

Prime Numbers Calculator:
(<https://melyaman.github.io/>)

Implementation details:

1. A first approach to the solution was tested using the definition of prime numbers and verifying whether every number less than n is prime or not , and then return the number of primes found. This solution proved to be quite slow (runtime of $O(n^2)$) , even with the optimization to test only the numbers up to \sqrt{n} (because a composite number will be a product of two number p and q , one of which must be less than or equal than the square root of n .)
2. The approach implemented uses the sieve of Eratosthenes, which gives an efficient way to find the prime numbers less than n , with the following optimizations:
 - a. If the current number is k , we can mark off multiples of k starting at k^2 in order not to mark off already marked off numbers.
 - b. Terminating the loop condition at the \sqrt{n} .

This method runs in $O(n \log(\log(n)))$ time , which is a good improvement , but it has the **limitation** of having $O(n)$ space complexity because we have to declare the array of elements up to n , which can be problematic for very large numbers as the array may not fit in memory.

In practise , this method is used for numbers up to 10^8 . For larger numbers , an asymptotic approximation is used ($n/\log(n)$)[1].

Further improvements:

The program performance can be improved by including a list of prime numbers in a lookup table in an increasing order which should be faster for large numbers.

This task has the property of being **embarrassingly parallel** , so one could use a parallel approach using a MapReduce programming model , the reduce function would be just a sum . For this , the sieve has to be modified to output the number of primes in a range $[a,b]$ instead of primes up to n .

References:

- [1] https://en.wikipedia.org/wiki/Prime-counting_function
[2] <http://www.w3schools.com/bootstrap/> (some elements of the graphical interface were taken from examples of this website)