50.012 Networks (2023 Term 6)

Homework 2

Hand-out: 16 Feb

Due: 7 Mar 23:59

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1. (2019 midterm exam question) Consider data communication over a link of RTT 100ms and transmission bandwidth 1Gbit/s. Assume 1G=10^9. Consider a pipelined transport protocol that uses ACKs to decide if packets were received successfully. Answer the following three questions:

1.1 After the protocol has sent a packet, what is the minimum amount of time needed for the protocol to infer that the packet was lost?

At least 1 RTT as it requires the packet to be sent, and a reply from the receiver. Distance is twice of 1 link, and hence total time is twice of 1 trip. Which is 1 RTT = 100ms.

1.2 If the protocol uses a window size of 6 packets (each of size 1000 bytes), what is the maximum achievable data throughput?

$$\text{Throughput} = \frac{MSS}{RTT}$$

$$= \frac{6 \times 1000 \text{ bytes}}{100 \times 10^{-3} \text{ s}}$$

$$= 60\,000 \text{ bytes} / \text{s}$$

$$= 480\,000 \text{ bps}$$

$$= 480 \text{ kbps}$$

1.3 To fully use the transmission bandwidth, estimate the minimum window size (in bytes) needed.

$$\text{Throughput} = \frac{MSS}{RTT}$$

$$MSS = RTT\,(\text{Throughput})$$

$$= (100 \times 10^{-3}\, s)\,(1 \times 10^{9}\ bps)$$

$$= 100 \times 10^{6}\ \text{bits}$$

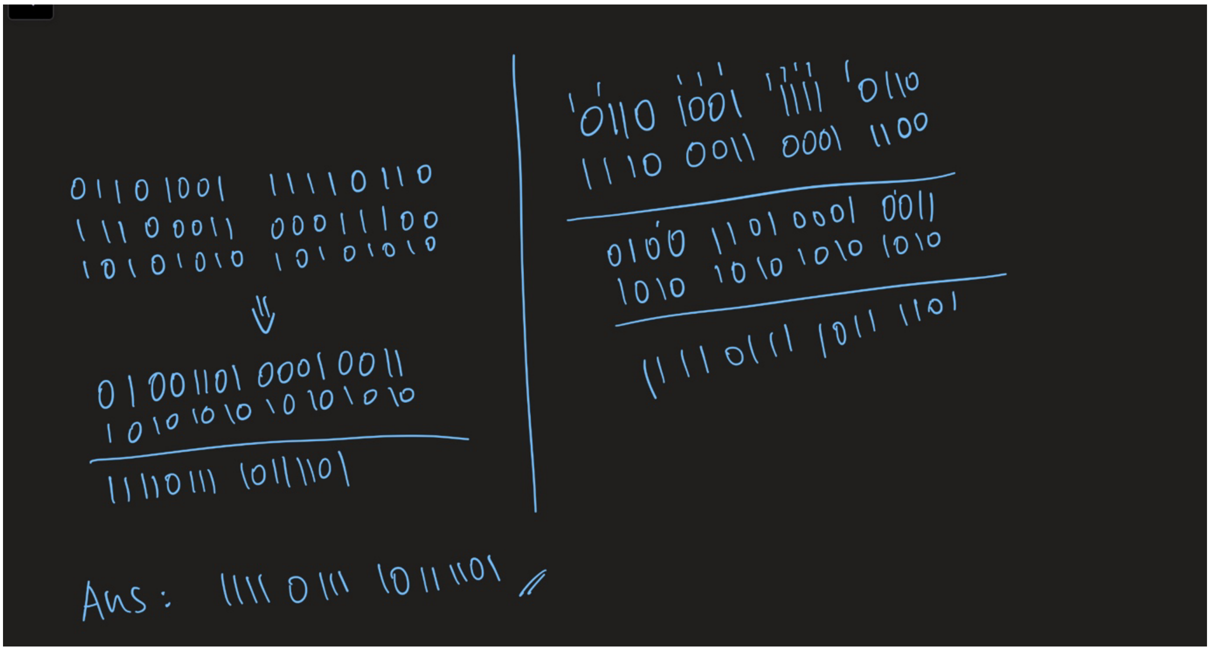
$$= 12.5 \times 10^{6}\ \text{bytes}$$

$$= 12.5\ Mb$$

2. Consider the three 16-bit words (shown in binary) below.

01101001 11110110
11100011 00011100
10101010 10101010

What is the Internet checksum value for these three16-bit words?



3. (textbook chapter 3, problem P44): Consider sending a large file from a host to another over a TCP connection that has no loss.

