Probability, Statistics and Computational Mathematics

Notes MCA 101

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Syllabus

1. Probability Theory: Sample Space, Events, Different Approaches to probability, Addition and Multiplication theorems on probability, Independent Events, conditional probability, Bayes Theorem
2. Random Variables and Distribution: Random Variables, probability density Functions and distribution functions, Marginal density Functions, Joint density Functions, mathematical expectations, moments and moment generating functions. Discrete probability distributions - Binomial, poisson distribution, continuous probability distributions - uniform distribution and normal distribution.
3. Basic Statistics: Measures of central tendency :- mean, median, mode; Measures of dispersion: Range, Mean deviation, Quartile deviation and standard deviation; Moments, skewness and kurtosis, Linear correlation, karl pearson's coeffcient of correlation, Rank correlation and linear regression.
4. Mathematical Logic: propositional and predicate Logic, propositional Equivalences, Normal forms predicates and Quantifiers, Nested Quantifiers, Rule of Interface.
5. Counting, Mathematical Induction: Basics of counting pigeonhole principle, permutations and combinations, Inclusion - Exclusion principle, Mathematical Induction.

Example for Random Experiment: Tossing of a coin, Throwing a die, Drawing a card from a pack of playing card

probability

probability Is a mathematical concept which to calculate degree of certainity & uncertainity of an event to occur. probability Is the measure of the likelihood that an event will Occur In a Random Experiment. probability Is quantified as a number between 0 and 1, where, loosely Speaking 0 Indicates Impossibility and 1 Indicates certainty. The higher the probability of an Event, the more likely It Is that the event will occur.

Eg: Tossing of a fair coin. Since the coin Is fair, the two outcomes ("heads" and "tails") are both equally probable; the probability of "heads" Equals the probability of "tails"; and since no other Outcomes are possible, the probability of either "heads" or "tails" is $1/2$.

Random Experiment

Experiments of any type where the outcome cannot be predicted are called random Experiments.

Sample Space

A set of all possible Outcomes from an experiment Is called a Samplespace. It Is denoted by S or U

Eg: Consider a random experiment, If a coin tossed the $S = \{H, T\}$

*) Each outcome Is called a sample point.

*) If the number of samplepoint of S Is finite It Is known as finite Sample space.

*) If the number of sample point of S Is Infinite It Is known as Infinite Sample space.

Event : It is a collection or set of one or more simple event in a sample space.
An individual outcome in sample space called simple event.

Event
The result of an experiment are known as Event
eg :- (i) If a coin is tossed head (H) and tail (T) are two different Events
(ii) 1,2,3,4,5,6 are different Events when a dice is thrown

Exhaustive Events
If all possible outcomes of an experiment are considered, the outcomes are said to be exhaustive. The exhaustive events are nothing but all the sample points in the sample points. In throwing a die 1,2,3,4,5,6 are exhaustive event.

Mutually exclusive event
Events are said to be mutually exclusive if they can not occur together. That is the occurence of any one of them prevents the occurence of the remaining. If A and B are two mutually exclusive events then $A \cap B = \phi$
Head and tail are mutually exclusive events when a coin is tossed

Equally likely Events
Events are said to be equally likely if we have no reason to believe that one event is preferable to the other. Head and tail are equally likely Events in tossing a coin

Favourable cases
The number of sample points favourable to the happening of an event A are known as

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favoureable cases of A. for example In drawing a card from a pack of cards, the favoureable cases for getting a spade are 13.

Independent Events

Events are said to be Independent If the happening of one event does not depend upon the happening or non happening of the other event.

eg

when a coin is tossed two times, the event of getting head in the first throw and that of getting head in the second throw are Independent Events.

Definition

The probability of happening of an event A is defined as the ratio of m to n.

The probability of an event A is denoted by p(A)

$p(A) = $ favoureable cases / total cases

$= m/n$

The probability of an event is always between 0 and 1.

