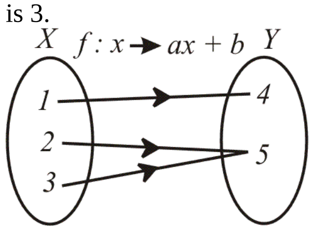
BRIEF NOTES
ADDITIONAL MATHEMATICS
FORM 4

CHAPTER 1: FUNCTION

1. $f : x \rightarrow x + 3$
   x is the object, $x + 3$ is the image

   $f : x \rightarrow x + 3$ can be written as
   $f(x) = x + 3$.
   To find the image for 2 means
   $f(2) = 2 + 3 = 5$
   Image for 2 is 5.
   Find the object for 8 means $f(x) = 8$ what is the value of x ?
   $x + 3 = 8$; $x = 5$
   The object is 5.
   If the function is written in the form of ordered pairs (x, y) , x is the object and y is the image.
   E.g. (2, 4), (2, 6), (3, 7)
   Image for 2 is 4 and 6 whereas object for 7 is 3.



   In the arrow diagram, the set of object is {1, 2, 3} and the set of image is {4, 5}

2. For $f : x \rightarrow \dfrac{5}{x-3}$, $x - 3 \neq 0$, i.e. $x \neq 3$

   because $\dfrac{5}{0}$ is undefined.

   Thus, if $f : x \rightarrow \dfrac{5}{x-3}$, $x \neq k$ then k is 3.

3. Function which maps into itself means $f(x) = x$

   If $f : x \rightarrow \dfrac{3}{x-2}$, find the value of x which is mapped into itself.

   $\dfrac{3}{x-2} = x$
   $3 = x(x - 2) = x^2 - 2x$
   Thus, $x^2 - 2x - 3 = 0$
   $(x - 3)(x + 1) = 0$
   $x = 3$ or -1

3. Inverse Function

   Symbol : f^{-1}
   To find the inverse function, change $f(x)$ to y and find x in tems of y.

   Given $f : x \rightarrow \dfrac{x}{3-x}$, find f^{-1}
   Let $f(x) = y$

   $y = \dfrac{x}{3-x}$ $y(3 - x) = x$
   $3y - xy = x$
   $3y = x + xy$
   $= x(1 + y)$
   $x = \dfrac{3y}{1+y}$, thus $f^{-1}(x) = \dfrac{3x}{1+x}$

4. Composite Function
   Given $f : x \rightarrow 3x - 4$ and $g : x \rightarrow 2 - 3x$, find
   (a) $fg(x)$
   (b) $gf(x)$
   (c) $f^2(3)$
   (d) $gf^{-1}(4)$

   (a) $fg(x) = f(2 - 3x) = 3(2 - 3x) - 4$
   $= 6 - 9x - 4 = 2 - 9x$
   (b) $gf(x) = g(3x - 4) = 2 - 3(3x - 4)$
   $= 2 - 9x + 12 = 14 - 9x$
   (c) $f^2(3) = ff(3) = f(9 - 4) = f(5)$
   $= 15 - 4 = 11$.
   (d) Let $y = 3x - 4$, $x = \dfrac{y+4}{3}$

   Thus $f^{-1}(4) = \dfrac{8}{3}$

   $gf^{-1}(4) = g(\dfrac{8}{3}) = 2 - 3 \times \dfrac{8}{3} = -6$

5. To find $f(x)$ or $g(x)$ given the composite function.
   Given $f(x) = 2x + 8$ and $fg(x) = 6x + 12$, find $g(x)$.
   $f(x) = 2x + 8$
   $f[g(x)] = 2g(x) + 8$
   $= 6x + 12$
   $2g(x) = 6x + 12 - 8$
   $= 6x + 4$
   $g(x) = 3x + 2$

Given $f(x) = 3x - 5$ and $gf(x) = 9x^2 - 30x + 30$, find $g(x)$

gf(x) = $9x^2 - 30x + 30$
g(3x – 5) = $9x^2 - 30x + 30$
Let y = 3x – 5, x = $\frac{y+5}{3}$

g (y) = $9\left(\frac{y+5}{3}\right)^2 - 30\left(\frac{y+5}{3}\right) + 30$
= $y^2 + 10y + 25 - 10y - 50 + 30$
= $y^2 + 5$
Thus, g(x) = $x^2 + 5$

