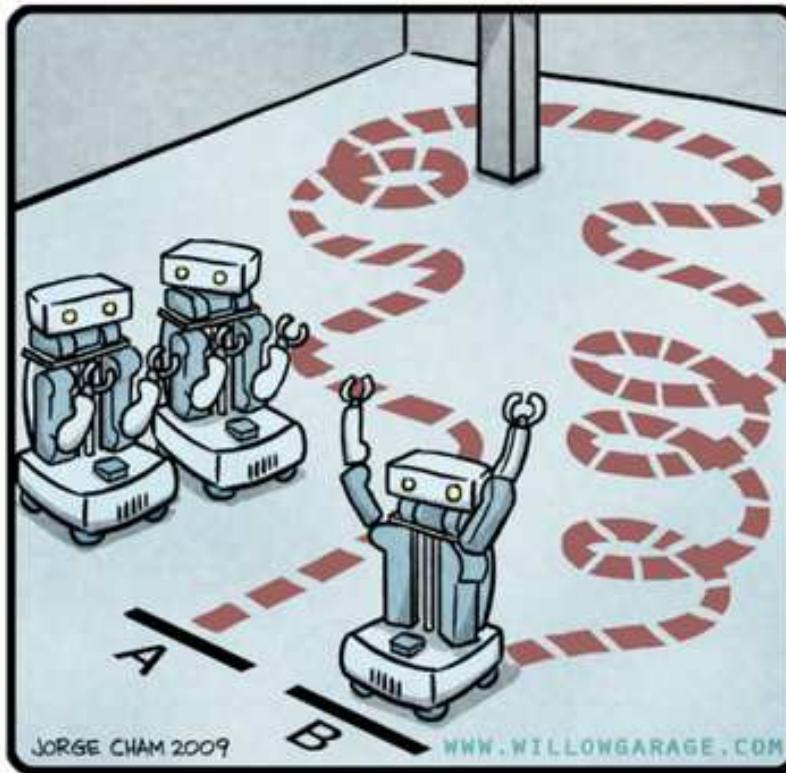


CS 249r: Special Topics in Edge Computing

Intro to Autonomous Systems / Robotics Part 2



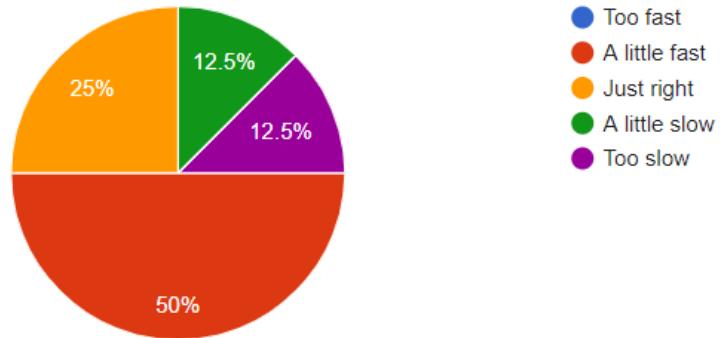
"HIS PATH-PLANNING MAY BE
SUB-OPTIMAL, BUT IT'S GOT FLAIR."

Brian Plancher
Fall 2019

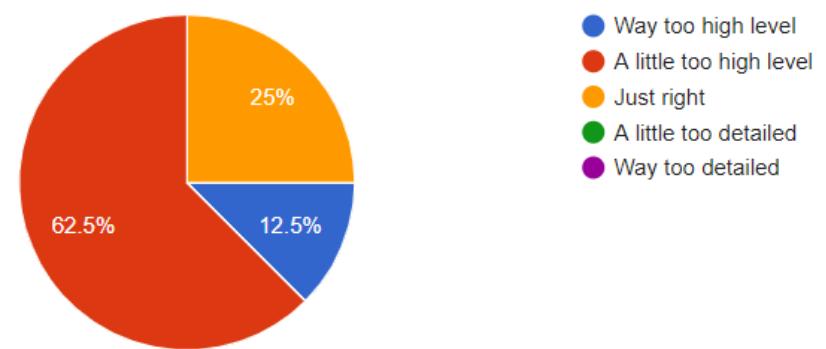


Feedback from last class

How was the pace of class today?



How was the depth of the content covered today?

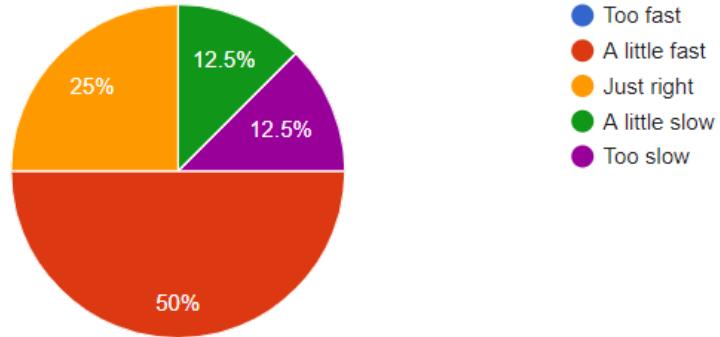


1. Pace is a tad fast
2. Get more technical/depth

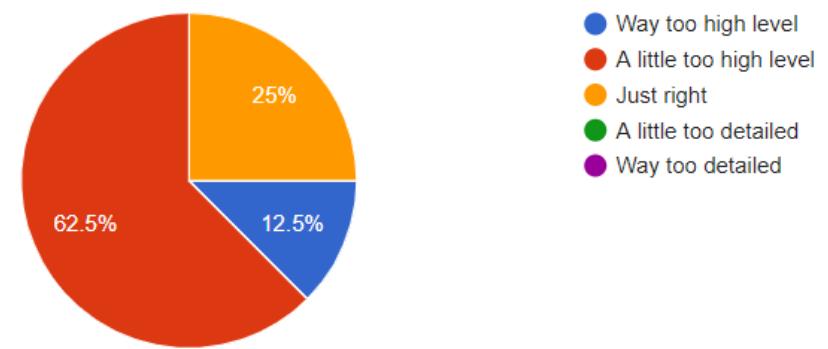
Feedback from last class

Also thanks for the open ended feedback!

How was the pace of class today?



How was the depth of the content covered today?



1. Pace is a tad fast
2. Get more technical/depth

Your homework for next class

Pre-Reads for Intro to Domain Specific Architectures

Is dark silicon useful? Harnessing the four horsemen of the coming dark apocalypse: <https://ieeexplore.ieee.org/document/6241647> ↗

Turing Lecture: A New Golden Age for Computer Architecture:
<https://californiaconsultants.org/wp-content/uploads/2018/04/CNSV-1806-Patterson.pdf> ↗ (watch the lecture its great!)



We have posted a tentative paper list to Canvas (along with PDFs and links)

Start to think about which papers you want – I will send a link to vote for preferences in a week or so!

If you have an idea for a paper not on the list please run it by us and we may be willing to swap it in!

Your homework for next class

Pre-Reads for Intro to Domain Specific Architectures

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Turing Lecture: A New Golden Age for Computer Architecture:
<https://californiaconsultants.org/wp-content/uploads/2018/04/CNSV-1806-Patterson.pdf> ↗ (watch the lecture its great!)



We're going to use **HOTCRP** (linked on Canvas and <https://www.eecs.harvard.edu/cs249r/>) for these for Monday – you will get an email from **Glenn Holloway** with a Password to access the site. (I am giving him the full roster as of today)

Your homework for next class

Click on a paper to access that paper's page

The screenshot shows a web-based conference management system for CS 249. At the top left is a pink header bar with the text "CS 249". On the right, the user is identified as "brian_plancher@g.harvard.edu" with links for "Profile", "Help", and "Sign out". Below the header is a search bar with "Search: (All)" and a dropdown menu set to "Active papers". A "Search" button is to the right. To the right of the search bar is a blue sidebar containing "Administration" with links to "Settings", "Users", "Assignments", "Mail", and "Action log", and "Conference information" with a link to "Program committee". The main content area displays recent activity and submissions. Under "Recent activity", it says "Your Submissions: Start new paper (No deadline)". It lists two submissions: "#1 Is Dark Silicon Useful? Harnessing the Four Horsemen of the Coming Dark Silicon Apocalypse" and "#2 A New Golden Age for Computer Architecture: WATCH THE VIDEO LINKED ON SLIDE 1". Both submissions are marked as "Submitted". A red box highlights these two submission entries.

CS 249

brian_plancher@g.harvard.edu [Profile](#) · [Help](#) · [Sign out](#)

(All)

Search: (All) in Active papers

Reviews: The average PC member has submitted 0.0 reviews. ([details](#) · [graphs](#))
As an administrator, you may review [any submitted paper](#).
[Review preferences](#) · [Offline reviewing](#)

▶ Recent activity

Your Submissions: [Start new paper](#) (No deadline)
As an administrator, you can start a paper regardless of deadlines and on behalf of others.

#1 Is Dark Silicon Useful? Harnessing the Four Horsemen of the Coming Dark Silicon Apocalypse Submitted
#2 A New Golden Age for Computer Architecture: WATCH THE VIDEO LINKED ON SLIDE 1 Submitted

HolCRP v2.100

Your homework for next class

The screenshot shows a web interface for a conference submission system. At the top, it says "CS 249r Home" and "brian_plancher@g.harvard.edu". Below that, the title of the submission is "#1 Is Dark Silicon Useful? Harnessing the Four Horsemen of the Coming Dark Silicon Apocalypse". The main content area shows the submission details: "Submitted" (10 Sep 2019 2:37:22am EDT), "Abstract" (N/A), and "Authors" (+ Hidden for blind review). A note says "You have used administrator privileges to view and edit reviews for this paper. (Unprivileged view)". Below this, it says "You are an author of this paper." and has two buttons: "Write review" (highlighted with a red box) and "Add comment". At the bottom, there is a "+ Add Comment" button and a "HotCRP v2.100" footer.

Then click “Write review” to open up the form to submit a “review”

Your homework for next class

Are you a(n):

What is your field of expertise?