When $p(A)=0$, the event is Impossible

When $p(A)=1$, the event is certain to happen

The total probability of happening an event and not happening an event is 1.

ie, $p(A) + p(A') = 1$.

Addition rule for mutually exclusive Events

If A and B are 2 mutually exclusive Events then, the probability for A or B to happen is

$$\boxed{p(A \cup B) = p(A) + p(B)}$$

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Addition rule for not mutually exclusive Event

If A and B are any two Events then,

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Conditional probability

The probability of occurence of an event B when it is known that A has occured is known as conditional probability of B under the condition that A has occured and it is denotes

$$\boxed{P(B/A) = P(A \cap B) / P(A)}$$

or

$$\boxed{P(A|B) = P(A \cap B) / P(B)}$$

proof: $P(A) = \dfrac{n(A)}{n(S)}$ $\quad P(B) = \dfrac{n(B)}{n(S)}$

$$P(A \cap B) = n(A \cap B) / n(S)$$

Thus, under the condition that the event B has occured, the outcome in sample space will be reduced to $n(B)$ only and outcomes in A will be reduced to $n(A \cap B)$.

So that $P(A|B) = n(A \cap B) / n(B)$

$$P(A|B) = \frac{n(A \cap B) / n(S)}{n(B) / n(S)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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Multiplication Rule of probability

If A and B are any two Events then the probability for both A and B to take place together $p(A \cap B)$, from the equation of conditional probability,

$$\boxed{p(A \cap B) = p(A|B) \cdot p(B)}$$

Multiplication Rule for 2 Independent Event

$$\boxed{p(A \cap B) = p(A|B) \cdot p(B)}$$

If A and B are Independent Events,

$$\boxed{p(A|B) = p(A)} \quad \therefore \quad \boxed{p(A \cap B) = p(A) \cdot p(B)}$$

Theorem of Total probability

let $\{E_1, E_2 \ldots E_n\}$ be a partition of the sample space S and suppose that each of the Event $E_1, E_2 \ldots E_n$ has non zero probability of occurence. let A be any Event associated with the sample space S. Then probability of A $p(A) = p(E_1)\, p(A|E_1) + p(E_2)\, p(A|E_2) + \ldots + p(E_n)\, p(A|E_n)$

$$\boxed{p(A) = \sum_{j=1}^{n} p(E_j)\, p(A|E_j)}$$

Baye's Theorem

If $B_1, B_2 \ldots B_n$ are n non empty Events which constitutes a partition of sample space S. ie.) $B_1, B_2 \ldots B_n$ are pair wise disjoint and $B_1 \cup B_2 \cup \ldots B_n = S$ and A is any event of Non zero probability. Then the theorem

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Says that probability is,

$$\boxed{P(B_i/A) = \frac{P(A|B_i)\,P(B_i)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \cdots P(A|B_n)P(B_n)}}$$

[ie:, conditional probability + Law of total probability]

proof of Baye's Theorem

Based on conditional probability

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \quad \text{——} \quad ①$$

$$P(B_i/A) = \frac{P(A\cap B_i)}{P(A)} \quad \text{——} \quad ②$$

$$= \frac{P(A|B_i)\,P(B_i)}{P(A)}$$

$$\boxed{P(B_i/A) = \frac{P(A|B_i)\,P(B_i)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \cdots + P(A|B_n)P(B_n)}}$$

Requirements for Assigning Probabilities

Given a samplespace $S = \{O_1, O_2 \cdots\}$ the probabilities assigned to events must satisfy these requirements.

1) The probability of any event must be

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nonnegative, eg; $p(O_i) \geq 0$ for each i

2) The probability of the entire sample space must be 1, ie; $p(S) = 1$

3) for two disjoint events A and B, the probability of the union of A and B is equal to the sum of the probabilities of A and B, ie;

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

Approaches

There are three ways to assign probabilities to events:

[A] classical Approach

If an experiment has n simple outcomes, this method would assign a probability of $1/n$ to each outcome. This method also known as axiomatic approach

example1:- Roll of a Die.

