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1) What is Random experiment and sample space?

In probability theory, an experiment a trial is any procedure that can be infinitely repeated and has a well-defined set of positive outcomes, known as the sample space. An experiment is said to be random if it has more than one possible outcome, and deterministic if it has only one. A random experiment that has exactly two [mutually exclusive] possible outcomes is known as Bernoulli Trial.

In probability theory, the sample space also called sample description space or possibility space of an experiment or random trial is the set of all possible outcomes or results of that experiments. A sample space is usually denoted using set notation.

2) A coin is tossed twice at random. What is the probability of getting
   i) At least one head.
   ii) the same face.

Ans] Sample space $= S = \{HH, HT, TH, TT\} = 4$

Getting at least one head $= \{HH, HT, TH\} = 3$

Probability of getting atleast one head $= 3/4$

Probability of getting same face $= \{HH, TT\}$

$= 2/4$

3) Three fair coins are tossed simultaneously. What is the probability of getting at least 2 tails?

Ans] $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

Total outcomes = 8

Getting at least 2 tails = {TTH, THT, HTT, TTT}

Probability of getting atleast 2 tails = $4/8 = 1/2$ //

4) A Bag contains balls of numbered from 1 to 50. A ball is drawn randomly. Find the probability that it is the multiple of 6.

Ans] Possible outcome is 1 to 50.

Sample Space, $S = \{1, 2, 3 \cdots\cdots 50\}$

Multiple of 6 from 1 to 50 = {6, 12, 18, 24, 30, 36, 42, 48}

Total no. of outcomes = 50.

Probability that it is a multiple of 6 = $8/50$

$= 4/25$ //

5) If rolling a single die. Determine the probability of rolling an even number or a number greater than ~~0~~ two.

Ans] $S = \{1, 2, 3, 4, 5, 6\}$ // Sample Space

$P(A) = P(\text{getting an even number}) = \{2, 4, 6\} = 3/6$

P(B) = P(getting number greater than two)

= {3,4,5,6}

= 4/6

P(A∩B) = {4,6} = 2/6

P(A∪B) = P(A)+P(B) - P(A∩B)

= 3/6 + 4/6 - 2/6

= 7/6 - 2/6

= 5/6 = ~~0.83~~ 0.83

6) A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared atleast once.

Ans) S = {(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

= 36 Possibilities

A : Number 4 has appeared atleast once

B : Sum of numbers appearing to be 6.

$$P(A) = \{(1,4)(2,4)(3,4)(4,4)(5,4)(6,4)(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)\} = 11/36$$

$$P(B) = \{(1,5)(2,4)(3,3)(4,2)(5,1)\} = 5/36.$$

$$P(A \cap B) = \{(2,4)(4,2)\} = 2/36$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{5/36} = 2/5 //$$

$$\therefore P(A|B) = 2/5 //$$

7) If I have 3 bags that each contain 100 marbles. Bag 1 has 75 red and 25 blue. Bag 2 has 20 red and 80 blue. Bag 3 has 45 red and 55 blue. I choose one of the bag at random then pick a marble from the chosen bag; Also at random. What is the Probability that the chosen marble is red?

Ans] Let B_1, B_2 and B_3 be bag 1, bag 2 and bag 3.

Probability of getting $B_1 = P(B_1) = 1/3$

Probability of getting $B_2 = P(B_2) = 1/3$

Probability of getting $B_3 = P(B_3) = 1/3$.

A be the event getting red marbles

Probability of getting red marbles from $B_1 = P(A|B_1)$
$= 75/100$

Probability of getting red marbles from $B_2 = P(A|B_2)$
$= 20/100$

Probability of getting red marbles from $B_3 = P(A|B_3)$
$= 45/100$

$$P(A) = P(A|B_1) \times P(B_1) + P(A|B_2)\,P(B_2) + P(A|B_3)\,P(B_3)$$

$$= \frac{75}{100} \times \frac{1}{3} + \frac{20}{100} \times \frac{1}{3} + \frac{45}{100} \times \frac{1}{3}$$

$$= \frac{1}{3}\left[\frac{75+20+45}{100}\right]$$

$$= 140/100 \times \frac{1}{3} = 140/300$$

$$= \underline{\underline{0.466}}$$

8) As you know, Covid-19 tests are common nowadays but some results of tests are not true. Let's assume; a diagnostic test has 99% accuracy and 60% of all people have Covid-19. If a patient tests positive, what is the probability that they actually have the disease?