CHAPATER 2 : QUADRATIC EQUATION

1. Find the roots of quadratic equation
   (a) Factorisation
   (b) formula x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   (a) Solve $6x^2 - 7x - 3 = 0$

$$\begin{array}{lll} 2x & -3 & = -9x \\ 3x & +1 & = +2x \\ \hline & -9x + 2x & = -7x \end{array}$$

(2x – 3)(3x + 1) = 0

2x – 3 = 0, x = $\frac{3}{2}$

3x + 1 = 0, x = $-\frac{1}{3}$

   (b) If it cannot be factorised, use the formula.
   Solve $2x^2 - 4x - 5 = 0$
   a = 2, b = –4 and c = –5

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 2 \times (-5)}}{4}$$

$$= \frac{4 \pm \sqrt{16 + 40}}{4} = \frac{4 \pm \sqrt{56}}{4}$$

$$x = \frac{4 + \sqrt{56}}{4} = 2.871$$

$$x = \frac{4 - \sqrt{56}}{4} = -0.8708$$

2. Form equation form roots.
Use the reverse of factorisaton

Find the quadratic equation with roots $\frac{1}{2}$ and 3

x = $\frac{1}{2}$,
×2, 2x = 1, (2x – 1) = 0
x = 3, (x – 3) = 0
The equation is
(2x – 1)(x – 3) = 0
$2x^2 - 7x + 3 = 0$

2. Using SOR and POR and the formula x^2 – (SOR)x + POR = 0
Cari persamaan kuadratik dengan punca $\frac{1}{2}$ dan 3

SOR = $\frac{1}{2} + 3 = \frac{7}{2}$

POR = $\frac{1}{2} \times 3 = \frac{3}{2}$

Equation is

$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

× 2, $2x^2 - 7x + 3 = 0$

3. If $ax^2 + bx + c = 0$ is the general form of the quadratic equation,

SOR = $\alpha + \beta = \frac{-b}{a}$

POR = $\alpha\beta = \frac{c}{a}$

Given that one root is twice the other root for the quadratic equation $x^2 + mx + 18 = 0$, find the postive value of m.

The roots are α and 2α

SOR = $\alpha + 2\alpha = 3\alpha = \frac{-m}{1} = -m$

POR = $\alpha \times 2\alpha = 2\alpha^2 = 18$

$\alpha^2 = 9$ $\alpha = \sqrt{9} = \pm 3$

When α = 3, 3α = 9 = –m, m = –9 (not accepted)

When α = –3, 3α = –9 = –m, thus m = 9

4. Types of roots
   (a) 2 real and distinct roots.
   $b^2 - 4ac > 0$

   (b) 2 real and equal roots
   $b^2 - 4ac = 0$

   (c) No real root
   $b^2 - 4ac < 0$

   (d) Real root (distinct or same)
   $b^2 - 4ac \geq 0$

Find the range of values of k in which the equation $2x^2 - 3x + k = 0$ has two real and distinct roots.

For two real and distinct roots
$b^2 - 4ac > 0$

$(-3)^2 - 4(2)k > 0$
$9 - 8k > 0$
$8k < 9 \qquad k < \frac{9}{8}$

CHAPTER 3: QUADRATIC FUNCTIONS

1. To find the maximum/minimum value by completing the square.

   Given $f(x) = 2x^2 - 6x + 8$, find the maximum or minimum value and state the corresponding value of x.

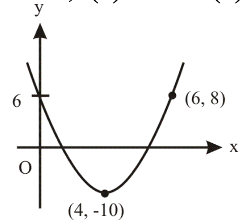
   $$f(x) = 2x^2 - 6x + 8$$
   $$= 2[x^2 - 3x] + 8$$
   $$= 2[x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2] + 8$$
   $$=2[(x - \frac{3}{2})^2 - \frac{9}{4}] + 8$$
   $$= 2 (x - \frac{3}{2})^2 - \frac{9}{2} + 8$$
   $$= 2(x - \frac{3}{2})^2 + \frac{7}{2}$$

   The minimum value (the coefficient of x^2 is positive and the graph is 'u' shaped) is $\frac{7}{2}$ when $x - \frac{3}{2} = 0$, or $x = \frac{3}{2}$.

2. To sketch quadratic function
   (a) Determine the y-intercept and the x-intercept (if available)
   (b) Determine the maximum or minimum value.
   (c) Determine the third point opposite to the y-intercept.

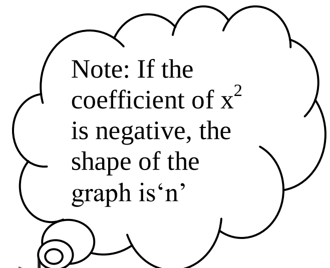
   Sketch the graph $f(x) = x^2 - 8x + 6$

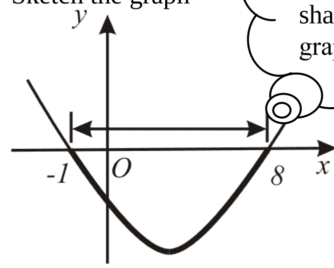
   (a) Y-intercept = 6
   (b) $f(x) = x^2 - 8x + 4^2 - 4^2 + 6$
   $= (x - 4)^2 - 16 + 6$
   $= (x - 4)^2 - 10$
   Min value = -10 when $x - 4 = 0$, $x = 4$. Min point $(4, -10)$
   (c) when $x = 8$, $f(8) = 8^2 - 8(8) + 6 = 6$



3. Quadratic Inequality
   (a) Factorise
   (b) Find the roots
   (c) Sketch the graph and determine the range of x from the graph.

   Find the range of value of x for which $x^2 - 7x - 8 < 0$
   $x^2 - 7x - 8 < 0$
   $(x - 8)(x + 1) < 0$
   $x = 8$, $x = -1$
   Sketch the graph

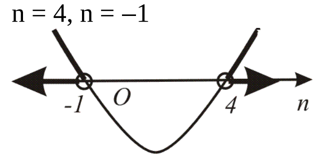




   From the sketch, $(x - 8)(x + 1) < 0$
   $-1 < x < 8$

4. Types of Roots
   (a) If the graph intersects the x-axis at two different points→ 2 real and distinct roots → $b^2 - 4ac > 0$
   (b) If the graph touches the x-axis, → 2 equal roots→ $b^2 - 4ac = 0$
   (c) If the graph does not intersect the x-axis,(or the graph is always positiv or always negative.) → no real root → $b^2 - 4ac < 0$

   The graph $y = nx^2 + 4x + n - 3$ does not intersect the x-axis for $n < a$ and $n > b$, find the value of a and b.
   $y = nx^2 + 4x + n - 3$ does not intersect the x-axis → no real root → $b^2 - 4ac < 0$
   $4^2 - 4n(n - 3) < 0$
   $16 - 4n^2 + 12n < 0$
   $0 < 4n^2 - 12n - 16$
   $\div 4$
   $n^2 - 3n - 4 > 0$
   $(n - 4)(n + 1) > 0$
   $n = 4$, $n = -1$



   From the graph, for $(n - 4)(n + 1) > 0$, $n < -1$ and $n > 4$

$\therefore$ a = –1 and b = 4

CHAPTER 4: SIMULTANEOUS EQUATIONS

To solve between one linear and one non-linear equation.
Method : Substitution

Example : Solve

$x + 2y = 4$ --------(1)

$\frac{2x}{y} + \frac{2y}{x} = 5$ -----(2)

from (2), × xy
$2x^2 + 2y^2 = 5xy$ ------------(3)
from (1), $x = 4 - 2y$
substitute in (3)
$2(4 - 2y)^2 + 2y^2 = 5(4 - 2y)y$
$2(16 - 16y + 4y^2) + 2y^2 = 20y - 10y^2$
$8y^2 + 10y^2 + 2y^2 - 32y - 20y + 32 = 0$
$20y^2 - 52y + 32 = 0$
÷ 4

$5y^2 - 13y + 8 = 0$
$(5y - 8)(y - 1) = 0$
$y = \frac{8}{5}$ or 1
$y = \frac{8}{5}$, $x = 4 - 2(\frac{8}{5}) = 4 - \frac{16}{5} = \frac{4}{5}$
$y = 1$, $x = 4 - 2 = 2$
Thus, $x = 2$, $y = 1$ and $x = \frac{4}{5}$, $y = \frac{8}{5}$.

!Note Be careful not to make the mistake
$(4 - 2y)^2 = 16 + 4y^2$ ***wrong***

If the equations are joined, you have to separate them.

Solve $x^2 + y^2 = x + 2y = 3$
$x^2 + y^2 = 3$
and $x + 2y = 3$

CHAPTER 5: INDEX AND LOGARTHM

Index form:

$b = a^x$

Logarithm form

$\log_a b = x$

Logarithm Law :

1. $\log_a x + \log_a y = \log_a xy$
2. $\log_a x - \log_a y = \log_a \frac{x}{y}$
3. $\log_a x^n = n\log_a x$
4. $\log_a b = \frac{\log_c a}{\log_c b}$
5. $\log_a a = 1$
6. $\log_a 1 = 0$

Example: Find the value of $\frac{5}{3}\log_4 8 - 2\log_4 3 + \log_4 18$

$\frac{5}{3}\log_4 8 - 2\log_4 3 + \log_4 18$

$= \log_4 \frac{8^{\frac{5}{3}} \times 18}{3^2}$

$= \log_4 \frac{32 \times 18}{9} = \log_4 64 = \log_4 4^3$

$= 3\log_4 4 = 3 \times 1 = 3$

To solve index equations, change to the same base if possible. If not possible to change to the same base take logarithm on both sides of the equation.

Example: Solve $3.27^{x-1} = 9^{3x}$

$3.27^{x-1} = 9^{3x}$
$3 \times 3^{3(x-1)} = 3^{2(3x)}$
$3^{1 + 3x - 3} = 3^{6x}$
$1 + 3x - 3 = 6x$
$-2 = 3x$
$x = -\frac{2}{3}$

Example: Solve $5^{x+3} - 7 = 0$
$5^{x+3} - 7 = 0$
$5^{x+3} = 7$
$\log 5^{x+3} = \log 7$
$(x + 3)\log 5 = \log 7$
$x + 3 = \frac{\log 7}{\log 5} = 1.209$
$x = 1.209 - 3 = -1.791$

Example: Solve

$\log_{\sqrt{a}} 384 - \log_{\sqrt{a}} 144 + \log_{\sqrt{a}} 6 = 4$

$\log_{\sqrt{a}} \frac{384 \times 6}{144} = 4$

$\log_{\sqrt{a}} 16 = 4$

$16 = \left(\sqrt{a}\right)^4 = a^2$

$a = \pm\ 4$

CHAPTER 6: COORDINATE GEOMETRY

1. Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$

   $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example: If M(2k, k) and N(2k + 1, k – 3) are two points equidistant from the origin O. Find the value of k.

MO = ON

$$\sqrt{(2k)^2 + k^2} = \sqrt{(2k+1)^2 + (k-3)^2}$$

Square,

$4k^2 + k^2 = 4k^2 + 4k + 1 + k^2 - 6k + 9$

$0 = -2k + 9$

$2k = 9 \qquad k = \frac{9}{2}$

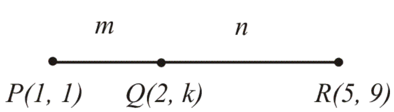
2. Point which divides a line segment in the ratio m : n

   $$\left(\frac{nx_1 + mx_2}{n+m}, \frac{ny_1 + my_2}{n+m}\right)$$

Example: Given Q(2, k) divides the line which joins P(1, 1) and R(5, 9) in the ratio m : n. Find

(a) the ratio m : n

(b) the value of k



(a) $\frac{n + 5m}{n + m} = 2$

$n + 5m = 2n + 2m$

$5m - 2m = 2n - n$

$3m = n$

$\frac{m}{n} = \frac{1}{3}$ thus, m : n = 1 : 3

(b) $\frac{3\times 1 + 1\times 9}{1+3} = k$

$\frac{12}{4} = 3 = k$

2. Equation of a straight line

   Gradient form: $y = mx + c$

   Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$

   Graident = m = $-\frac{y-\text{intercept}}{x-\text{intercept}} = -\frac{b}{a}$

   General form: $ax + by + c = 0$

   The equation of straight line given the gradient, m, and passes through the point (x_1, y_1) :

   $y - y_1 = m(x - x_1)$

   Equation of a straight line passing throug two points (x_1, y_1) and (x_2, y_2) is

   $$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Find the equatioon of the straight line

(a) with gradient 3 and passes through (1, –2)