$S = \{1, 2, \ldots 6\}$

probabilities : Each simple Event has a $1/6$ chance of occuring

example2 :- Two Rolls of a Die

$S = \{(1,1), (1,2), \ldots (6,6)\}$

Assumption: The two rolls are Independent

probabilities: Each Simple Event has $(1/6) \cdot (1/6) = 1/36$ chance of occuring.

[B] Relative- frequency Approach

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probabilities are assigned on the basis of experimentation or historical data.
formally, let A be an event of Interest, and assume that you have performed the same experiment n times so that n is the number of times A could have occured. further, let n_A be the number of times that A did occur. consider the relative frequency n_A/n. $p(A) = \lim_{n \to \infty} \frac{n_A}{n}$

Example 1: Roll of a Die.

$S = \{1, 2, ---6\}$

probabilities: Roll the given die 100 times and suppose the number of times the outcome 1 is observed is 15.
Thus $A = \{1\}$, $n_A = 15$ and $n = 100$.
$\therefore p(A) = 15/100 = 0.15$

C Subjective Approach

Here, we define probability as the degree of belief that we hold in the occurence of an event. Thus judgment is used as the basis for assigning probabilities.

Example: Horse Race

Consider a horse race with 8 horses running. What is the probabilities for a particular ~~approach~~ horse to win? Is it reasonable to assume that

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the probability is 1/8 ?
So, here There are diffrent Judgement that make for how likely It's for a particular horse to win.

⇒ problems — Event

1) What is the probability that a die is an Even number

A) $S = \{1, 2, 3, 4, 5, 6\}$

$p(\text{even no}) = 3/6 = 1/3$

2) Two dies are thrown Simultaneously. What is the probability that sum of their faces shown 6

A) $S = \{ (1,1)\ (1,2) \ldots\ (1,6),$
$(2,1)$
$(3,1)$
$\vdots$
$(6,1) \ldots\ (6,6) \}$

$n(S) = 36$. Elements are present in the sample space

$p(A) = n(A) / n(S)$

$n(A) = \{ (1,5), (2,4), (3,3), (4,2), (5,1) \}$

$p(A) = 5/36$.

3) A bag contains 4 white and 3 red balls.

a) What is the probability that drawing a white ball.

b) What is the probability that drawing 2 white balls.

Ans) a) p (drawing a white balls) $= 4/7$

$= {}^4C_1 / {}^7C_1$

$nC_r = \frac{n!}{(n-r)!\,r!}$

$= 4/7$

3

b) $\frac{4C_2}{7C_2}$

4) A bag contain 3 W and 2 R balls. randomly you are picking 3 balls.

a) 2W & 1R. What is the probability.

Ans) a) → $p(2W \,\&\, 1R) = \frac{3C_2 \times 2C_1}{5C_3} = \frac{6}{10}$

5) A bag contains 6 white and 4 black 2 balls are drawn at random. what is the probability that are of the same colour.

A) Total No. of balls $= 6+4 = 10$ Balls.

$10C_2 = \frac{10\times 9}{2\times 1} = 45$

E = Event of getting both balls of same colour.

$n(E)$ = no. of ways (2 balls out of six) or (2 balls out of 4)

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$= 6C_2 + 4C_2$

$= \frac{6\times5}{2\times1} + \frac{4\times3}{2\times1} = 15 + 6 = 21$

$p(E) = \frac{n(E)}{n(S)} = \frac{21}{45} = 7/15$

6) From a pack of 52 playing cards. two card are drawn together at random. What is the probability of both the cards being king

A) $n(S) = 52C_2 = 1326.$

let E = Event of getting 2 kings out of 4.

