Spectral Shape Analysis for 3D matching

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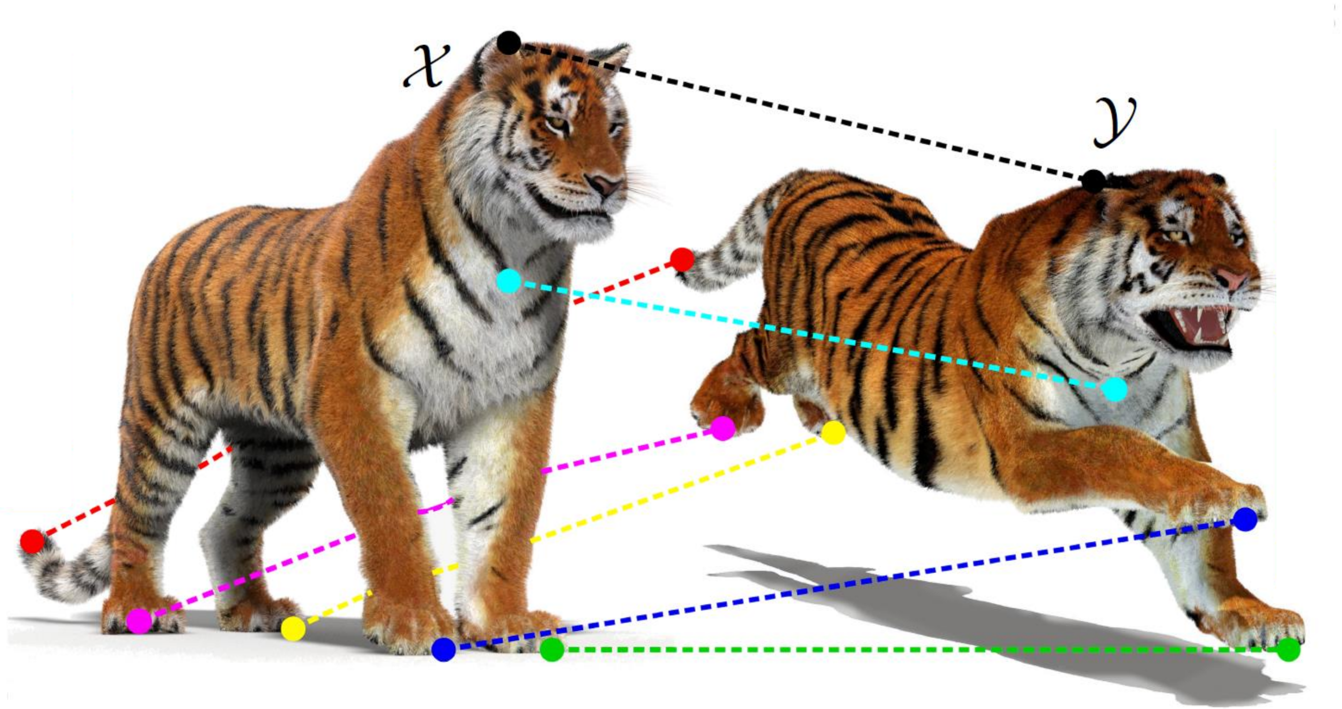
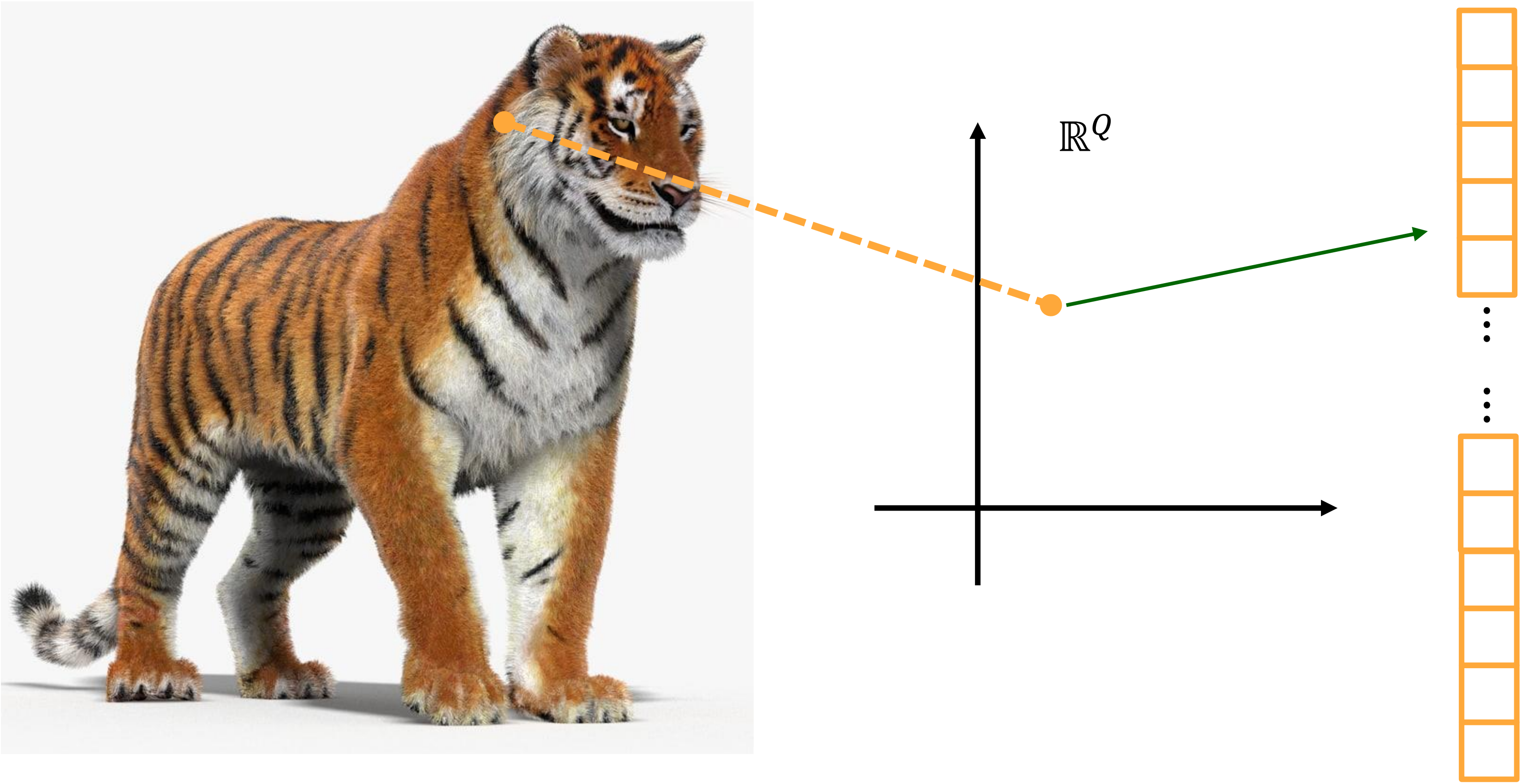
Descriptor for shape matching

