ECEN5154: Homework #3, Problem 1

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I. Introduction/Purpose of Paper

There are a few goals mentioned, including:

- To discuss the choice of expansion functions for MoM
- To discuss the mathematical requirements involved of expansion functions
- To discuss choices of methods in weighting functions
- To give simple examples illustrating points and a complex example at the end

II. Relevance Towards Class

We discussed the general overview of MoM but this paper gave more detail on when/how to choose weighting and expansion functions, which we didn't discuss in class. So this was good supplementary material to go over for the class. This paper will be particularly useful in solving the next homework, which will involve using the Method of Moments to solve for a dipole. The final example in this paper discusses how to solve this problem and includes a discussion on the solution using the three different methods discussed in the paper. Their benefits and drawbacks are quite useful and discussed further in the other sections of this report.

III. What Have I learned From it

Besides the main points of the paper, the point matching method breaks down for the dipole example if the antenna is broken into parts that have small fields. The approximate kernel in the dipole example was pretty similar to the exact kernel, the least squares method had low residuals, and the radar cross section was pretty insensitive to poor solutions. Point Matching Method is the good approach for dipole but leads to complications because choosing matching points can be difficult - if chosen poorly will lead to incorrect/diverging solutions. Galerkin's method yields correct results for the dipole wire but is mathematically unsound (technically it's acting like there's a large excitation at the end of the dipole, but the overall current distribution isn't effected so you get the proper solution).

IV. Paper's Procedure

The paper first goes through the general steps involved in solving a linear operator equation using the Method of Moments, the general idea being that you can use expansion functions to expand the unknown I in the formula AI = Y. Once I is expanded, the residuals are calculated and then suppressed using weighting functions.

The next section talks about the choice of expansion functions. A simple example of a differential equation with a known solution is done that illustrates what happens when the chosen expansion functions don't make Ax_i form a complete set (the result is that you get the wrong answer).

The next section discusses your choice of weighting functions. If you decide to use the Point Matching Method, your weighting functions will involve the Dirac delta and matching points where you evaluate your excitations (Y). At the end of the section two theorems are discussed, the first being that there is always a way to obtain a divergent solution and the second being that there is always a uniformly converging solution. The paper goes back to discussing weighting functions based on which method you choose.

Galerkin's method often uses piecewise continuous functions over subintervals that are small (because the smaller the subinterval the more accurate the solution is), but it has certain requirements that must be met in terms of the operator in order to be used. Also, the expansion functions must span both the domain and range of the operator. The method of least squares is preselected as Ax_i. Once again, Ax_i must form a complete set and Y must be in the domain of the adjoint operator. It is the safest to use when little is known about the operator and the exact solution. The last section of the paper goes through the numerical example of the dipole, which is discussed more in depth in the other sections.

V. Paper's Main Points

- Two main rules for choice of expansion functions: they must satisfy the boundary/differentiability conditions and Ax_i must be a complete set.
- When choosing expansion functions make sure they are continuous in the first derivative
- Choosing matching points is in general difficult and can't be determined *a priori*.
- The point matching method is good choice for the cylindrical dipole example
- Choosing how to weight your residuals depends on your problem – point matching method is usually chosen for its simplicity, Galerkin's method needs a certain type of operator or it isn't applicable, and the method of least squares is the safest choice if you don't know much about the solution or the operator.

References

[1] T. K. Sarkar, A. R. Djordjevic, and E. Arvas, "On the Choice of Expansion and Weighting Functions in the Numerical Solution of Operator Equations," in IEEE Antennas and Propagation Magazine, vol. 33, no. 9, September 1985.

HW3 Problem2 Melissa Mantey

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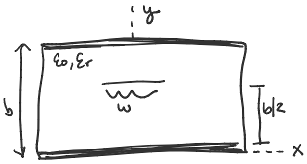
Step 1: Set up governing equation

For TEM mode of propogation in stripline, $\Psi(x,y)$ satisfies Poisson Equation

$$\nabla_t^2 \Psi(x,y) = -\frac{\rho(x,y)}{\varepsilon}$$



Boundary Conditions

$\Psi = 0$ at $y = 0, b$ and $x = \pm\infty$

$\Psi = V_0$ on the strip

Green's function + Superposition Results in

$$\Psi(x,y) = \iint G(x,y;x',y')\rho(x',y')\,dx'dy' \text{ where } \nabla_t^2 G(x,y;x',y') = -\frac{\delta(x-x')\delta(y-y')}{\varepsilon} \text{ and } G=0 \text{ at } y=0,b \text{ and } x=\pm\infty$$

and so Integral Equation (IE) can be found as

$$V_0 = \iint_{\text{strip}} G(x,\tfrac{b}{2};x',y')\rho(x',y')\,dx'dy' \text{ for } -\frac{w}{2} \le x \le \frac{w}{2}$$

Step 2: Discretization

We need to expand our $\rho(x',y')$ term

Assume $V_0 = 1V$ to reduce IE

$$1 = \int_{-w/2}^{w/2} G(x;\tfrac{b}{2};x',\tfrac{b}{2})\rho(x')\,dx'$$

expanding: $\rho(x') = \sum_{m=1}^{M} \alpha_m f_m(x')$ so that we get:

$$1 \approx \frac{1}{\pi\varepsilon_0\varepsilon_r} \sum_{n,\text{odd}}^{\infty} \frac{1}{n} \int_{-w/2}^{w/2} e^{-n\pi|x-x'|/b} \sum_{m=1}^{M} \alpha_m f_m(x')\,dx'$$

(need to divide strip into M segments)

Step 3: Application of WRM - use point matching at midpoint x_p

important pulse function property: have unit amplitude + are nonzero only over length of segment

↑ use to reduce

so that

$$I(x_p, x_m) = \frac{1}{\pi\varepsilon_0\varepsilon_r} \sum_{n,\text{odd}}^{\infty} \frac{1}{n} \int_{\Delta x_m} e^{-n\pi|x_p-x'|/b}\,dx' \quad \text{and} \quad \sum_{m=1}^{M} \alpha_m I(x_p,x_m) \approx 1$$

and in Matrix Form

$[I][\alpha] = 1$ (MxM square matrix and M size column vectors)

and approx. solution of IE is $\rho(x) = [\alpha]^t[\rho]$

Step 4: Fill System Matrix

Matrix elements are I_{pm}

$$I_{pm} = \frac{1}{\pi\varepsilon_0\varepsilon_r} \frac{2b}{\pi} \sum_{n,\text{odd}}^{\infty} \frac{1}{n^2} e^{-n\pi|x_p-x_m|/b} \sinh\left(\frac{n\pi}{2b} hx\right)$$

Step 5 Solve!

as mentioned above, approx. solution of IE is $\rho(x) = [\alpha]^t[\rho]$

Step 6: Extraction of parameters

example, $C_0 = \frac{Q}{V_0} = Q$, $Z_0 = \frac{1}{C_0 v}$