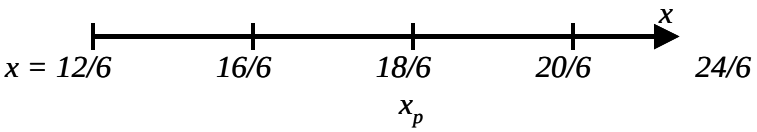
ECEN5154: Homework #1 – *due 2-21-20*

1. Evaluate $\int_0^{2\pi} x\cos 10x \sin 20x dx$ analytically and then use N=10, 50, 100, 500 equal intervals to numerically compute the same integral using:
   a) midpoint rule
   b) trapezoidal rule
   c) Simpson's rule

Provide as many comments as you can on your observation (suggestion use Matlab for programming).

2. Consider the integral:

$$I = \int_a^b \ln\left|\frac{x - x_k}{x_p - x}\right| dx$$



   a) Given that x_p=18/6 and x_k=-18/6 evaluate integral from a=12/6 to b=16/6 using: midpoint, 1pt Gauss quadrature and 2pt Gauss quadrature rules. Perform these integrations to fill the table below:

Integration Type	Exact Result	1 segment	Error	2 segments	Error
Midpoint					
1pt Gauss					
2pt Gauss					

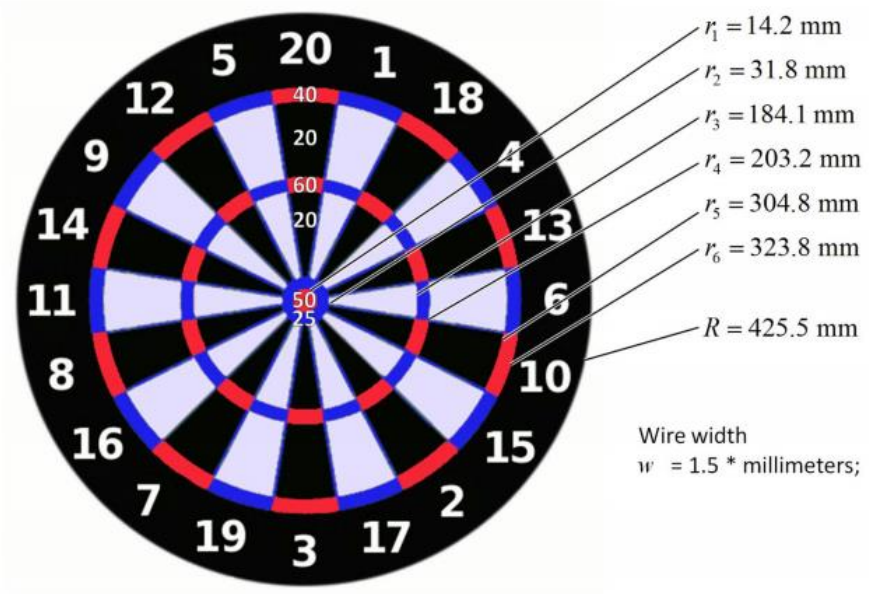
   b) Again take x_p=18/6 and x_k=-18/6. However, we now want to perform integration from a=16/6 to b=20/6. Clearly, the integral now goes to infinity over the integration interval, a CLASSICAL SITUATION IN INTEGRAL EQUATION METHODS. A possible fix to this situation is to rewrite the I as:

$$I = \tilde{I} - \int_{16/6}^{20/6} \ln|x - 18/6| dx = \{u = x - 18/6\} = \tilde{I} - \int_{-2/6}^{2/6} \ln|u| du \Rightarrow I = \tilde{I} - \Delta(\ln\Delta - (\ln 2 + 1)),$$ where

$\Delta = b - a = 4/6$ and $\tilde{I}$ is a regularized integral: $\tilde{I} = \int_{16/6}^{20/6} \ln\left|\frac{x - x_k}{x_p - x}\right| dx - \int_{16/6}^{20/6} \ln\left|\frac{1}{x_p - x}\right| dx$

Use this approach to again evaluate *I* and fill the above table.

3. The standard English dartboard is shown below. It is composed of the numbers 1 to 20 arranged randomly as wedges in a pie so each wedge spans 18°. The inner bullseye is worth 50 points while the outer bullseye is worth 25 points. Within the outermost ring, the number score is doubled. Inside the middle ring, the number score is tripled. The number scores in each of these regions for the 20-point wedge are shown in the figure. The approximate dimensions of the radii describing these regions are listed to the right.



Write a MATLAB program to build a standard English dartboard onto a Cartesian grid with 512×512 points. This should be a single array with values assigned to each point in a manner that describes the dartboard. You do not have to account for the wires between regions on the board. Image the data in your array to a figure window and include the image in your homework solution. Show in steps how you performed your coding and include your Matlab code in appendix.

Note: Your solution should look like this: