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Dynamic Programming Problems
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17. Consider the following problem. The input is an undirected graph G and an integer k. The problem is to determine if G contains a clique of size k AND an independent set of size k. Recall that a clique is a collection of mutually adjacent vertices, and an independent set is a collection of mutually nonadjacent vertices. Show by reduction that if this problem has a polynomial time algorithm then the clique problem has a polynomial time algorithm.

18. Consider the following problem. The input is an undirected graph G and an integer k. The problem is to determine if G contains a clique of size k OR an independent set of size k. Show by reduction that if this problem has a polynomial time algorithm then the clique problem has a polynomial time algorithm.

22. Consider the following problem. The input is a graph $G = (V, E)$, a subset R of vertices of G, and a positive integer k. The problem is to determine if there is a subset U of V such that

1. All the vertices in R are contained in U, and
2. the number of vertices in U is at most k, and
3. for every pair of vertices x and y in R, one can walk from x to y in G only traversing vertices that are in U.

Show that this problem is NP-hard using a reduction from Vertex Cover. Recall that the input for the vertex cover problem is a graph H and an integer ℓ, and the problem is to determine whether H has a vertex cover of size ℓ or not. A vertex cover S is a collection of vertices with the property that every edge is incident on at least one vertex in S.

1. Consider the problem of computing the AND of n bits.

    1. Give an algorithm that runs in time O(log n) using n processors on an EREW PRAM. What is the endciency of this algorithm?
    2. Give an algorithm that runs in time O(log n) using n= log n processors on an EREW PRAM. What is the endciency of this algorithm?
    3. Give an algorithm that runs in time O(1) using n processors on a CRCW PRAM.