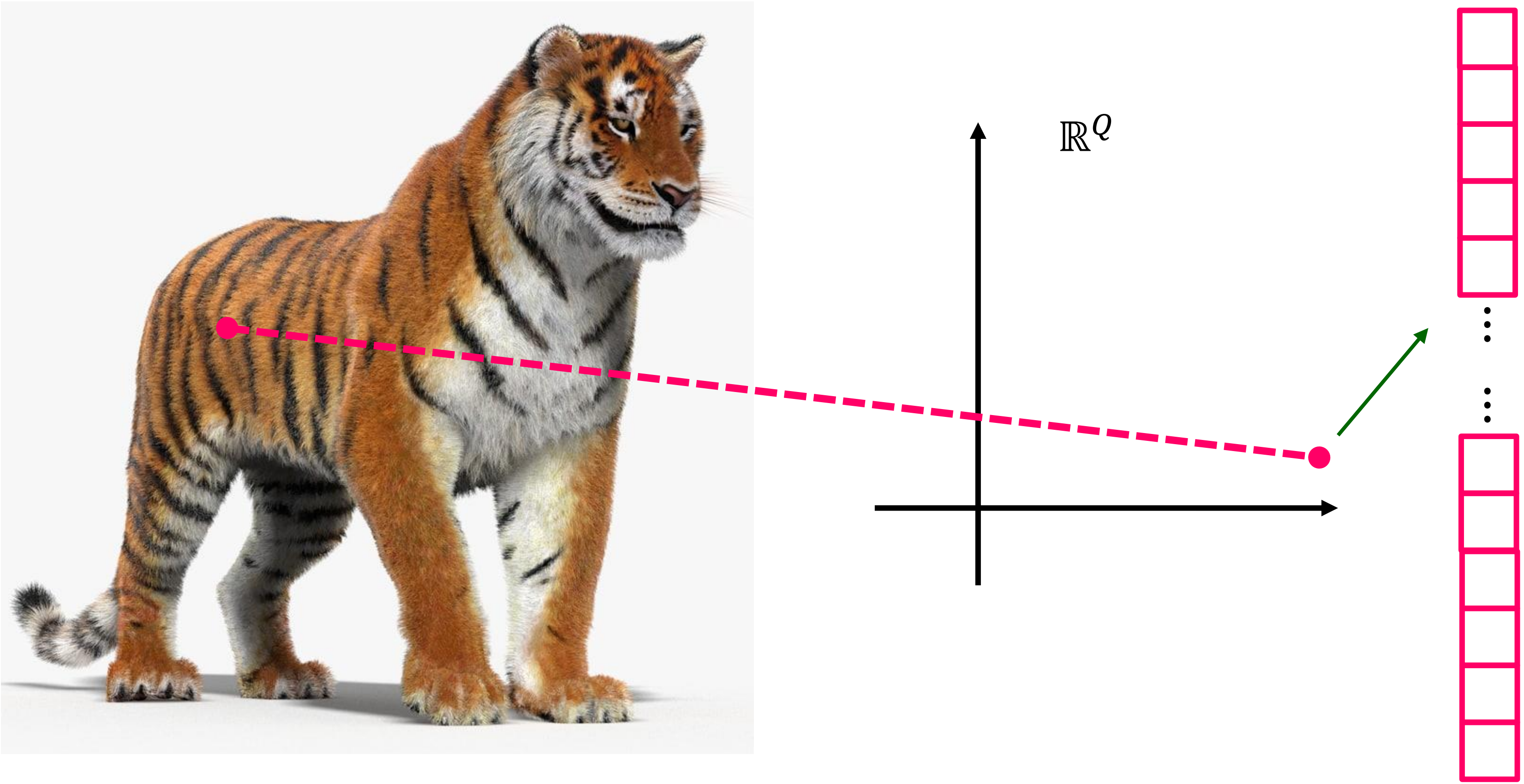
Motivations: point-to-point matching



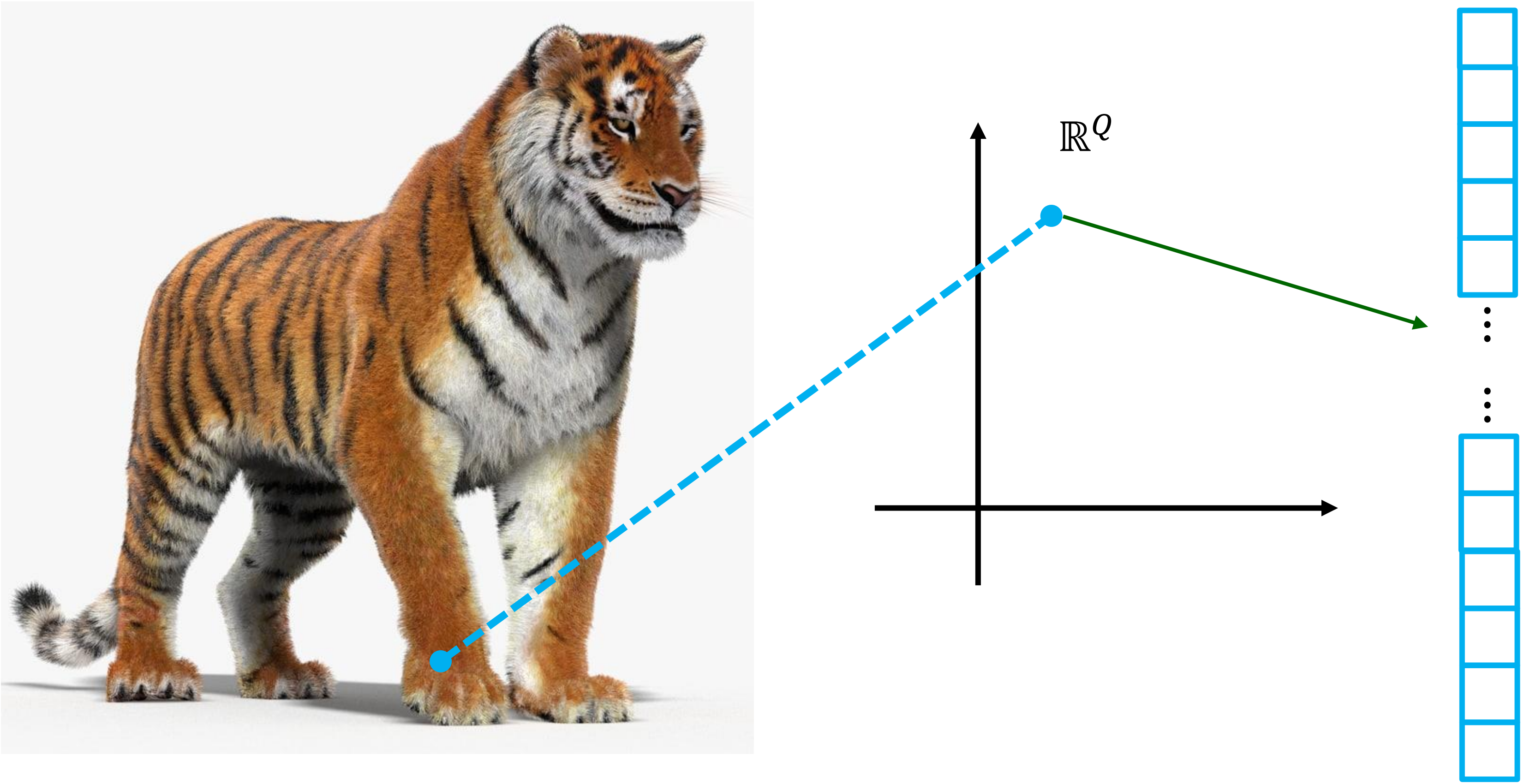
Pointwise signature



Pointwise signature

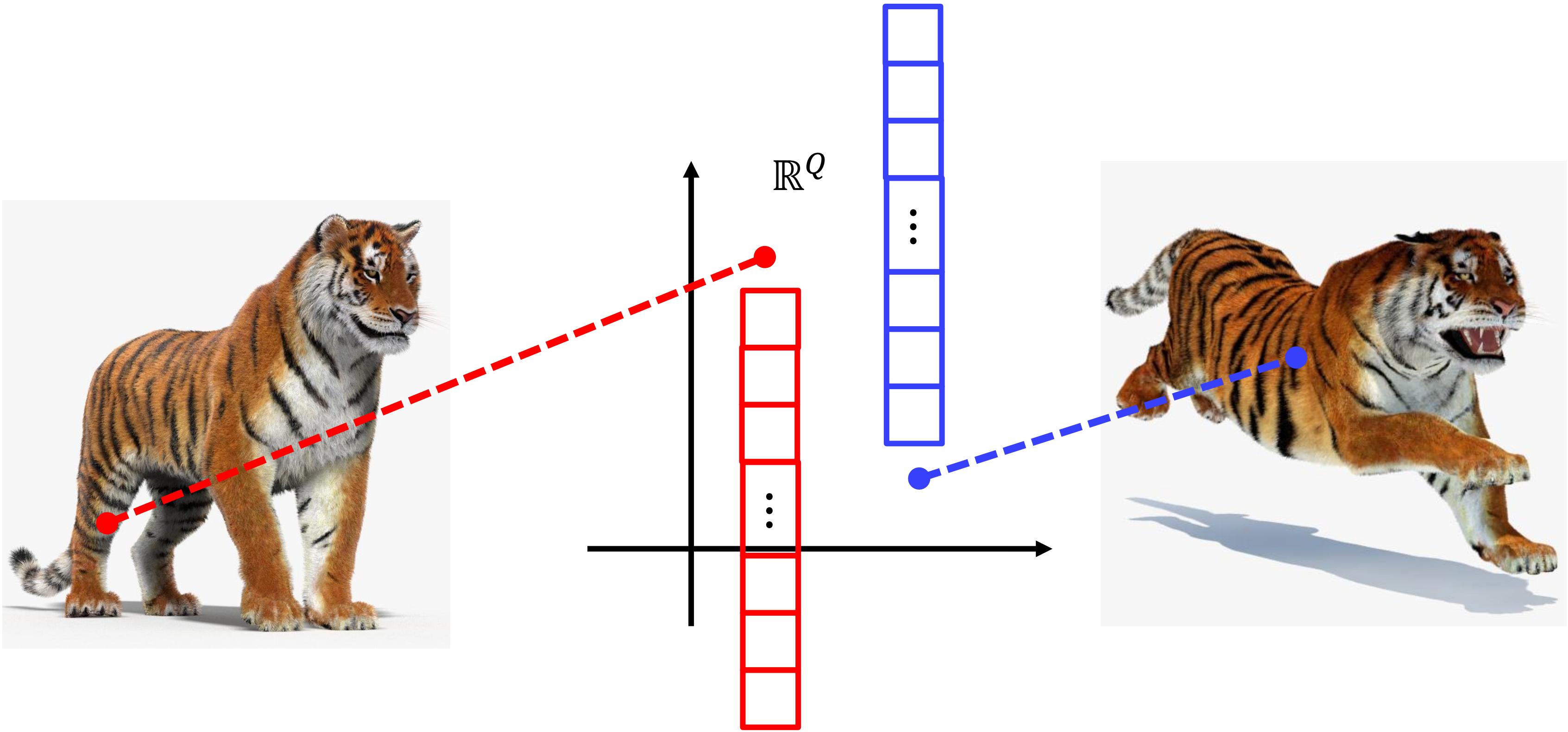


Pointwise signature

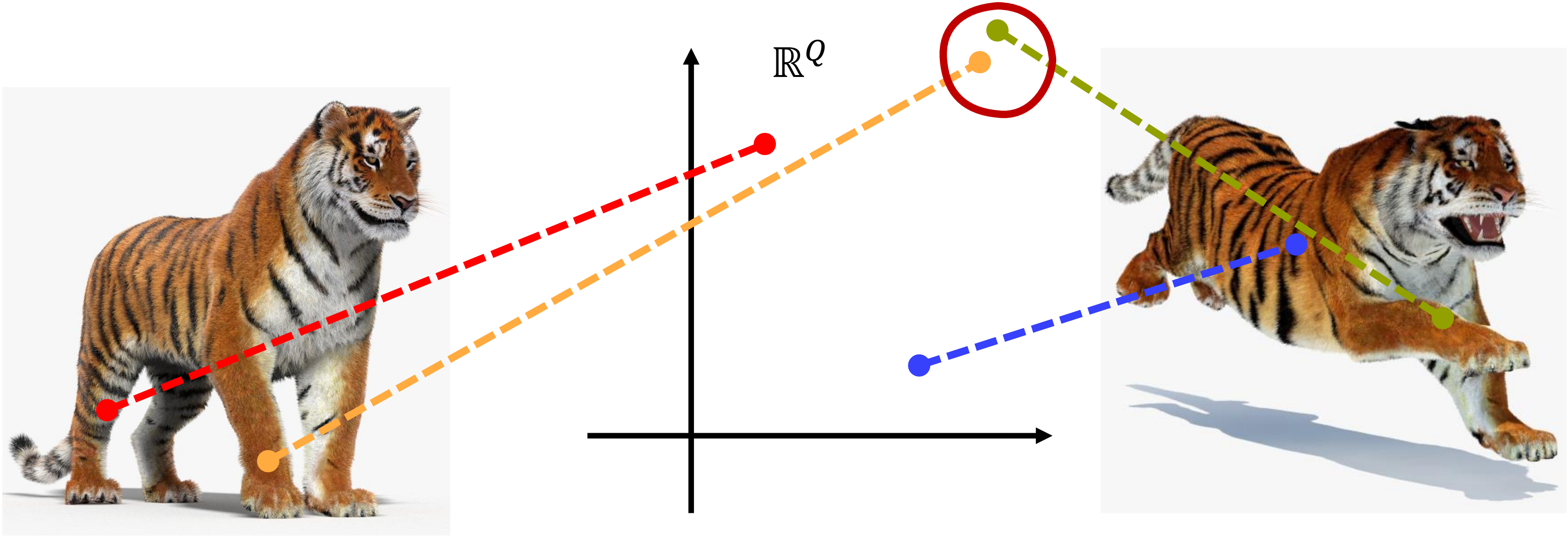


Descriptor for shape matching

Pointwise signature

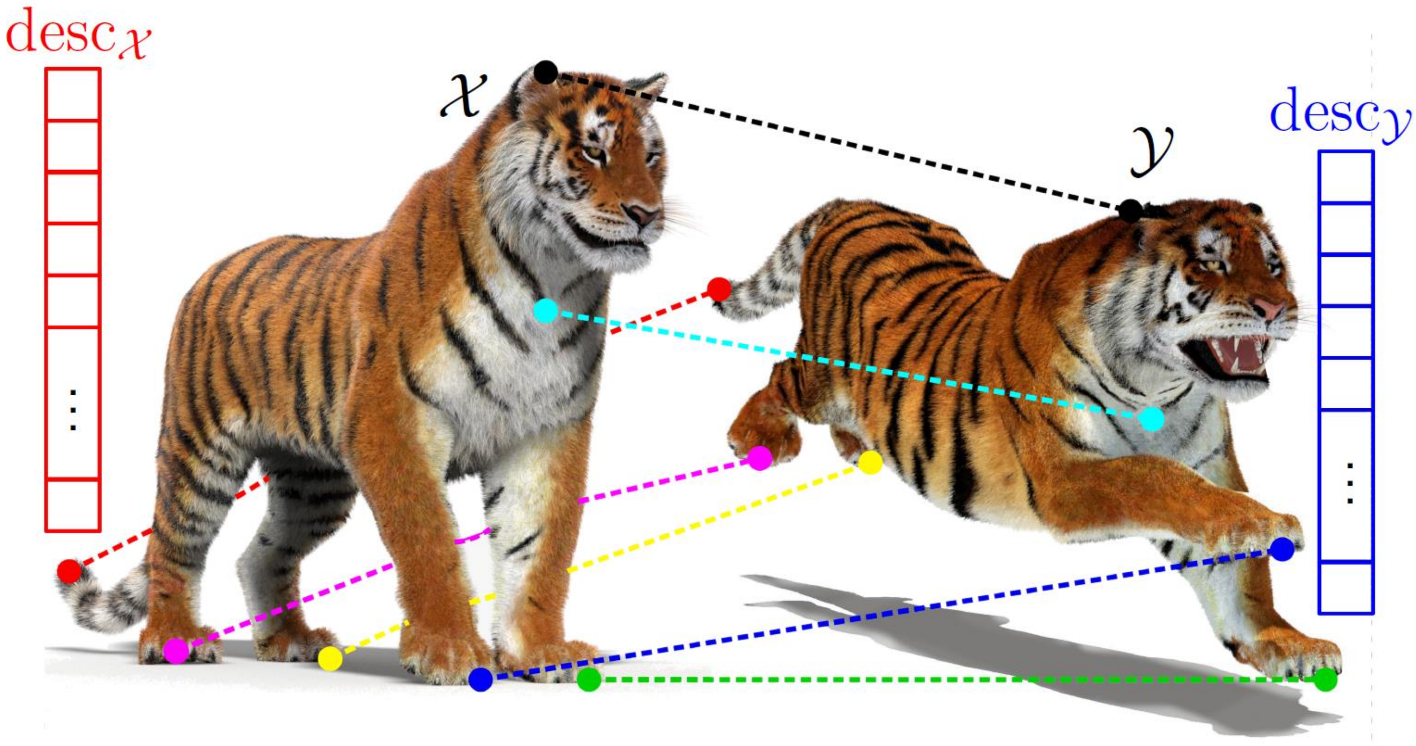


Pointwise signature



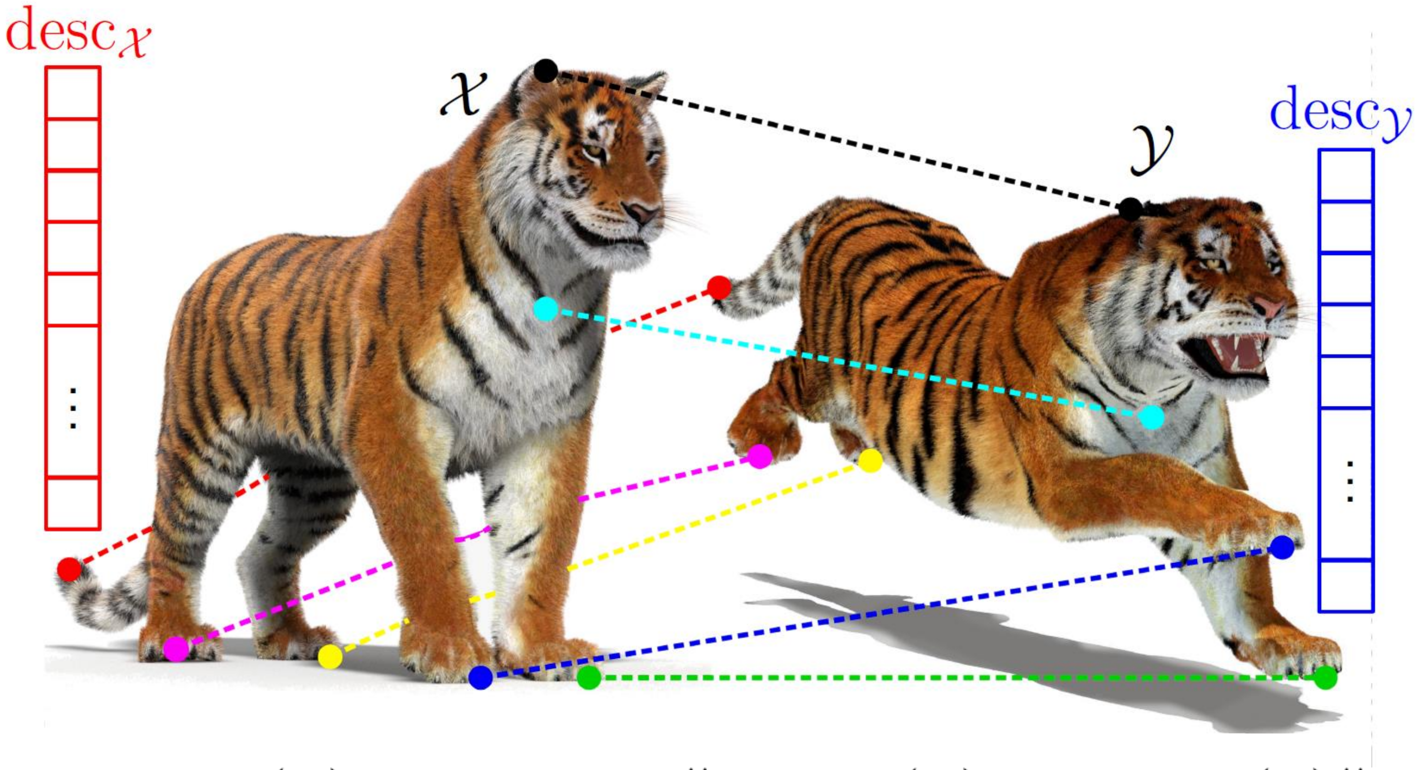
How do you suggest to find the most similar point to the orange one?

Pointwise signature



$$\text{distance} = \mathcal{D}(\mathrm{desc}_{\mathcal{X}}, \mathrm{desc}_{\mathcal{Y}}) = \|\mathrm{desc}_{\mathcal{X}} - \mathrm{desc}_{\mathcal{Y}}\|$$

Pointwise signature



$$y = \Pi(x) = \underset{y \in \mathcal{Y}}{argmin} \| desc_{\mathcal{X}}(x) - desc_{\mathcal{Y}}(y) \|$$

Desired properties

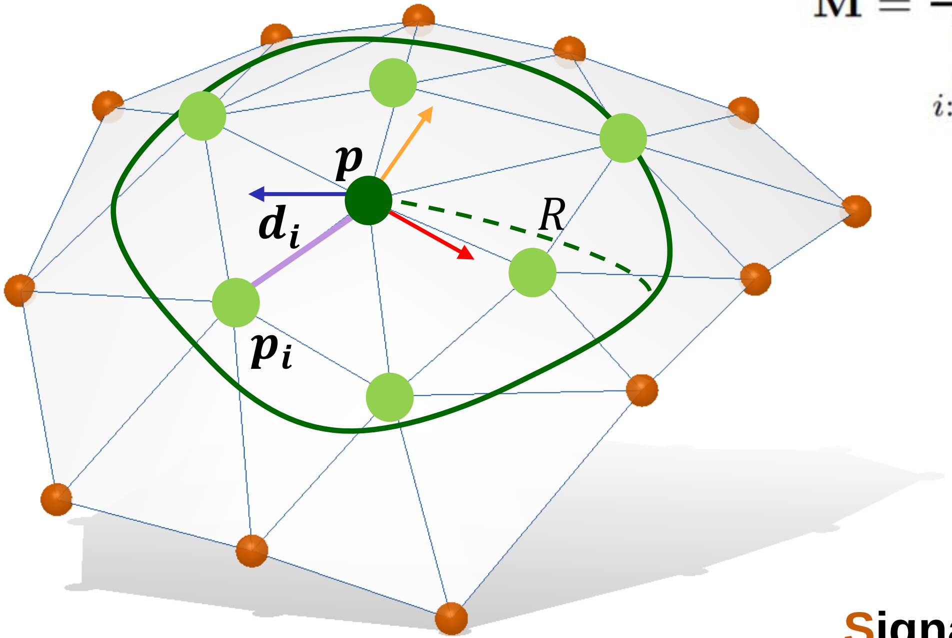
A **descriptor (signature)** should be:

- Effective
- Concise
- Repetable

The **properties of the descriptor** should be evaluated w.r.t. the kind of **deformations** that would be matched (**near isometric tiger deformation**)

SHOT: an example of descriptor

For all $\boldsymbol{p}$ we define the covariance matrix:



$$\mathbf{M} = \frac{1}{\sum_{i:d_i \le R} (R-d_i)} \sum_{i:d_i \le R} (R-d_i)(\mathbf{p}_i - \mathbf{p})(\mathbf{p}_i - \mathbf{p})^T$$

From the eigenvetors of M we obtaine a LRF (x, y, z) that is then used to define:

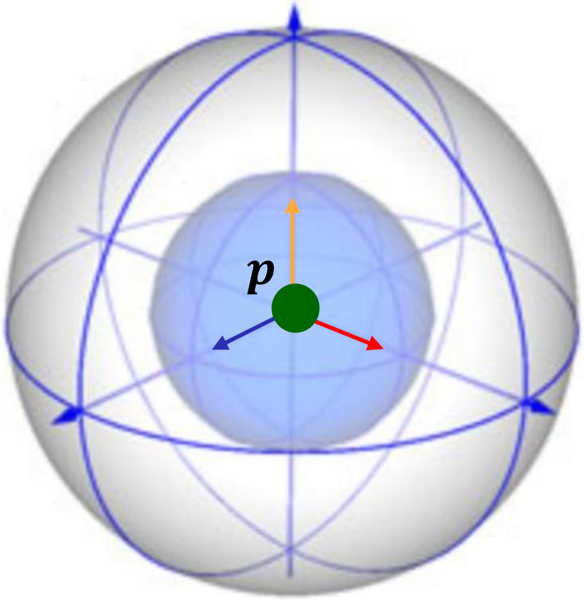
SHOT

Signature of Histograms of OrienTations

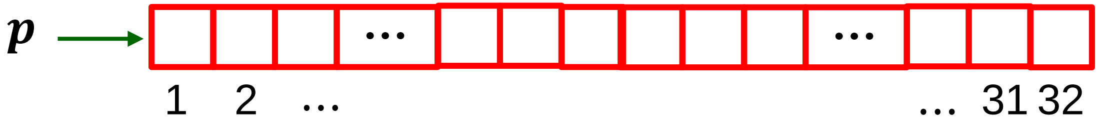
Unique Signatures of Histograms for Local Surface Description, *Tombari et al.*, ECCV 2010

SHOT: Signature of Histograms of OrienTations

Once we have the LRF for every point $\boldsymbol{p}$ we can define a **coherent 3D grid**



The 3D space around $\boldsymbol{p}$ is subdivided in 32 regions each of wich is a different bin of the histogram that describes the point.

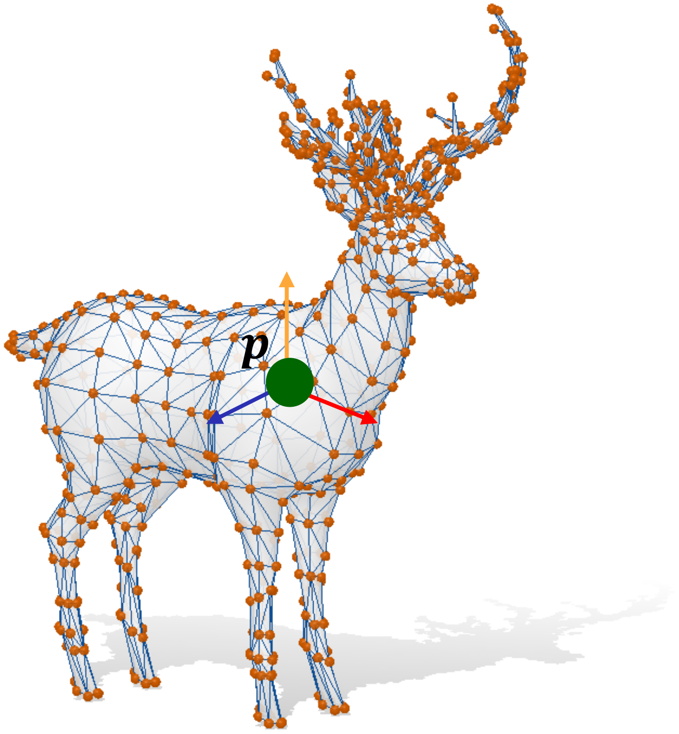


The value of each bin is a weighted sum of $cos\theta_i$ where θ_i is the angle between the normals of the point $\boldsymbol{p}$ and the point within each region of the 3D grid.

Unique Signatures of Histograms for Local Surface Description, *Tombari et al.*, ECCV 2010

SHOT: a comment

SHOT is an extrinsic descriptor: it depends on the 3D embedding of the shape



The analysis for the point $\boldsymbol{p}$ is performed looking at how the shape behaves around the point.

To obtain a coherent description of similar points and to be invariant to rigid deformations the LRF is necessary.

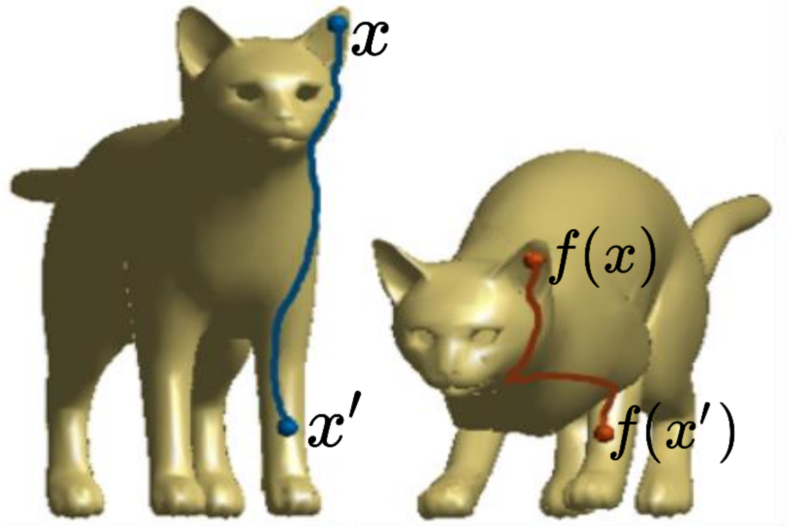
The SHOT descriptors is not invariant to non-rigid deformations.

Unique Signatures of Histograms for Local Surface Description, *Tombari et al.*, ECCV 2010

LBO and isometry invariance

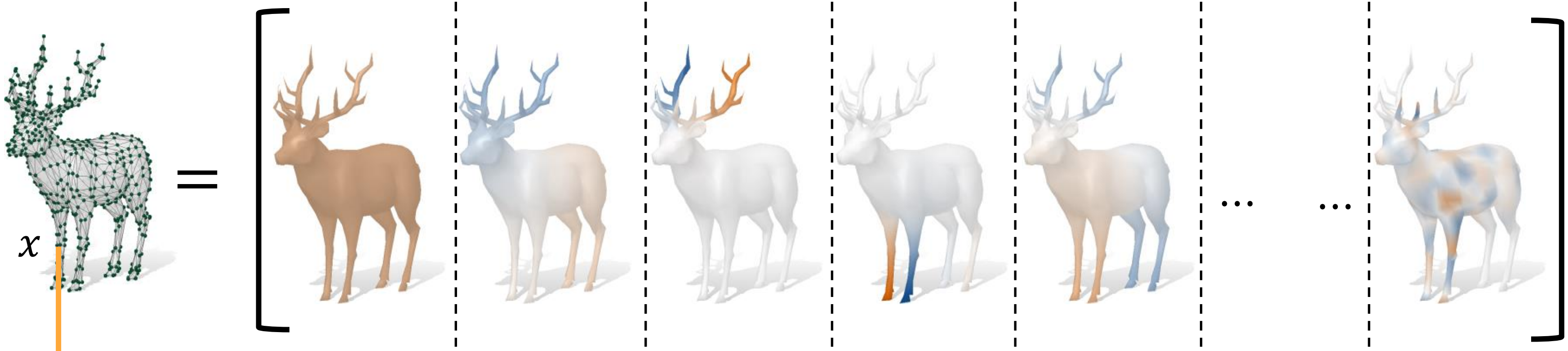
Two shapes are isometric $\Longleftrightarrow$ their LBO agree

$$d_{\mathcal{X}}(x, x') = d_{\mathcal{Y}}(f(x), f(x')),\ \forall\, x, x'$$



Any quantity derived from the LBO is invariant to isometry

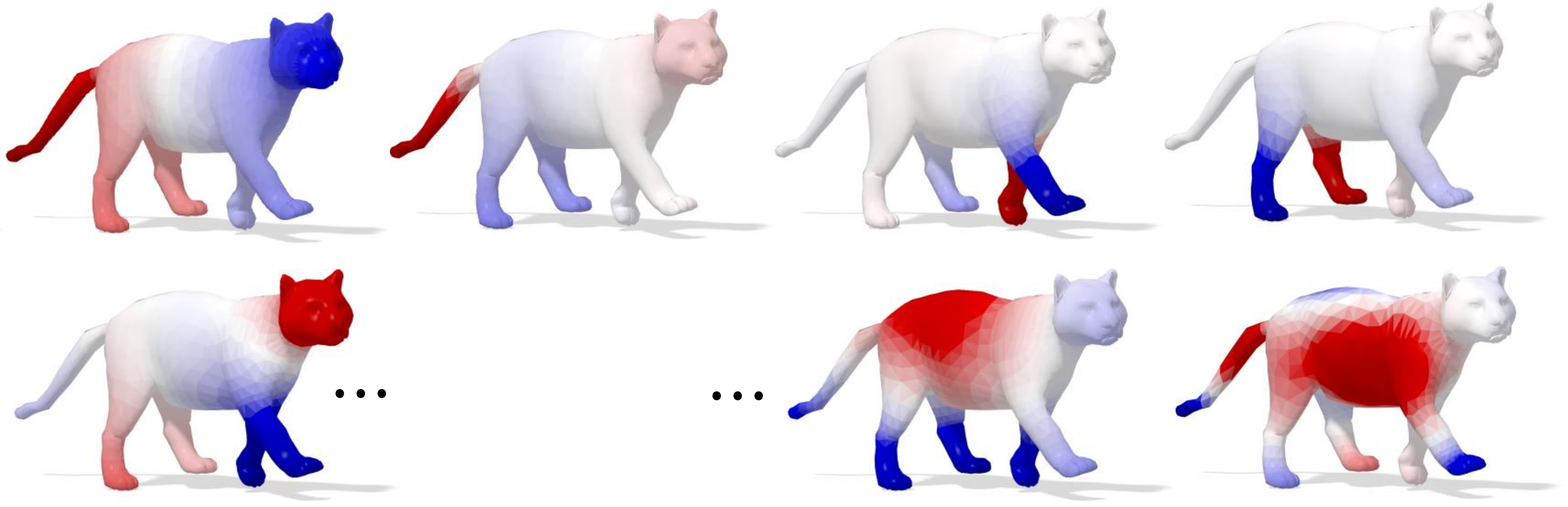
Spectral embedding



$$\boldsymbol{\Phi}(x) = \left[\ \phi_1(x) \ \vdots\ \phi_2(x) \ \vdots\ \phi_3(x) \ \vdots\ \phi_4(x) \ \vdots\ \phi_5(x) \ \vdots\ \cdots \quad \cdots \ \vdots\ \phi_k(x)\ \right]$$

Descriptor for shape matching

GPS = Global Point Signature



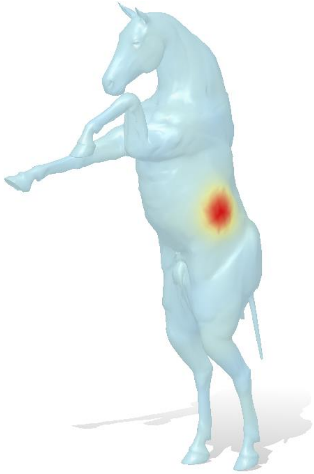
$$GPS(x) = \left[-\frac{1}{\sqrt{\lambda_1}}\phi_1(x), -\frac{1}{\sqrt{\lambda_2}}\phi_2(x), \ldots, -\frac{1}{\sqrt{\lambda_Q}}\phi_Q(x)\right]$$

Descriptor for shape matching

R. Rustamov, Laplace-Beltrami eigenfunctions for deformation invariant shape representation, SGP, 2007

Heat diffusion

$\mathcal{X}$ is a Riemannian surface, $u(x,t)$ is the amount of heat in a point $x \in \mathcal{X}$ at time $t \in \mathbb{R}$



$t = 0$

Given a initial distribution u_0 of heat on $\mathcal{X}$ at time $t = 0$, ($u_0(x) = u(x,0)$)

How is it diffused over time on the surface?

A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion, *Sun et al.*, SGP 2009.

Heat equation

From physics that the heat diffusion is governed by the **heat equation**:

$$\Delta_{\mathcal{X}} u(x,t) = -\frac{\partial u(x,t)}{\partial t}$$

The LBO
=
derivatives in space

derivative in time

$u(x,t)$ solution of the heat equation is a function of $x \in \mathcal{X}$ and time $t \in \mathbb{R}$

which satisfies the **heat equation** for a given initial condition $u_0(x) = u(x,0)$

A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion, *Sun et al.*, SGP 2009.

Heat diffusion solution

Given an initial heat distribution f on $\mathcal{X}$

The solution of the heat diffusion at time $t \in \mathbb{R}$ is given by the **heat operator** H_t

$$H_t = e^{-t\boldsymbol{\Delta}_{\mathcal{X}}}$$

$$e^{-t\lambda_l}$$

$$\boldsymbol{\Delta}_{\mathcal{X}} : \mathcal{F}(\mathcal{X}, \mathbb{R}) \longrightarrow \mathcal{F}(\mathcal{X}, \mathbb{R}) \quad H_t : \mathcal{F}(\mathcal{X}, \mathbb{R}) \longrightarrow \mathcal{F}(\mathcal{X}, \mathbb{R})$$

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Heat diffusion solution

Given an initial heat distribution f on $\mathcal{X}$

$$H_t f(x) = \int_{\mathcal{X}} k_t(x, y) f(y) d\mu(y)$$

$H_t(f)$ is the heat distribution at time $t \in \mathbb{R}$ and H_t is the **heat operator**

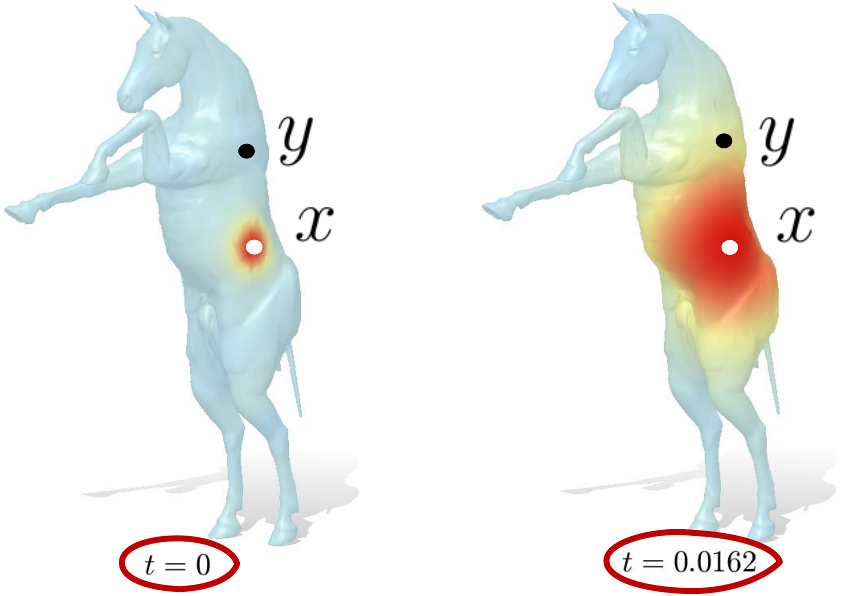
A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion, *Sun et al.*, SGP 2009.

Heat diffusion solution

There is a function $k_t : \mathcal{X} \times \mathcal{X} \longrightarrow \mathbb{R}$ such that:

$$H_t f(x) = \int_{\mathcal{X}} k_t(x, y) f(y) d\mu(y)$$

k_t is the heat kernel and $k_t(x, y)$ corresponds to the heat transferred from x to y in time $t \in \mathbb{R}$

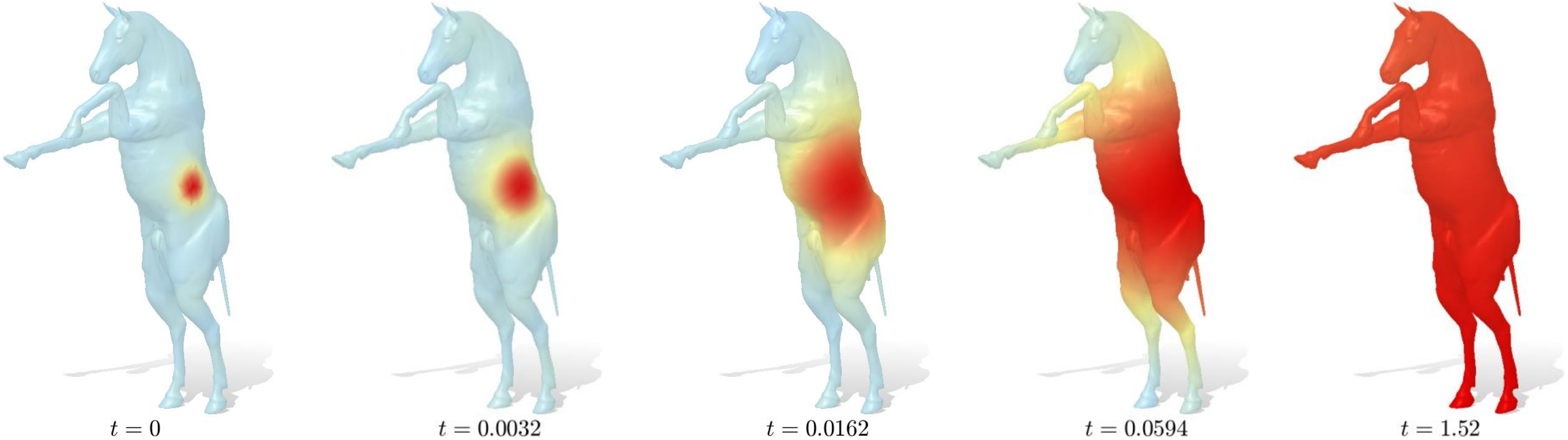


A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion, *Sun et al.*, SGP 2009.

Heat diffusion solution

For an initial delta distribution of heat $\delta_x, \quad x \in \mathcal{X}$

the heat kernel $$k_t(x,y) = \sum_{l=0}^{\infty} e^{-t\lambda_l} \phi_l(x)\phi_l(y)$$



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Heat Kernel signature

For an initial delta distribution of heat $\delta_x, \quad x \in \mathcal{X}$

$$k_t(x, x) = \sum_{l=0}^{\infty} e^{-t\lambda_l} \phi_l(x) \phi_l(x)$$

Is the amount of heat remaining at x after the time $t \in \mathbb{R}$

$$\boldsymbol{HKS}(x) = [k_{t_1}(x, x), k_{t_2}(x, x), \ldots, k_{t_Q}(x, x)] \quad t_1 < t_2 < \ldots t_Q \in \mathbb{R}$$

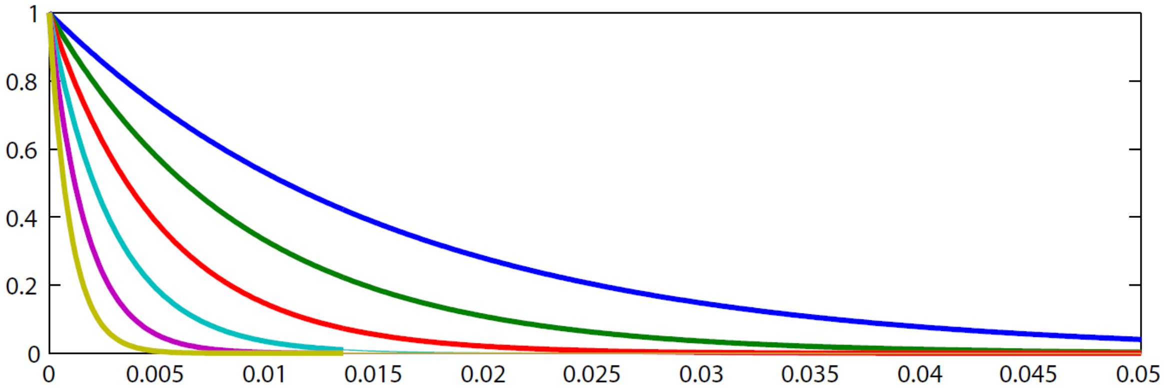
is the heat kernel signature (HKS) at the point $x \in \mathcal{X}$ for a given set of time scales $t_1, \ldots, t_Q$

A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion, *Sun et al.*, SGP 2009.

HKS as a filter on the frequencies

$$k_t(x,x) = \sum_{l=1}^{\infty} e^{-t\lambda_l}\phi_l(x)\phi_l(x) = \sum_{l=1}^{\infty} e^{-t\lambda_l}\phi_l(x)^2$$

$$g_t(\lambda_l) = e^{-t\lambda_l}$$



A low-pass filter applied to the frequencies to produce the HKS

A Concise and Provably Informative Multi-scale Signature Based on Heat Diffusion, *Sun et al.*, SGP 2009.

The wave equation (Schrödinger)

Heat equation: $$\Delta_{\mathcal{X}} u(x,t) = -\frac{\partial u(x,t)}{\partial t}$$

It governs the temporal evolution of a quantum particle

Wave equation: $$i\Delta_{\mathcal{X}} u(x,t) = \frac{\partial u(x,t)}{\partial t}$$

presence of the *i*

missing a minus

It encodes oscillation rather than dissipation as done by the heat equation

Idea: point $x \longleftrightarrow$ the average probabilities of quantum particles of different energies to be measured at x

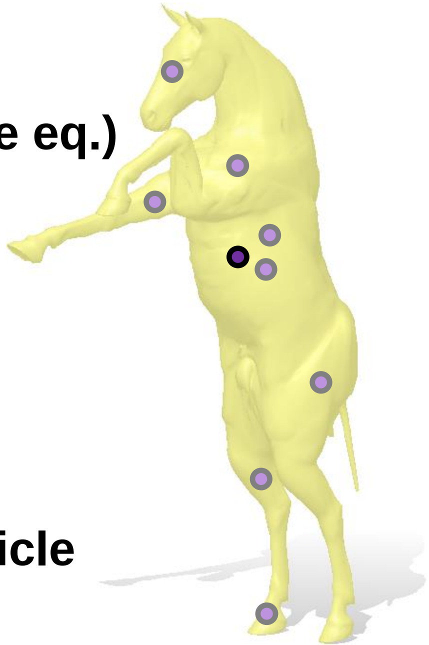
The wave kernel signature: A quantum mechanical approach to shape analysis, *Aubry et al.*, ICCV 2011.

WKS = wave kernel signature

- a quantum particle with unknown position on the surface

f_E^2 the probability distribution with expectation value E estimated at time $t = 0$

$$\psi_E(x,t) = \sum_{k=0}^{\infty} e^{iE_k t}\phi_k(x) f_E(E_k)$$ **= the wave function (solution of the wave eq.)**



$$|\psi_E(x,t)|^2$$ **= the probability to find the particle at (x, t)**

$$\mathrm{WKS}(E,x) = \lim_{T\to\infty} \frac{1}{T}\int_0^T |\psi_E(x,t)|^2$$

= the average probability over the time to find the particle at position $x \in \mathcal{X}$ given the initial energy E

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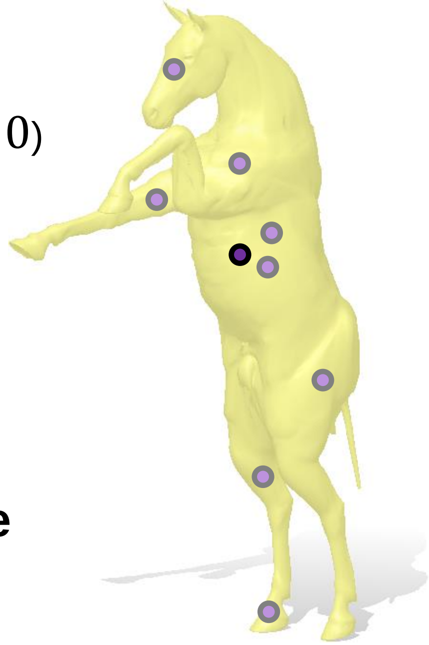
WKS = wave kernel signature

- a quantum particle with unknown position on the surface

f_E^2 the probability distribution with expectation value E (at $t = 0$)

$$WKS(E, x) = \sum_{l=1}^{\infty} f_E(E_l)^2 \phi_l(x)^2$$

= the average probability over the time to find the particle at position $x \in \mathcal{X}$ given the initial energy E



$$WKS(x) = [WKS(E_1, x), WKS(E_2, x), \ldots, WKS(E_Q, x)]$$

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The wave kernel

$$f_E(E_l)^2 = f_E(\lambda_l)^2 = e^{-\frac{(log(E) - log(\lambda_l))^2}{2\sigma^2}}$$

$$k_E(x,x) = WKS(E,x) = \sum_{l=1}^{\infty} f_E(E_l)^2 \phi_l(x)^2$$

$$k_E(x,x) = \sum_{l=1}^{\infty} e^{-\frac{(log(E) - log(\lambda_l))^2}{2\sigma^2}} \phi_l(x)^2$$

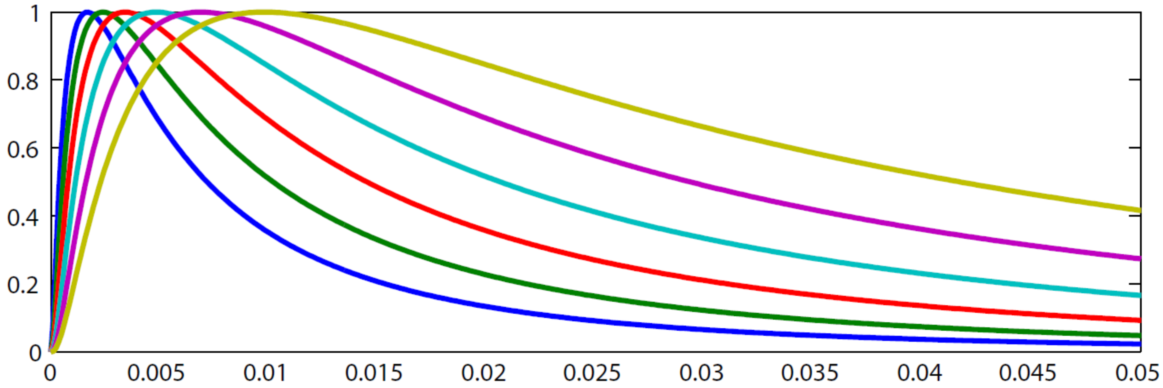
The wave kernel signature: A quantum mechanical approach to shape analysis, *Aubry et al.*, ICCV 2011.

WKS as a filter on the frequencies

$$k_E(x,x) = \sum_{l=1}^{\infty} e^{-\frac{(log(E) - log(\lambda_l))^2}{2\sigma^2}} \phi_l(x)^2$$

$$g_t(\lambda_l) = e^{-\frac{(log(E) - log(\lambda_l))^2}{2\sigma^2}}$$

A band-pass filter applied to the frequencies to produce the WKS



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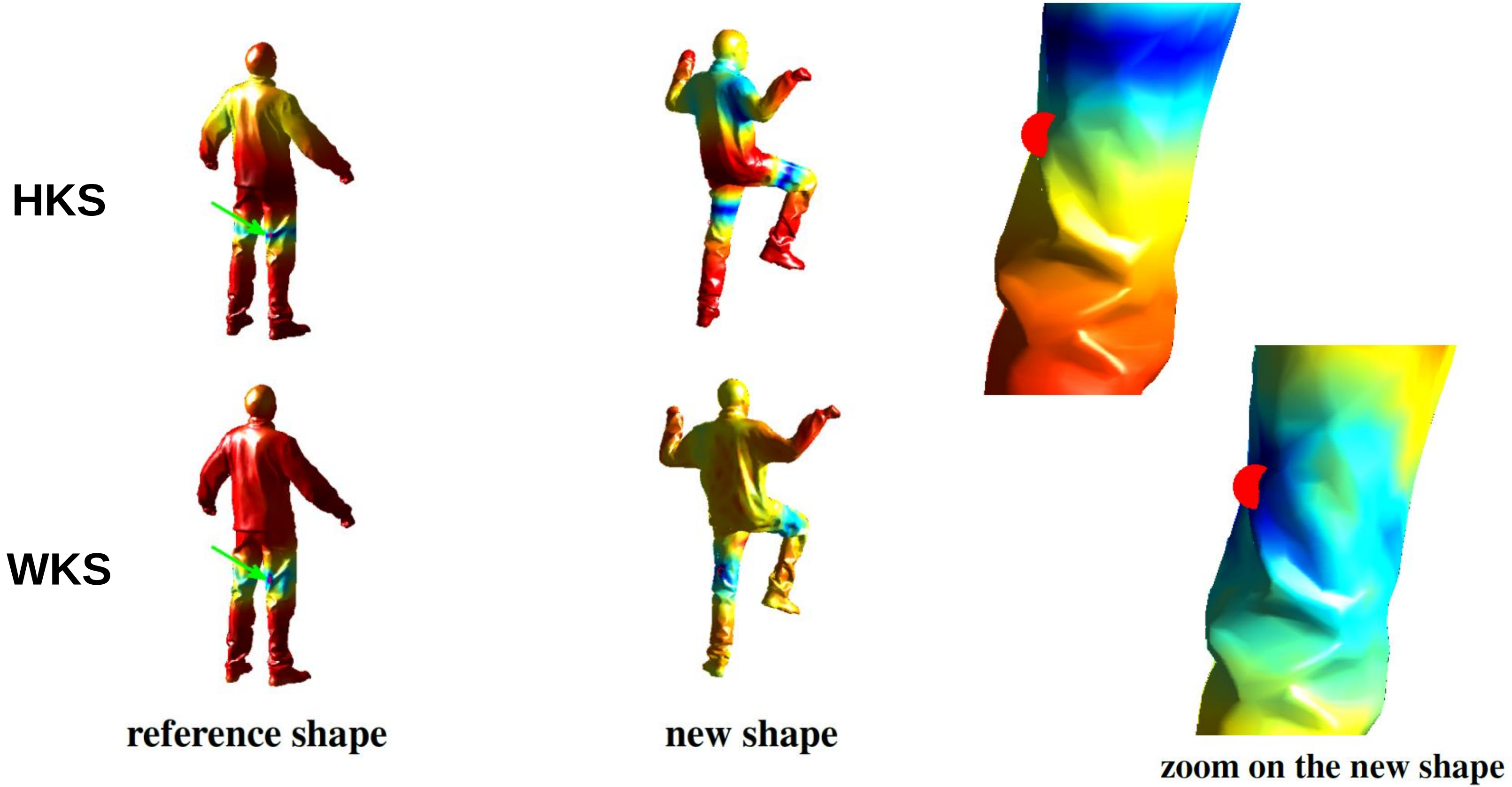
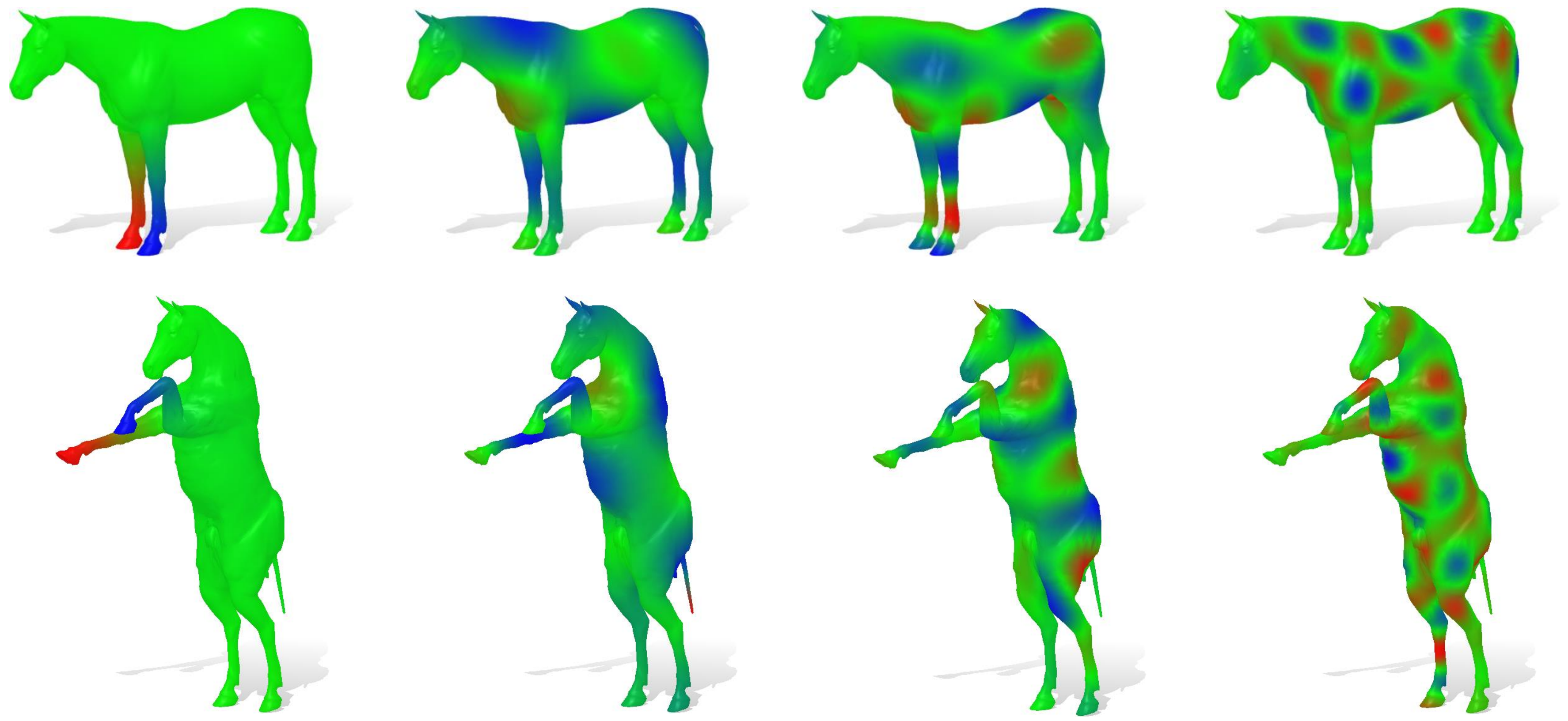
HKS vs WKS



The wave kernel signature: A quantum mechanical approach to shape analysis, *Aubry et al.*, ICCV 2011.

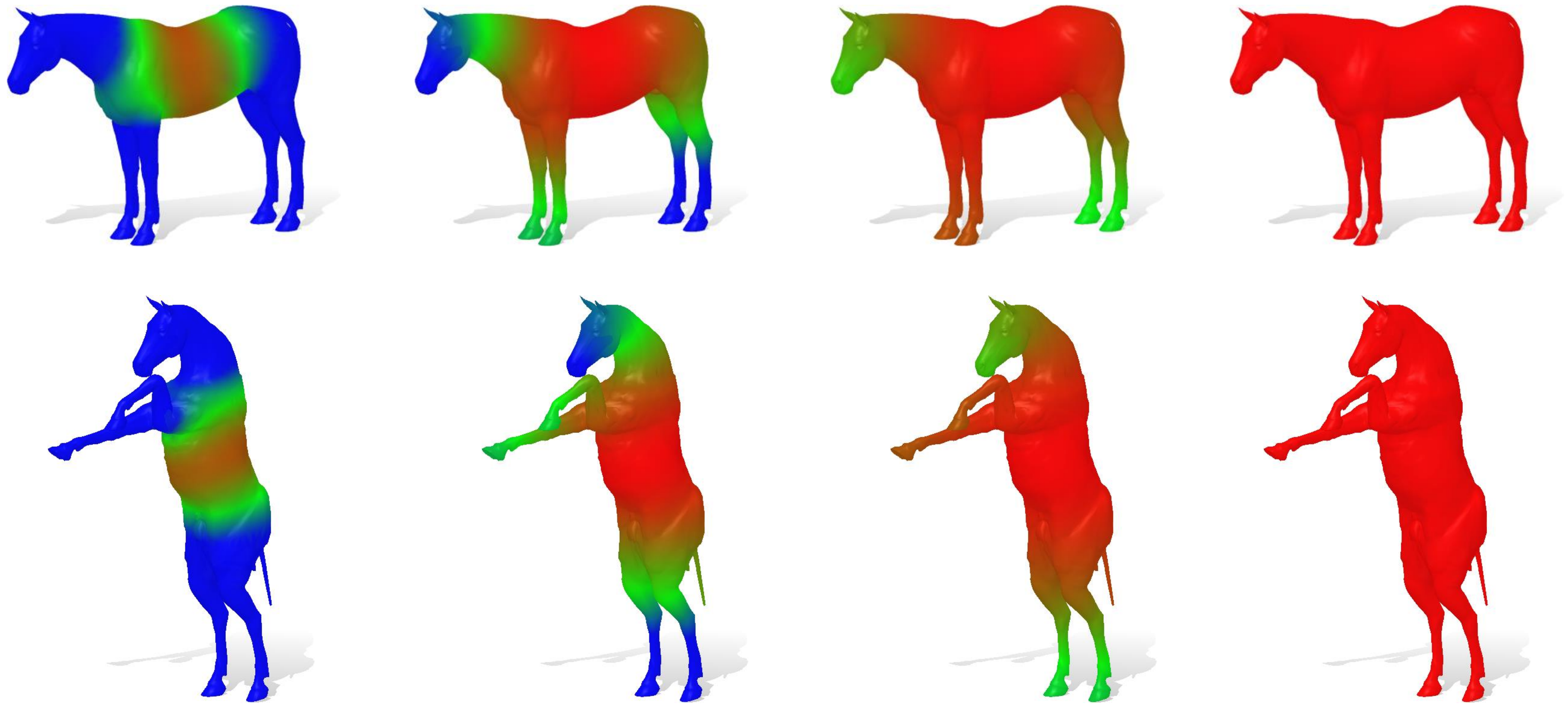
GPS visualization



Descriptor for shape matching

R. Rustamov, Laplace-Beltrami eigenfunctions for deformation invariant shape representation, SGP, 2007

HKS visualization



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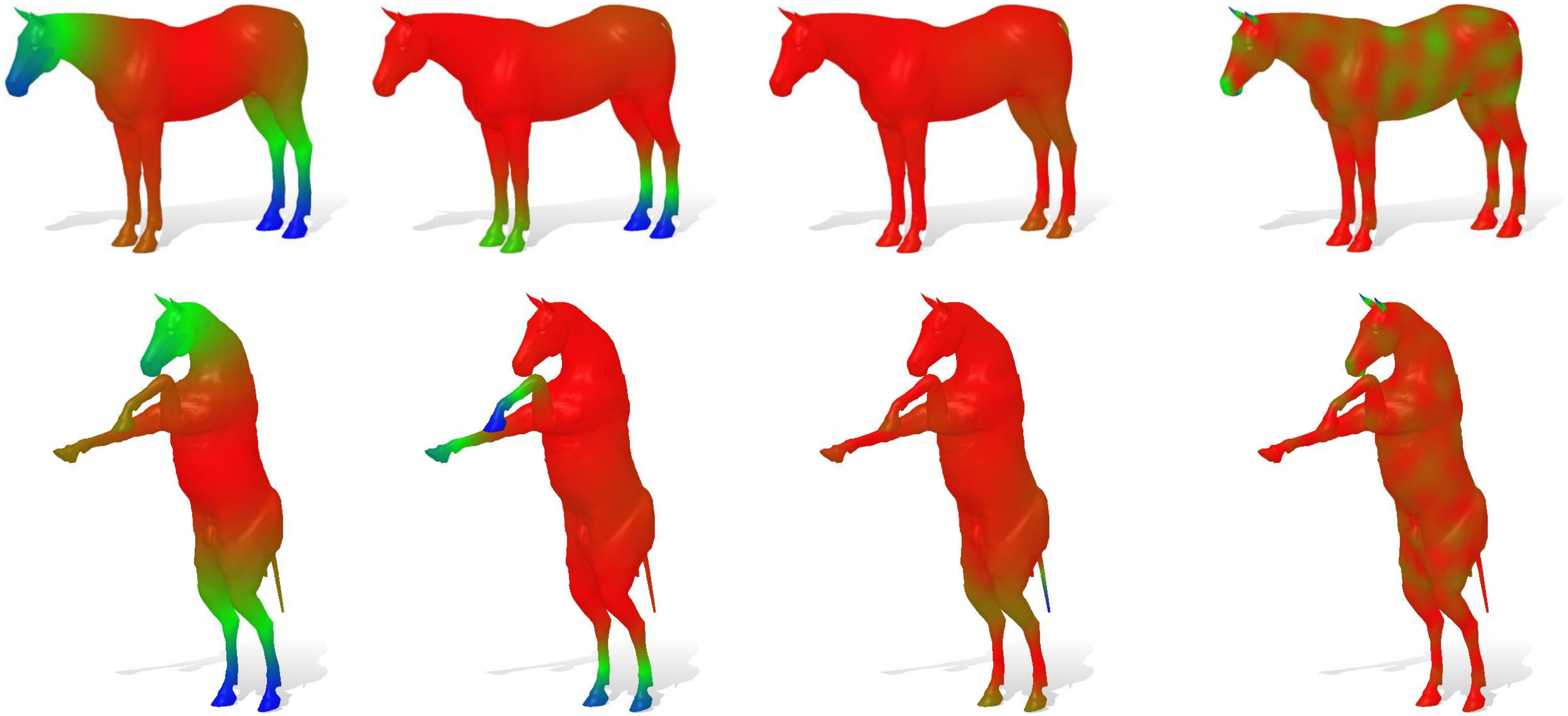
WKS visualization



Descriptor for shape matching

The wave kernel signature: A quantum mechanical approach to shape analysis, *Aubry et al.*, ICCV 2011.

Spectral descriptors

A common structure is shared by the spectral descriptors **HKS** and **WKS**

$$desc_q(x) = \sum_{l=1}^{k} g_{t_q}(\lambda_l)\phi_l^2(x), \qquad \forall q \in 1, \ldots, Q$$

A set of filters on the frequencies
=
functions of the eigenvalues

The square of each dimension of the spectral embedding

A signal processing overview of spectral descriptors

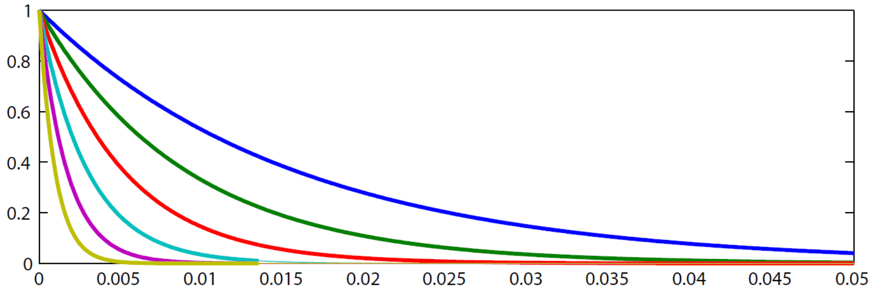
$$desc_q(x) = \sum_{l=1}^{k} g_{t_q}(\lambda_l)\phi_l^2(x), \quad \forall q \in 1, \ldots, Q$$

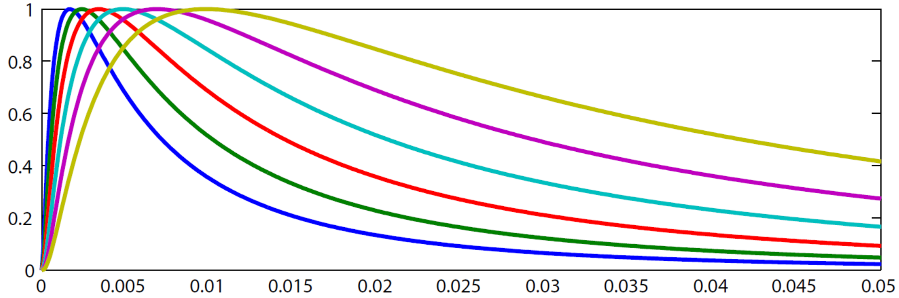
HKS:

$$g_t(\lambda_l) = e^{-t\lambda_l}$$

WKS:

$$g_t(\lambda_l) = e^{-\frac{(log(E) - log(\lambda_l))^2}{2\sigma^2}}$$





Learning spectral descriptors for deformable shape correspondence, *Litman et al.*, PAMI 2014.

How could we obtain stronger spectral descriptors

$$desc_q(x) = \sum_{l=1}^{k} g_{t_q}(\lambda_l)\phi_l^2(x), \quad \forall q \in 1, \ldots, Q$$

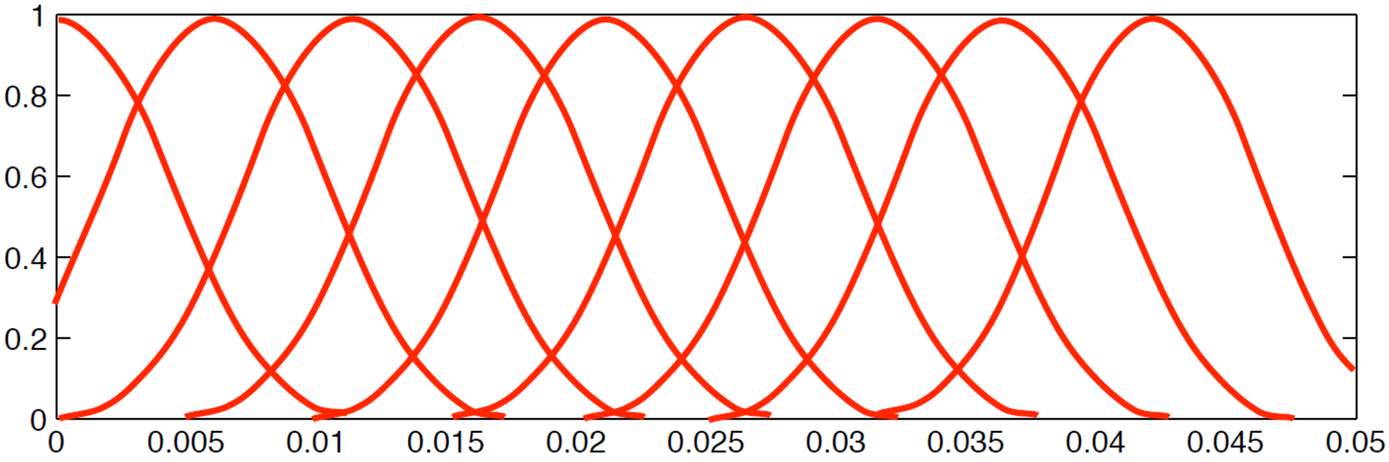
What are the best filters to apply in this equation to obtain the best descriptors?

Can we learn them?

Learning spectral descriptors for deformable shape correspondence, *Litman et al.*, PAMI 2014.

Learn filter for spectral descriptors

Given a set of basis functions : $\beta_1(\lambda), \beta_2(\lambda), \ldots, \beta_Z(\lambda)$



We can learn the best coefficients to linearly combine them to obtain the best filters.

Learning spectral descriptors for deformable shape correspondence, *Litman et al.*, PAMI 2014.

What do we need to learn?

The q-th filter is obtained as a linear combination of the basis functions $\{\beta_z(\lambda_l)\}_{z=1}^{Z}$

$$g_q(\lambda_l) = \left(\sum_{z=1}^{Z} a_z^q \beta_z(\lambda_l) \right)$$

$$desc_q(x) = \sum_{l=1}^{k} g_{t_q}(\lambda_l)\phi_l^2(x), \quad \forall q \in 1, \ldots, Q$$

$$desc_q(x) = \sum_{l=1}^{k} \left(\sum_{z=1}^{Z} a_z^q \beta_z(\lambda_l) \right) \phi_l^2(x)$$

Learning spectral descriptors for deformable shape correspondence, *Litman et al.*, PAMI 2014.

What do we need to learn?

$$desc_q(x) = \sum_{l=1}^{k} \left(\sum_{z=1}^{Z} a_z^q \beta_z(\lambda_l) \right) \phi_l^2(x)$$

we should learn the set of coefficients:

$$a_z^q \ \forall q = 1, \dots, Q \text{ and } z = 1, \dots, Z$$

that is equivalent to learn a matrix:

$$\boldsymbol{A} \in \mathbb{R}^{Q \times Z} \text{ s.t. } \boldsymbol{A}_{q,z} = a_z^q$$

Learning spectral descriptors for deformable shape correspondence, *Litman et al.*, PAMI 2014.

Learned descriptors

We can compute a learned kernel signature by learning the matrix $\boldsymbol{A} \in \mathbb{R}^{Q \times Z}$

$$\boldsymbol{LKS}(x) = [desc_1^{\boldsymbol{A}}(x), desc_2^{\boldsymbol{A}}(x), \ldots, desc_q^{\boldsymbol{A}}(x)]$$

These explixitly depend on the learned matrix

How could we learn this matrix A?

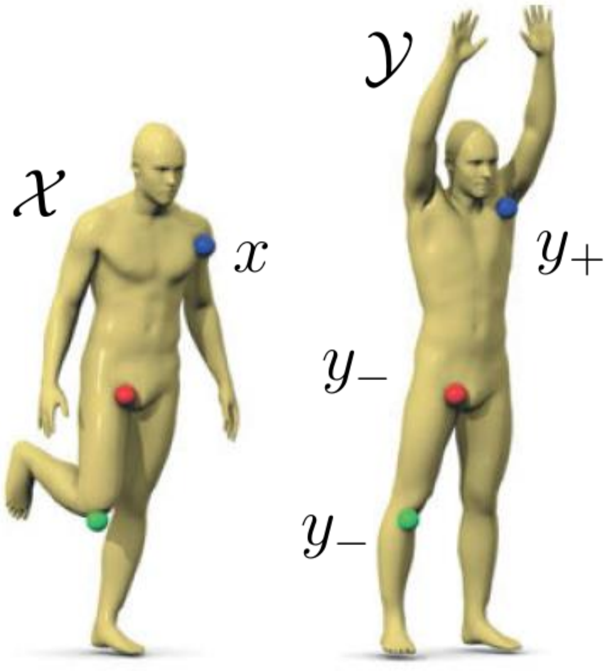
Learning spectral descriptors for deformable shape correspondence, *Litman et al.*, PAMI 2014.

Loss definition

Given a pair of shapes $\mathcal{X}$ and $\mathcal{Y}$

We consider a set of points X on $\mathcal{X}$ such that $\forall x \in X$ we can define a set of points Y on $\mathcal{Y}$ that is composed by:

- similar points (**positive**) y_+
- dissimilar points (**negative**) y_-

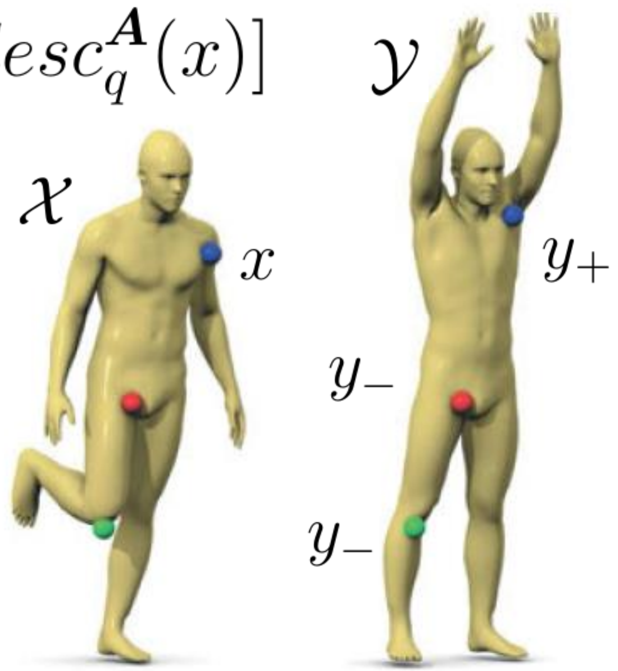


Learning spectral descriptors for deformable shape correspondence, *Litman et al.*, PAMI 2014.

Loss definition

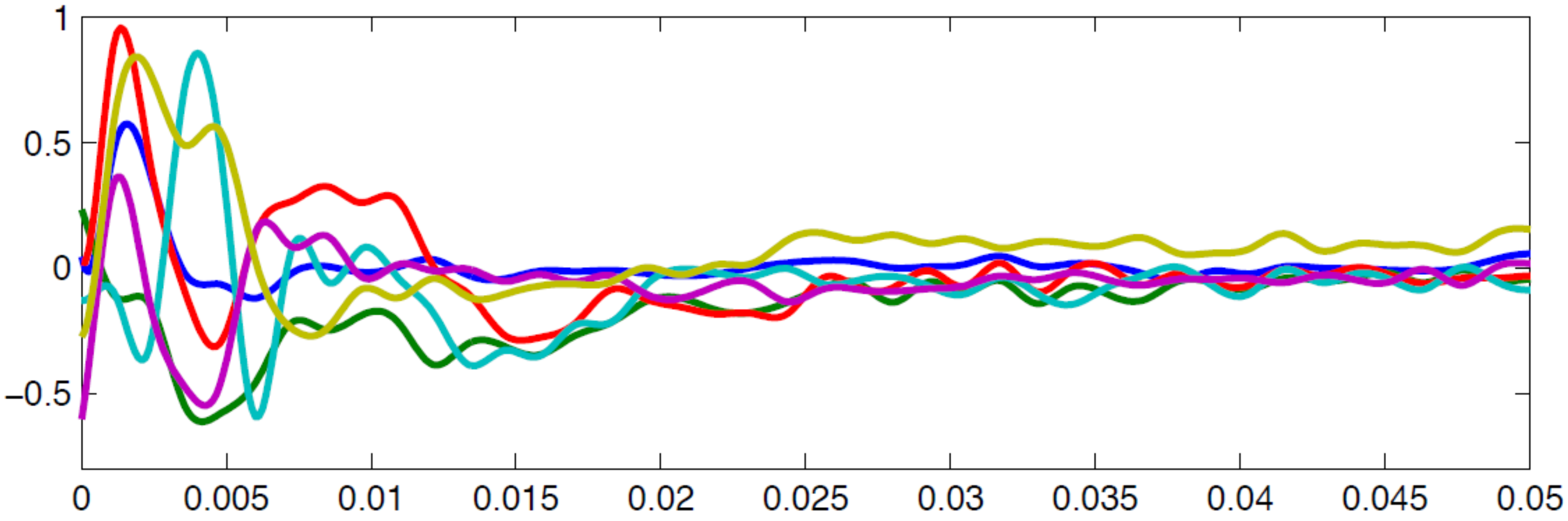
$$\boldsymbol{LKS}(x) = [desc_1^{\boldsymbol{A}}(x), desc_2^{\boldsymbol{A}}(x), \ldots, desc_q^{\boldsymbol{A}}(x)]$$

$$\underset{\boldsymbol{A}}{argmin} \sum_{x \in X} \gamma(\|LKS(x) - LKS(y_+)\|^2)$$
$$-(1-\gamma)(\|LKS(x) - LKS(y_-)\|^2)$$



Learning spectral descriptors for deformable shape correspondence, *Litman et al.*, PAMI 2014.

Learned filter



Learning spectral descriptors for deformable shape correspondence, *Litman et al.*, PAMI 2014.

Fourier analysis

$$f(x) = \sum_{k\geq 1} \underbrace{\int_X f(\xi)\phi_k(\xi)d\xi}_{\mathcal{F}(f)_k = \langle f, \phi_k \rangle_{L^2(X)}} \phi_k(x)$$

The Fourier coefficients depend on the Global gerometry of the surface

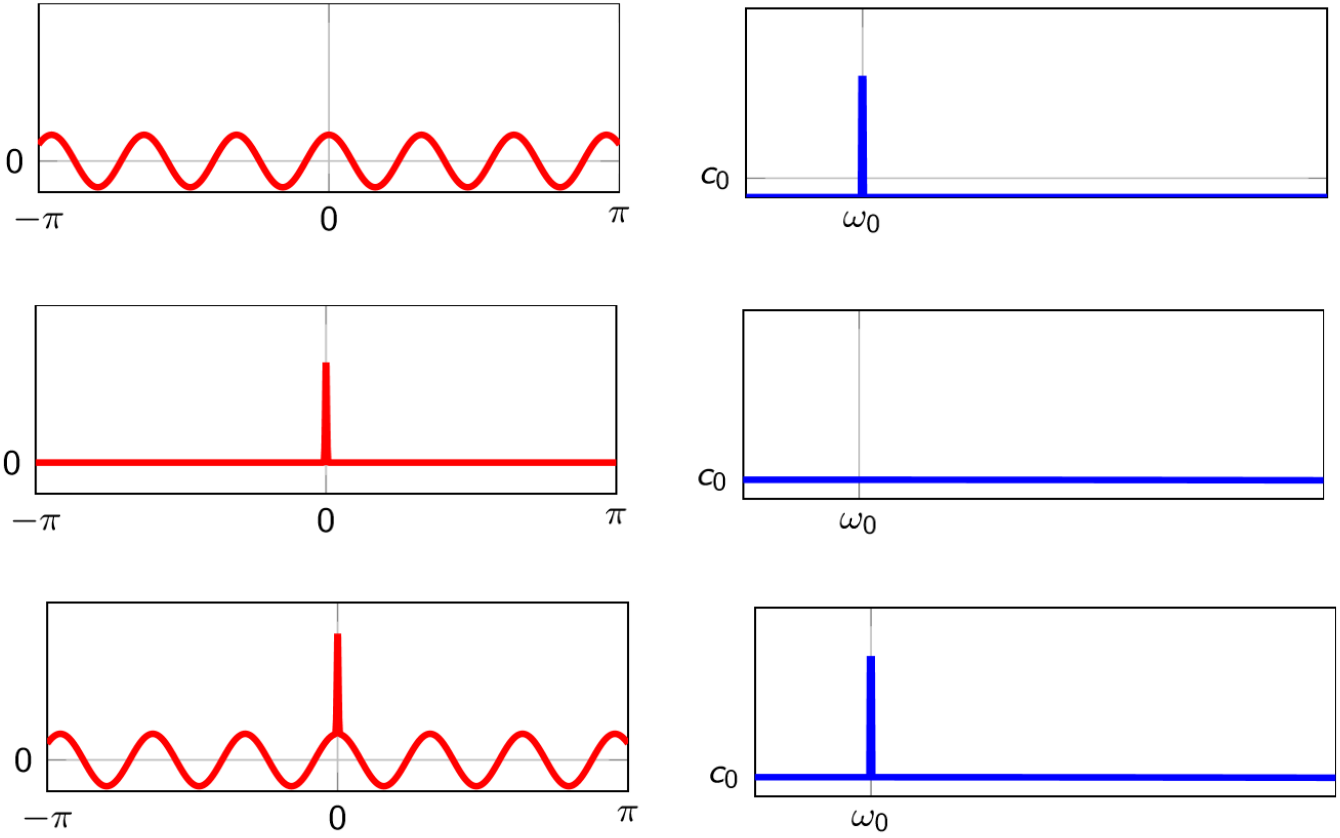
The only characterization of these coefficients is the frequency that each of them is representing

We want to compute pointwise descriptors

$\Downarrow$

We would like to enforce LOCALIZATION of the Fourier analysis

Fourier and the need for localization



Poor spatial localization

$\Updownarrow$

Good spectral localization

Good spatial localization

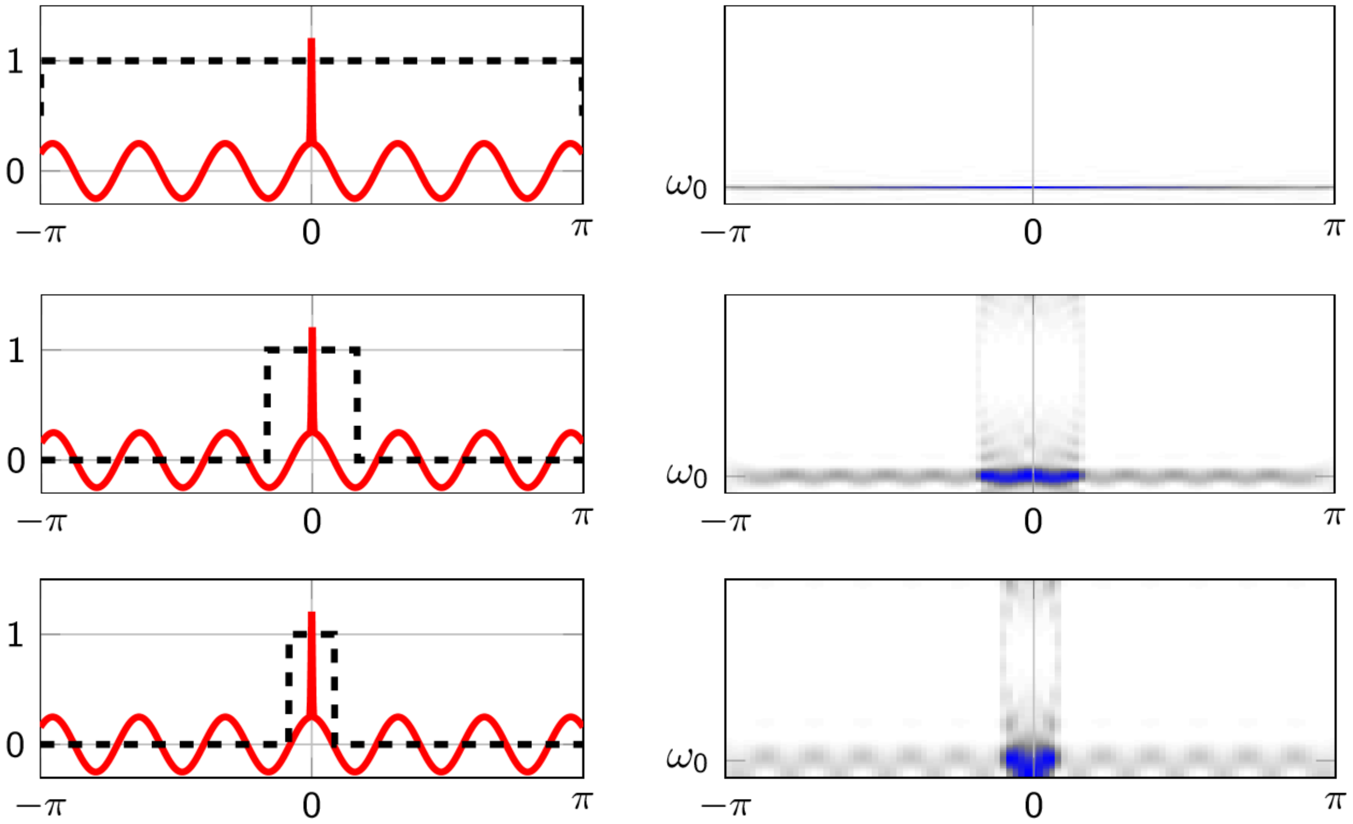
$\Updownarrow$

Poor spectral localization

QUESTION:
Do you know how this issue has been solved In standard signal processing?

Learning class-specific descriptors for deformable shapes using localized spectral convolutional networks, *Boscaini et al. SGP 2015*

WFT is the standard solution



Windowed Fourier Transform of a signal enveloped by a **window** g

$$WFT(f)(\xi,\omega) = \underbrace{\int_D f(x)g(x-\xi)e^{-i\omega x}}_{\langle f(x), g_{\xi,\omega}(x)\rangle_{L^2(D)}}$$

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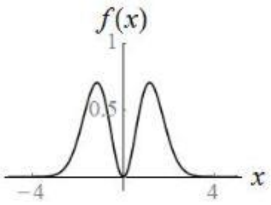
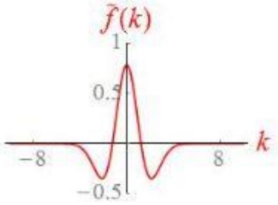
WFT

In some case the signal drastically changes in the space-time domain. → *It is desirable to localize signal and analyze it locally.*

The **Windowed Fourier Transform (WFT)** is the solution for this problem

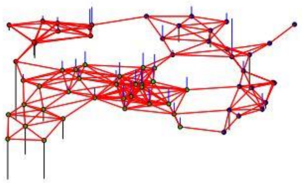
Euclidean domains

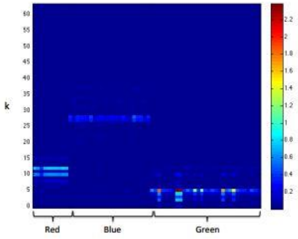




→

Graphs





→ **And on manifolds ?**

Vertex-frequency analysis on graphs, Shuman et al., 2011

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Theorem of convolution

Convolution on Euclidean domain : $[-\pi, \pi]$ of two functions $f, g: [-\pi, \pi] \to \mathbb{R}$
Is defined as:

$$(f \star g)(x) = \int_{-\pi}^{\pi} f(x')g(x - x')dx'$$

Convolution Theorem: Fourier transform diagonalizes the convolution operator.

$$\Rightarrow \boldsymbol{Convolution\ can\ be\ computed\ in\ the\ spectral\ domain}$$

$$\widehat{(f \star g)} = \hat{f} \cdot \hat{g}$$

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WFT on non-Euclidean manifolds

Euclidean domains	Manifolds
Fourier Transform: $\hat{f}_\omega = \langle f,\ e^{2\pi i \omega x} \rangle$	Fourier Transform: $\hat{f}_k = \langle f, \phi_k \rangle_{L^2(X)}$
Convolution: $(f * g)(x) = \int_{-\infty}^{\infty} \hat{f}_\omega \hat{g}_\omega e^{2\pi i \omega x} dx$	Convolution: $(f \star g)(x) = \sum_{k \geq 1} \hat{f}_k \hat{g}_k \phi_k(x)$
Translation: $(T_u f)(x) = (f * \delta_u)(x) = f(x - u)$	Translation: $(T_{x'} f)(x) = (f \star \delta_{x'})(x) = \sum_{k \geq 1} \hat{f}_k \phi_k(x') \phi_k(x)$
Modulation: $(M_\omega f)(x) = e^{i \omega x} f(x)$	Modulation: $(M_k f)(x) = \phi_k(x) f(x)$

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WFT on non-Euclidean manifolds

Euclidean domains

Basic Atom: $g_{u,\ \omega}(x) = (M_\omega T_u g)(x) = g(x-u)e^{2\pi i\omega x}$

Windowed Fourier Transform: $Sf(u,\omega) = \langle\ f,\ g_{u,\ \omega}\ \rangle$

Manifolds

Basic Atom: $g_{x',k}(x) = (M_k T_{x'} g)(x) = \phi_k(x) \sum_{l\geq 1} \hat{g}_l \phi_l(x')\phi_l(x)$

Windowed Fourier Transform: $(Sf)_{x,k} = \langle f, g_{x,k}\rangle_{L^2(X)}$

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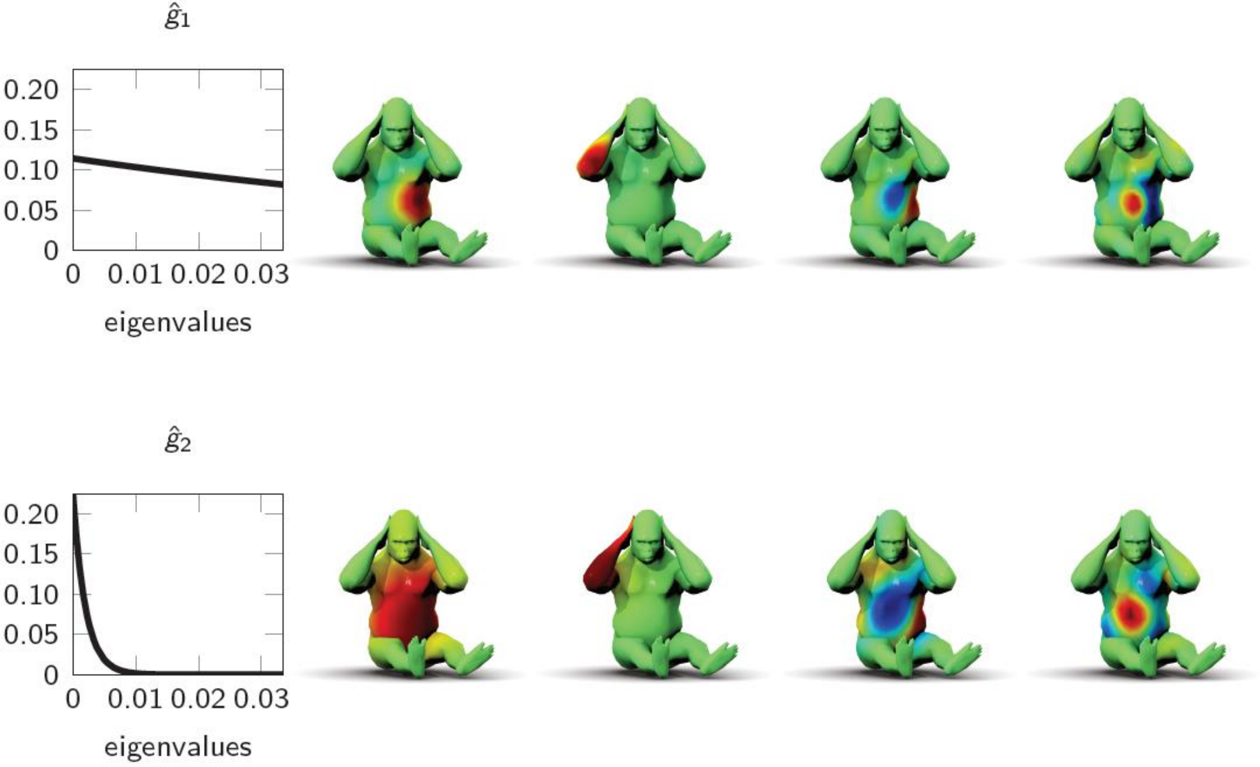
WFT atoms



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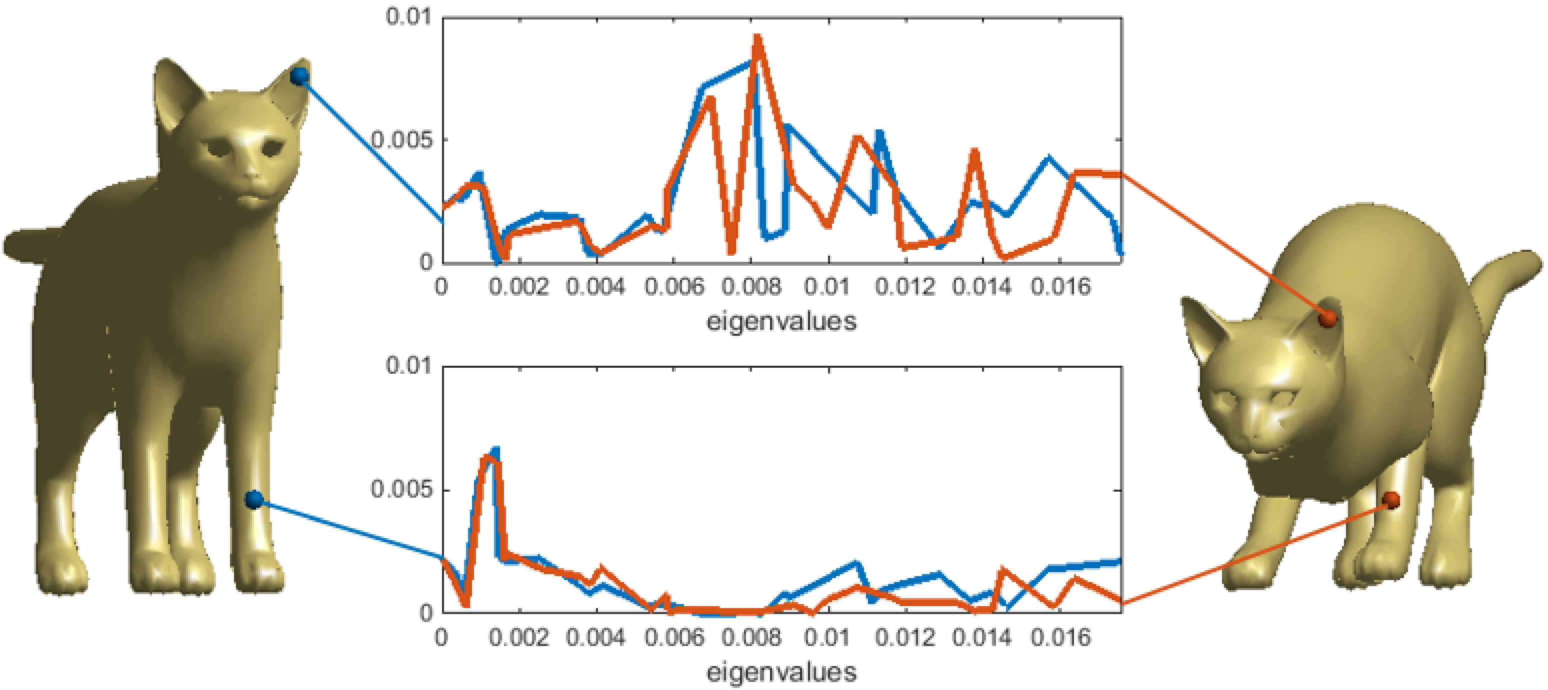
WFT problems



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Learn a new spectral descriptor

Given a set of $P \in \mathbb{N}$ functions $f_1, \ldots, f_P : \mathcal{X} \longrightarrow \mathbb{R}$

$$LSCNN_q(x) = \mathfrak{F}(f_1, \ldots, f_P), \ \forall q = 1, \ldots, Q$$

$$LSCNN_q(x) = \sum_{p=1}^{P} \left(\sum_{k=1}^{K} a_{q,p,k} (Sf_p)_{x,k} \right)$$

The Windowed Fourier atoms for the function *f* with translation in *x* and modulation at frequency *k*

QUESTION:
what is not defined?

Learning class-specific descriptors for deformable shapes using localized spectral convolutional networks, *Boscaini et al. SGP 2015*

Learn the window for each input function

It is easier to learn it in the spectral domain!

We already did something similar, do you remeber where?

In the definition of the optimal shape descriptor!

The windows are obtained

as a linear combination of the

basis functions $\{\beta_z(\lambda_l)\}_{z=1}^{Z}$

$$g_p(\lambda_k) = \sum_{z=1}^{Z} b_z^p \beta_z \lambda_k$$

Learning class-specific descriptors for deformable shapes using localized spectral convolutional networks, *Boscaini et al. SGP 2015*

Localized Spectral CNN descriptor (LSCNN)

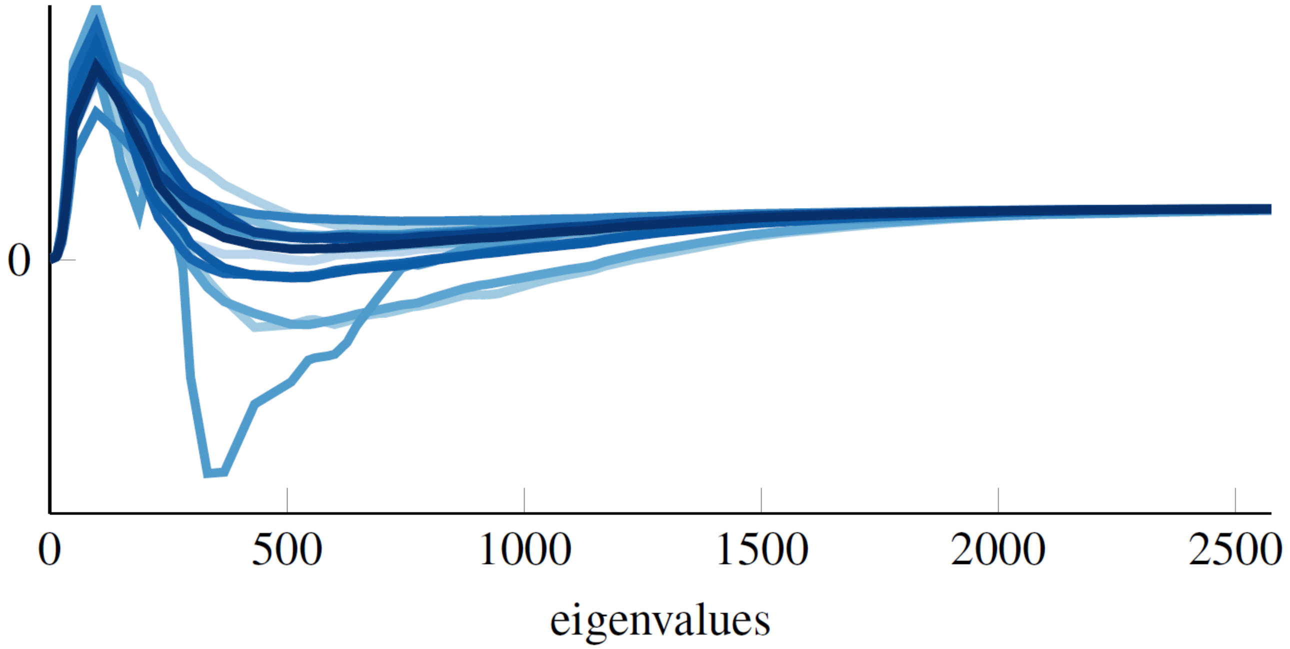
$$desc(x) = [LSCNN_1(x), LSCNN_2(x), \ldots, LSCNN_Q(x)]$$

$$LSCNN_q(x) = \sum_{p=1}^{P} \left(\sum_{k=1}^{K} a_{q,p,k} (Sf_p)_{x,k} \right)$$

$$(Sf_p)_{x,k} = \langle f_p, \sum_{l=1}^{K} \Big(g_p(\lambda_l) \phi_l(x) \phi_k(x) \Big) \phi_l \rangle_{\mathcal{X}}$$

Learning class-specific descriptors for deformable shapes using localized spectral convolutional networks, *Boscaini et al. SGP 2015*

Learned windows



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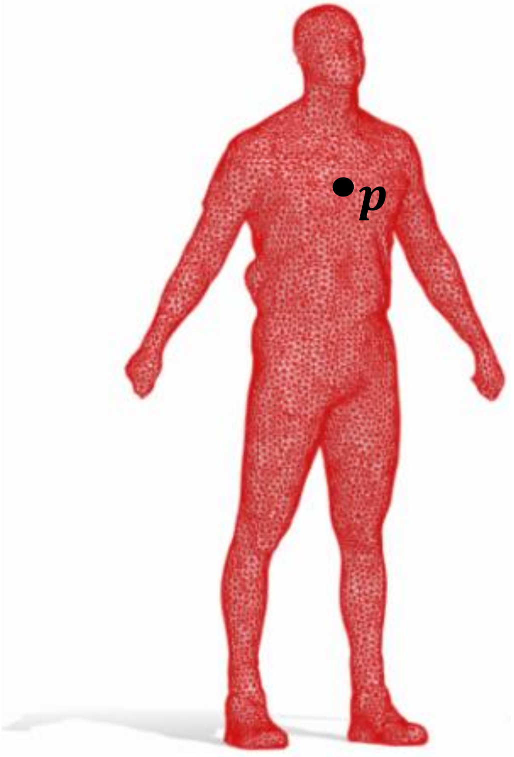
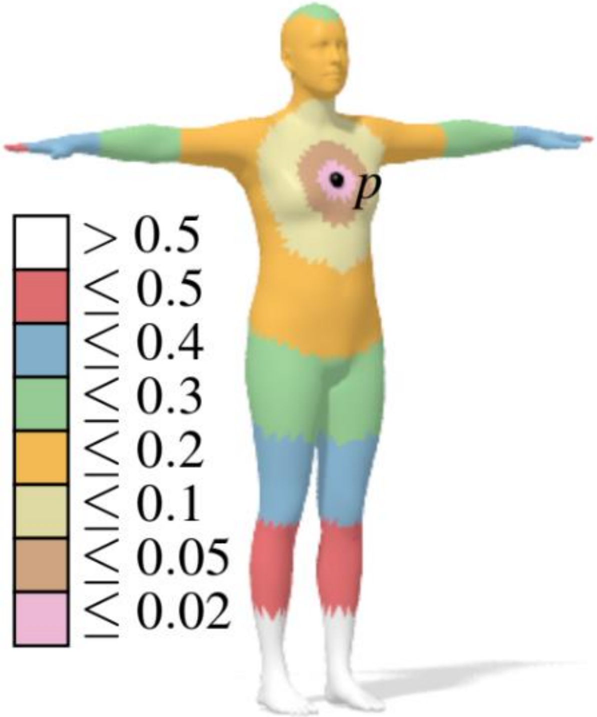
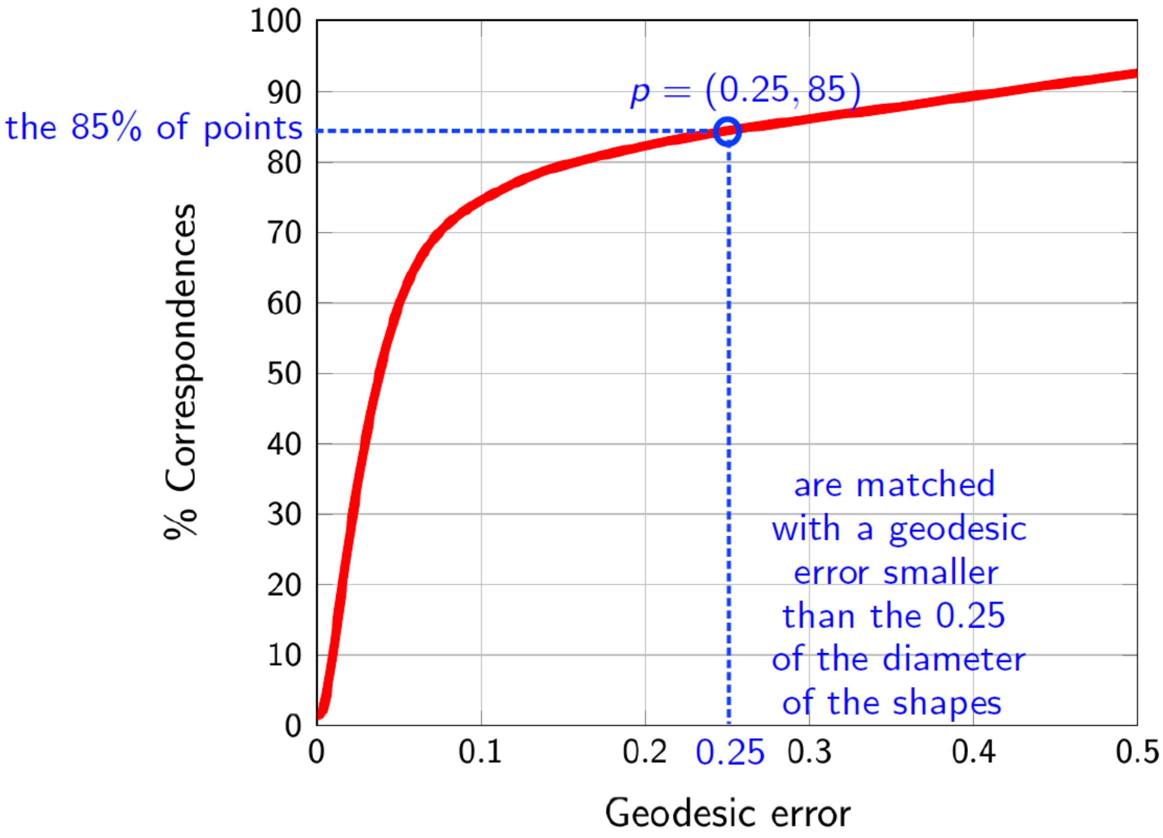
Geodesic error



