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Dynamic Programming Problems
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5. Show that if one of the following three problems has a polynomial time algorithm then they all do.

- The Independent Set Problem: The input is a graph G. The problem is to find the largest independent set in G. In an independent set all vertices are mutually nonadjacent.
- The Clique Problem: The input is a graph G. The problem is to find the largest clique in G. In a clique all vertices are mutually adjacent.
- The Vertex Cover Problem: The input is a graph G. The problem is to find the smallest vertex cover in G. A set S is a vertex cover if each edge in G is incident to a vertex in S.

Independent Set $\leq$ Clique

Program Independent Set:
    read G
    Construct G' with the same vertices
    For each vertex pair: $(v_1, v_2) \ni v_1 \neq v_2$
        If there is no edge from v_1 to v_2 in G:
            Create edge from v_1 to v_2 in G'
    output Clique(G')

8. Show that the subset sum problem is self-reducible. The decision problem is to take a collection of positive integers $x_1, \ldots, x_n$ and an integer L and decide if there is a subset of the x_is that sum to L. The optimization problem asks you to return the actual subset if it exits. So you must show that if the decision problem has a polynomial time algorithm then the optimization problem also has a polynomial time algorithm.

subset sum optimization $\leq$ subset sum decision.

program subset sub optimization:
    read $x_1, \ldots, x_n$, L
    If $x_1, \ldots, x_n$, L decision is 1 then:
        S = $[x_1, \ldots, x_n]$
        foreach x in S:
            if $S \backslash x$, L decision is 1 then: *if the subset sum is still possible without x remove it*
                remove x from S
    output S

12. Consider the following 2Clique problem:
INPUT: A undirected graph G and an integer k.
OUTPUT: 1 if G has two vertex disjoint cliques of size k, and 0 otherwise.
Show that this problem is NP-hard. Use the fact that the clique problem in NP-complete. The input to the clique problem is an undirected graph H and an integer j. The output should be 1 if H contains a clique of size j and 0 otherwise. Note that a clique is a mutually adjacent collection of vertices. Two cliques are disjoint if they do not share any vertices in common.

Clique $\leq$ 2-Clique

Program Clique:
    read H, j
    G = H + H, where both copies of H are disjoint
    output 2-Clique(G,j)

Therefore, 2Clique is NPH.