Spectral shape analysis for 3D matching

Umberto Castellani, Riccardo Marin, Simone Melzi

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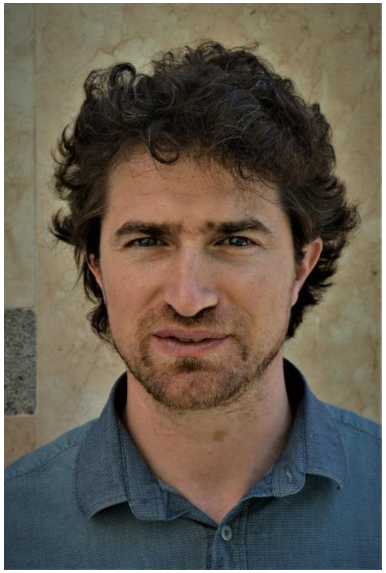
PhD School A.A. 2019-2020

Instructor



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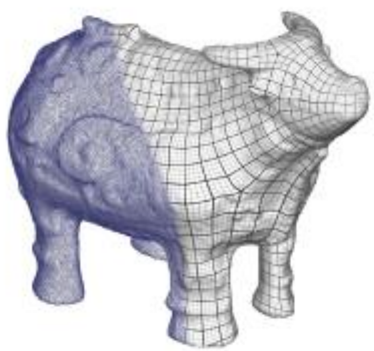
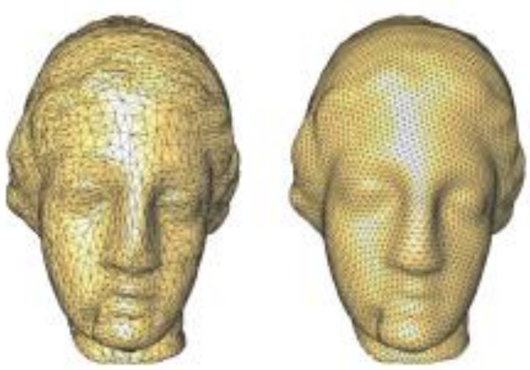
Outline

