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Reduction and Parallel Problems
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17. Consider the following problem. The input is an undirected graph $G$ and an integer $k$. The problem is to determine if $G$ contains a clique of size $k$ **AND** an independent set of size $k$. Recall that a clique is a collection of mutually adjacent vertices, and an independent set is a collection of mutually nonadjacent vertices. Show by reduction that if this problem has a polynomial time algorithm then the clique problem has a polynomial time algorithm.

18. Consider the following problem. The input is an undirected graph $G$ and an integer $k$. The problem is to determine if $G$ contains a clique of size $k$ **OR** an independent set of size $k$. Show by reduction that if this problem has a polynomial time algorithm then the clique problem has a polynomial time algorithm.

22. Consider the following problem. The input is a graph $G = (V, E)$, a subset $R$ of vertices of $G$, and a positive integer $k$. The problem is to determine if there is a subset $U$ of $V$ such that

1. All the vertices in $R$ are contained in $U$, and
2. the number of vertices in $U$ is at most $k$, and
3. for every pair of vertices $x$ and $y$ in $R$, one can walk from $x$ to $y$ in $G$ only traversing vertices that are in $U$.

Show that this problem is NP-hard using a reduction from Vertex Cover. Recall that the input for the vertex cover problem is a graph $H$ and an integer $\ell$, and the problem is to determine whether $H$ has a vertex cover of size $\ell$ or not. A vertex cover $S$ is a collection of vertices with the property that every edge is incident on at least one vertex in $S$.

1. Consider the problem of computing the AND of $n$ bits.

   1. Give an algorithm that runs in time O(log $n$) using n processors on an EREW PRAM. What is the endciency of this algorithm?
   2. Give an algorithm that runs in time O(log $n$) using n= log n processors on an EREW PRAM. What is the endciency of this algorithm?
   3. Give an algorithm that runs in time O(1) using n processors on a CRCW PRAM.