Homework 2 Suggested Solutions

Question 1

Part 1

1 RTT

Part 2

With a window size of 6 packets, that means there can be a maximum of 6 packets in flight. The maximum throughput is thus

$$R = \frac{W}{RTT} = \frac{6 * 1000 * 8}{100e^{-3}} = 480e^{3} = 480\text{Kbps}$$

Part 3

Rearranging the equation in part 2,

$$W = R * RTT = 1e^{9} * 100e^{-3} = 100e^{6} = 100\text{Mbits} = 12.5\text{MB}$$

Question 2

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 01101001 11110110
+11100011 00011100
=01001101 00010010
+00000000 00000001 (wrap around bit)
=01001101 00010011

 01001101 00010011
+10101010 10101010
=11110111 10111101

One’s complement: 00001000 01000010
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Question 3

Part 1

Since this is arithmetic increase, MSS increases at a constant rate. It will take 6 batches of ACKs, i.e. 6 RTTs, to go from 6 MSS to 12 MSS.

Part 2

$$\text{throughput} = \frac{\text{data sent}}{\text{duration}} = \frac{\sum_{MSS=6}^{12} MSS}{6 * RTT} = \frac{63}{6}\frac{MSS}{RTT} = 10.5\frac{MSS}{RTT}$$

Question 4

The loss rate, L, is the ratio of the number of packets lost over the number of packets sent. In a cycle, 1 packet is lost. The number of packets sent in a cycle is

$$\frac{W}{2}+\left(\frac{W}{2}+1\right)+\cdots+W=\sum_{n=0}^{W/2}\left(\frac{W}{2}+n\right)$$

$$=\left(\frac{W}{2}+1\right)\frac{W}{2}+\sum_{n=0}^{W/2}n$$

$$=\left(\frac{W}{2}+1\right)\frac{W}{2}+\frac{W/2(W/2+1)}{2}$$

$$=\frac{W^2}{4}+\frac{W}{2}+\frac{W^2}{8}+\frac{W}{4}$$

$$=\frac{3}{8}W^2+\frac{3}{4}W$$

Thus, the loss rate is

$$L=\frac{1}{\frac{3}{8}W^2+\frac{3}{4}W}$$

b) For W large, $\frac{3}{8}W^2 >> \frac{3}{4}W$. Thus $L \approx 8/3W^2$ or $W \approx \sqrt{\frac{8}{3L}}$. From the text, we therefore have

$$\text{average throughput } = \frac{3}{4}\sqrt{\frac{8}{3L}}\cdot\frac{MSS}{RTT}$$

$$=\frac{1.22\cdot MSS}{RTT\cdot\sqrt{L}}$$

The tolerable loss rates are 2.14e -8 and 2.14e-12 respectively. (substitute 1Gbps for the average throughput formula above with MSS = packet size and RTT = 100ms)

Question 5

Part 1

It is in the congestion avoidance state because of the linear increase.

Connection 1 CWND = 43KB (half + 3 MSS)
Connection 2 CWND = 23KB (half + 3 MSS)

The slow start (+3) is optional

Part 2

Both RTT is 100ms. Find the average window size divided by *one* RTT.

$$\bar{w}_1 = \frac{50+80}{2}\text{KB} = 65\text{KB}$$
$$\bar{R}_1 = \frac{\bar{w}_1}{RTT} = \frac{65\text{KB}}{100e^{-3}} = 650\text{KBps} = 5200\text{Kbps}$$
$$\bar{w}_2 = \frac{10+40}{2}\text{KB} = 25\text{KB}$$
$$\bar{R}_2 = \frac{\bar{w}_2}{RTT} = \frac{25\text{KB}}{100e^{-3}} = 250\text{KBps} = 2000\text{Kbps}$$

Part 3

In this problem, because of the assumption that loss events only occur when the sum of congestion windows exceeds a certain threshold, equal TCP connections (with the same RTT) will eventually converge to a state where the available resources (congestion window) will be split equally between the connections. Use the steady-state TCP throughput formula:

$$W_{max} = W_{total} \div N = 120\text{KB} \div 2 = 60\text{KB}$$
$$R_{stable} = \frac{0.75 * W_{max}}{RTT} = \frac{0.75 * 60\text{KB}}{100e^{-3}} = 450\text{KBps} = 3600\text{Kbps}$$

Part 4

As a more general case of part 3, long-running connections will eventually converge to a state where the max congestion window is split according to the *geometric* ratio of the RTTs (i.e. x2 smaller RTT = x2 congestion window size), *regardless of starting congestion window size*. This can be proven graphically by plotting points on the graph or algebraically (see addendum).

$$W_{max_i} = W_{total} \times \frac{\left(\frac{RTT_i}{\max(\{RTT\})}\right)^{-1}}{\sum_{k=1}^{N}\left(\frac{RTT_k}{\max(\{RTT\})}\right)^{-1}}$$

$$W_{max_1} = 120\text{KB} \times \frac{\left(\frac{0.05}{\max(\{0.05, 0.1\})}\right)^{-1}}{\sum_{k=1}^{N}\left(\frac{RTT_k}{\max(\{0.05, 0.1\})}\right)^{-1}} = 120\text{KB} \times \frac{\frac{0.1}{0.05}}{\frac{0.1}{0.05}+\frac{0.1}{0.1}} = 120\text{KB} \times \frac{2}{3} = 80\text{KB}$$

$$W_{max_2} = 120\text{KB} \times \frac{\left(\frac{0.1}{\max(\{0.05, 0.1\})}\right)^{-1}}{\sum_{k=1}^{N}\left(\frac{RTT_k}{\max(\{0.05, 0.1\})}\right)^{-1}} = 120\text{KB} \times \frac{\frac{0.1}{0.1}}{\frac{0.1}{0.05}+\frac{0.1}{0.1}} = 120\text{KB} \times \frac{1}{3} = 40\text{KB}$$

$$R_{stable_1} = \frac{0.75 * W_{max_1}}{RTT} = \frac{0.75 * 80\text{KB}}{50e^{-3}} = 1200\text{KBps} = 9600\text{Kbps}$$
$$R_{stable_2} = \frac{0.75 * W_{max_1}}{RTT} = \frac{0.75 * 40\text{KB}}{100e^{-3}} = 300\text{KBps} = 2400\text{Kbps}$$