- Introduction,
- 3D Matching,
- 3D Shape,
- Spectral Shape Analysis,
- Schedule

Introduction

Digital representations of 3D objects and scene are ubiquitus

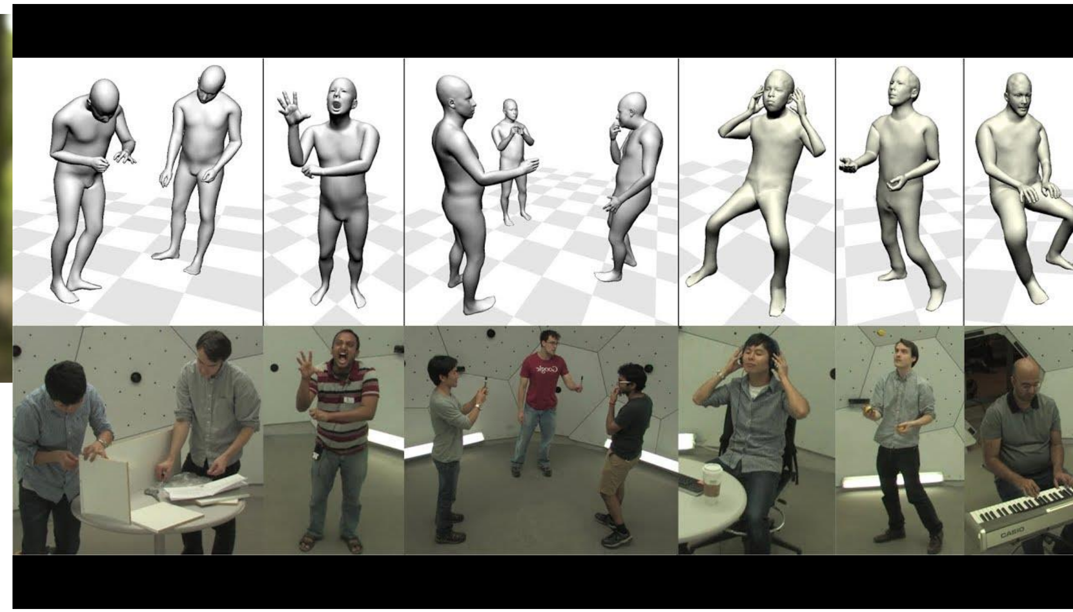


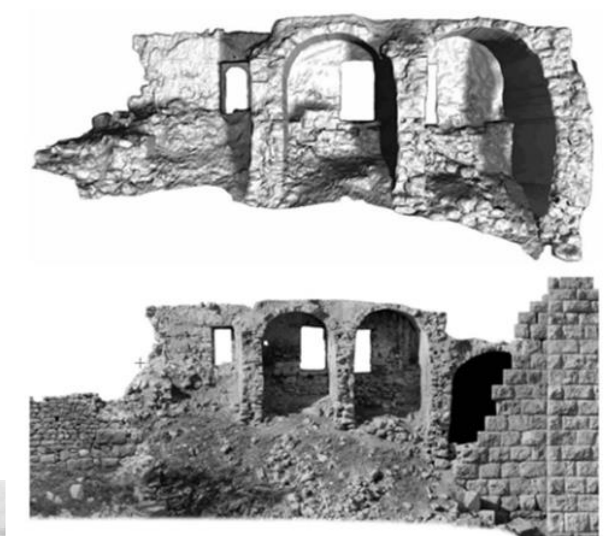


https://giphy.com/

Introduction

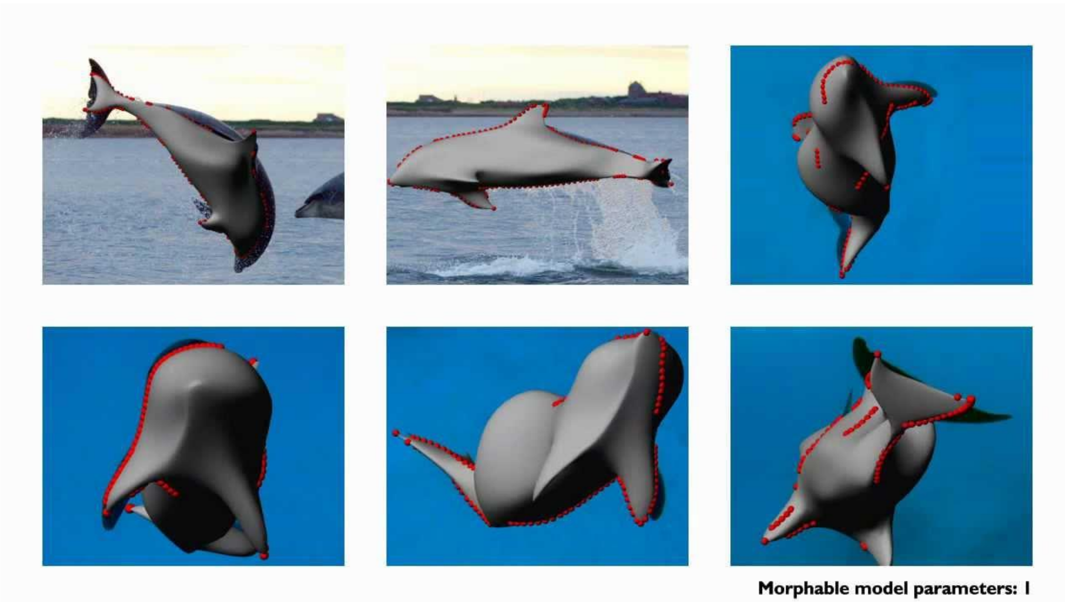
Virtual objects from real world

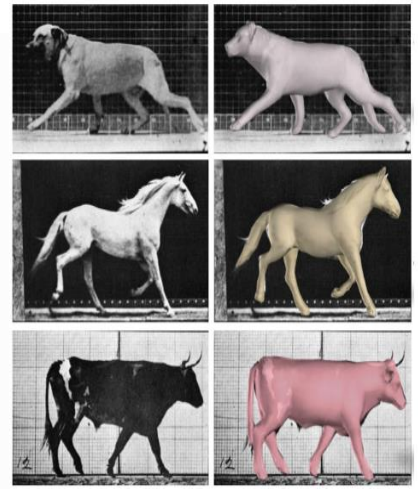


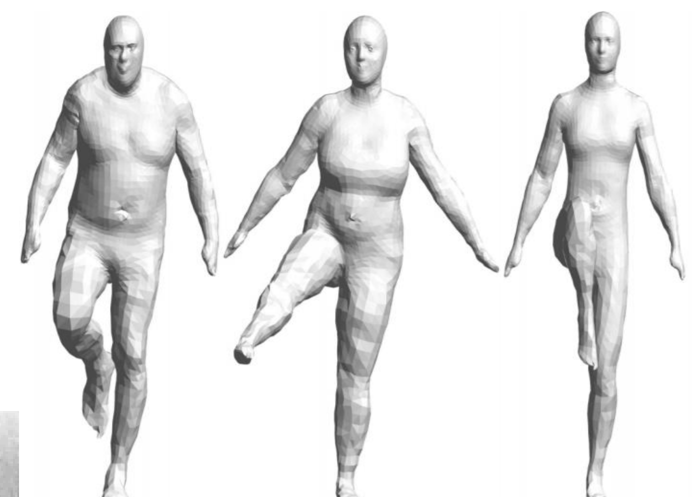


Introduction

Non-rigid world



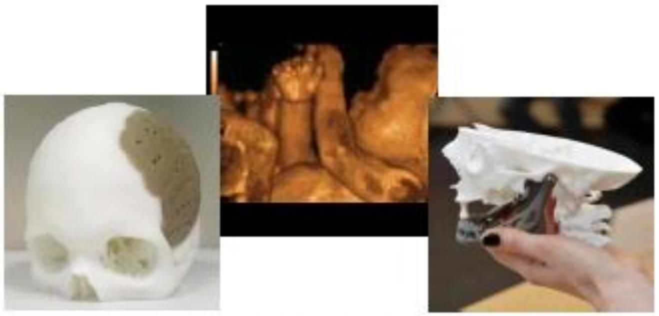




What shape are dolphin?, Cashman et. al.,’12
3D Menagerie: Modeling the 3D Shape and Pose of Animals, Zu et. al, ’17
Total Capture: A 3D Deformation Model for Tracking Faces, Joo et. al, ‘18
Expressive Body Capture: 3D Hands, Face, and Body, Pavlakos et. al, ‘19
Multi-chart Generative Surface Modeling, Ben-Hamu et. al, ’19
Learning an Infant Body Model from RGB-D Data, Hesse et .al, ‘18

Introduction

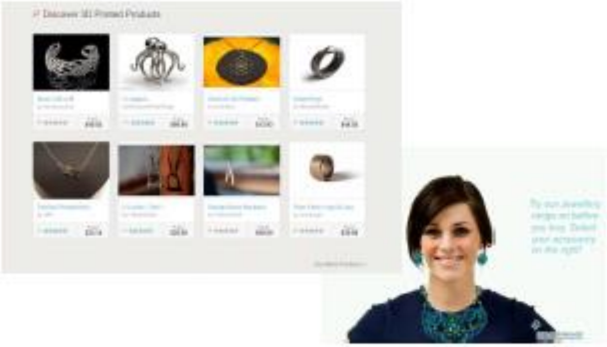
Where digital representation are important