If you were scoring this paper for a conference how would you rank it's overall merit?

What was the main contribution of this paper?

What did you find confusing about this paper?

What did you like about the writing?

What did you dislike about the writing?

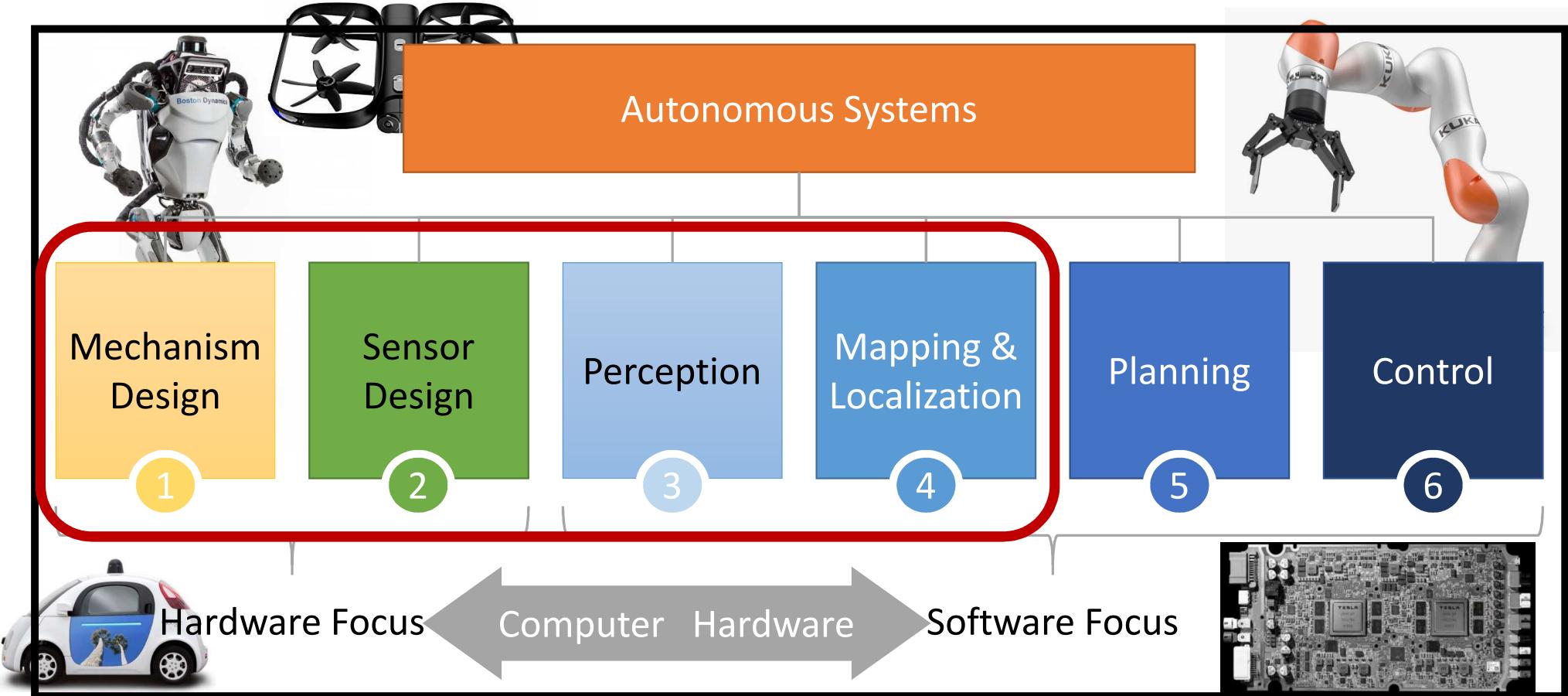
Any other comments?

Then just fill it
out and submit
and you'll be
good to go!

The goal for the next couple of lectures is to develop a **high level** understanding of:

1. What is an autonomous system
 2. Key **problems** for autonomous systems
 3. Some of the most important (classes of) **algorithms** in robotics
 4. The **model based** vs. **model free** tradeoff
 5. The **online** vs **offline** tradeoff
 6. The **no free lunch** theorem and the need for **approximations**
 7. How **computer systems / architecture** design has and can play a role in improving autonomous systems
-

Autonomous Systems / Robotics is a BIG space



1 2

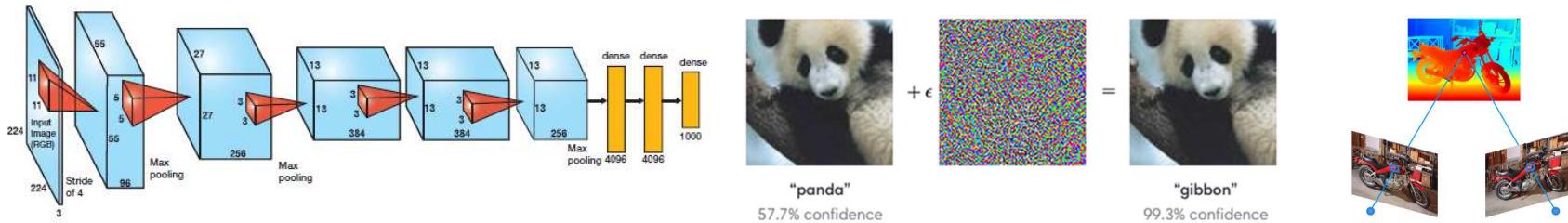
Key Takeaways:



1. When designing algorithms for robots you need to understand the physical capabilities of the robot and you (potentially) need to understand how to model its physical behaviors
 2. Different kinds of systems will have different power, weight, and performance budgets for computer hardware
-

3

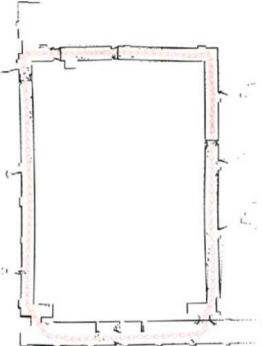
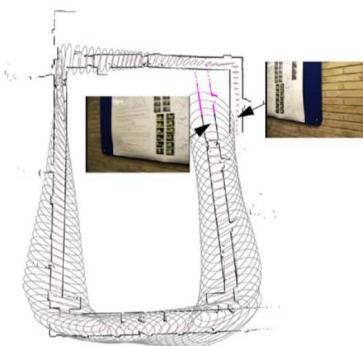
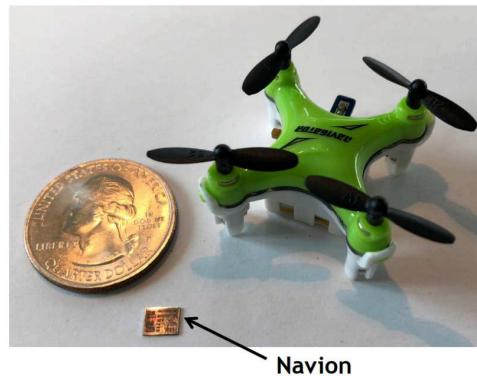
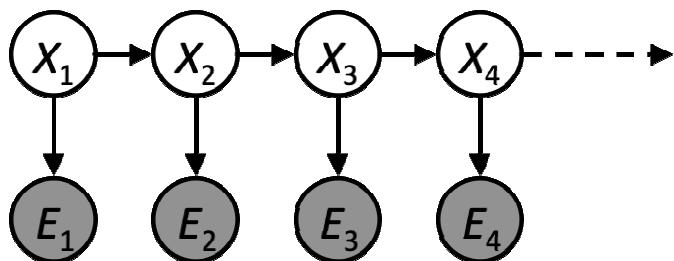
Key Takeaways:



1. As of today it seems like **CNNs** that automate the design and summary of salient features via convolution are the way to go
 - But/and will need specialized NN running on **specialized accelerator chips** to get them small enough to fit on small power constrained autonomous systems (e.g., small drones)
 - And we will need to find ways to **secure them against attacks!**
2. Also, other more targeted problems such as **Stereo Depth** seem to need accelerators!

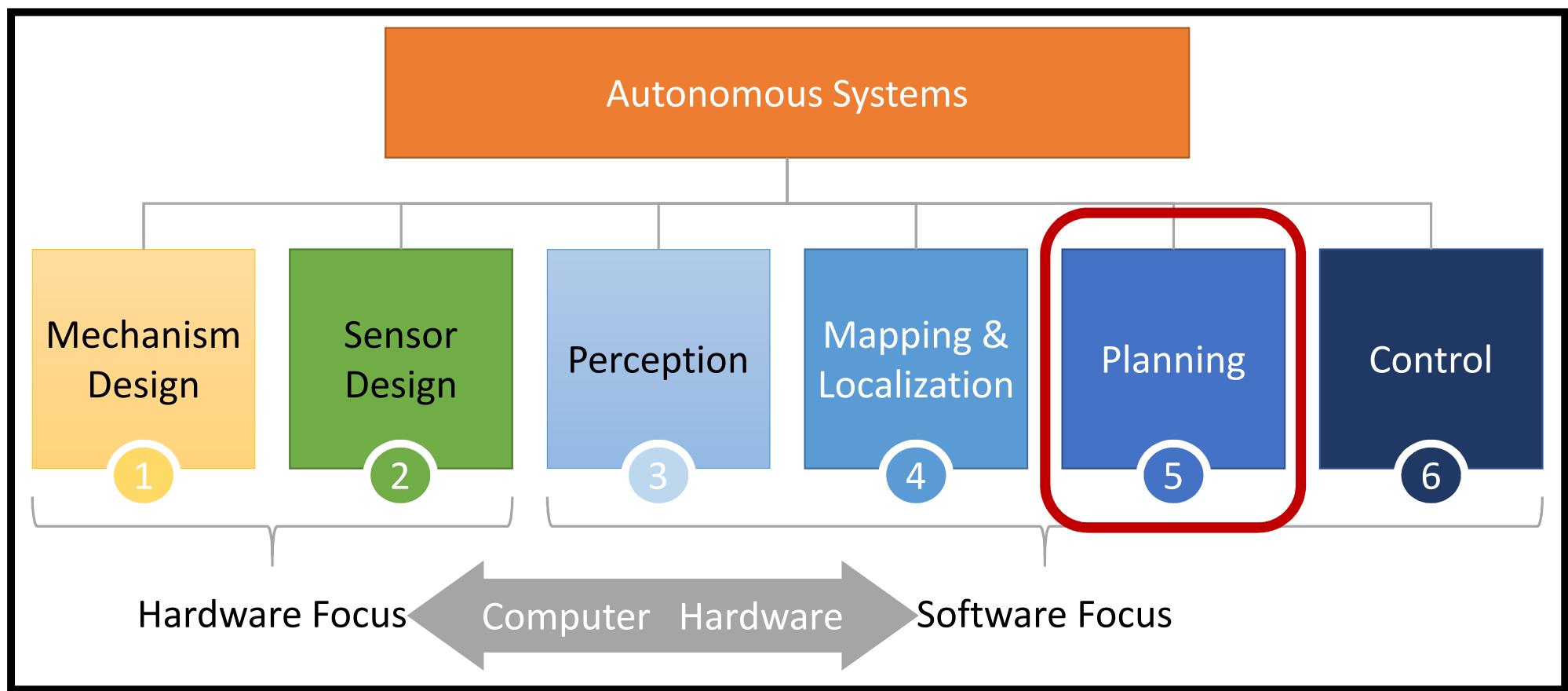
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Key Takeaways:



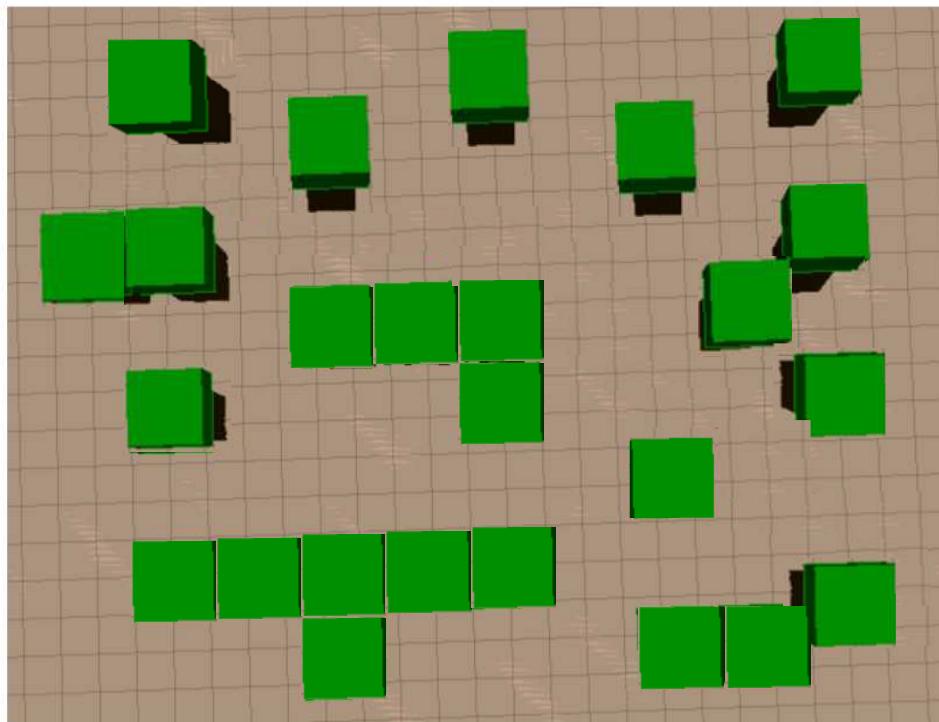
1. The **Kalman/Particle Filter** uses probability to solve the localization problem but **modeling and/or approximations** are needed for it to run efficiently online
2. Mapping quickly becomes a **memory storage problem**
3. Constrained form factors (aka **tiny drones**) will need **novel accelerators** to allow for autonomy

Autonomous Systems / Robotics is a BIG space



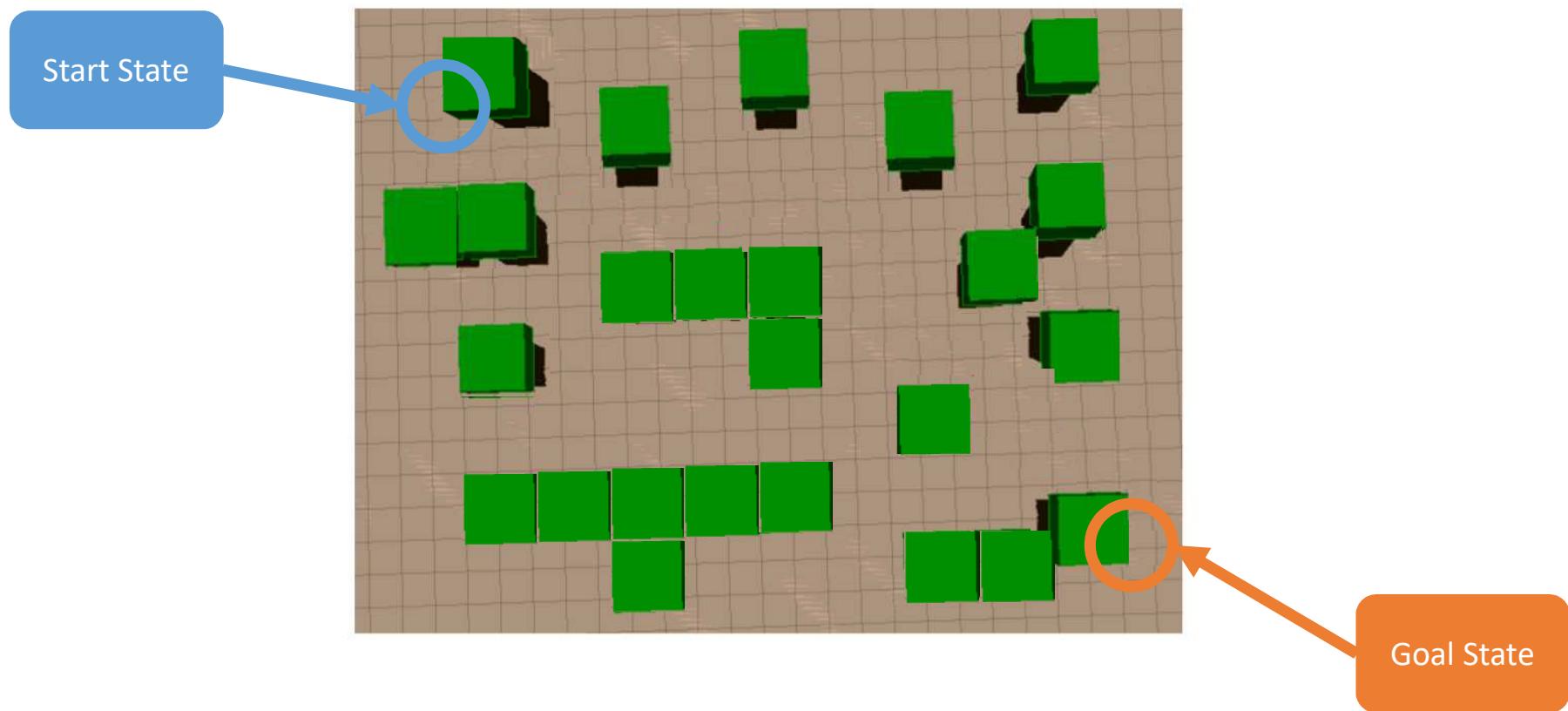
5

Planning is the process of computing an action plan for a robot based on the previously computed map