$n(E) = \frac{52\times51}{2\times1} = 6$

$4C_2 \Rightarrow \frac{4*3}{2*1}$

$p(E) = \frac{n(E)}{n(S)} = \frac{6}{1326} = \frac{1}{221}$

7) A bag contains 4 red, 6 yellow, 5 green balls 3 balls are randomly taken. What is the probability that the ball drawn from the bag exactly 2 green balls.

A) $\frac{(5C_2 * 6C_1) + (5C_2 + 4C_1)}{15C_3} = \frac{10*6+10*4}{455}$

$= \frac{60+40}{455} = \frac{100}{455}$

8) A man and his wife appear in an Interview for 2 vaccanaces in the same post

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The probability of husband selection is 1/7. and probability of wifes selection is 1/5. What is the probability that only one of them is selected.

A) Total no of Vaccancies

hus	wife
1/7	1/5

$p(A) = 1/5 \quad p(B) = 1/7$

$p(C) = p(A \text{ \& not} B) + p(B \text{ \& not} A)$

$= p(A)\, p(\text{not } B) + p(B)\, p(\text{not } A)$

$= p(\text{not } B) = 1 - p(B) = 1 - 1/7 = 6/7$

$= p(\text{not } A) = 1 - 1/5 = 4/5$

$p(C) = 1/5 * 6/7 + 1/7 * 4/5$

$= 0.284 = 2/7$

Q) What is the probability of rolling a die and getting either a one or six.

A) $p(1 \text{ or } 6) = 1/6 + 1/6$

$= 0.16 + 0.16 = 0.32$

⇒ problems – Addition Rule for any 2 events

example problems

$S = \{1, 2 \cdots 10\}$

$A = \{2, 3, 7, 8\} \quad B = \{7, 2, 3, 9, 10\}$

$p(A) = 4/10 \quad p(B) = 5/10$

$A \cup B = \{2, 3, 7, 8, 9, 10\}$

$p(A \cup B) = 6/10$

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$A \cap B = \{2, 3, 7\}$

$P(A \cap B) = 3/10$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$6/10 = 4/10 + 5/10 - 3/10$

$6/10 = 9/10 - 3/10$

$= 6/10$

9) What is the probability that a card selected from a deck will be either a ace or a spade?

A) There are 13 spade cards in the deck of 52 cards

$P_1 = 13/52 = 1/4$

There are 4 aces is 52 cards

$P_2 = 4/52 = 1/13$

There is only one Ace spade card

$P_{12} = 1/52$

$P_3 = P_1 + P_2 - P_{12}$

$= 1/4 + 1/13 - 1/52$

$= 4/13$

10) If rolling a single die determining the probability of rolling an even number or a number greater than 2.

A) $S = \{1, 2, --- 6\}$

2, 4, 6, 3 and 5 $= 5/6$

probability $= 5/6$

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11) A coin is tossed twice at random What is the probability of getting
(i) atleast one head
(ii) The same face

A) (i) When a coin is tossed twice.
total possible outcome is $(2)^2 = 4$
$S = \{H,T\}, \{T,H\}, \{H,H\} = 3$
$\therefore$ probability of getting atleast one head
$= 3/4 = 0.75$

(ii) $\{H,H\}, \{H,T\}, \{T,H\}, \{T,T\}, \{H,H\}, \{T,T\}$. There are only 2 possibilities
probability = favourable cases / exhaustive cases
$= 2/4 = 0.5$

12) Three fair coins are tossed Simultaneously what is the probability of getting atleast 2 tails?

A) If we toss 3 coins then the possible outcomes are $\{H,H,H\}$ $\{H,H,T\}$ $\{H,T,H\}$ $\{T,H,H\}$
$\{H,T,T\}$ $\{T,H,T\}$ $\{T,T,H\}$ $\{T,T,T\}$
No. of possible Outcomes = 8
No. of favourable Outcomes = 4

$$P(\text{atleast 2 tails}) = \frac{\text{No. of favourable Outcomes}}{\text{Total no. of Outcomes}}$$

$$P(\text{atleast 2 tails}) = 4/8$$
$$= 1/2$$

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13) A bag contains balls of numbered from 1 to 50. A ball is drawn randomly find the probability that it is multiple of six

A) There are 8 numbers which are multiple of 6.