Ans) B : People who have Covid $= \frac{60}{100} = 0.6$

B' : People not have Covid $= 1 - 0.6$
$= \underline{\underline{0.4}}$

A be the event that they actually have the disease

$$P(A|B) = \frac{99}{100}$$
$$= 0.99$$

$$P(A|B') = \frac{1}{100}$$
$$= 0.01$$

$$P(A) = P(B) \times P(A|B) + P(B') \times P(A|B')$$
$$= 0.6 \times 0.99 + 0.01 \times 0.4$$
$$= 0.598 //$$

$$P(B/A) = \frac{P(A|B) \times P(B)}{P(A|B) \times P(B) + P(A|B') \times P(B')}$$

$$= \frac{0.99 \times 0.6}{0.598}$$

$$= \underline{\underline{0.993}}$$

9) It is estimated that 50% of e-mails are spam emails. Some software has been applied to filter these spam emails before they reached your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is determined as spam, then what is the probability that it is in fact a non-spam email?

Ans] A : Event that an email is detected as spam.

B : Event that an email is spam.

$\bar{B}$: Event that an email is not spam.

$P(B) = P(\bar{B}) = \frac{1}{2} = 0.5$

$P(A|B) = 99/100 = 0.99$

$P(A|B') = 5/100 = 0.05$

$$P(B'/A) = \frac{P(A/B') \times P(B')}{P(A|B) \times P(B) + P(A|B') \times P(B)}$$

$$= \frac{0.05 \times 0.5}{0.99 \times 0.5 + 0.05 \times 0.5}$$

$$= \frac{0.025}{0.495 + 0.025}$$

$$= \frac{0.025}{0.52} = 0.0480$$

10) A bag contains 22 yellow, 33 green and 22 blue balls. Two balls are drawn at random. What is the probability that none of the ball drawn is blue?

Ans] Total number of balls = 77 balls

Let s be the sample space Then,

n(s) = Number of ways of drawing 2 balls out of 77.

$n(S) = {}^{77}C_2$

$$n(S) = \frac{n!}{2!(n-2!)} = \frac{77!}{2!\,(77-2)!}$$

$$= \underline{\underline{2926}}$$

Let E = Event of 2 colour balls, none of which is blue.

$\therefore$ n(E) = No. of ways of drawing 2 balls out of (22+33) 55 balls.

$n(E) = {}^{55}C_2$

$$= \frac{55!}{2!\,(53)!} = 1485$$

$n(E) = 1485$

$n(S) = 2926$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1485}{2926}$$

$$= \underline{\underline{0.5075}}$$

11) You are given 3 coins one has heads on both faces. The second has tail on both faces and the third has head on one face and tail on other face. You choose a coin at random and toss it and it come up heads. The probability that the other face of it is tail.

Ans] E_1 = Event that we got head on both side [1st coin]

E_2 = Event that getting head on both side [IInd coin]

$S = \{(1,H)(1,H)(2,T)(2,T)(3,H)(3,T)\}$

E_3 = Event that getting 3rd coin head & tail.

A = Event of Head comes up.

Now the probability of choosing E_3 when A has happened.

$$P(E_3/A) = \frac{P(A|E_3)\,P(E_3)}{P(E_1)\times P(A|E_1)+P(E_2)\times P(A|E_2)P(E_3)\times P(A|E_3)}$$

$P(E_1) = P(E_2) = P(E_3) = 1/3.$

$P(A|E_1) = 1$

$P(A|E_2) = 0$

$P(A|E_3) = 1/2.$

$$P(E_3/A) = \frac{1/3 \times \frac{1}{2}}{1/3\times 1 + 0\times 1/3 + 1/3\times 1/2} = \frac{1/6}{1/3 + 0 + 1/6}$$

$$= \frac{1/6}{9/18}$$

$$= 1/6 \times 18/9 = 18/34 = 1/3$$