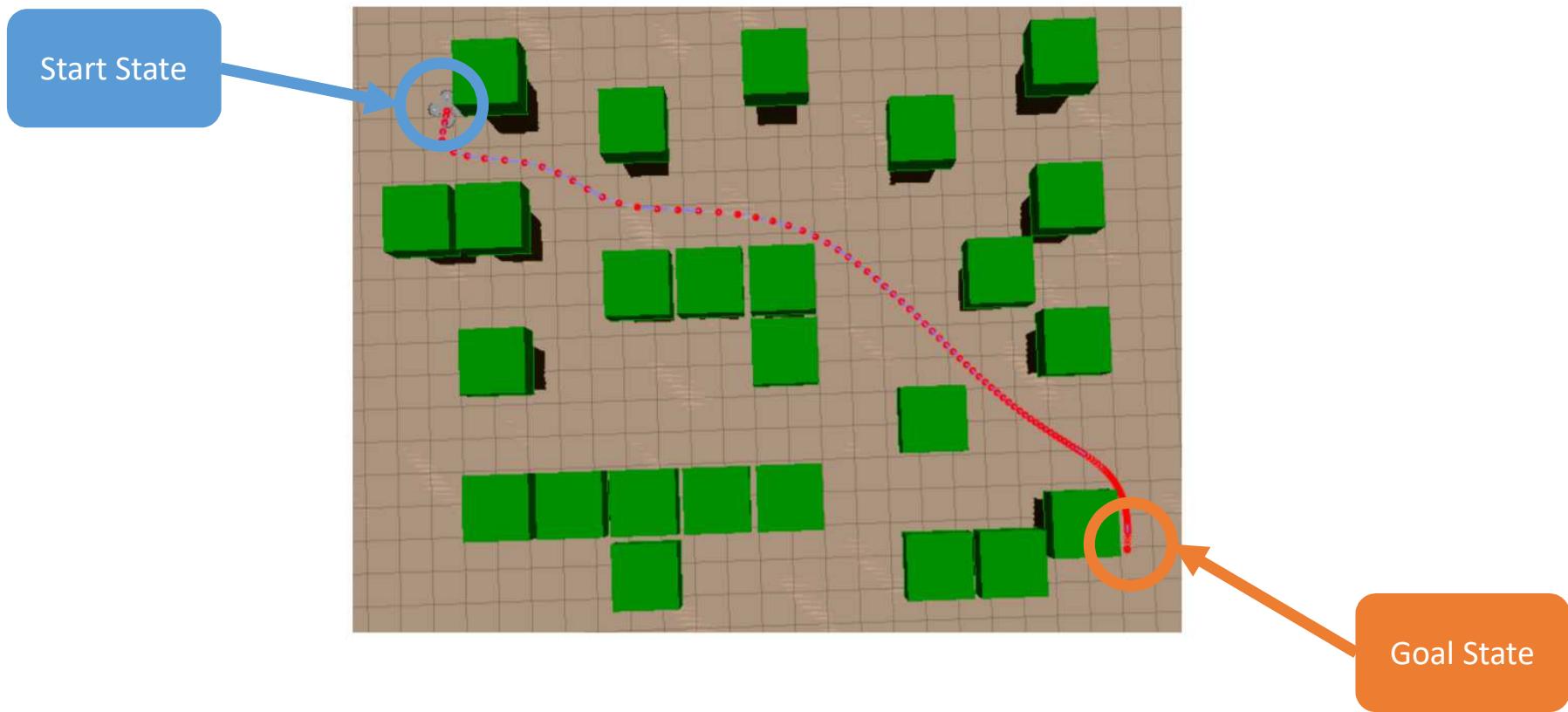
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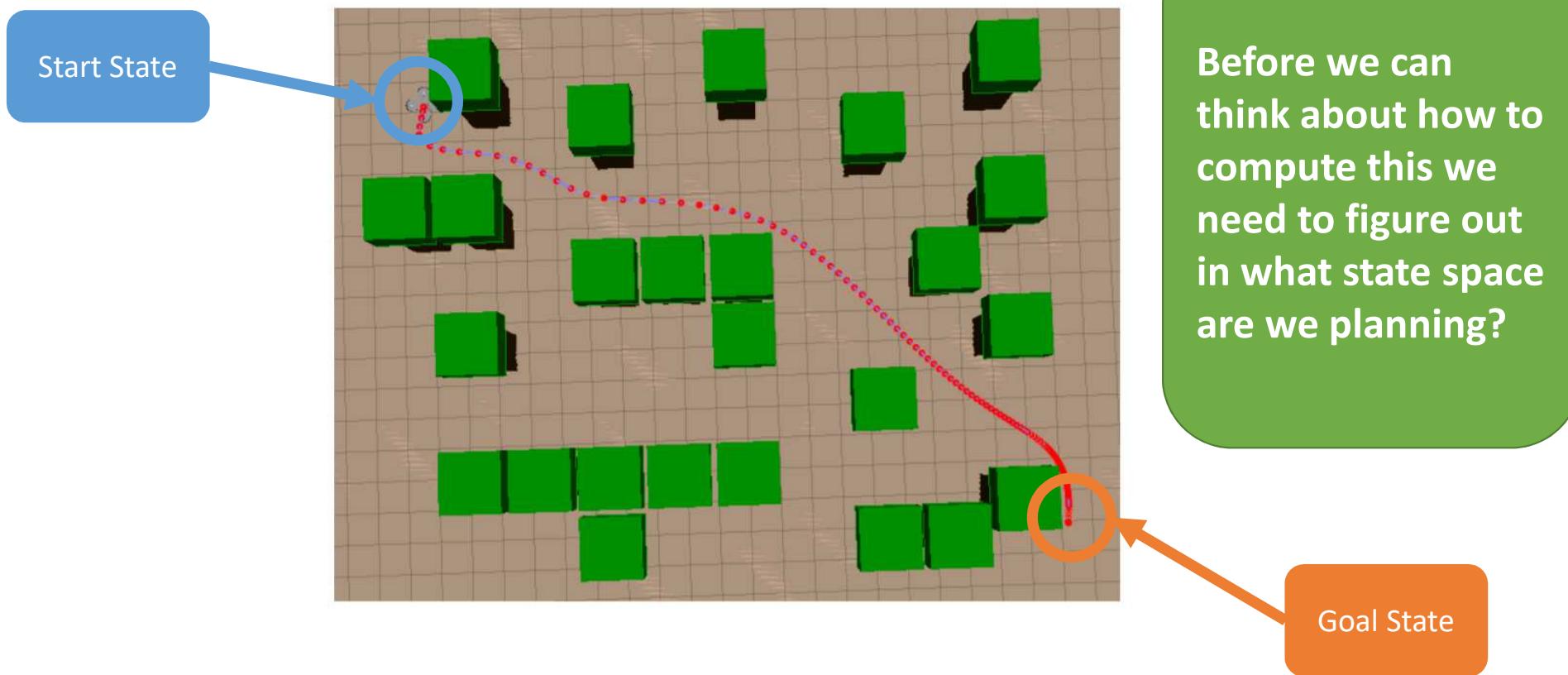
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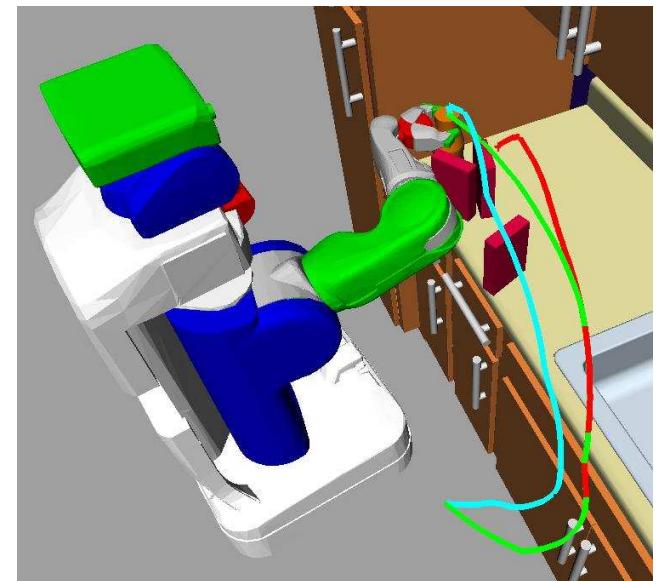
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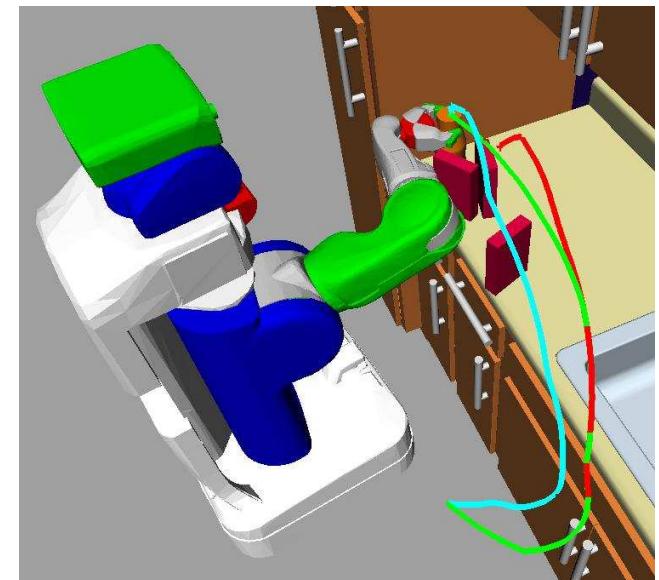
5 Spaces and Transformations (aka where are we planning?)

- **Task space**: the 6D workspace of the robot
 - E.g., the **pose** ($x,y,z,\text{roll},\text{pitch},\text{yaw}$) of the robot's hand or an object



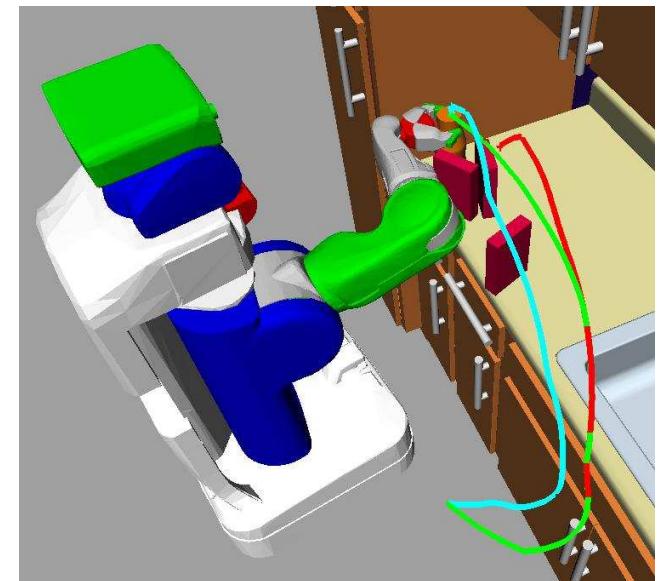
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 - Vector $q \in \mathbb{R}^n$



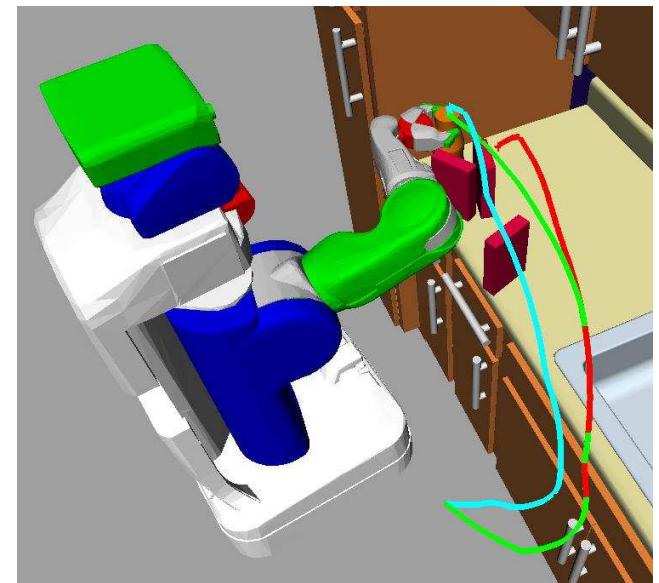
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 - **Inverse kinematics**: maps task space poses to configuration space
-



