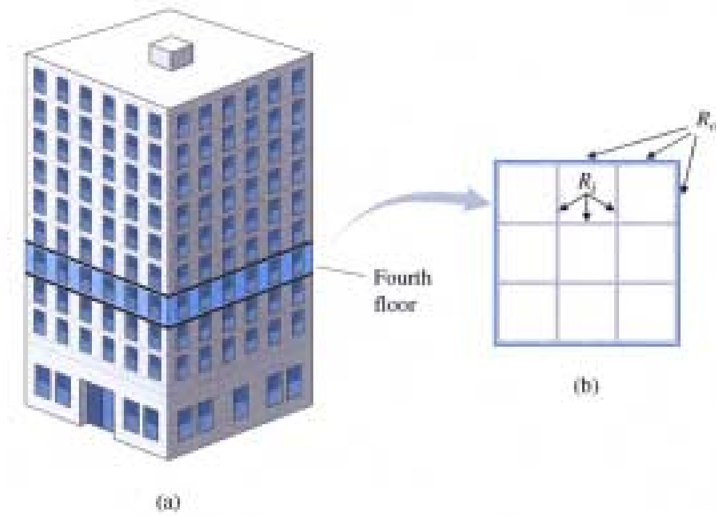


Figure 2.52:

25. An air conditioner supplies cold air at the same temperature to each room on the fourth floor of the high-rise building shown in Fig. 2.52(a). The floor plan is shown in Fig. 2.52(b). The cold air flow produces an equal amount of heat flow q out of each room. Write a set of differential equations governing the temperature in each room, where

$$T_o = \text{temperature outside the building,}$$
$$R_o = \text{resistance to heat flow through the outer walls,}$$
$$R_i = \text{resistance to heat flow through the inner walls.}$$

Assume that (1) all rooms are perfect squares, (2) there is no heat flow through the floors or ceilings, and (3) the temperature in each room is uniform throughout the room. Take advantage of symmetry to reduce the number of differential equations to three.

Solution:

We can classify 9 rooms to 3 types by the number of outer walls they have.

Type 1	Type 2	Type 1
Type 2	Type 3	Type 2
Type 1	Type 2	Type 1

We can expect the hotest rooms on the outside and the corners hotest of all, but solving the equations would confirm this intuitive result. That is,

$$T_o > T_1 > T_2 > T_3$$

and, with a same cold air flow into every room, the ones with some sun load will be hotest.

Let's redefince the resistances

$$\begin{aligned} R_o &= \text{resistance to heat flow through one unit of outer wall} \\ R_i &= \text{resistance to heat flow through one unit of inner wall} \end{aligned}$$

Room type 1:

$$\begin{aligned} q_{out} &= \frac{2}{R_i}(T_1 - T_2) + q \\ q_{in} &= \frac{2}{R_o}(T_o - T_1) \end{aligned}$$

$$\begin{aligned} \dot{T}_1 &= \frac{1}{C}(q_{in} - q_{out}) \\ &= \frac{1}{C}\left[\frac{2}{R_o}(T_o - T_1) - \frac{2}{R_i}(T_1 - T_2) - q\right] \end{aligned}$$

Room type 2:

$$\begin{aligned} q_{in} &= \frac{1}{R_o}(T_o - T_2) + \frac{2}{R_i}(T_1 - T_2) \\ q_{out} &= \frac{1}{R_i}(T_2 - T_3) + q \end{aligned}$$

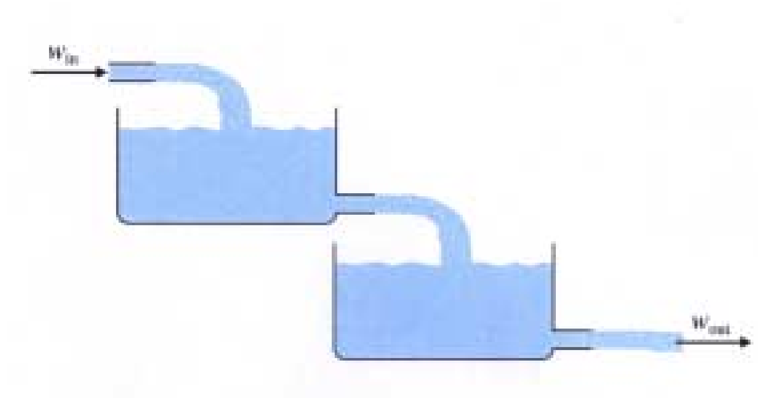
$$\dot{T}_2 = \frac{1}{C}\left[\frac{1}{R_o}(T_o - T_2) + \frac{2}{R_i}(T_1 - T_2) - \frac{1}{R_i}(T_2 - T_3) - q\right]$$

Room type 3:

$$\begin{aligned} q_{in} &= \frac{4}{R_i}(T_2 - T_3) \\ q_{out} &= q \end{aligned}$$

$$\dot{T}_3 = \frac{1}{C}\left[\frac{4}{R_i}(T_2 - T_3) - q\right]$$

Figure 2.53: Two-tank fluid-flow system for Problem 26



26. For the two-tank fluid-flow system shown in Fig. 2.53, find the differential equations relating the flow into the first tank to the flow out of the second tank.

Solution:

This is a variation on the problem solved in Example 2.20 and the definitions of terms is taken from that. From the relation between the height of the water and mass flow rate, the continuity equations are

$$\begin{aligned} \dot{m}_1 &= \rho A_1 \dot{h}_1 = w_{in} - w \\ \dot{m}_2 &= \rho A_2 \dot{h}_2 = w - w_{out} \end{aligned}$$

Also from the relation between the pressure and outgoing mass flow rate,

$$\begin{aligned} w &= \frac{1}{R_1} (\rho g h_1)^{\frac{1}{2}} \\ w_{out} &= \frac{1}{R_2} (\rho g h_2)^{\frac{1}{2}} \end{aligned}$$

Finally,

$$\begin{aligned} \dot{h}_1 &= -\frac{1}{\rho A_1 R_1} (\rho g h_1)^{\frac{1}{2}} + \frac{1}{\rho A_1} w_{in} \\ \dot{h}_2 &= \frac{1}{\rho A_2 R_1} (\rho g h_1)^{\frac{1}{2}} - \frac{1}{\rho A_2 R_2} (\rho g h_2)^{\frac{1}{2}} . \end{aligned}$$

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27. A laboratory experiment in the flow of water through two tanks is sketched in Fig. 2.54. Assume that Eq. (2.86) describes flow through the equal-sized holes at points A, B, or C.