$= 6, 12, 18, 24, 30, 36, 42, 48$

total sample space $= 50$

probability $= 8/50 = \underline{\underline{0.16}}$

⇒ **problems – Conditional probability**

14) 10 cards numbered 1 to 10 are placed in a box mixed up throwlly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3. what is the probability that it is an even number?

A) $S = \{1, 2, \ldots 10\}$

Suppose,

A : no. of card is even

B : no. on the card drawn greater than 3.

$A = \{2, 4, 6, 8, 10\}$

$B = \{4, 5, 6, 7, 8, 9, 10\}$

$A \cap B = \{4, 6, 8, 10\} = 4/10 = 2/5$

$P(A) = 5/10$

$P(B) = 7/10$

$P(A \cap B) = 4/10$

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$$p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{4/10}{7/10} = \underline{\underline{4/7}}$$

15) A die is thrown twice and the sum of the numbers appearing is observed to be six. What is the conditional probability that number 4 has a appeared atleast once.

A) A dice is thrown twice.

$S = \{ (1,1), (1,2) \text{ - - - - } (1,6)$

$\vdots$

$(6,1), (6,2) \text{ - - - } (6,6) \}$

$\therefore$ let F : Sum of numbers is 6

E : 4 has appeared atleast once

$p(E/F) = ?$

$E = \{(1,4)(2,4),(3,4),(4,4),(5,4),(6,4),$
$(4,1),(4,2),(4,3),(4,5),(4,6)\}$

$p(E) = 11/36$

$F = \{(1,5),(5,1),(2,4),(4,2),(3,3)\}$

$p(F) = 5/36$

$E \cap F = \{(2,4),(4,2)\}$

$p(E \cap F) = 2/36$

$p(E/F) = p(E \cap F) / p(F)$

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$$= \frac{2/36}{5/36} = 2/5$$

$\therefore$ Required probability is $2/5$.

⇒ problems – Multiplication Rule

16) A university has to select an examiner from a list of 50 persons. Twenty of them are women and 30 men. 10 of them are know hindi and 40 donot. 15 of them are teachers and remaining are not. what is the probability of the university selecting a hindi knowing women teacher.

A) Events E_1, E_2, E_3 such that
E_1 = A women is selected
E_2 = Hindi knowing person is selected
E_3 = A teacher is selected.
Required probability $P(E_1 \cap E_2 \cap E_3)$
Where E_1, E_2 and E_3 are Independent Events

$P(E_1) = 20/50 = 2/5$
$P(E_2) = 10/50 = 1/5$
$P(E_3) = 15/50 = 3/10$

Required probability $= P(E_1 \cap E_2 \cap E_3)$
$= P(E_1) \cdot P(E_2) \cdot P(E_3)$
$= 2/5 \times 1/5 \times 3/10$
$= 6/250 = \underline{3/125}$

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17) Two persons A and B attempt Independently to solve a puzzle the probability that $p(A)$ will solve $3/5$ and probability for B to solve is $p(B) = 1/3$. Find the probability that showing the puzzle will be solved by atleast one of them.

A)

$$p(A) = 3/5$$
$$p(A') = 1 - 3/5 = 2/5$$
$$p(B) = 1/3$$
$$p(B') = 2/3$$
$$p(A \cap B') + p(A' \cap B) + p(A \cap B)$$
$$= 3/5 * 2/3 + 2/5 * 1/3 + 1/3$$

probability $= 11/15 = \underline{0.73}$

OR

$p(\text{Atleast one solve})$
$= 1 - p(\text{none solve})$
$= p(A' \cap B')$
$= 1 - 2/5 * 2/3$
$= 1 - 4/15$
$= \underline{11/15}$

18) You are given 3 coins. One has heads on both faces. The second has tail on both faces. and the third has head on one face and tail on other face. you choose a coin random and toss it and it come up heads. the probability

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that the other is tail.