5 Spaces and Transformations (aka where are we planning?)

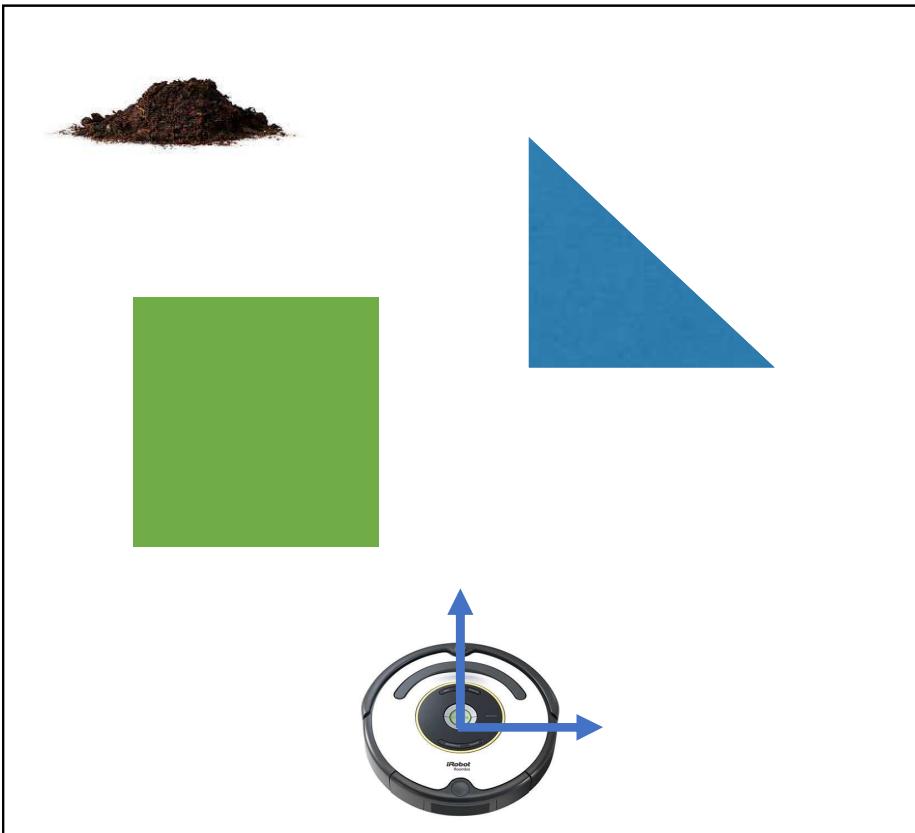
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- **Forward kinematics**: maps q to outputs in task space (e.g. hand position)
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Q: Are forward and inverse kinematics 1 to 1 operations?

5

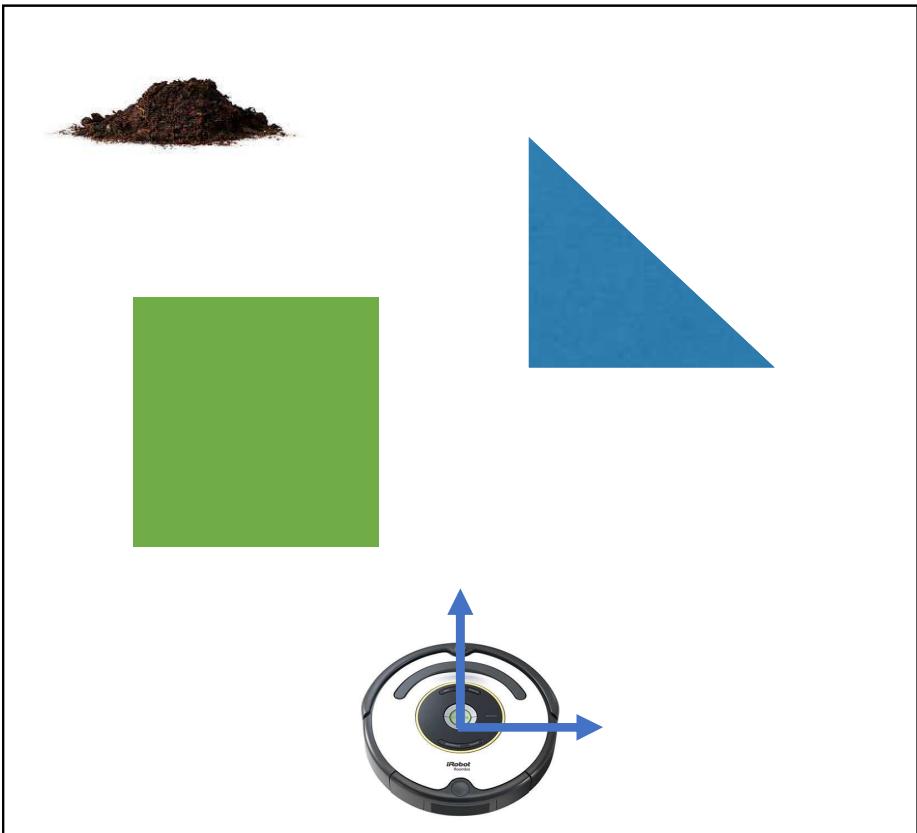
Configuration Space



Q1: What is the configuration space state for this omnidirectional robot?

5

Configuration Space

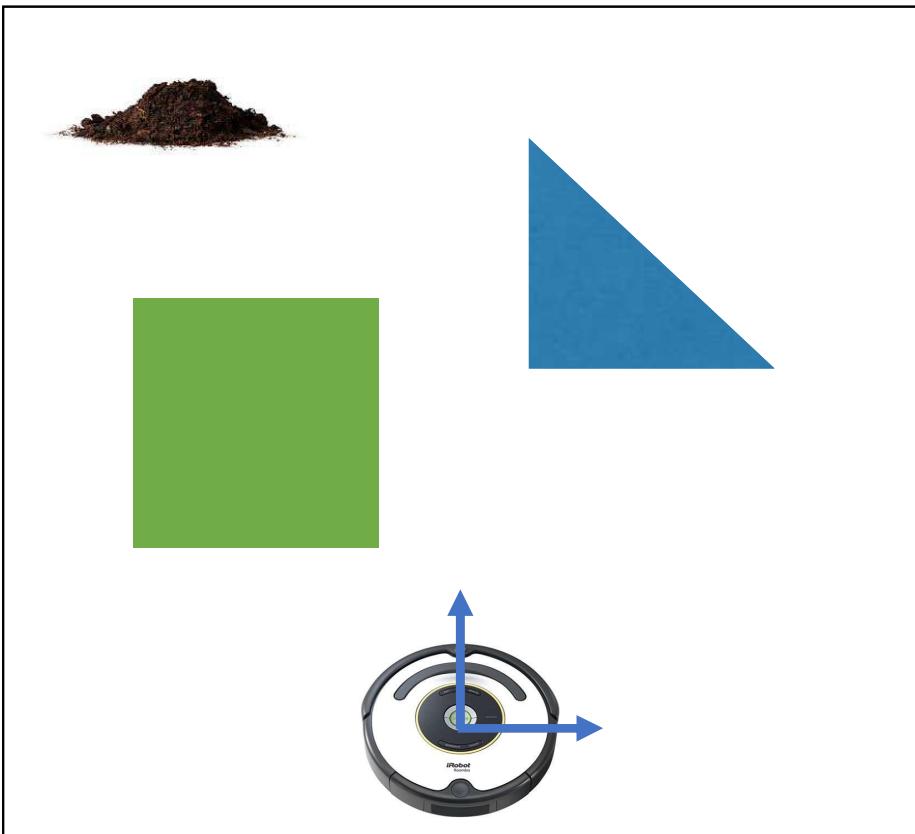


Q1: What is the configuration space state for this omnidirectional robot?

