Finding a Unit Vector on a Plane having
Cartesian Vector Co-ordinates (Points / Vertices)

- by Melwyn F. Carlo

Let there be a 3-D Plane formed by the Points A(1,0,1), B(0,2,2) and C(3,3,0).
Let $\vec{a}, \vec{b}$, and $\vec{c}$ be the position vectors of points A, B and C respectively.

$$\vec{a} = 1\hat{i} + 0\hat{j} + 1\hat{k}$$
$$\vec{b} = 0\hat{i} + 2\hat{j} + 2\hat{k}$$
$$\vec{c} = 3\hat{i} + 3\hat{j} + 0\hat{k}$$

$$\vec{v_1} = \overrightarrow{AB} = \vec{b} - \vec{a} = -1\hat{i} + 2\hat{j} + 1\hat{k}$$
$$\vec{v_2} = \overrightarrow{AC} = \vec{c} - \vec{a} = 2\hat{i} + 3\hat{j} - 1\hat{k}$$

Let $\vec{u}$ be a vector that is perpendicular to the plane.

$$\vec{u} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_{1x} & v_{1y} & v_{1z} \\ v_{2x} & v_{2y} & v_{2z} \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= [(v_{1y} \cdot v_{2z}) - (v_{1z} \cdot v_{2y})]\ \hat{i} - [(v_{1x} \cdot v_{2z}) - (v_{1z} \cdot v_{2x})]\ \hat{j} + [(v_{1x} \cdot v_{2y}) - (v_{1y} \cdot v_{2x})]\ \hat{k}$$

$$= -5\hat{i} + 1\hat{j} - 7\hat{k}$$

$$= (-5,\ 1,\ -7)$$

Let $\hat{n}$ be the unit vector.

$$\left\|\frac{\vec{u}}{u}\right\| = \frac{\|\vec{u}\|}{u} = \frac{u}{u} = 1$$

$$\|\vec{u}\| = \sqrt{u_x^2 + u_y^2 + u_z^2} = \sqrt{75} = 5\sqrt{3}$$

$$\therefore\ \hat{n} = \frac{\vec{u}}{\|\vec{u}\|} = \frac{(-5,\ 1,\ -7)}{5\sqrt{3}} = \left(-\frac{1}{\sqrt{3}},\ \frac{1}{5\sqrt{3}},\ -\frac{7}{5\sqrt{3}}\right)$$