Medical

Engineering

Games



E-Commerce

Culture

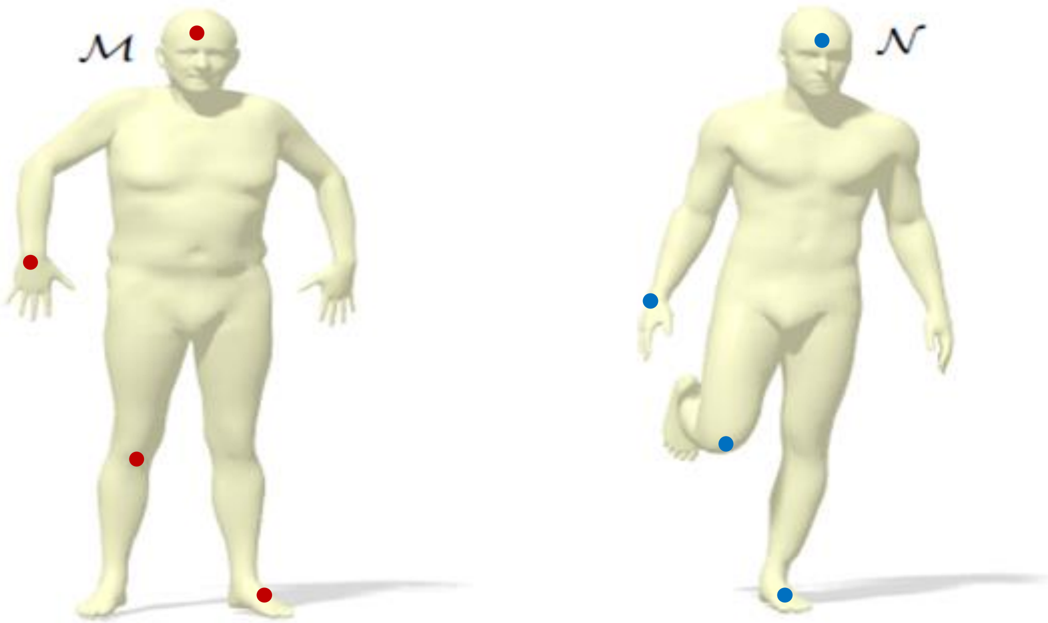
Simulation

3D printing

In all these contexts there is a fundamental issue to solve:

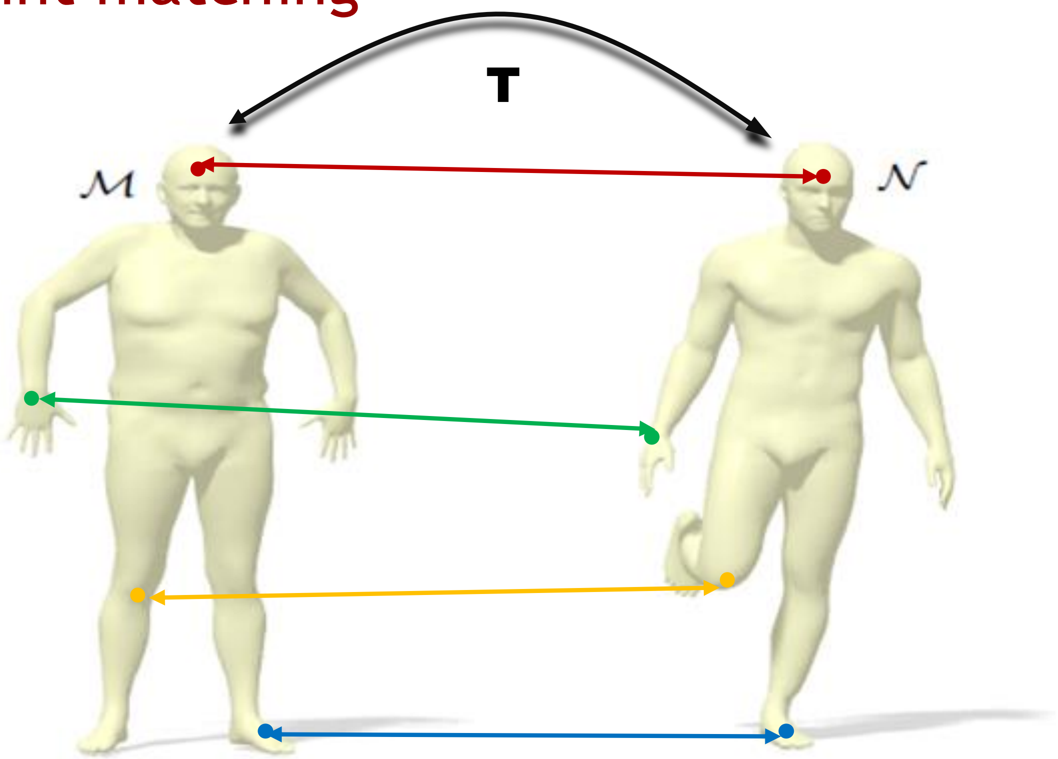
3D Shape matching

3D Shape matching



3D Shape matching

Point to point matching



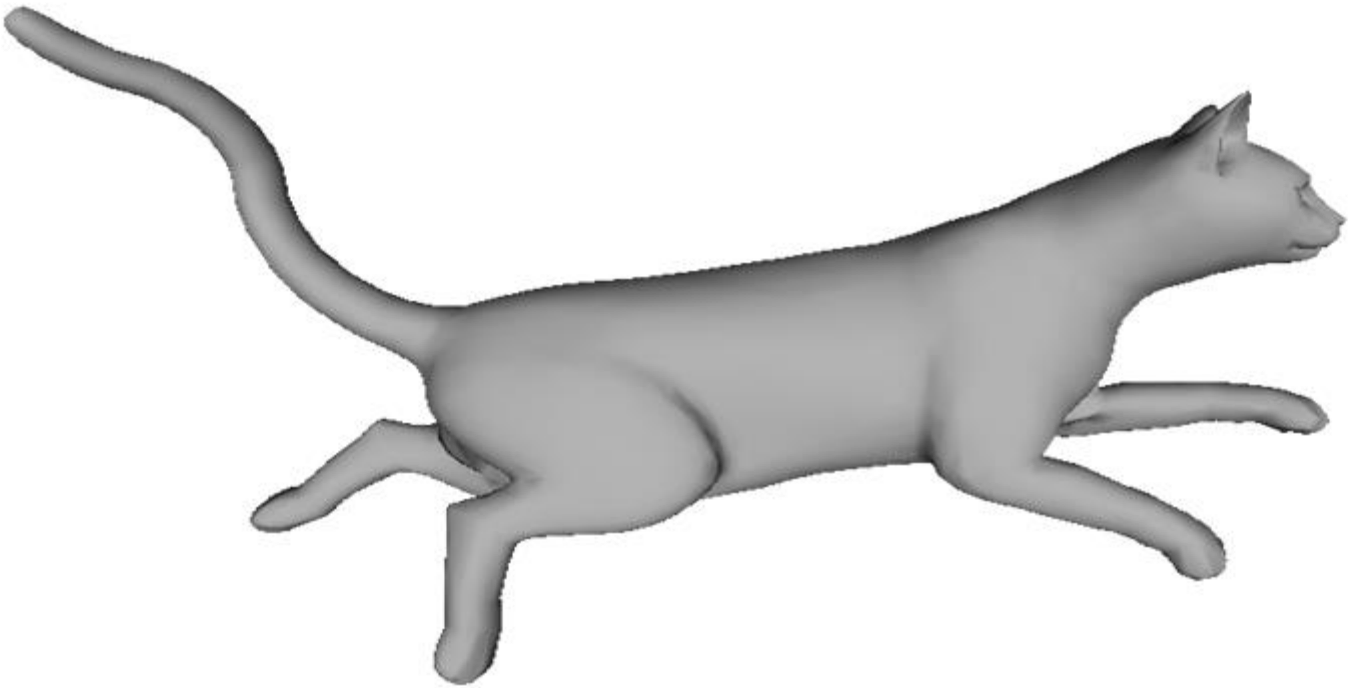
3D Shape matching

Open problems:

- Reliability and accuracy of matching,
- Robustness to heterogeneous data representation,
- Matching between non rigid objects,
- Computational complexity,
- Robustness to noise and missing parts,

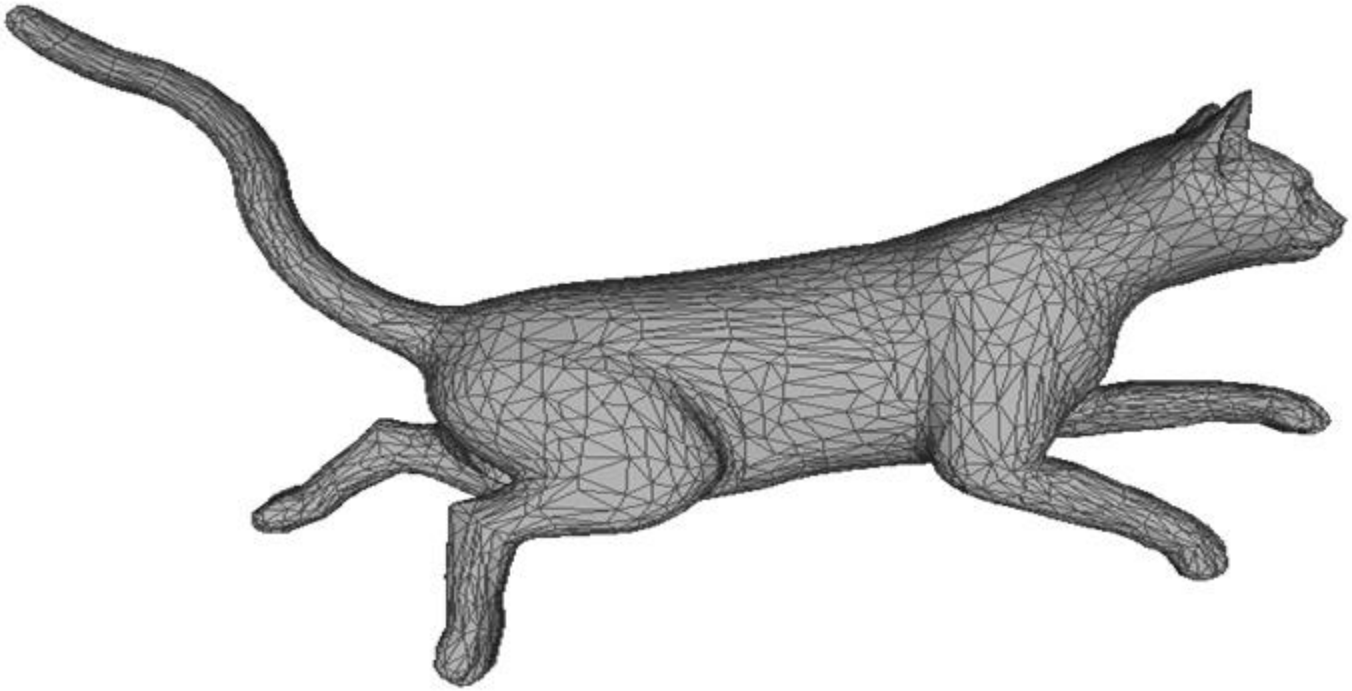
What is a shape?

- **Continuous:** a surface embedded in 3D



What is a shape?

