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All the code snippets are captured with the line numbers of that block, these line numbers will match the python file submitted with this pdf

### **Task 1:**

The task 1 is defined as a function with name task1() and is called at the end of the script A1.py submitted along with this report, the snippet below shows the initial few lines of task 1

```
8   from ortools.sat.python import cp_model
9   import numpy as np
10  import pandas as pd
11  import copy
12
13  def task1():
14      names = ["James", "Daniel", "Emily", "Sophie"]
15      starters = ["Carpaccio", "Prawn_Cocktail", "Onion_Soup", "Mushroom_Tart"]
16      mainCourse = ["Filet_Steak", "Vegan_Pie", "Baked_Mackerel", "Fried_Chicken"]
17      drinks = ["Beer", "Coke", "Red_Wine", "White_Wine"]
18      deserts = ["Ice_Cream", "Chocolate_Cake", "Apple_Crumble", "Tiramisu"]
19
```

The task is to identify objects predicates and attributes and solve using CT-SAT solver using necessary decision variables

The snippet below is where decision variables are created for each possible pair

```
51      #creating necessary decision variables
52      person_starter = {}
53      for name in names:
54          variables = {}
55          for starter in starters:
56              variables[starter] = model.NewBoolVar(name + starter)
57              person_starter[name] = variables
58
```

The snippet below is to create at least one item from each course for each person

```

80     for name in names:
81         variables = []
82         for starter in starters:
83             variables.append(person_starter[name][starter])
84         model.AddBoolOr(variables)
85
86         variables = []
87         for mains in mainCourse:
88             variables.append(person_mainCourse[name][mains])
89         model.AddBoolOr(variables)
90
91         variables = []
92         for desert in deserts:
93             variables.append(person_deserts[name][desert])
94         model.AddBoolOr(variables)
95
96         variables = []
97         for drink in drinks:
98             variables.append(person_drinks[name][drink])
99         model.AddBoolOr(variables)

```

A person can only have one item from each course, hence maximum is one item as shown in below snippet

```

102     """
103     Max one item per course per person
104     """
105     for i in range(4):
106         for j in range(i+1,4):
107             model.AddBoolOr([
108                 person_drinks[name][drinks[i]].Not(),
109                 person_drinks[name][drinks[j]].Not()])
110             model.AddBoolOr([
111                 person_starter[name][starters[i]].Not(),
112                 person_starter[name][starters[j]].Not()])
113             model.AddBoolOr([
114                 person_mainCourse[name][mainCourse[i]].Not(),
115                 person_mainCourse[name][mainCourse[j]].Not()])
116             model.AddBoolOr([
117                 person_deserts[name][deserts[i]].Not(),
118                 person_deserts[name][deserts[j]].Not()])
119

```

My assumption from task 1 description is that each person eats an item from each course, the variables for this are scripted as shown below

```

120     """
121     Every person has a different item
122     """
123     for i in range(4):
124         for j in range(i+1,4):
125             for k in range(4):
126                 model.AddBoolOr([
127                     person_starter[names[i]][starters[k]].Not(),
128                     person_starter[names[j]][starters[k]].Not()])
129                 model.AddBoolOr([person_mainCourse[names[i]][mainCourse[k]].Not(),
130                                     person_mainCourse[names[j]][mainCourse[k]].Not()])
131                 model.AddBoolOr([person_deserts[names[i]][deserts[k]].Not(),
132                                     person_deserts[names[j]][deserts[k]].Not()])
133                 model.AddBoolOr([person_drinks[names[i]][drinks[k]].Not(),
134                                     person_drinks[names[j]][drinks[k]].Not()])
135

```

Sentence 1 constraints are as follows

```

137     -----Constraints-----
138
139     1.The carpaccio starter is not combined with the vegan pie as main course
140     and the filet steak main course is not followed by ice cream as desert
141     a. The carpaccio starter is not combined with the vegan pie as main course
142     b. Filet steak main course is not followed by ice cream as desert
143     """
144     for name in names:
145         model.AddBoolAnd([person_starter[name]["Carpaccio"].Not()]).\
146             OnlyEnforceIf([person_mainCourse[name]["Vegan_Pie"]])
147         model.AddBoolAnd([person_mainCourse[name]["Filet_Steak"].Not()]).\
148             OnlyEnforceIf([person_deserts[name]["Ice_Cream"]])
149