(b) passes through (2, 5) and (4, 8)

(a) Equation of straight line

$y - (-2) = 3(x - 1)$

$y + 2 = 3x - 3$

$y = 3x - 5$

(b) Equation of straight line

$\frac{y-5}{x-2} = \frac{8-5}{4-2}$

$\frac{y-5}{x-2} = \frac{3}{2}$

$2(y - 5) = 3(x - 2)$

$2y - 10 = 3x - 6$

$2y = 3x + 4$

3. Parallel and Perpendicular Line

   Parallel lines,

   $m_1 = m_2$

   Perpendicular lines,

   $m_1 \times m_2 = -1$

Example: Find the equation of the straight line which is parallel to the line 2y = 3x – 5 and passes through (1, 4)

$2y = 3x - 5$, $y = \frac{3}{2}x - \frac{5}{2}$

$m = \frac{3}{2}$, passes through (1, 4)

Persamaan garis lurus ialah

$y - 4 = \frac{3}{2}(x - 1)$
$2y - 8 = 3x - 3$
$2y = 3x + 5$

Example: Find the equation of the straight line which is perpendicular to the line $\frac{x}{3} - \frac{y}{4} = 1$ and passes through (2, 3)

$\frac{x}{3} - \frac{y}{4} = 1$, $m_1 = \frac{-(-4)}{3} = \frac{4}{3}$

$\frac{4}{3} \times m_2 = -1$

$m_2 = -\frac{3}{4}$, passes through (2, 3)

The equation of the straight line is

$y - 3 = -\frac{3}{4}(x - 2)$
$4y - 12 = -3x + 6$
$4y + 3x = 18$

4. Equation of Locus

Example: Find the equation of the locus for P which moves such that its distance from Q(1, 2) and R(–2, 3) is in the ratio 1 : 2

Let P(x, y), Q(1, 2), R(–2, 3)
PQ : PR = 1 : 2

$\frac{PQ}{PR} = \frac{1}{2}$

PR = 2PQ

$\sqrt{(x+2)^2 + (y-3)^2} = 2\sqrt{(x-1)^2 + (y-2)^2}$

Square,
$x^2 + 4x + 4 + y^2 - 6y + 9 =$
$4(x^2 - 2x + 1 + y^2 - 4y + 4) =$
$4x^2 + 4y^2 - 8x - 16y + 20$
$0 = 4x^2 - x^2 + 4y^2 - y^2 - 12x - 10y + 7$
$3x^2 + 3y^2 - 12x - 10y + 7 = 0$

CHAPTER 7: STATISTICS

1. Ungrouped Data

Mean, $\bar{x} = \frac{\sum x}{N}$

Variance, $\sigma^2 = \frac{\sum (x - \bar{x})^2}{N}$

$= \frac{\sum x^2}{N} - (\bar{x})^2$

Standard deviation = $\sqrt{\text{variance}}$

Example: For the data3, 5, 5, 6, 7, 8 find the
(a) mean
(b) variance
(c) standard deviation

(a) $\bar{x} = \frac{\sum x}{N} = \frac{3+5+5+6+7+8}{6} =$
= 5. 667

(b) variance, $\sigma^2 =$

$\frac{9+25+25+36+49+64}{6} - \left(\frac{34}{6}\right)^2$

$= \frac{208}{6} - \left(\frac{34}{6}\right)^2 = 2.556$

(c) standard deviation = $\sigma = \sqrt{2.556} = 1.599$

2. **Grouped Data**

Mean, $\bar{x} = \frac{\sum f x_i}{\sum f}$ x_i = mid-point

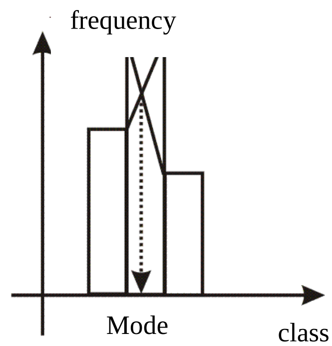
f = frequency

Median,

$M = L + \frac{\frac{1}{2}N - F_{cu}}{f_m} \times c$

L = lower boundary of the median class
N = total frequency
F_{cu} = cumulative frequency before the median class
f_m = frequency of median class
c = class interval size

Mode is obtained from a histogram



Standard deviation, $\sigma =$

$\sqrt{\frac{\sum f x_i^2}{\sum f} - (\bar{x})^2}$

Example:

The table shows the marks obtained in a test.