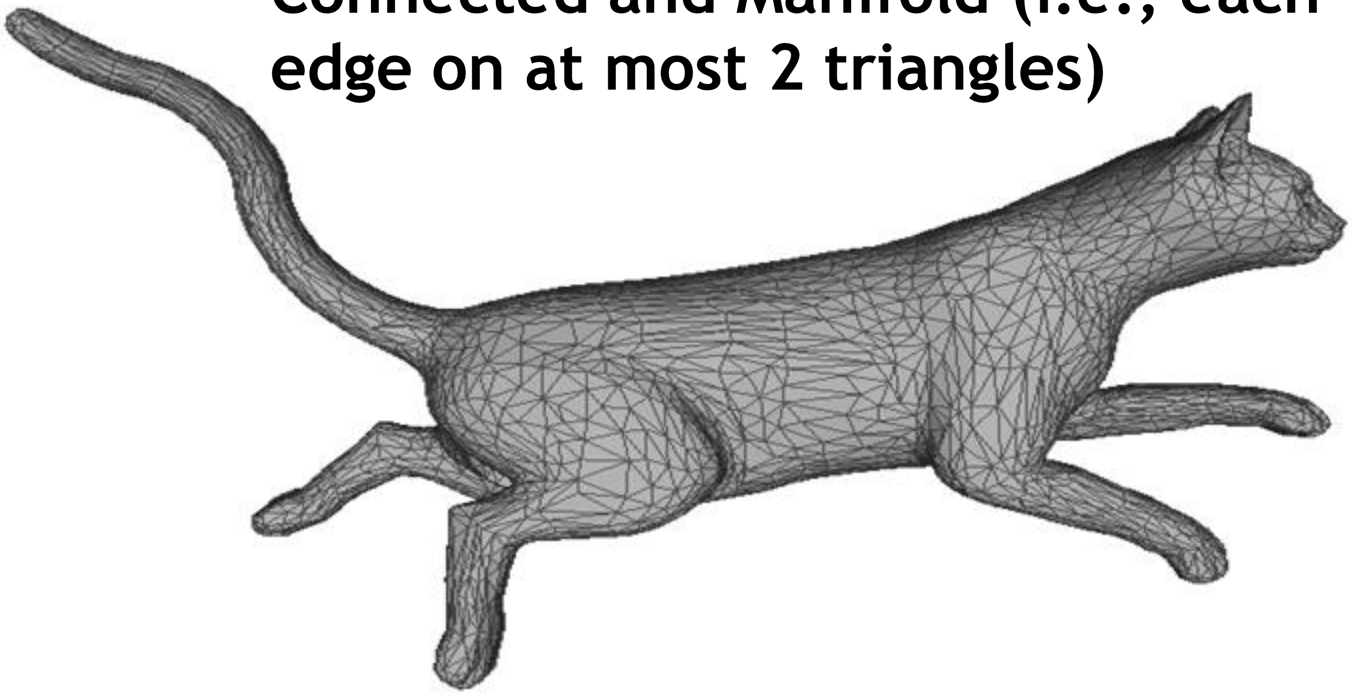
- **Continuous**: a surface embedded in 3D
- **Discrete**: a graph embedded in 3D (triangle mesh).



What is a shape?

- **Continuous**: a surface embedded in 3D
- **Discrete**: a graph embedded in 3D (triangle mesh).

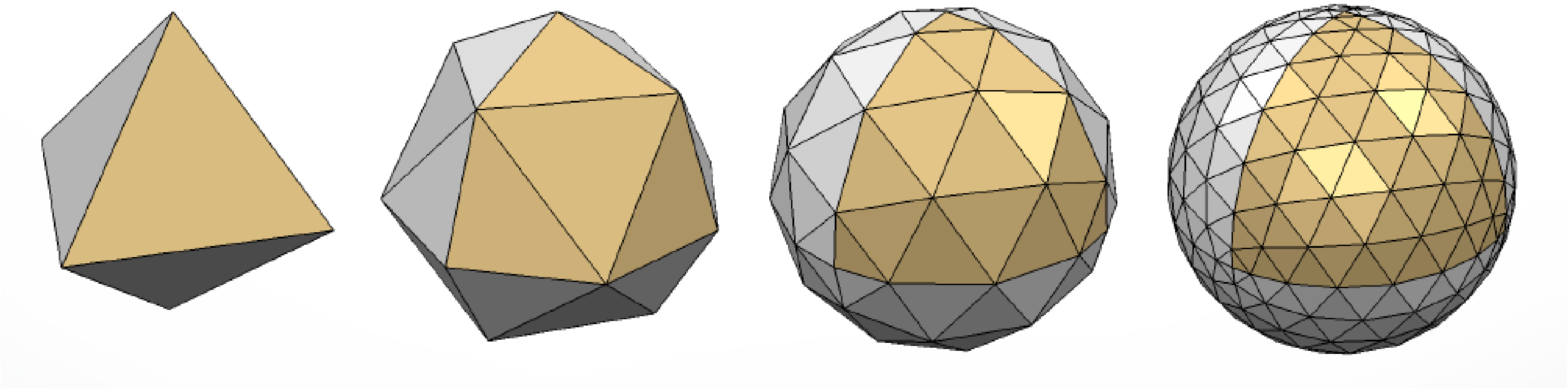
Connected and Manifold (i.e., each edge on at most 2 triangles)



What is a shape?

- Polygonal meshes are the most used representation for 3D shapes:

"Piecewise linear approximation --> error is $O(h^2)$"

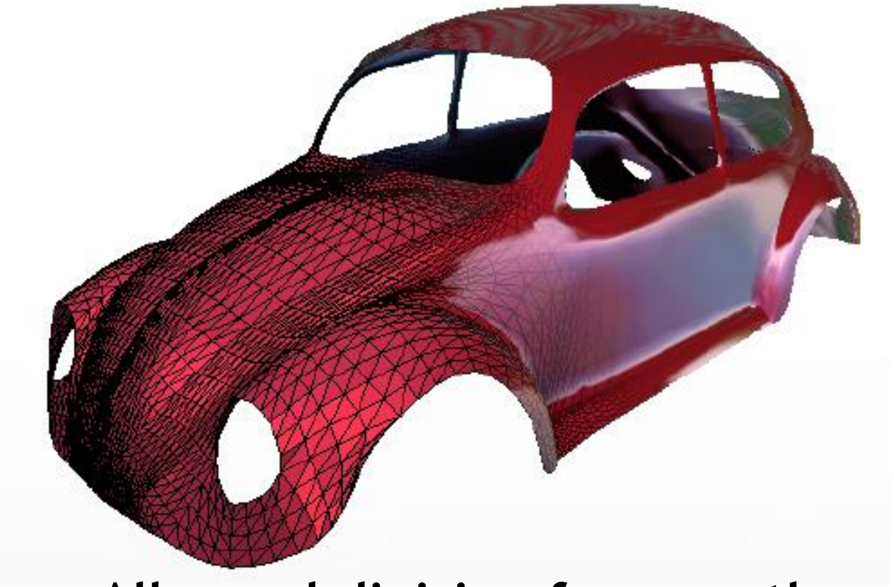


Error inversely proportional to #faces

What is a shape?