```

Sentence 2 constraints are as follows:

```

151     2. Emily does not have prawn cocktail or onion soup as starter
152     none of the men has beer or coke to drink
153     a.Emily doesnt have praws or onions
154     b.James and Daniel wont have coke or beer
155     """
156     model.AddBoolAnd([person_starter["Emily"]["Prawn_Cocktail"].Not(),
157                         person_starter["Emily"]["Onion_Soup"].Not()])
158     model.AddBoolAnd([person_drinks["James"]["Beer"].Not(),
159                         person_drinks["James"]["Coke"].Not(),
160                         person_drinks["Daniel"]["Beer"].Not(),
161                         person_drinks["Daniel"]["Coke"].Not()])

```

Sentence 3, 4, 5 and 6 constraints are as shown in the snippet below

```

164     3. The person having prawn cocktail as starter has baked mackerel as main course and the
165     filet steak main course works well with the red wine.
166     """
167     for name in names:
168         model.AddBoolAnd([person_starter[name]["Prawn_Cocktail"]]).\
169             OnlyEnforceIf([person_mainCourse[name]["Baked_Mackerel"]])
170         model.AddBoolAnd([person_mainCourse[name]["Filet_Steak"]]).\
171             OnlyEnforceIf([person_drinks[name]["Red_Wine"]])
172
173     """
174     4. One of the men has white wine as drink and one of the women drinks coke
175     """
176     model.AddBoolOr([person_drinks["James"]["White_Wine"],
177                     person_drinks["Daniel"]["White_Wine"]])
178     model.AddBoolOr([person_drinks["Emily"]["Coke"],
179                     person_drinks["Sophie"]["Coke"]])
180
181     """
182     5. The vegan pie main always comes with mushroom tart as starter and vice versa;
183     also, the onion soup and filet steak are always served together.
184     """
185     for name in names:
186         model.AddBoolAnd([person_mainCourse[name]["Vegan_Pie"]]).\
187             OnlyEnforceIf([person_starter[name]["Mushroom_Tart"]])
188         model.AddBoolAnd([person_starter[name]["Onion_Soup"]]).\
189             OnlyEnforceIf([person_mainCourse[name]["Filet_Steak"]])
190
191     """
192     6. Emily orders beer as drink or has fried chicken as main and ice cream as desert;
193     James orders coke as drink or has onion soup as starter and filet steak as main.
194     """
195     model.AddBoolOr([person_drinks["Emily"]["Beer"],
196                     person_mainCourse["Emily"]["Fried_Chicken"]])
197     model.AddBoolAnd([person_deserts["Emily"]["Ice_Cream"]])
198
199
200     model.AddBoolOr([person_drinks["James"]["Coke"],
201                     person_starter["James"]["Onion_Soup"]])
202     model.AddBoolAnd([person_mainCourse["James"]["Filet_Steak"]])
203

```

Likewise for sentence 7 constraints are as shown below

```

204     """
205     7. Sophie orders chocolate cake but does not drink beer nor likes fried chicken;
206     Daniel orders apple crumble for dessert but has neither carpaccio nor mushroom tart as sta
207     """
208     model.AddBoolAnd([person_drinks["Sophie"]["Beer"].Not(),
209                     person_mainCourse["Sophie"]["Fried_Chicken"].Not()])
210     model.AddBoolAnd([person_deserts["Sophie"]["Chocolate_Cake"]])
211     model.AddBoolAnd([person_starter["Daniel"]["Carpaccio"].Not(),
212                     person_starter["Daniel"]["Mushroom_Tart"].Not()])
213     model.AddBoolAnd([person_deserts["Daniel"]["Apple_Crumble"]])
214