3.1 Suppose TCP uses AIMD for its congestion control without slow start. Assuming cwnd increases by 1 MSS every time a batch of ACKs is received and assuming approximately constant round-trip times, how long does it take for cwnd increase from 6 MSS to 12 MSS (assuming no loss events)?

AIMD increases cwnd by 1 MSS every RTT until loss is detected. If there are no loss events, it will take 6 RTTs for cwnd to increase from 6 MSS to 12 MSS.

3.2 Again, assume in the first RTT 6 MSS was sent, what is the average throughput (in terms of MSS and RTT) for this connection up through time = 6 RTT?

RTT	MSS
1	6
2	7
3	8
4	9
5	10
6	11

$$\text{Throughput} = \frac{\Sigma \text{MSS}}{\text{RTT}} = \frac{6+7+8+9+10+11}{6} = \frac{51}{6} = 8.5 \text{ MSS/RTT}$$

4. (textbook Chapter 3, problem 45 and 53) Recall the macroscopic description of TCP throughput. In the period of time from when the connection's rate varies from W/(2 · RTT) to W/RTT, only one packet is lost (at the very end of the period). W is measured in terms of the number of segments.

4.1 Show that the loss rate (fraction of packets lost) is equal to

$$L = \text{loss rate} = \frac{1}{\frac{3}{8}W^2 + \frac{3}{4}W}$$

$$\text{total pkts} = \frac{W}{2} + \frac{W}{2}+1 + \frac{W}{2}+2 + \ldots + W$$

$$= \sum_{n=0}^{W/2}\left(\frac{W}{2}+n\right)$$

$$= \frac{W}{2}\left(\frac{W}{2}+1\right) + \sum_{n=0}^{W/2}(n)$$

$$= \frac{W^2}{4} + \frac{W}{2} + \frac{\frac{W}{2}\left(\frac{W}{2}+1\right)}{2}$$

$$= \frac{W^2}{4} + \frac{W}{2} + \frac{\frac{W^2}{4} + \frac{W}{2}}{2}$$

$$= \frac{W^2}{4} + \frac{W}{2} + \frac{W^2}{8} + \frac{W}{4}$$

$$= \frac{3W^2}{8} + \frac{3W}{4},$$

$$\text{Loss rate} = \frac{\text{loss}}{\text{total pkts}}$$

$$= \frac{1}{\frac{3}{8}W^2 + \frac{3}{4}W} \quad \text{(shown)}$$

4.2 Use the result above to show that if a connection has loss rate L, then its average rate is approximately given by

$$\approx \frac{1.22 \cdot MSS}{RTT\sqrt{L}}$$

Total packets $= \frac{3}{8}W^2 + \frac{3}{4}W$,

$= \frac{1}{L}$

$\frac{1}{L} = \frac{3W^2+6W}{8}$

$\frac{1}{L} \approx \frac{3W^2}{8}$ for large W,

$3W^2 = \frac{8}{L}$

$W = \sqrt{\frac{8}{3L}}$ bytes $= \sqrt{\frac{8}{3L}}(MSS)$

Avg Throughput $= \frac{3}{4}\left(\frac{W}{RTT}\right)$

$= \frac{3}{4}\left(\sqrt{\frac{8}{3L}}\right)\left(\frac{MSS}{RTT}\right)$

$= \frac{3}{4}\sqrt{\frac{8}{3}}\left(\frac{MSS}{RTT\sqrt{L}}\right)$

$= \frac{1.22\ MSS}{RTT\sqrt{L}}$ (shown)

4.3 Let's assume 1500-byte packets and a 100 ms round-trip time. If TCP needed to support a 1Gbps connection, what would the tolerable loss rate be? How about 100Gbps?