- Polygonal meshes are the most used representation for 3D shapes:

"Arbitrary topology surfaces"

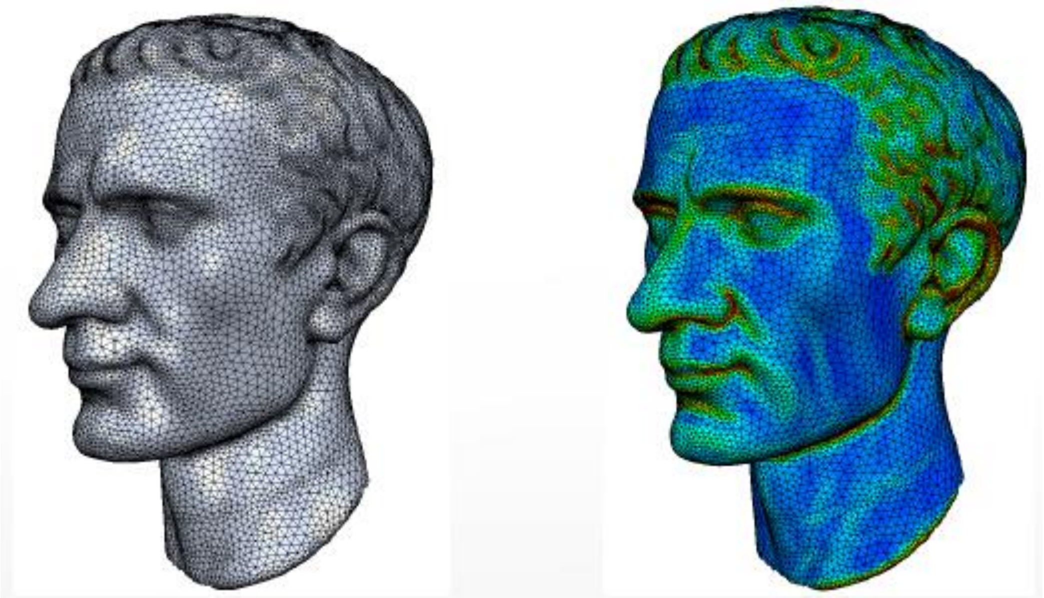


Allow subdivision for smoothness

What is a shape?

- Polygonal meshes are the most used representation for 3D shapes:

"Adaptive sampling"

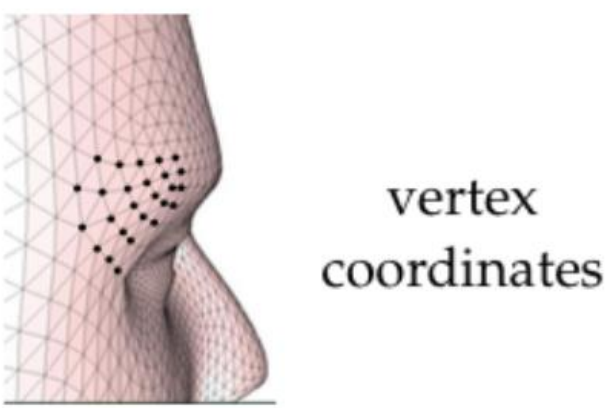


Can add resolution only where necessary

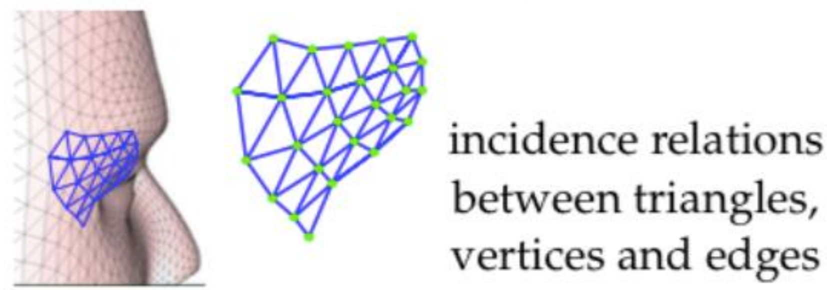
What is a shape?

- Two main components:

Geometry



"Connectivity": the underlying triangulation



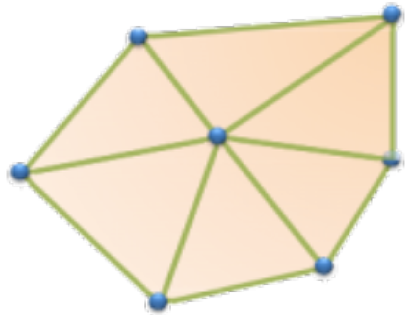
geometric structure

combinatorial structure

What is a shape?

- Two main components:
  - Geometry: vertex positions

$$\mathcal{P} = \{p_1, p_2, \ldots, p_n\}, \quad p_i \in \mathbb{R}^3$$

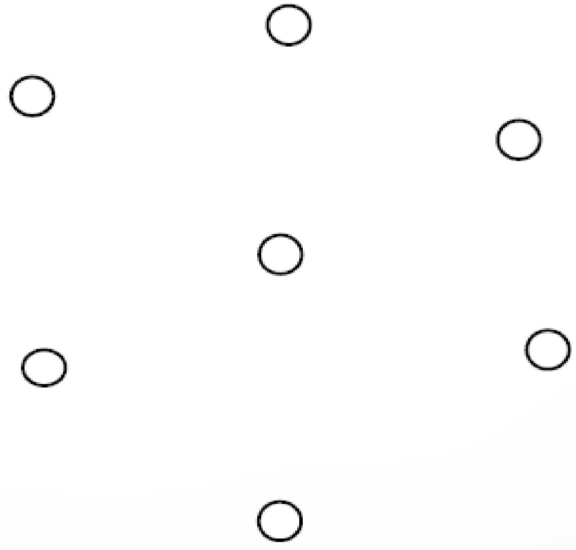


  - Connectivity:
    - Vertices: $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$
    - Edges: $\mathcal{E} = \{e_1, e_2, \ldots, e_m\}, \quad e_i \in \mathcal{V} \times \mathcal{V}$
    - Faces: $\mathcal{F} = \{f_1, f_2, \ldots, f_k\}, \quad f_i \in \mathcal{V} \times \mathcal{V} \times \mathcal{V}$