A1: (x,y) position of the center of the robot

5

Configuration Space

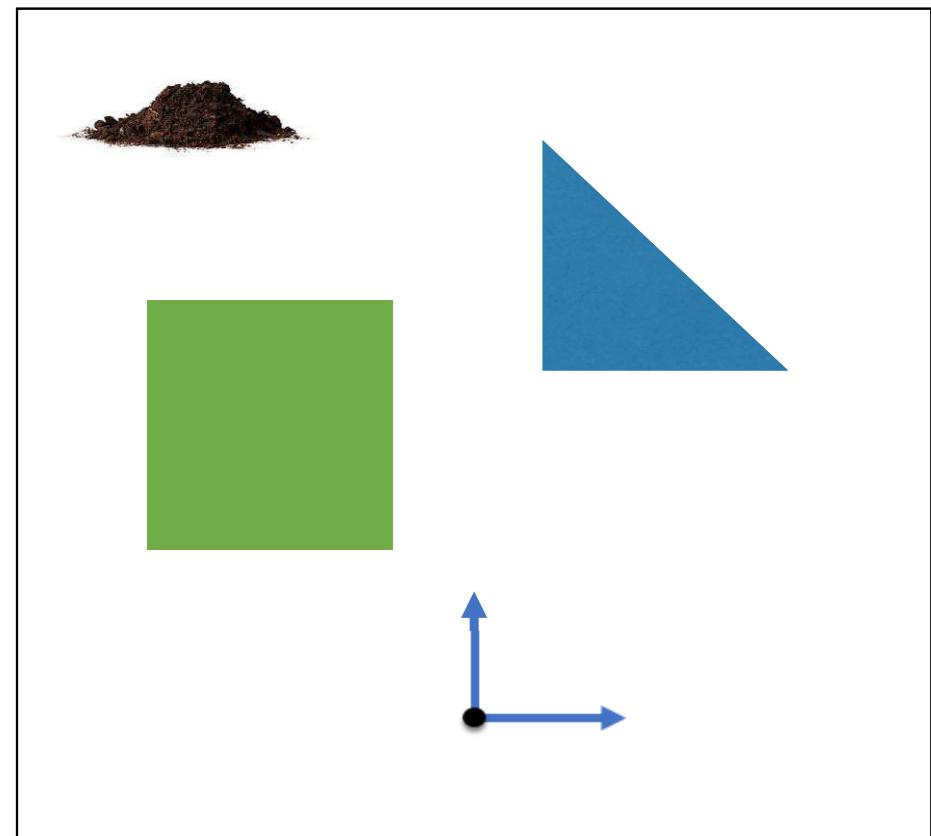
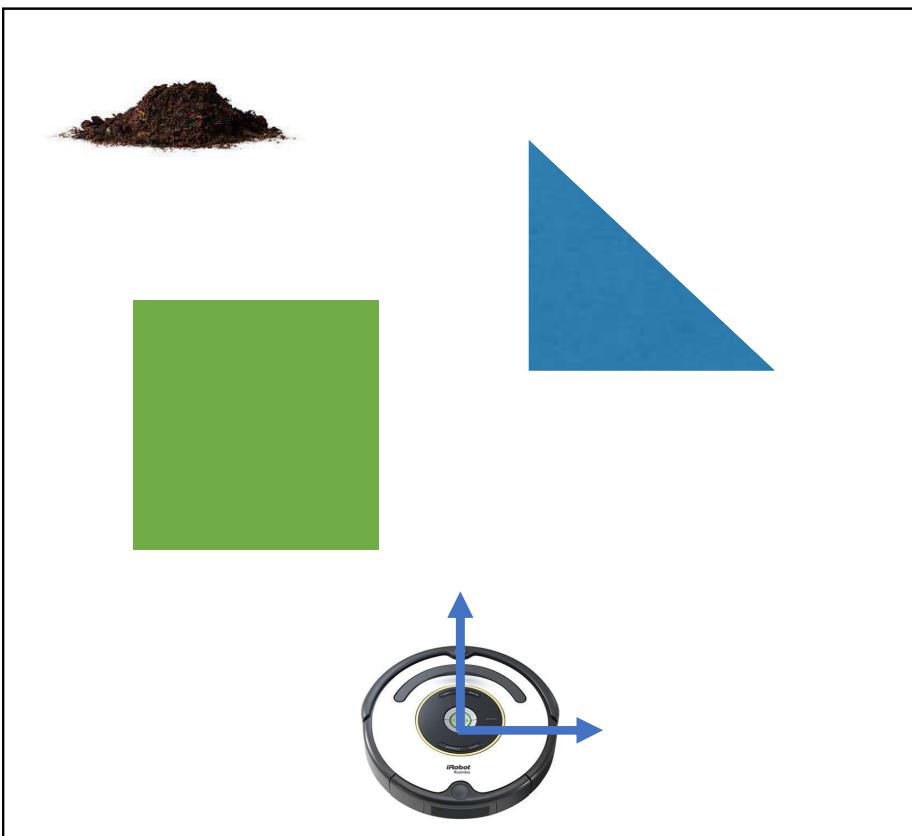


Q2: How can we map this robot's world into configuration space?

5

Configuration Space

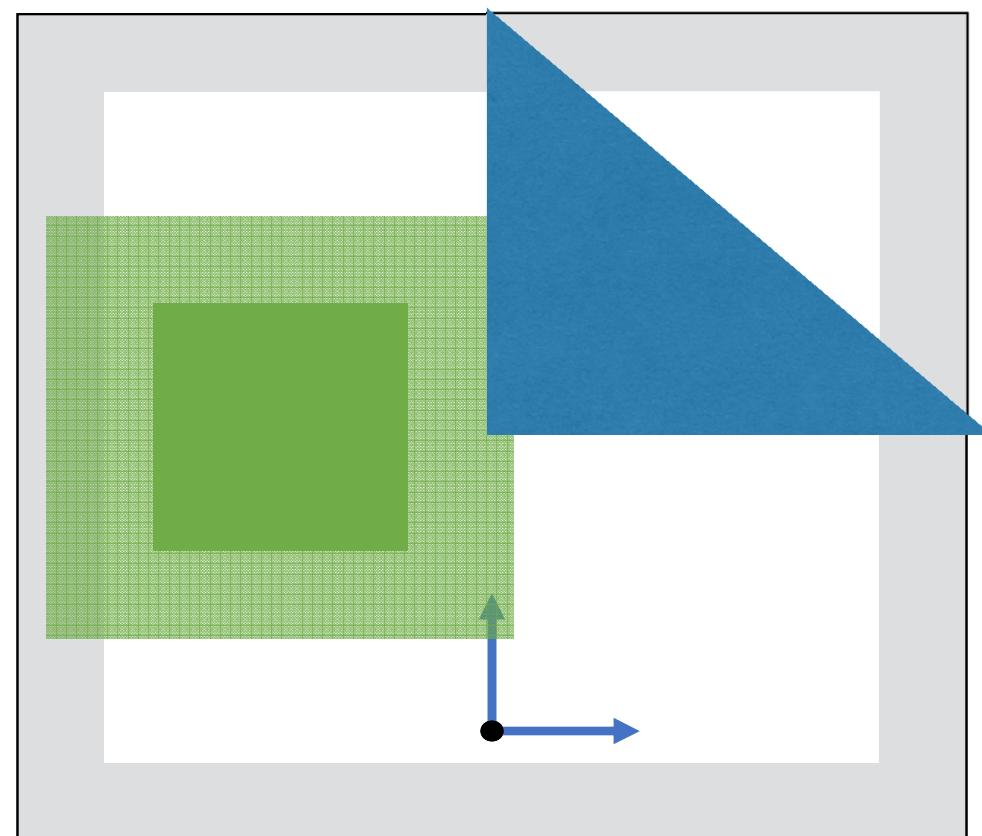
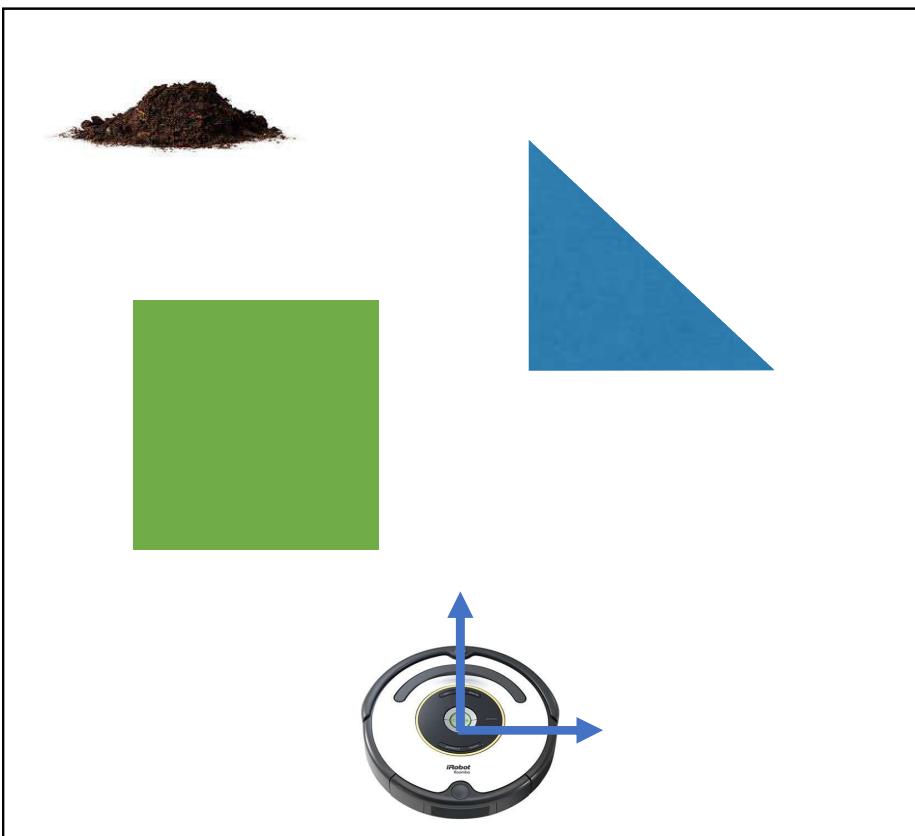
Well we want the robot to become a single (x,y) point



5

Configuration Space

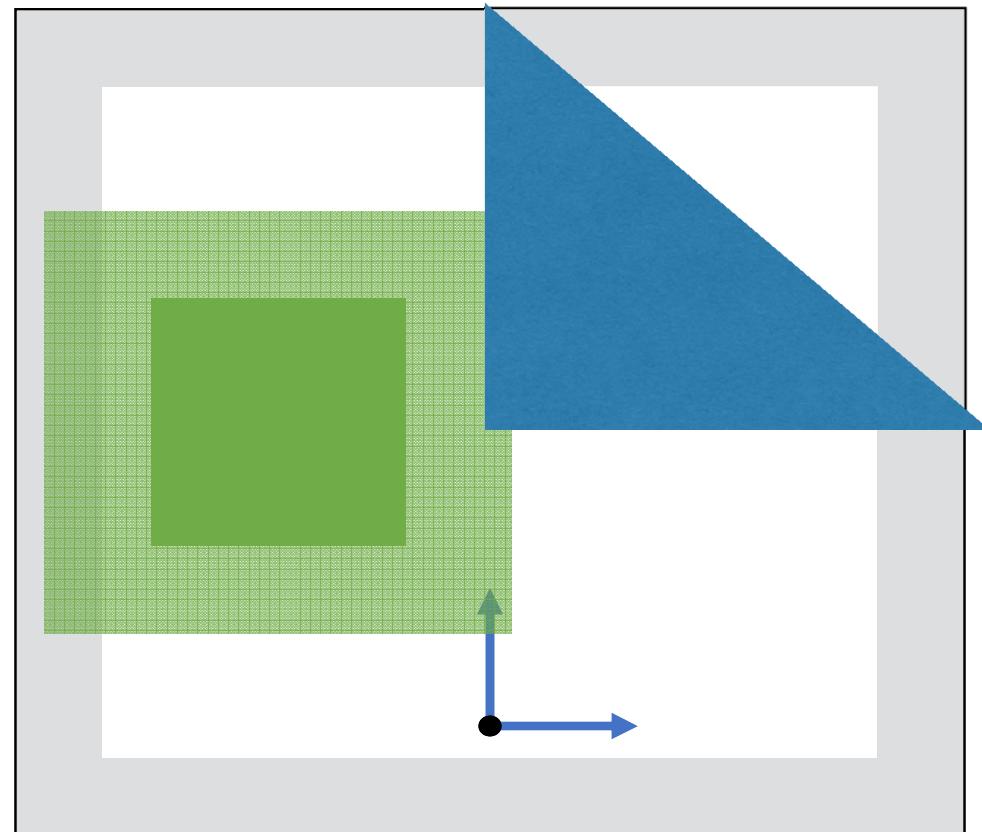
So we need to inflate the obstacles accordingly



