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Dynamic Programming Problems
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11. Show that the Vertex Cover Problem is self-reducible. The decision problem is to take a graph G and an integer k and decide if G has a vertex cover of size k or not. The optimization problem takes a graph G, and returns a smallest vertex cover in G. So you must show that if the decision problem has a polynomial time algorithm then the optimization problem also has a polynomial time algorithm. Recall that a vertex cover is a collection S of vertices with the property that every edge is incident to a vertex in S.

14. Consider the problem where the input is a collection of linear inequalities. For example, the input might look like $3x - 2y = 3$ and $2x - 3x = 9$. The problem is to determine if there is an integer solution that simultaneously satisfies all the inequalities. Show that this problem is NP-hard using the fact that it is NP-hard to determine if a Boolean formula in conjunctive normal form is satisfiable.

16. The input to the three coloring problem is a graph G, and the problem is to decide whether the vertices of G can be colored with three colors such that no pair of adjacent vertices are colored the same color. The input to the four coloring problem is a graph G, and the problem is to decide whether the vertices of G can be colored with four colors such that no pair of adjacent vertices are colored the same color. Show by reduction that if the four coloring problem has a polynomial time algorithm then so does the three coloring problem.