What is a shape?

$$\mathcal{M} = (\{\mathbf{v}_i\}, \{e_j\}, \{f_k\})$$

geometry $\mathbf{v}_i \in \mathbb{R}^3$

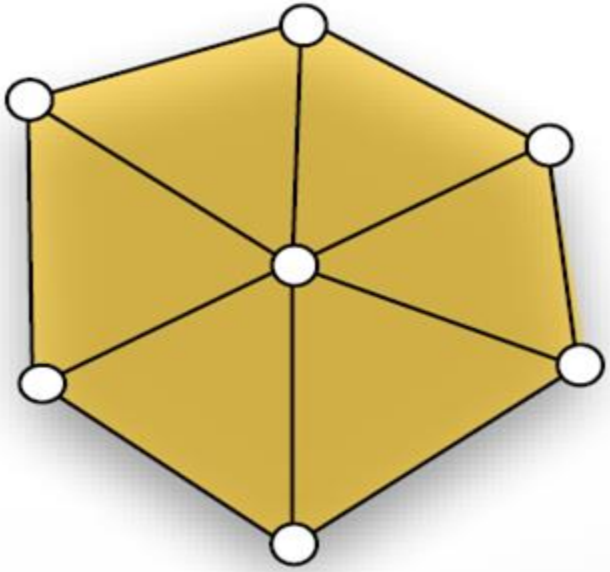


What is a shape?

$$\mathcal{M} = (\{\mathbf{v}_i\}, \{e_j\}, \{f_k\})$$

geometry $\mathbf{v}_i \in \mathbb{R}^3$

topology $e_i, f_i \subset \mathbb{R}^3$



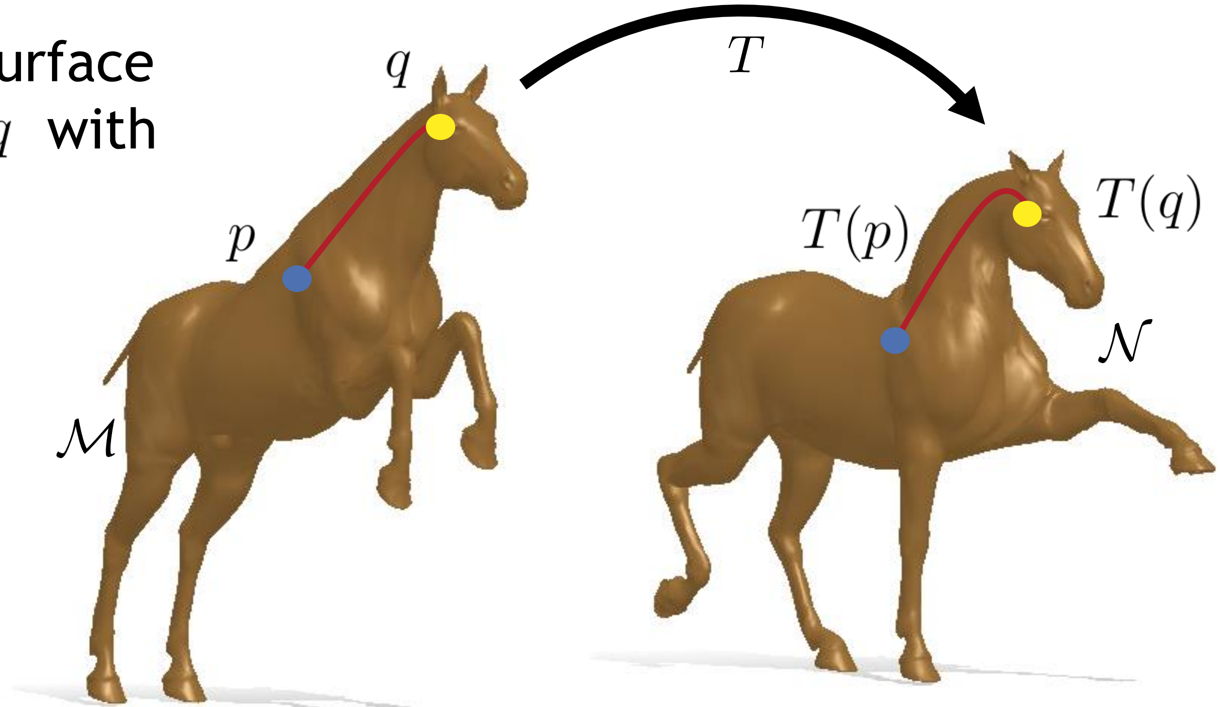
Why spectral shape analysis?

- Representation independent,
- Intrinsic, hence deformation-invariant,
- Computationally efficient,
- Can be interpreted in terms of classical signal processing.

Why spectral shape analysis?

The geodesics

The path on the surface connecting p to q with minimum length

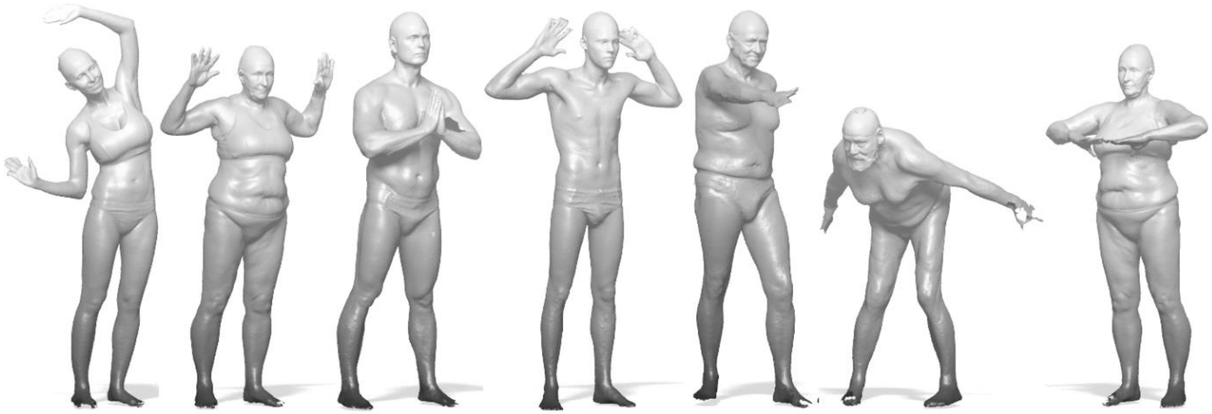


T is an isometry $\Longleftrightarrow d_{\mathcal{M}}(p,q) = d_{\mathcal{N}}(T(p),T(q)) \; \forall p,q \in \mathcal{M}$

Why spectral shape analysis?