5

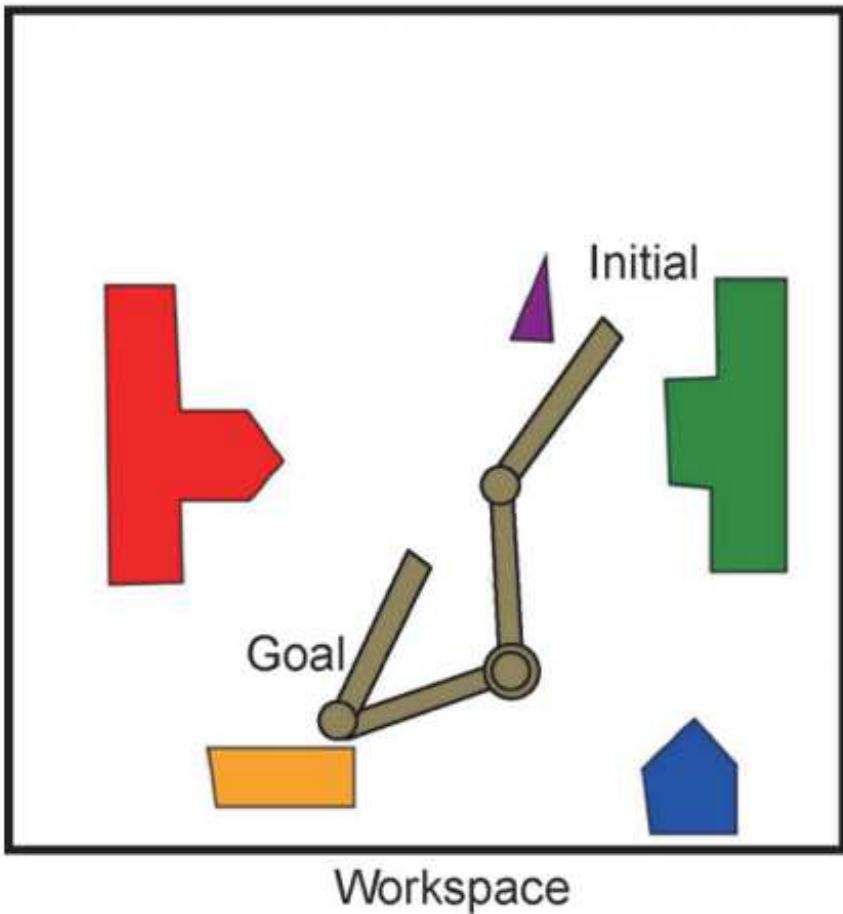
Configuration Space

- **Insight:** mapping task space obstacles and goals into configuration space allows us to **plan a path for a single point** instead of worrying about a full robot



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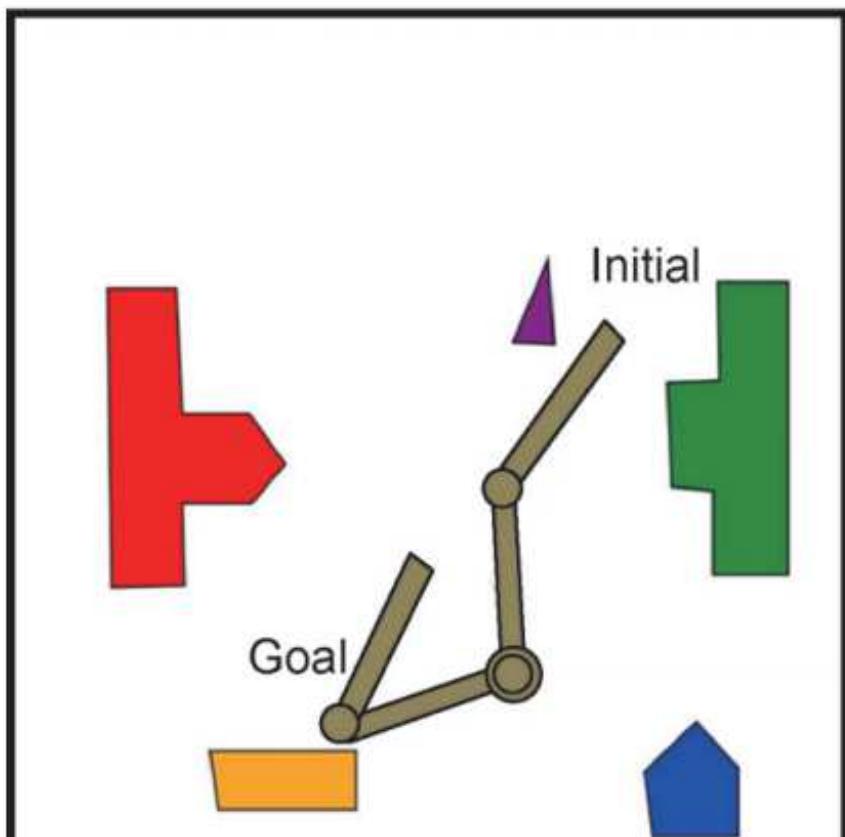
Configuration Space



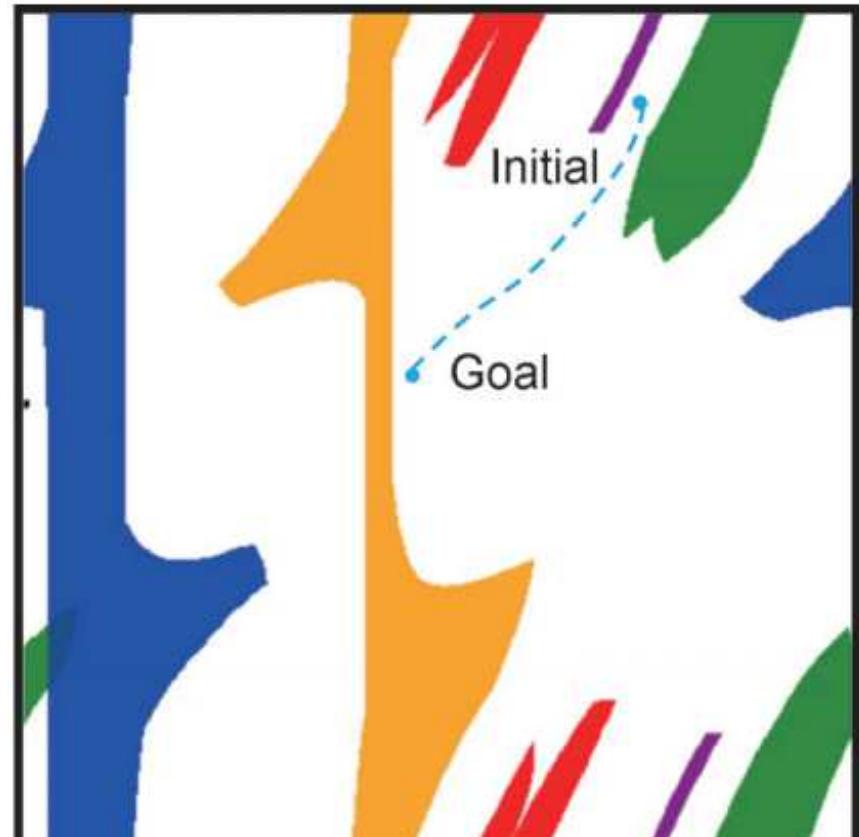
How can we map this robot and its world into configuration space?

5

Configuration Space



Workspace



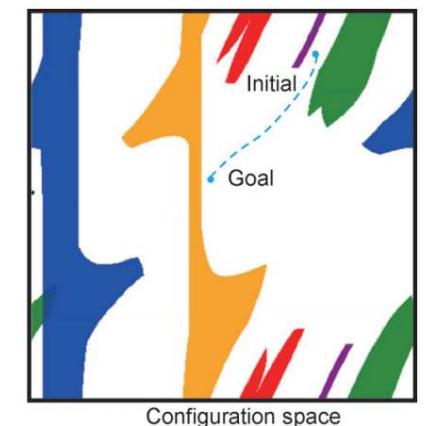
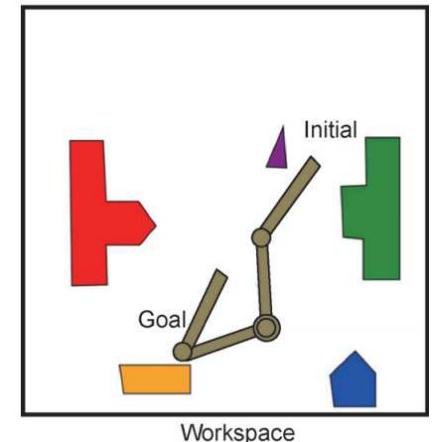
Configuration space

5

How to use configuration space in practice

If we map the obstacles into configuration space we can check whether the configuration point, q , is in an obstacle and we have a **unique plan** for the robot

- **Problem:** mapping obstacles into configuration space is hard



5

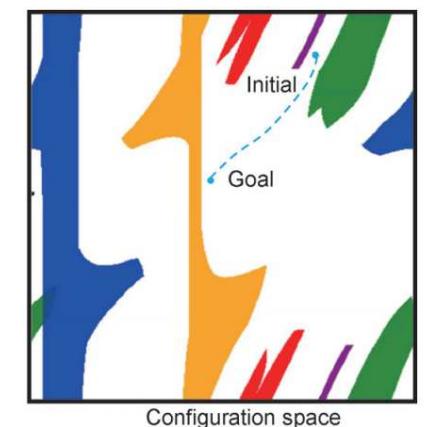
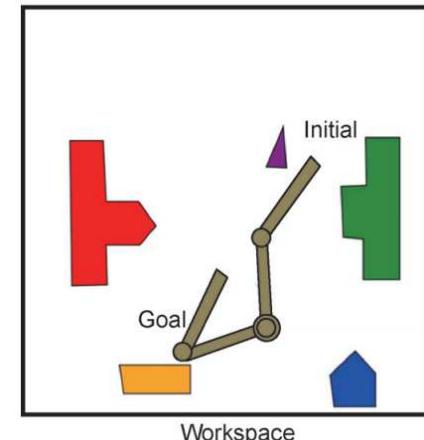
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- **Problem:** mapping obstacles into configuration space is hard

Better approach: **use forward kinematics** to check task space obstacle collisions!

