A and B are on the parallel of latitude $5°30'$ north to the equator, their longitude are $100°15'E$ and $103°E$ respectively. Find the distance between two cities along the parallel of latitude.

5. Find the circumference of the parallel of latitude $35°30'S$.

6. Find the radius of the parallel of latitude $60'N$.

7. A ship set sail from $P(20°E)$ and sail $600NM$ due east along $42°N$ parallel of latitude. Find the longitude and latitude of the destination.

8. A ship sails from port $P(48°N, 12°W)$ $1000NM$ due west to another port Q, find the longitude and latitude of Q.

9. Given that A is located at the east of Paris$(49°N, 2°30'E)$ with a distance of $2200km$. Find the longitude and latitude of A.

10. A plane flies from $X(40°N, 2°30'E)$ $9265km$ due east to Y, find the longitude and latitude of Y.

11. Given that the earth takes $24hrs$ to rotate once. Find the speed of Kuala Lumpur$(3°15'N, 102°E)$ to rotate once. (Express your answer in NM/hr)

12. Given that the longitude of P and Q are 50° and $100c°$ respectively. If P and Q both located at the west of $R(55°S)$ and $PR = PQ$, find:

(a) The longitude of R.

(b) Th distance between Q and R along the parallel of latitude.

13. A plane flies from $F(50°S, 50°E)$ due west to $H(50°S, 45°W)$, then flies from H due north $4800NM$ to K. Given that the average speed of the plane is $480NM/hr$ throughout the journey, find:

(a) The latitude of K.

(b) The distance between F and H along the parallel of latitude.

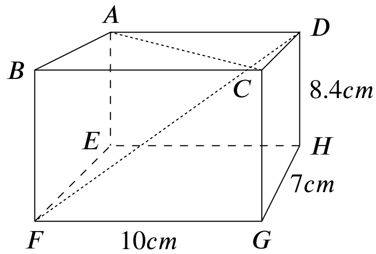
(c) The flight duration for the whole journey.

17.7 Revision Exercise 17

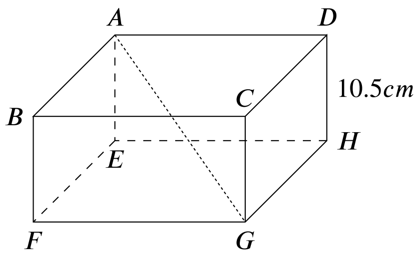
1. In the cuboid shown below, $FG = 10cm$, $GH = 7cm$, $DH = 8.4cm$, find:

(a) The angle formed by angle AC and plane $BFGC$.

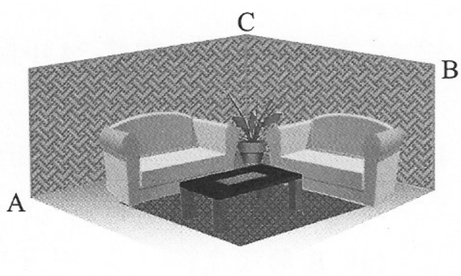
(b) The angle formed by angle FD and plane $EFGH$.



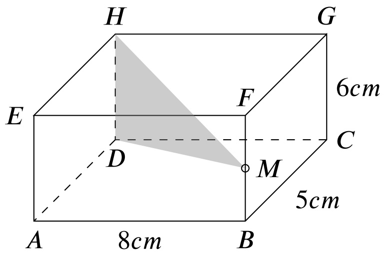
2. The diagram below shows a cuboid with volume of $400cm^3$, height of $10.5cm$, $AD = 2DC$. Find the angle formed by angle AG and plane $ADHE$.



3. The diagram below shows a reception room with a square floor with side length of $6m$. Given that the elevation angle of corner C measured from corner A is 30°, find the angle formed by the line connecting corner A and B with the floor.



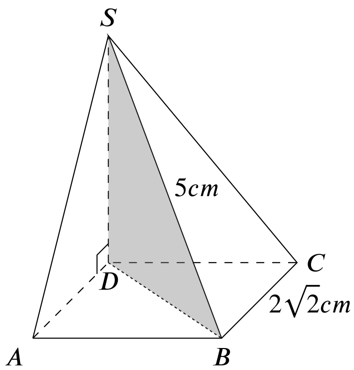
4. The diagram below shows a cuboid with length of $8cm$, width of $5cm$ and height of $6cm$, M is the midpoint of BF. Find the angle formed by plane HDM and plane $ADHE$.