```

CT\_SAT solver is used to solve for tiramisu question

```

215     solver = cp_model.CpSolver()
216     status = solver.SearchForAllSolutions(model, SolutionPrinter(person_starter, person_mainCo
217     print(solver.StatusName(status))
218
219     for name in names:
220         if solver.Value(person_deserts[name]["Tiramisu"]):
221             print(name + ' has Tiramisu for dessert')
222

```

The CP-SAT solver finds optimal solution and the tiramisu question is hence answered, James has tiramisu for dessert, the solution is printed in the console as shown below

```
Solution: 1
- James:
  - Onion_Soup
  - Filet_Steak
  - Tiramisu
  - Red_Wine
- Daniel:
  - Prawn_Cocktail
  - Baked_Mackerel
  - Apple_Crumble
  - White_Wine
- Emily:
  - Carpaccio
  - Fried_Chicken
  - Ice_Cream
  - Beer
- Sophie:
  - Mushroom_Tart
  - Vegan_Pie
  - Chocolate_Cake
  - Coke

OPTIMAL
James has Tiramisu for dessert
```

## Task 2:

Sudoku solver using CP-SAT

Task 2 is defined as a function with name `task2_sudoku(sud)`:

It takes one argument which is the sudoku values matrix (numpy array)

```
224     def task2_sudoku(sud):
225         model = cp_model.CpModel()
226         sud_size = sud.shape[0]
227         sud_dict = {}
```

The code snippet below and the comments explain the steps followed

```

246
247 #create Int from 1,9 for non-zero sudoku slots
248 for i in range(sud_size):
249     for j in range(sud_size):
250         if sud[i][j] != 0:
251             sud_dict[i,j] = sud[i][j]
252         else:
253             sud_dict[i,j] = model.NewIntVar(1, sud_size, f"sudoku_{i}_{j}")
254
255 #different numbers in row
256 for i in range(sud_size):
257     model.AddAllDifferent([sud_dict[i,j] for j in range(sud_size)])
258
259 #different number in column
260 for j in range(sud_size):
261     model.AddAllDifferent([sud_dict[i,j] for i in range(sud_size)])
262
263 grid_size = 3
264 for i in range(0,sud_size,grid_size):
265     for j in range(0,sud_size,grid_size):
266         model.AddAllDifferent([sud_dict[i+m,j+n] for m in range(3) for n in range(3)])
267
268 solver = cp_model.CpSolver()
269 solver.SearchForAllSolutions(model, SolutionPrinter(sud_size, sud_dict))
270

```

The solver finds 5 solutions for this problem, and they are printed in console as shown below

Solution: 1

```
[[2 8 6 7 4 9 5 3 1]
 [7 4 5 1 2 3 6 8 9]
 [1 9 3 5 8 6 4 2 7]
 [8 1 7 9 5 4 3 6 2]
 [4 5 9 6 3 2 7 1 8]
 [3 6 2 8 1 7 9 5 4]
 [5 7 4 2 6 8 1 9 3]
 [9 2 1 3 7 5 8 4 6]
 [6 3 8 4 9 1 2 7 5]]
```

Solution: 2

```
[[2 6 8 7 4 9 5 3 1]
 [7 4 5 1 2 3 6 8 9]
 [1 9 3 5 8 6 4 2 7]
 [8 1 7 9 5 4 3 6 2]
 [4 5 9 6 3 2 7 1 8]
 [3 2 6 8 1 7 9 5 4]
 [5 7 4 2 6 8 1 9 3]
 [9 8 1 3 7 5 2 4 6]
 [6 3 2 4 9 1 8 7 5]]
```

Solution: 3

```
[[2 6 1 9 4 8 5 3 7]
 [7 4 5 1 2 3 6 8 9]
 [8 9 3 7 5 6 4 2 1]
 [1 8 7 5 9 4 3 6 2]
 [4 5 9 6 3 2 7 1 8]
 [3 2 6 8 1 7 9 5 4]
 [5 7 8 2 6 9 1 4 3]
 [9 1 4 3 8 5 2 7 6]
 [6 3 2 4 7 1 8 9 5]]
```