Marks	Frequency
10 – 14	2
15 – 19	5
20 – 24	8
25 – 29	12
30 – 34	10
35 – 39	7
40 – 44	6

Find,

(a) mean mark
(b) median
(c) mode
(d) standard devition

Mark	f	x_i	fx_i	fx_i^2	C.F.
10 – 14	2	12	24	288	2
15 – 19	5	17	85	1445	7
20 – 24	8	22	176	3872	15
25 – 29	12	27	324	8748	27
30 – 34	10	32	320	10240	37
35 – 39	7	37	259	9583	44
40 – 44	6	42	252	10584	50

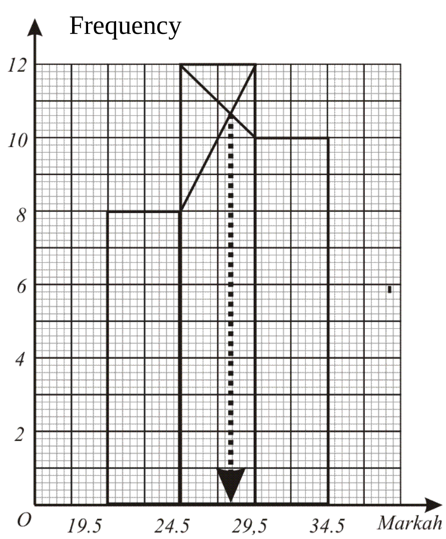
(a) Mean = $\bar{x} = \sqrt{\dfrac{\sum fx_i}{\sum f}} = \dfrac{1440}{50} = 28.8$

(b) $\dfrac{1}{2}N = \dfrac{1}{2} \times 50 = 25$

Median class = 25 – 29

M = $24.5 + \dfrac{25-15}{12} \times 5 = 28.67$

(c)



From the graph, mode = 28 mark

(d) $\sigma = \sqrt{\dfrac{\sum fx_i^2}{\sum f} - (\bar{x})^2}$

$= \sqrt{\dfrac{44760}{50} - 28.8^2} = \sqrt{65.76}$

= 8.109

CHAPTER 8: DIFFERENTIATION

$\dfrac{dy}{dx}$ represents the gradient of a curve at a point.

$\dfrac{dy}{dx} = f'(x)$ = first derivative

= gradient function.

$\dfrac{d}{dx}(ax^n) = anx^{n-1}$

Differentiation of Polynomials

1. Differentiate with respect to x:

(a) $y = 3x^4 + 2x^3 - 5x - 2$

(b) $y = \sqrt{x}$

(c) $y = \dfrac{2}{x^2}$

(a) $y = 3x^4 + 2x^3 - 5x - 2$

$\dfrac{dy}{dx} = 12x^3 + 6x^2 - 5$

(b) $y = \sqrt{x} = x^{\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{1}{2}x^{\frac{1}{2}-1} = \dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

(c) $y = \dfrac{2}{x^2} = 2x^{-2}$

$\dfrac{dy}{dx} = -4x^{-3} = \dfrac{-4}{x^3}$

Differentiation of Product

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

2. Differentiate with respect to x:
y = (3x + 2)(4 – 5x)

$\dfrac{dy}{dx} = (3x + 2) \times -5 + (4 - 5x) \times 3$

$= -15x - 10 + 12 - 15x$

$= 2 - 30x$

Differentiation of Quotient

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

3. Differentiate $\frac{3x+4}{2x-5}$ with respect to x

y = $\frac{3x+4}{2x-5}$

$$\frac{dy}{dx}=\frac{(2x-5)3-(3x+4)2}{(2x-5)^2}$$

$$= \frac{6x-15-6x-8}{(2x-5)^2} = -\frac{23}{(2x-5)^2}$$

Differentiation of Composite Function

$\frac{d}{dx}(ax+b)^n$ = $n(ax + b)^{n-1} \times a$

Note:
you must differentiate the function in the brackets.