- Spectral shape analysis deals with intrinsic information which is invariant to **isometric variation**

We can deal with non-rigid shapes



Why spectral shape analysis?

- **Rough intuition**

What can you learn about its shape from
vibration frequencies and oscillation patterns?

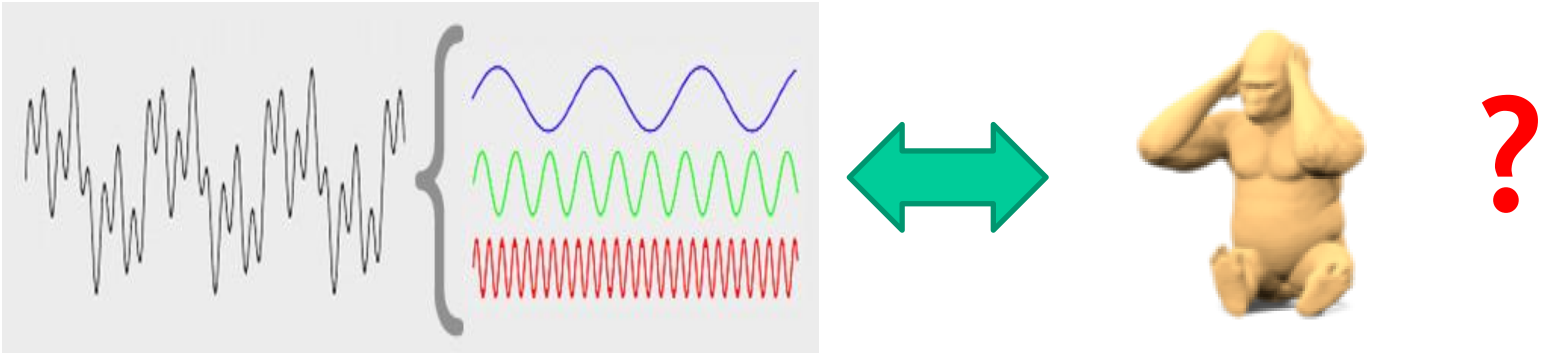
You can learn a lot about a shape by **hitting it** (lightly) **with a hammer**!

Solomon et al., “Laplace-Beltrami: The Swiss Army Knife of Geometry Processing” - SGP School 2014

Why spectral shape analysis?

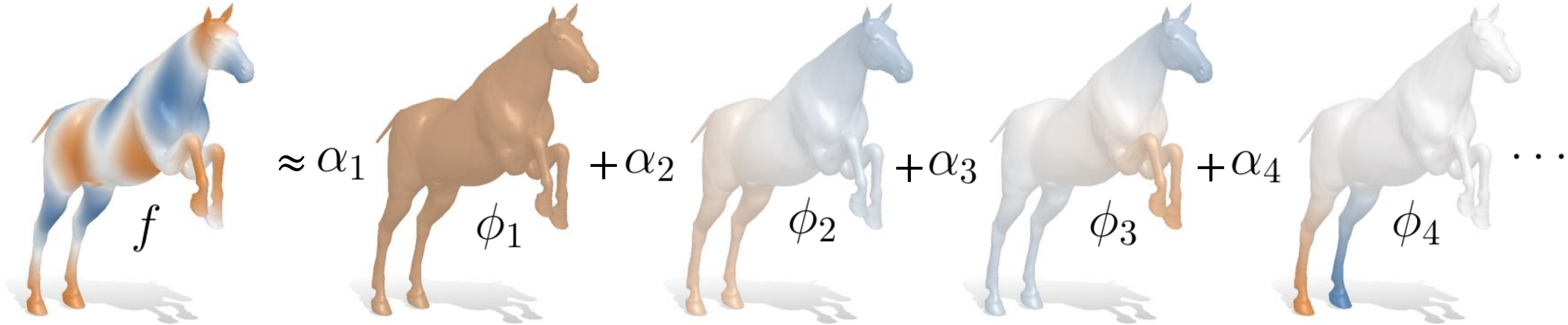
«Spectral shape analysis aims at porting standard signal processing tools to the setting of 3D mesh models» [Levy and Zhang09].

- The main motivation is the pursuit of Fourier analysis in the manifold setting (i.e., 3D meshes)



Why spectral shape analysis?

- **Fourier representation on 3D shapes**



$$f \approx \alpha_1 \phi_1 + \alpha_2 \phi_2 + \alpha_3 \phi_3 + \alpha_4 \phi_4 \cdots$$

Laplace-beltrami eigenfunctions towards an algorithm that understands geometry, *Levy*, SMI, 2006

Why spectral shape analysis?

Image from the Michael Bronstein’s SGP school 2012 on Spectral geometry of shape

Why spectral shape analysis?

Laplace-Beltrami: *"The Swiss Army Knife of Geometry Processing"*

Solomon et al., "Laplace-Beltrami: The Swiss Army Knife of Geometry Processing" – SGP School 2014

Schedule

Tuesday 28

10-13

- Introduction (1h)
- Differential geometry (2h)

15-17

- Harmonic analysis (2h)

Wednesday 29

10-13

- Spectral analysis on shape (2h)
- Exercise (1h)

15-17

- Descriptors for shape matching (2h)

Friday 31

10-13

- Functional map(2h)
- Exercise(1h)

15-18

- Advanced methods(2h)
- Conclusions(1h)

At the end of the school (last day) we will discuss the assignments for credits.