Solution: 4

```
[[2 6 1 7 4 8 5 3 9]
 [7 4 5 9 2 3 6 8 1]
 [8 9 3 1 5 6 4 2 7]
 [1 8 7 5 9 4 3 6 2]
 [4 5 9 6 3 2 7 1 8]
 [3 2 6 8 1 7 9 5 4]
 [5 7 8 2 6 9 1 4 3]
 [9 1 4 3 8 5 2 7 6]
 [6 3 2 4 7 1 8 9 5]]
```

Solution: 5

```
[[2 6 1 8 4 7 5 3 9]
 [7 4 5 9 2 3 6 8 1]
 [8 9 3 1 5 6 4 2 7]
 [1 8 7 5 9 4 3 6 2]
 [4 5 9 6 3 2 7 1 8]
 [3 2 6 7 1 8 9 5 4]
 [5 7 8 2 6 9 1 4 3]
 [9 1 4 3 8 5 2 7 6]
 [6 3 2 4 7 1 8 9 5]]
```

### Task 3:

The task was to find the optimal way of assigning the project jobs to contractors to finish different projects on time while keeping the profit margin 2160

The task 3 is defined as a function called task3(data, min\_profit\_margin = 2160)

Argument data is read from the xlsx file as dataframes projects, quotes, dependencies and values

```
274 def task3(data, min_profit_margin = 2160):
275
276     #part a. Loading data
277     projects_df, quotes_df, dependencies_df, value = data['Projects'], data['Quotes'], data['Dependencies'], data['Value']
278     #cost of all jobs in projects completed
279     cost = 0
280
281     model = cp_model.CpModel()
282
```

Main decision variables are created for projects

Project contractor pair, contractor project month pairs are created which are used for different blocks later in the script

```
283     #part b. creating decision variables
284     proj_dict = {}
285     for p in projects_df.index.values:
286         proj_dict[p] = model.NewBoolVar(p)
287
288
289     pc_pair = {}
290     #contractor >> month >> jobs during this month belonging to a project dict
291     contractor_project_month = {}
292     #project >> month >> contractors eligible to work on the project dict
293     project_month_contractors = {}
```

This is the main loop in the snippet below, it loops on all the dataframes to add many Boolean constraints

```
307     #main loop creates various decision variables (which contractor is working on which project and when)
308     #project/contractor/month/job decision variables
309     for contractor in quotes_df.index.values:
310         for job in quotes_df.columns.values:
311             if str(quotes_df.loc[contractor][job]) == 'nan':
312                 #contractor not qualified, so pass
313                 pass
314             else:
315                 for project in projects_df.index.values:
316                     for month in projects_df.columns.values:
317                         if str(projects_df.loc[project][month]) == 'nan':
318                             # no project in this month so pass
319                             pass
320                         else:
321                             if projects_df.loc[project][month] == job:
322                                 #boolean var for job which can be done by a contractor
323                                 pc_pair[project+'_'+contractor+'_'+month+'_'+job] = model.NewBoolVar(project+'_'+contractor+'_'+month+'_'+job)
324                                 #contractors monthly job availability belonging to different projects
325                                 contractor_project_month[contractor][month].append(pc_pair[project+'_'+contractor+'_'+month+'_'+job])
326                                 #project jobs of every month and the contractors eligible to do this job
327                                 project_month_contractors[project][month].append(pc_pair[project+'_'+contractor+'_'+month+'_'+job])
328                                 #cost calculation of the projects delivered
329                                 cost += int(quotes_df.loc[contractor][job])*pc_pair[project+'_'+contractor+'_'+month+'_'+job]
330
```

A contractor can only work on one project job every month, this is constructed as shown below



```

332     #a contractor cannot work on two jobs/projects at the same time
333     for contractor, month_projects_df in contractor_project_month.items():
334         # print(contractor, month_projects_df)
335         # print(f"{contractor} >>>>>>>>> {month_projects_df}")
336         for m,p in month_projects_df.items():
337             # print(m,p)
338             model.Add(sum(p) <= 1)