4. Differentiate with respect to x :
   (a) $(3x + 5)^8$
   (b) $(2x – 1)^4(3x + 2)^5$

   (a) $y = (3x + 5)^8$

   $\frac{dy}{dx}$ = $8(3x + 5)^7 \times 3$
   = $24(3x + 5)^7$

   (b) $y = (2x – 1)^4(3x + 2)^5$

   $\frac{dy}{dx}$ = $(2x – 1)^4 5(3x + 2)^4 \times 3 + (3x + 2)^5 4(2x – 1)^3 \times 2$
   = $15(2x – 1)^4(3x + 2)^4 + 8(2x – 1)^3(3x + 2)^5$
   = $(2x – 1)^3(3x + 2)^4[15(2x – 1) + 8(3x + 2)]$
   = $(2x – 1)^3(3x + 2)^4[30x – 15 + 24x + 16]$
   = $(2x – 1)^3(3x + 2)^4(54x + 1)$

Equation of Tangent and Normal

Gradient of tangent = gradient of curve = $\frac{dy}{dx}$

Example: Find the equation of the tangent to the curve $y = 3x^2 – 5x + 2$ at the point x = 1.

$y = 3x^2 – 5x + 2$

$\frac{dy}{dx}$ = 6x – 5

x = 1, y = 3 – 5 + 2 = 0

$\frac{dy}{dx}$ = 6 – 5 = 1

Equation of tangent :

y – 0 = 1(x – 1)

y = x – 1.

Maximum and Minimum Value

Given $y = 2x^2 – 8x + 3$. Find the coordinates of the turning point. Hence, determine if the turning point is maximum or minimum.

$y = 2x^2 – 8x + 3$

$\frac{dy}{dx}$ = 4x – 8

For turning point $\frac{dy}{dx}$ = 0

4x – 8 = 0

x = 2

x = 2, y = 2(4) – 16 + 3 = –5

$\frac{d^2y}{dx^2}$ = 4 > 0, thus the point (2, –5) is a minimum point.

Rate of Change of Related Quantities

Example: The radius of a circle increases which a rate of 0.2 cm s^{-1}, find the rate of change of the area of the circle when the radius is 5 cm.

$A = \pi r^2$

$\frac{dA}{dr}$ = $2\pi r$

$\frac{dr}{dt}$ = 0.2 cm s^{-1}

$$\frac{dA}{dt}=\frac{dA}{dr}\times\frac{dr}{dt}$$

= $2\pi r \times 0.2$

= $0.4\pi r$

When r = 5 cm,

$\frac{dA}{dt}$ = $0.4\pi \times 5$

= 2π cm^2 s^{-1}

Small Changes and Approximation

$$\delta y=\frac{dy}{dx}\times\delta x$$

Example: Given $y = 2x^2 – 5x + 3$, find the small change in y when x increases from 2 to 2.01

$y = 2x^2 – 5x + 3$

$\frac{dy}{dx}$ = 4x – 5

δx = 2.02 – 2 = 0.01

$\delta y = \frac{dy}{dx} \times \delta x$

$= (4x – 5) \times 0.01$

Substitute the original value, x = 2,

$\delta y = (8 – 5) \times 0.01$

$= 0.03$

Thus the small increment in y is 0.03.

CHAPTER 9: INDEX NUMBER

1. Price Index, I = $\frac{p_1}{p_0} \times 100$

   p_1 = price at a certain time
   p_0 = price in the base year

2. Composite index $\bar{I} = \frac{\sum Iw}{\sum w}$

   I = price index
   w = weightage

Example:

Item	Price index	Weightage
Book	100	6
Beg	x	2
Shirt	125	y
Shoes	140	3

The table above shows the price indices and the weightage for four items in the year 2004 based in the year 2000 as base year.
If the price of a beg in the year 2000 and 2004 are RM40 and RM44 respectively. The composite index for 2004 is 116. Find
(a) the value of x
(b) the value of y
(c) the price of a shirt in 2004, if the price in 2000 was RM60.

(a) x = $\frac{44}{40} \times 100 = 110$

(b) $\frac{6\times100+2\times110+125y+3\times140}{6+2+y+3} = 116$

$\frac{600+220+125y+420}{11+y} = 116$

1240 + 125y = 116(11 + y)
1240 + 125y = 1276 + 116y
125y – 116y = 1276 – 1240
9y = 36
y = 4

(c) $\frac{p_1}{60} \times 100 = 125$

$p_1 = 125 \times \frac{60}{100}$ = RM75