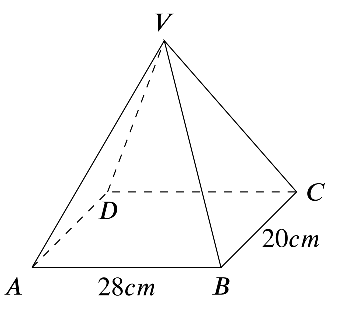
5. The diagram below shows a pyramid with a square base, its lateral edge SD is perpendicular to its base. Given that $BC = 2\sqrt{2}cm$, $SB = 5cm$. Find:

   (a) The angle formed by plane SAD and plane SBD.

   (b) The angle formed by lateral edge SA and base $ABCD$.



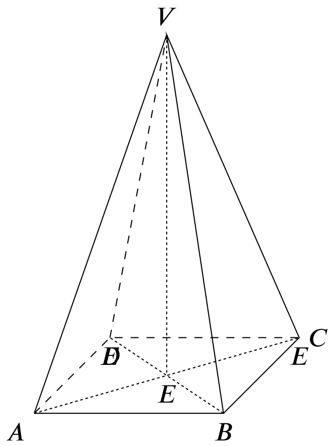
6. The diagram below shows a right prism with a rectangular base $ABCD$ with length of $28cm$ and width of $20cm$. Assume that plane VBC and the base of the pyramid forms a $60°$ angle. Find the angle formed by plane VAB and the base.



7. The diagram below shows a regular cuboid with a square base. Given that $VE = \frac{5}{2}AD$. Find:

   (a) The angle formed by the angle VA and the base $ABCD$.

   (b) The angle formed by plane VAD and the base.



8. Find the distance from the Panama City$(9°N, 79°30'W)$ to Toronto $(43°45'N, 79°30'W)$. (Express your answer in nautical miles)

9. Tokyo and Adelaide are located at the same longitude, their latitude are $35°45'N$ and $35°S$ respectively. Find the distance between two cities along the parallel of latitude.

10. A plane flies $2000NM$ along the equator, Find the difference of longitude between the point of departure and the destination.

11. Location M and N are both located at the parallel of latitude $45°$ north to the equator with a difference in longitude of $20°$. Find the distance between M and N along the parallel of latitude. (Express your answer in nautical miles)

12. Location X and Y are on the parallel of latitude $20°$ north to the equator, their longitude are $45°E$ and $80°E$ respectively. Find the distance between location X and Y along the parallel of latitude. (Express your answer in nautical miles)

13. A plane flies from $A(42°E)$ to $B(20°E)$ along the equator, then it flies from B due north to $C(30°N)$. Find the distance the plane flies in total.

14. Assume that A is located $1000NM$ due north of the equator, $600NM$ due east of the Greenwich Meridian, find the longitude and latitude of A.

15. A plane flies from $P(15°N, 30°E)$ $2000NM$ due south to B, find the longitude and latitude of B. Another plane flies from P $3000NM$ due east to C, find the longitude and latitude of C.

16. A plane flies from $A(130°E)$ along the equator to $B(120°30'E)$ along the equator, then flies from B due north to $C(20°45')$. Assume that the average speed of the plane is $300NM/hr$ throughout the journey, find the flight duration for the whole journey.

17. A plane flies from $A(50°N, 10°E)$ due east to $B(45°E)$.

    (a) Find the flight distance of the plane. (Express your answer in nautical miles)

(b) Assume that the speed of the plane is $420NM/hr$ in average, find the flight duration of the plane.

18. Given that three locations P, Q and R are located on the same parallel of latitude $40°$ north to the equator, The longitude of P and R are $10°30'W$ and $4°30'E$, Q is located at the middle of P and R.

    (a) Find the difference of longitude between P and R.

    (b) Find the longitude of Q.

    (c) Find the distance between P and R along the parallel of latitude.

    (d) A ship sails from P to Q along the parallel of latitude with a speed of $18NM/hr$, find the sailing duration of the ship.