```

Two contractors cannot work on the same project job, this is shown in the snippet below

```

340     #two contractors cant work on same project at same time
341     for p, mp in project_month_contractors.items():
342         # print(contractor, month_projects_df)
343         # print(f"{p} >>>>>>>>> {mp}")
344         for m,ps in mp.items():
345             #print(m,ps)
346             #If project is going ahead, exactly one contractor works on job
347             if len(ps)>0:
348                 model.Add(sum(ps) == 1).OnlyEnforceIf(proj_dict[p])
349             # Part E. Constraint #3 - If project is not taken on then 0 contractors work on any of the jobs
350             model.Add(sum(ps) == 0).OnlyEnforceIf(proj_dict[p].Not())
351

```

The dependencies of each project is checked and constraints are added as shown below

```

352     #dependencies bool variables
353     for project1 in dependencies_df.index.values:
354         for project2 in dependencies_df.columns.values:
355             if str(dependencies_df.loc[project1][project2]) == 'required':
356                 #Project B can only be taken on, if also Project A is taken on
357                 model.AddBoolAnd([proj_dict[project2])).OnlyEnforceIf(proj_dict[project1])
358             if str(dependencies_df.loc[project1][project2]) == 'conflict':
359                 #Project B and Project C are mutually exclusive and cannot be both taken on
360                 model.AddBoolAnd([proj_dict[project2].Not()]).OnlyEnforceIf(proj_dict[project1])
361

```

The value/cost of all the projects carried on by various contractors is calculated and the profit margin is found, the margin should be more than the minimum profit margin supplied to the script

```

366     #Value = sum of all projects_df being carried out
367     for p in value.index.values:
368         total_value += int(value.loc[p]['Value'])*proj_dict[p]
369
370     pm = total_value - cost
371     model.Add( pm >= min_profit_margin)
372

```

Finally, CP-SAT is called to solve the constraints and find the optimal solution

```

373
374     #CPSAT solver
375     solver = cp_model.CpSolver()
376     status = solver.Solve(model)
377     sp = SolutionPrinter_task3(proj_dict, pc_pair, pm)
378     status = solver.SearchForAllSolutions(model, sp)
379     print(f"There are {sp.solutions_} solutions")
380

```

There are 5 optimal solutions found by CP-SAT solver and they are as shown in the snippets below

Solution: 1

Projects Contracted:

--Project C--

- Job B was carried out in month M7 by Contractor E
- Job E was carried out in month M5 by Contractor E
- Job E was carried out in month M8 by Contractor E
- Job G was carried out in month M6 by Contractor G
- Job H was carried out in month M4 by Contractor H

--Project D--

- Job F was carried out in month M3 by Contractor F
- Job I was carried out in month M4 by Contractor G
- Job D was carried out in month M2 by Contractor H
- Job H was carried out in month M5 by Contractor H

--Project E--

- Job A was carried out in month M9 by Contractor A
- Job J was carried out in month M8 by Contractor C

--Project H--

- Job A was carried out in month M8 by Contractor A
- Job B was carried out in month M9 by Contractor E
- Job I was carried out in month M11 by Contractor G
- Job D was carried out in month M10 by Contractor H

--Project I--

- Job L was carried out in month M10 by Contractor D
- Job F was carried out in month M11 by Contractor F
- Job K was carried out in month M12 by Contractor K

Profit Margin is: 2165

Solution: 2

Projects Contracted:

--Project C--

- Job B was carried out in month M7 by Contractor E
- Job E was carried out in month M5 by Contractor E
- Job E was carried out in month M8 by Contractor E
- Job G was carried out in month M6 by Contractor G
- Job H was carried out in month M4 by Contractor H