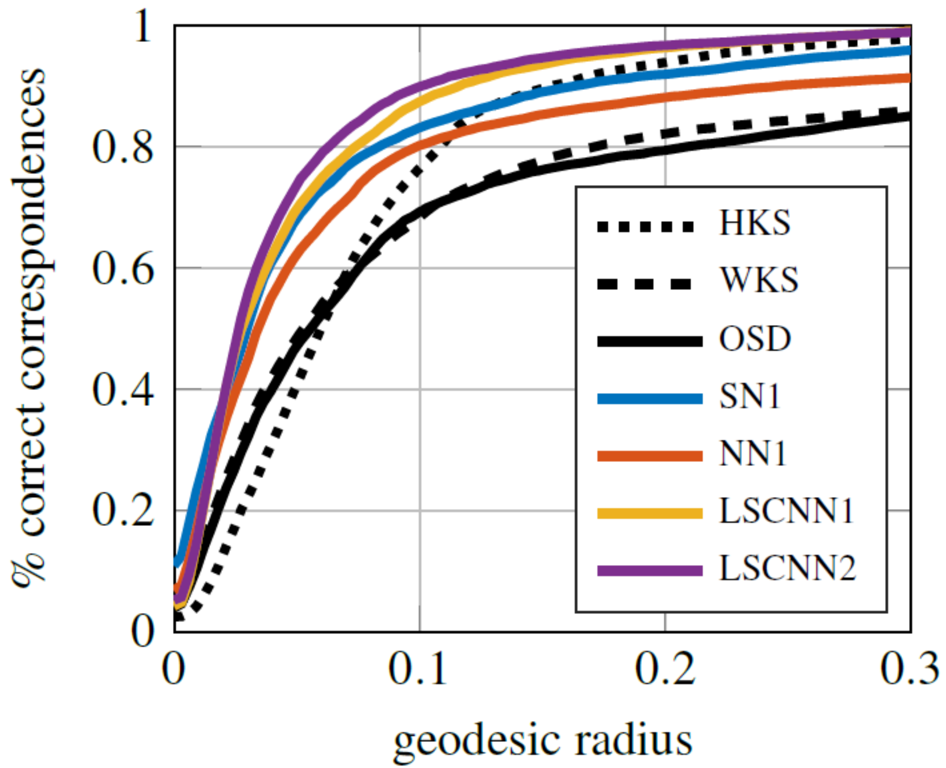


Evaluation: cumulative error curve



Blended intrinsic maps, *Kim*, *Lipman*, *Funkhouser*, TOG 2011

Quantitative comparison



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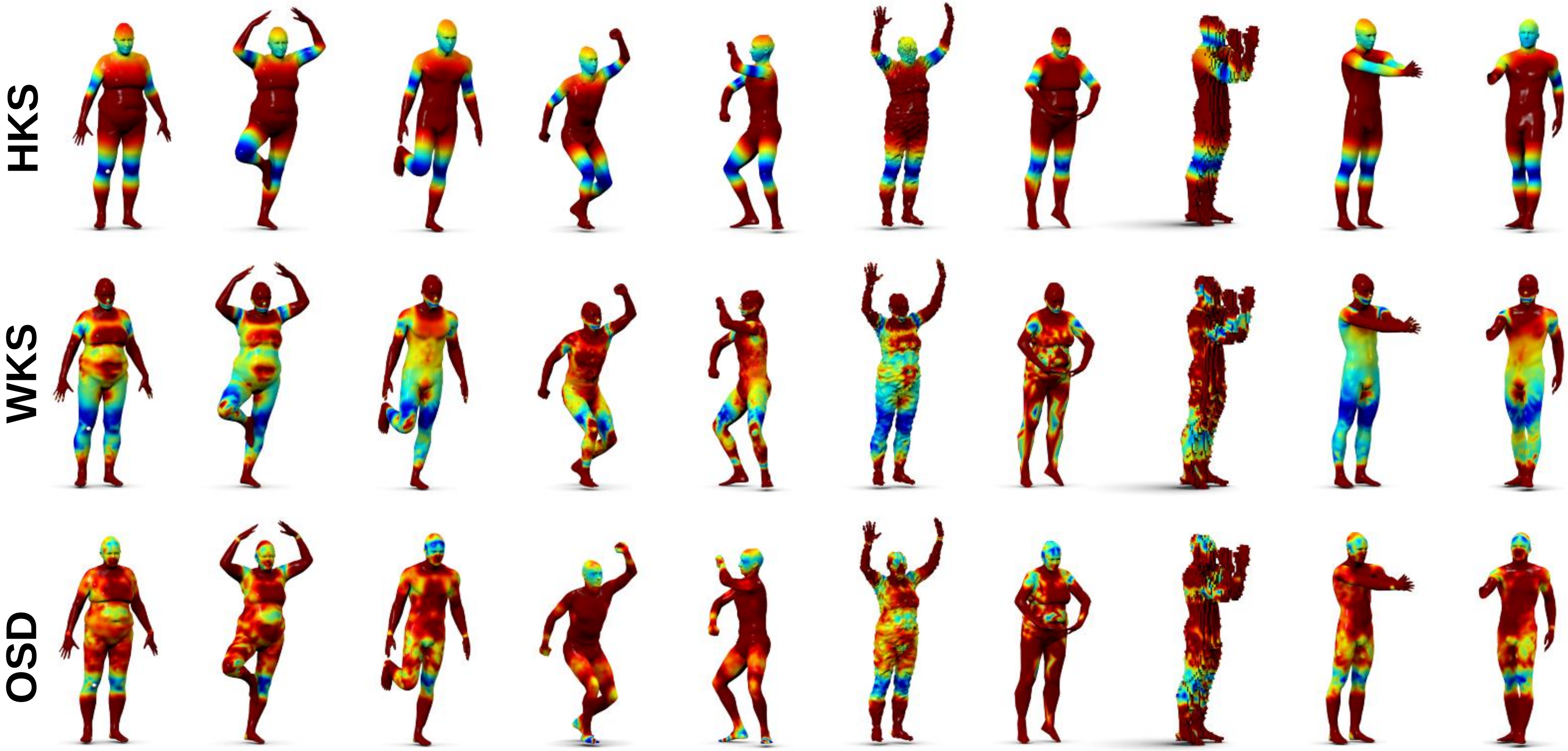
Qualitative comparison



Learning class-specific descriptors for deformable shapes using localized spectral convolutional networks, *Boscaini et al. SGP 2015*

Qualitative comparison

Learning class-specific descriptors for deformable shapes using localized spectral convolutional networks, *Boscaini et al. SGP 2015*

Some conclusions

- Spectral descriptors are invariant to isometric deformations
- Spectral descriptors do not solve the symmetries
- Spectral descriptors can be generalized via data-driven approaches
- WFT can characterize locally the shape
- The data-driven approaches outperform the standard spectral ones
- Other deformations (for from sidometries) can not be faced

Learning class-specific descriptors for deformable shapes using localized spectral convolutional networks, *Boscaini et al. SGP 2015*

Other data-driven approaches

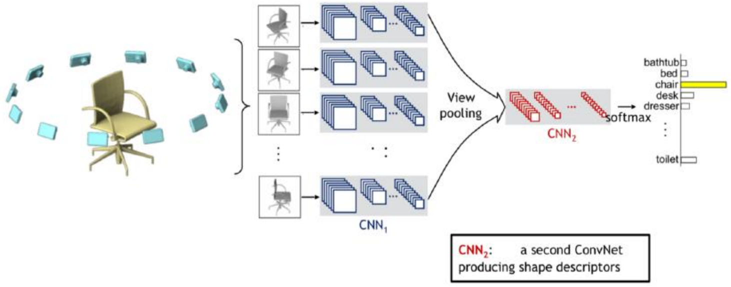
The data-driven approaches seem well-suited to solve the point-to-point mathcing problem

Recentrly this gives rise to a family of approaches that can be collected under the name of:

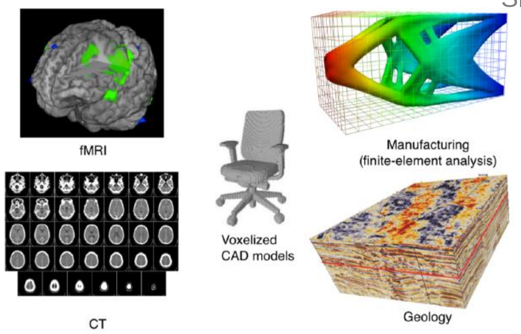
GEOMETRIC DEEP LEARNING

Geometric deep learning

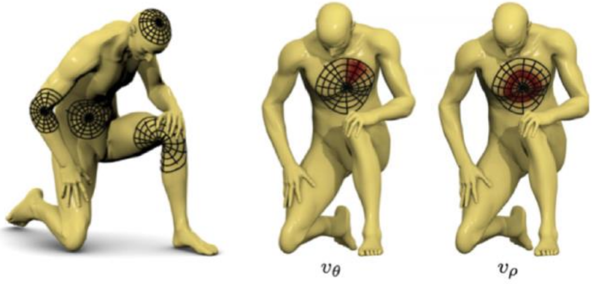
Slide credit M. *Ovsjanikov*



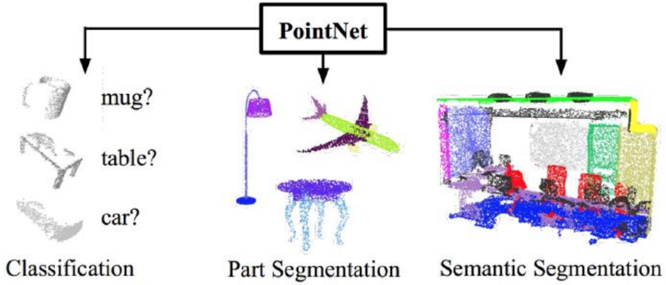
View-based



Volumetric



Intrinsic (surface-based)

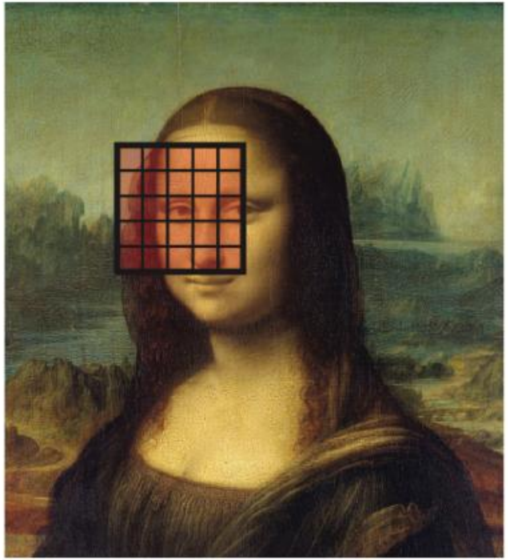


Point-based

Descriptor for shape matching

Alternatives convolutions

Slide credit M. *Ovsjanikov*



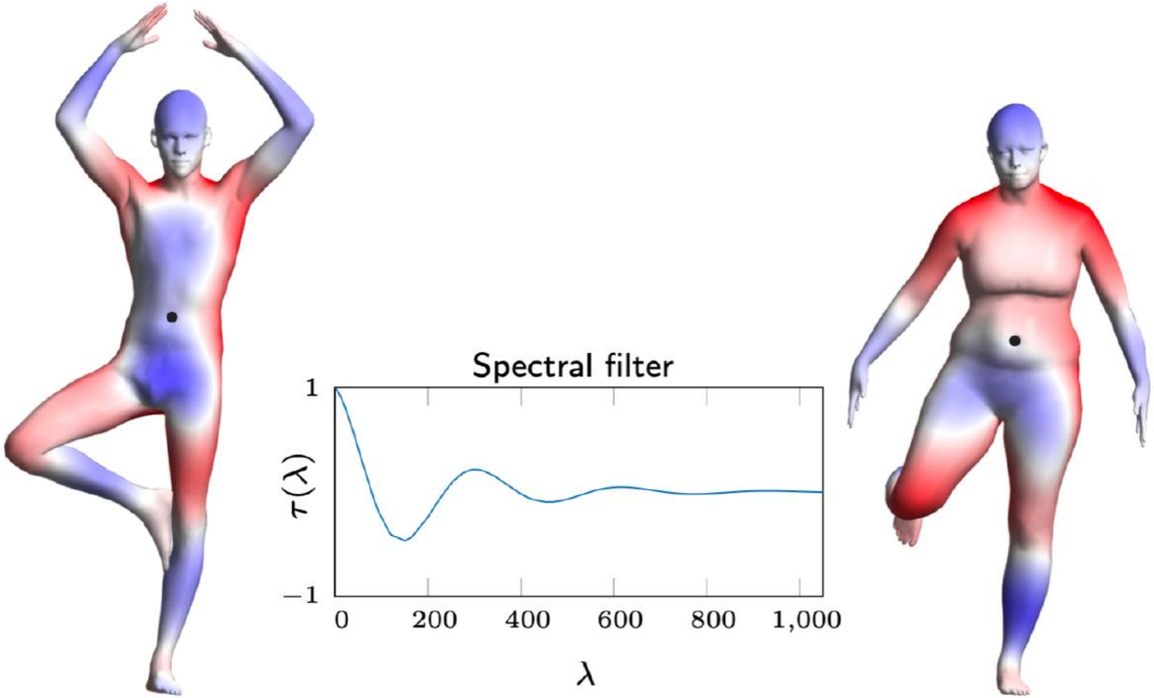
Euclidean

Non-Euclidean

Limits of the spectral convolution

Unfortunately spectral convolution as many limitations (shape, shift invariances).

Slide credit E. *Rodolà*



$\mathbf{\Phi}_1 \tau(\mathbf{\Lambda}_1) \mathbf{\Phi}_1^\top \boldsymbol{\delta}_0$ $\mathbf{\Phi}_2 \tau(\mathbf{\Lambda}_2) \mathbf{\Phi}_2^\top \boldsymbol{\delta}_0$

questions?

?