A) E_1 : Event that we get head as outcome

E_2 : Event that we have other side of outcome as tail

$S = \{(1,H), (1,H), (2,T), (2,T), (3,H), (3,T)\}$

$E_1 = (1,H)(1,H)(3,H)$

$E_2 = (2,T), (2,T)(3,T)$

$E_1 \cap E_2 = (3,H)$

$P(E_1 \cap E_2) = 1/6$

$P(E_1) = 3/6$

$P(E_2/E_1) = P(E_1 \cap E_2) / P(E_1)$

$= 1/6 / 3/6 = \underline{1/3}$

⇒ problems - Theorem of ~~law~~ total probability

Example

$S = \{1, 2, \ldots 10\}$

$E_1 = \{2,3,4\}$ $E_2 = \{5,6\}$ $E_3 = \{7\}$

$E_4 = \{8,9,10\}$ $A = \{2,6,7,10\}$

$P(A) = 4/10$

$A \cap E_1 = \{2\}$

$P(A \cap E_1) = 1/10$

$P(A \cap E_2) = 1/10$

$P(A \cap E_3) = 1/10$

$P(A \cap E_4) = 1/10$

$= \underline{\underline{4/10}}$

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19) A person has undertaken a construction Job. The probabilities are 0.65 that there will be strike 0.80 that the construction Job will be completed on time if there is no strike and 0.32 that the construction Job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.

A) $P(B) = 0.65$

$P(B') = 1 - P(B) = 1 - 0.65 = 0.35$

$P(A/B) = 0.32$

$P(A/B') = 0.80.$

By theorem of total probability

$P(A) = P(B)\,P(A/B) + P(B')\,P(A/B')$

$= 0.65 * 0.32 + 0.35 * 0.8$

$= \underline{\underline{0.488}}$

20) If I have 3 bags that each contain 100 marbles. Bag one contain 75 Red and 25 blue. Bag two contain 20 Red and 80 blue. Bag three has 45 red and 55 blue. I choose one of the bag at random and then pick a marble from the choosen bag. What is the probability that the choosen marble is red.

A) probability of choosing red ball from bag 1 is

$1/3 \times 75/100$

probability of choosing red ball from bag2 is

$1/3 \times 20/100$

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probability of choosing red ball from bag 3 is
$1/3 \times 45/100$

∴ The probability of choosing red ball is

$$= \frac{1}{3}\left(\frac{75+20+45}{100}\right)$$

$$= \frac{1}{3}\left(\frac{140}{100}\right) = \frac{1}{3} \times 1.4 = 0.462 = \frac{7}{15}$$

21) Bag one contains 3 Red and 4 black balls While another bag two contains 5 Red and 6 black balls. One ball is drawn at random from one of the bag and it is found to be red. find the probability that it was drawn from Bag two.

A) B_1 : the event of choosing Bag one
B_2 : the event of choosing Bag two
A : be the event of drawing red ball

$$P(B_1) = P(B_2) = 0.5$$
$$P(A/B_1) = 3/7$$
$$P(A/B_2) = 5/11$$

$$P(B_2/A) = \frac{P(A/B_2)\,P(B_2)}{P(A/B_1)\,P(B_1) + P(A/B_2)\,P(B_2)}$$

$$= \frac{5/11 * 1/2}{3/7 * 1/2 + 5/11 * 1/2} = 0.514$$

22) Box-I contain 2 gold coins, Box-II contains 1 gold & 1 silver coin. A person chose a box at random and takes out a coin. If the coin is of gold. what is the probability that the ~~[illegible]~~ order coin in the box is also of gold?

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A) Let B_1 = selecting Box one having two gold coins
B_2 = Box two having 2 silver coins
B_3 = Box three having one gold and one silver.