--Project D--

- Job F was carried out in month M3 by Contractor F
- Job I was carried out in month M4 by Contractor G
- Job D was carried out in month M2 by Contractor H
- Job H was carried out in month M5 by Contractor H

--Project E--

- Job A was carried out in month M9 by Contractor A
- Job J was carried out in month M8 by Contractor C

--Project H--

- Job A was carried out in month M8 by Contractor A
- Job B was carried out in month M9 by Contractor E
- Job I was carried out in month M11 by Contractor G
- Job D was carried out in month M10 by Contractor H

--Project I--

- Job K was carried out in month M12 by Contractor B
- Job L was carried out in month M10 by Contractor D
- Job F was carried out in month M11 by Contractor F

Profit Margin is: 2175

Solution: 3

Projects Contracted:

--Project C--

- Job E was carried out in month M5 by Contractor A
- Job B was carried out in month M7 by Contractor E
- Job E was carried out in month M8 by Contractor E
- Job G was carried out in month M6 by Contractor G
- Job H was carried out in month M4 by Contractor H

--Project D--

- Job F was carried out in month M3 by Contractor F
- Job I was carried out in month M4 by Contractor G
- Job D was carried out in month M2 by Contractor H
- Job H was carried out in month M5 by Contractor H

--Project E--

- Job A was carried out in month M9 by Contractor A
- Job J was carried out in month M8 by Contractor C

--Project H--

- Job A was carried out in month M8 by Contractor A
- Job B was carried out in month M9 by Contractor E
- Job I was carried out in month M11 by Contractor G
- Job D was carried out in month M10 by Contractor H

--Project I--

- Job K was carried out in month M12 by Contractor B
- Job L was carried out in month M10 by Contractor D
- Job F was carried out in month M11 by Contractor F

Profit Margin is: 2165

Solution: 4

Projects Contracted:

--Project C--

- Job B was carried out in month M7 by Contractor E
- Job E was carried out in month M5 by Contractor E
- Job E was carried out in month M8 by Contractor E
- Job G was carried out in month M6 by Contractor G
- Job H was carried out in month M4 by Contractor H

--Project D--

- Job F was carried out in month M3 by Contractor F
- Job I was carried out in month M4 by Contractor G
- Job D was carried out in month M2 by Contractor H
- Job H was carried out in month M5 by Contractor H

--Project E--

- Job A was carried out in month M9 by Contractor A
- Job J was carried out in month M8 by Contractor C

--Project H--

- Job A was carried out in month M8 by Contractor A
- Job B was carried out in month M9 by Contractor E
- Job I was carried out in month M11 by Contractor G
- Job D was carried out in month M10 by Contractor H

--Project I--

- Job L was carried out in month M10 by Contractor A
- Job K was carried out in month M12 by Contractor B
- Job F was carried out in month M11 by Contractor F

Profit Margin is: 2165

Solution: 5

Projects Contracted:

--Project A--

- Job A was carried out in month M1 by Contractor A
- Job B was carried out in month M2 by Contractor E
- Job C was carried out in month M3 by Contractor K

--Project C--

- Job B was carried out in month M7 by Contractor E
- Job E was carried out in month M5 by Contractor E
- Job E was carried out in month M8 by Contractor E
- Job G was carried out in month M6 by Contractor G
- Job H was carried out in month M4 by Contractor H

--Project D--

- Job F was carried out in month M3 by Contractor F
- Job I was carried out in month M4 by Contractor G
- Job D was carried out in month M2 by Contractor H
- Job H was carried out in month M5 by Contractor H

--Project E--

- Job A was carried out in month M9 by Contractor A
- Job J was carried out in month M8 by Contractor C

--Project H--

- Job A was carried out in month M8 by Contractor A
- Job B was carried out in month M9 by Contractor E
- Job I was carried out in month M11 by Contractor G
- Job D was carried out in month M10 by Contractor H

--Project I--

- Job K was carried out in month M12 by Contractor B
- Job L was carried out in month M10 by Contractor D
- Job F was carried out in month M11 by Contractor F

Profit Margin is: 2165

There are 5 solutions