- 19. Design a parallel algorithm that finds the maximum number in a sequence  $x_1, \ldots, x_n$  of (not necessarily distinct) integers in the range 1 to n. Your algorithm should run in constant time on a CRCW Common PRAM with n processors. Note that it is important here that the  $x_i$ 's have restricted range.
- 21. Give a parallel algorithm for the following problem that runs in time  $O(\log n)$  on an EREW PRAM. The input is a binary tree with n nodes. Assume that each processor has a pointer to a unique node in the tree. The problem is to number the leafs consecutively from left to right (that is in-order). Note that this algorithm is needed for the algorithm in the notes for computing arithmetic expressions.
- 23. Design a parallel algorithm that takes a binary expression tree, where the leaves are integers, and the internal nodes are the four standard arithmetic operators addition, subtraction, multiplication, and division, and computes the value of the expression. Your algorithm should run in  $O(\log n)$  time on a CREW PRAM with n processors, where n is the number of nodes in the tree. You may assume that each processor initially has a pointer to a unique node in the tree.

HINT: Following the technique used for subtraction in the class notes you need only find a collection of functions that contain the identity function and constant functions, and is closed under addition, subtraction, multiplication, division with constants, and composition.

We initially give each edge containing leaves the function x+xi, so the constant is converted into complex form. Complex numbers are closed under addition, subtraction, multiplication, division, and composition. The rest of the algorithm follows exactly as described in the class notes.