Calculating the Equation of a Plane having three Cartesian Vector Co-ordinates (Points / Vertices)

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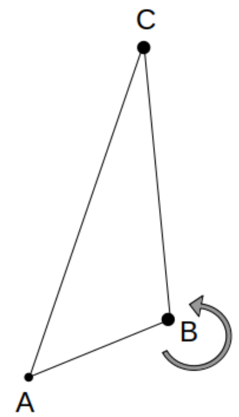
Let there be a 3-D Plane formed by the Points A(5, 0, 0.3), B(10, 0, 0.5) and C(10, 20, 0.5).
Let $\vec{a}, \vec{b},$ and $\vec{c}$ be the position vectors of points A, B and C respectively.

$$\vec{a} = 5\hat{i} + 0\hat{j} + 0.3\hat{k}$$
$$\vec{b} = 10\hat{i} + 0\hat{j} + 0.5\hat{k}$$
$$\vec{c} = 10\hat{i} + 20\hat{j} + 0.5\hat{k}$$



Note that we must solve the problem in a counter-clockwise direction in order to obtain the correct solution in terms of direction (sign).
Therefore, ABC, BCA and CAB are valid.
However, CBA, ACB and ABC are <u>NOT</u> valid.

To test the possibility of a solution, the determinant of the vectors must <u>NOT</u> be equal to zero (0).

$$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \begin{vmatrix} 5 & 0 & 0.3 \\ 10 & 0 & 0.5 \\ 10 & 20 & 0.5 \end{vmatrix} = 10 \neq 0$$

Therefore, a solution does exist.

To obtain the equation, solve the following determinant, equate it to zero and simplify the equation:

$$\begin{vmatrix} x-a_x & y-a_y & z-a_z \\ b_x-a_x & b_y-a_y & b_z-a_z \\ c_x-a_x & c_y-a_y & c_z-a_z \end{vmatrix} = 0$$

$$\begin{vmatrix} x-5 & y-0 & z-0.3 \\ 10-5 & 0-0 & 0.5-0.3 \\ 10-5 & 20-0 & 0.5-0.3 \end{vmatrix} = \begin{vmatrix} x-5 & y & z-0.3 \\ 5 & 0 & 0.2 \\ 5 & 20 & 0.2 \end{vmatrix} = 0$$

$$(x-5)(0-4) - (y)(1-1) + (z-0.3)(100-0) = 0$$
$$(x-5)(-4) - (y)(0) + (z-0.3)(100) = 0$$
$$(-4x + 20) - (0) + (100z - 30) = 0 \qquad -4x + 100z - 30 + 20 = 0$$
$$-4x + 100z - 10 = 0 \qquad \therefore\ -4x + 100z = 10$$

Arranging the equation into it’s standard form:

$$d_1 x + d_2 y + d_3 z = d_4$$

where, d_1 , d_2 , d_3 and d_4 are arbitrary constants/coefficients.

$$\therefore\ -4x + 0y + 100z = 10$$

Simplifying the generic equation:

Let L, M and N be arbitrary variables.

$$\begin{vmatrix} x-a_x & y-a_y & z-a_z \\ b_x-a_x & b_y-a_y & b_z-a_z \\ c_x-a_x & c_y-a_y & c_z-a_z \end{vmatrix} = 0$$

$$\begin{aligned} & \{ (x-a_x) \cdot [(b_y-a_y)(c_z-a_z) - (b_z-a_z)(c_y-a_y)] \} \\ - & \{ (y-a_y) \cdot [(b_x-a_x)(c_z-a_z) - (b_z-a_z)(c_x-a_x)] \} \\ + & \{ (z-a_z) \cdot [(b_x-a_x)(c_y-a_y) - (b_y-a_y)(c_x-a_x)] \} = 0 \end{aligned}$$

$$\begin{aligned} & \{ (x-a_x) \cdot [(b_y c_z - b_y a_z - a_y c_z + a_y a_z) - (b_z c_y - b_z a_y - a_z c_y + a_z a_y)] \} \\ - & \{ (y-a_y) \cdot [(b_x c_z - b_x a_z - a_x c_z + a_x a_z) - (b_z c_x - b_z a_x - a_z c_x + a_z a_x)] \} \\ + & \{ (z-a_z) \cdot [(b_x c_y - b_x a_y - a_x c_y + a_x a_y) - (b_y c_x - b_y a_x - a_y c_x + a_y a_x)] \} = 0 \end{aligned}$$

$$\begin{aligned} & \{ (x-a_x) \cdot [b_y c_z - b_y a_z - a_y c_z - b_z c_y + b_z a_y + a_z c_y] \} \\ - & \{ (y-a_y) \cdot [b_x c_z - b_x a_z - a_x c_z - b_z c_x + b_z a_x + a_z c_x] \} \\ + & \{ (z-a_z) \cdot [b_x c_y - b_x a_y - a_x c_y - b_y c_x + b_y a_x + a_y c_x] \} = 0 \end{aligned}$$

$$[\ L \cdot (x-a_x)\ ] - [\ M \cdot (y-a_y)\ ] + [\ N \cdot (z-a_z)\ ] = 0$$

$$Lx - La_x - My + Ma_y + Nz - Na_z = 0$$

$$\therefore\ Lx - My + Nz = [\ La_x - Ma_y + Na_z\ ]$$