A : The second coin is of gold

$$P(B_1/A) = \frac{P(B_1)\cdot P(A/B_1)}{P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2) + P(B_3)\cdot P(A/B_3)}$$

$P(B_1) = 1/3$ $\quad P(B_2) = 1/3$ $\quad P(B_3) = 1/3$

$P(A/B_1) = 1$ $\quad P(A/B_2) = 0$ $\quad P(A/B_3) = 1/2$

$$P(B_1/A) = \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times 0 + \frac{1}{3}\times\frac{1}{2}}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\left[1+0+\frac{1}{2}\right]} = \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}}$$

$$= 2/3$$

23) The probability that a doctor will diagnose a particular disease correctly is 0.6. The probability that a patient will die by his treatment after correct diagnosis is 0.4 and the probability of death due to wrong diagnosis is 0.7. A patient of the doctor who had the disease died. What

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is the probability that his disease was not correctly diagnosed.

A) $B_2 = p$ (not correctly diagnosed) = ?

B_1 = correctly diagnosed

$p(B_1) = 0.6$

A → patient died

$p(A/B_1) = 0.4$

$p(A/B_2) = 0.7$

$$p(B_2/A) = \frac{p(A/B_2) \cdot p(B_2)}{p(A/B_1) \cdot p(B_1) + p(A/B_2) \cdot p(B_2)}$$

$$= \frac{0.4 * 0.7}{(0.6 * 0.4) + (0.4 * 0.7)}$$

$$= \frac{0.28}{0.24 + 0.28} = \frac{0.28}{0.52} = 0.538$$

24) A die is tossed find the probability of getting an odd number or a number less than or equal to 3.

A) $S = \{1, 2, 3, 4, 5, 6\}$

Favoureable case $= \{1, 3, 5, 2\}$

$$P(A) = \frac{n(A)}{n(S)} = 4/6$$

$$= 2/3$$

25) A box contains 3 coins, two regular coins and one fake two headed coin. $p(CH) = 1$

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If you pick a coin at random at and toss What is the probability that it lands heads up

A) $p(\text{regular coin}) = 2/3$

$p(\text{fake coin}) = 1/3$

$p(\text{head} \mid \text{regular coin}) = 1/2$

$p(\text{head} \mid \text{fake coin}) = 1$

$$p(H) = p(H \mid c_1) \cdot p(c_1) + p(H \mid c_2) \cdot p(c_2)$$

$$= 0.5 \times 2/3 + 1 \times 1/3$$

$$= \underline{\underline{2/3}}$$

OR

$$p(H) = p(H/c_1) \cdot p(c_1) + p(H/c_2)\, p(c_2) + p(H/c_3)\, p(c_3)$$

$$= 0.5 \times \frac{1}{3} + 0.5 \times \frac{1}{3} + 1 \times \frac{1}{3}$$

$$= \underline{\underline{2/3}}$$

26) A man is known to speak truth 3 out of 4 times. He throws a di and reports that it is a six. find the probability that it actualy 6.

A) 3 out of 4 times.

$p(s1)$ = probability of six occurs $= 1/6$

$p(s2)$ = probability that six doesnt occurs $= 5/6$

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probability that he lies = 1/4
probability that he replies with 6 is 1/5

$p(E|S_1) = 3/4 \quad p(E|S_2) = 1/4$

$$S_1|E = \frac{p(E|S_1)\cdot p(S_1)}{p(E|S_1)\cdot p(S_1) + p(E|S_2)\cdot p(S_2)}$$

$$= \frac{3/4 \cdot 1/6}{3/4 \cdot 1/6 + 1/4 \cdot 5/6} = \frac{0.75 \times 0.16}{(0.75 \times 0.16) + (0.25 \times 0.83)}$$

$$= \frac{0.12}{0.12 + 1.08} = \frac{0.12}{1.2} =$$

$$= \underline{\underline{3/8}}$$

Required probability is $\underline{\underline{3/8}}$