$$\text{Avg Throughput} = \frac{1.22\ MSS}{RTT\sqrt{L}},$$

$$1\ Gbps = \frac{1.22\ (1500\ \text{bytes})}{100\times10^{-3}\,s\ (\sqrt{L})}$$

$$1\times10^{9} = \frac{1.22\ (1500\times8)}{100\times10^{-3}\ (\sqrt{L})}$$

$$\sqrt{L} = \frac{14640}{100\times10^{6}}$$

$$= 1.464\times10^{-4},$$

$$L = (1.464\times10^{-4})^{2}$$

$$= 2.143\times10^{-8},$$

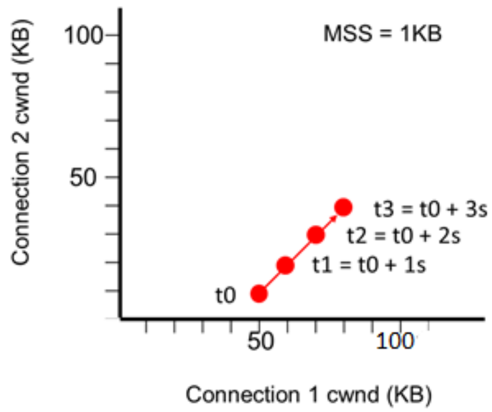
$$100\ Gbps = 1\ Gbps\times10^{2},$$

$$\sqrt{L} = \frac{14640}{100\times10^{8}}$$

$$\sqrt{L} = 1.464\times10^{-6}$$

$$L = 2.143\times10^{-12}\ //$$

5. (2020 midterm exam question) Consider two TCP Reno connections that share one link. The figure bellow shows the evolution of the size of their respective congestion window (cwnd) over time. As shown, at time t0, connection 1's cwnd = 50KB and connection 2's cwnd=10KB. At time t1=t0+1s, connection 1's cwnd = 60KB and connection 2's cwnd=20KB. At time t2=t0+2s, connection 1's cwnd = 70KB and connection 2's cwnd=30KB. At time t3=t0+3s, connection 1's cwnd = 80KB and connection 2's cwnd=40KB. Assume the maximum segment size (MSS) for both connections is 1KB and both connections have constant round-trip time (RTT). We further assume that when the sum of the cwnd of the two connections reaches 120KB, both connections experience a packet loss event as indicated by triple duplicate ACKs. We also assume these are the only moments that the two connections experience packet losses.



5.1 From time t0 to t3, the two connections are in which state of the TCP congestion control? After the packet loss event at t3, what will be the cwnd size of connection 1 and connection 2 respectively?

The two connections are in the fast recovery state as slow start state will multiply cwnd by 2, which is not the case in this figure. After the packet loss, both cwnd will be halved. Hence, cwnd size of connection 1 is 80/2 = 40 KB and that of connection 2 is 40/2 = 20 KB.

5.2 What is the RTT for the two connections respectively? What is the respective average throughput of these two connections from t0 to t3?

Both cwnd will increment by 1 MSS of 1KB for 1 RTT, both connections increased by 10 KB in 1 second, which is 10 MSS, implying 10 RTTs.

$$10 \text{ RTT} = 1\text{s}$$

$$1 \text{ RTT} = \frac{1}{10} = 0.1\text{s}$$

$$\text{Avg}_1 = \frac{3}{4}\left(\sum_{i=0}^{3} \frac{W_i}{\text{RTT}}\right)/4$$

$$= \frac{3}{4}\left(\frac{50}{0.1} + \frac{60}{0.1} + \frac{70}{0.1} + \frac{80}{0.1}\right)/4$$

$$= 487.5 \text{ KB/s}$$

$$= 3900 \text{ Kbps}$$

$$= 3.9 \text{ Mbps}$$

$$\text{Avg}_2 = \frac{3}{4}\left(\sum_{i=0}^{3} \frac{W_i}{\text{RTT}}\right)/4$$

$$= \frac{3}{4}\left(\frac{10}{0.1} + \frac{20}{0.1} + \frac{30}{0.1} + \frac{40}{0.1}\right)/4$$

$$= 187.5 \text{ KB/s}$$

$$= 1500 \text{ Kbps}$$

$$= 1.5 \text{ Mbps}$$

5.3 Assume the two connections run for a long time. What will these two connections' respective average throughput converge to?

Assuming the cycle continues, both cwnd will follow the cycle of packet loss and reducing by half. Since both sizes will increment at the same rate of 1KB/RTT, the gap will get smaller and converge at 60KB after n cycles where n is large.

Cycles	$cwnd_1$	$cwnd_2$
0	80	40
1	70	50
2	65	55
3	64.5	55.5
4	62.25	57.75
5	61.125	58.875
⋮	⋮	
n	60	60

$$\text{Avg Throughput} = \frac{3}{4}\frac{W}{RTT} = \frac{3}{4}\left(\frac{60kB}{0.1s}\right) = 450\,kB/s = 3600\,kbps = 3.6\,Mbps$$

Both connections will reach a cwnd of 60KB, hence

average throughput = ¾ (cwnd/RTT) = 3.6Mbps.

5.4 Assume now connection 1's RTT reduces by 50% and connection 2's RTT remains unchanged. After a long time, what will these two connections' respective average throughput converge to?

The convergence will still happen at each connection taking up 60KB cwnd, and average throughput = ¾ (cwnd/RTT), RTT reduced by 50% implies an increase by 50%.

Average Throughput of connection 1 = 7.2Mbps

Average Throughput of connection 2 = 3.6Mbps