Treat the collision checker as a black box function evaluator!



5

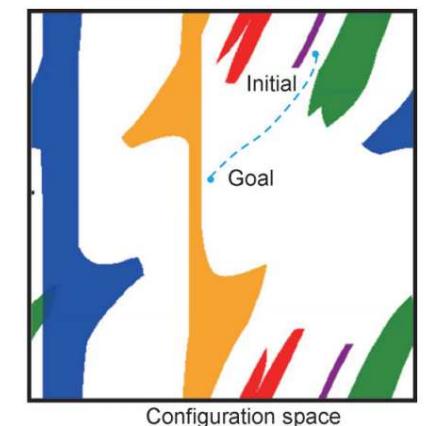
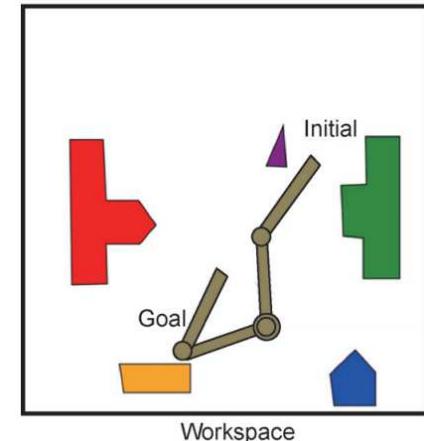
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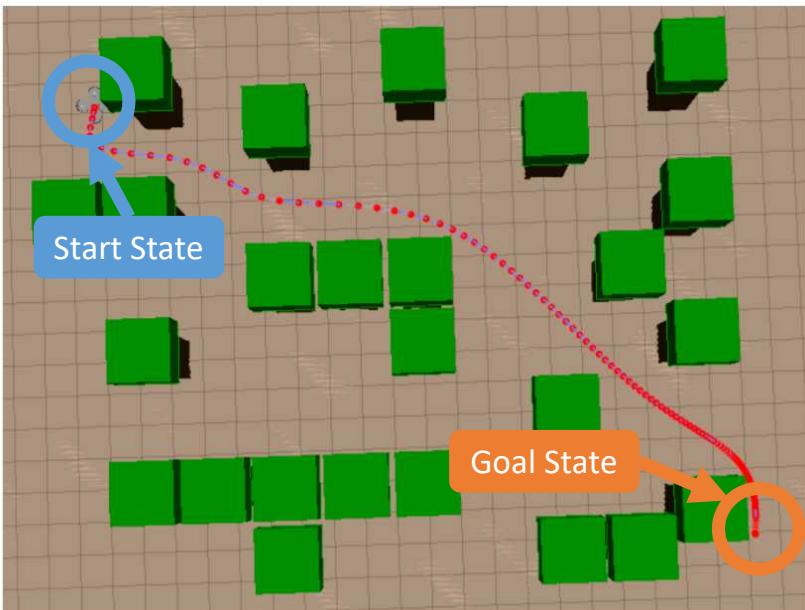
Better approach: **use forward kinematics** to check task space obstacle collisions!

- **No free lunch** – Now each collision check requires full kinematics and not a simple lookup



5

Planning in Configuration Space

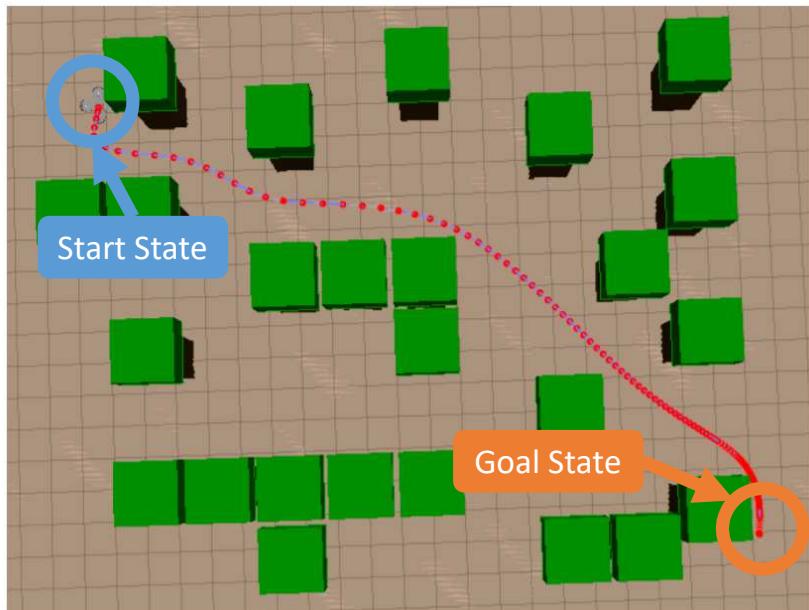


Goal: Find shortest collision-free path from start to goal

States: configurations $q \in \mathcal{R}^6$ **Actions:** Δq **Transition:** $q' \leftarrow q + \Delta q$

5

Planning in Configuration Space



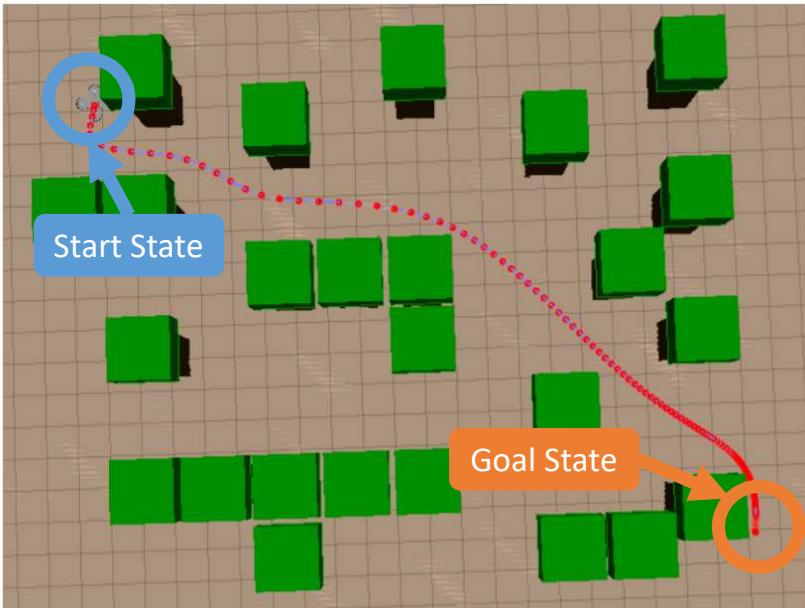
$$q \in \mathcal{R}^6: (x, y, z, \theta, \phi, \varphi)$$

Goal: Find shortest collision-free path from start to goal

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Planning in Configuration Space



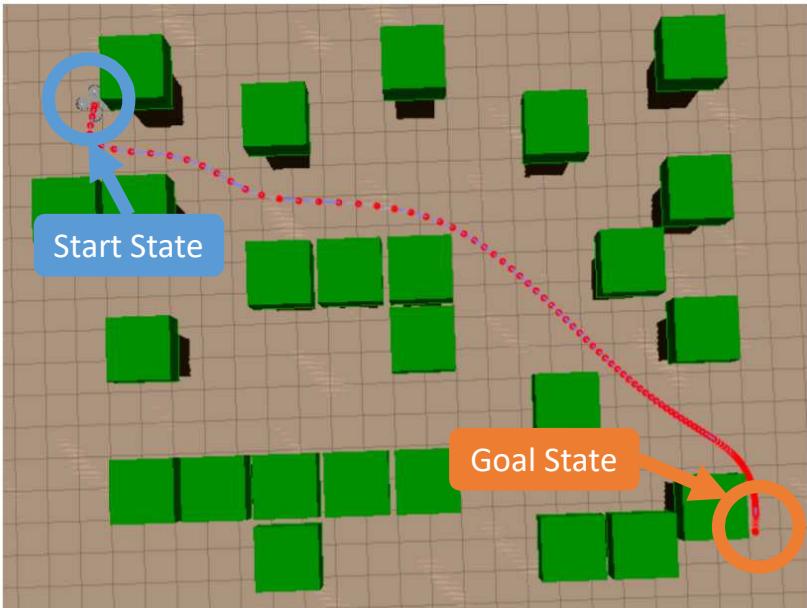
One approach is to *discretize* the statespace (grid it) and use graph search (think A* which is known fast)

Goal: Find shortest collision-free path from start to goal

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5

Planning in Configuration Space



One approach is to *discretize* the statespace (grid it) and use graph search (think A* which is known fast)

Unfortunately if we use say 100 discrete steps in each direction we get:

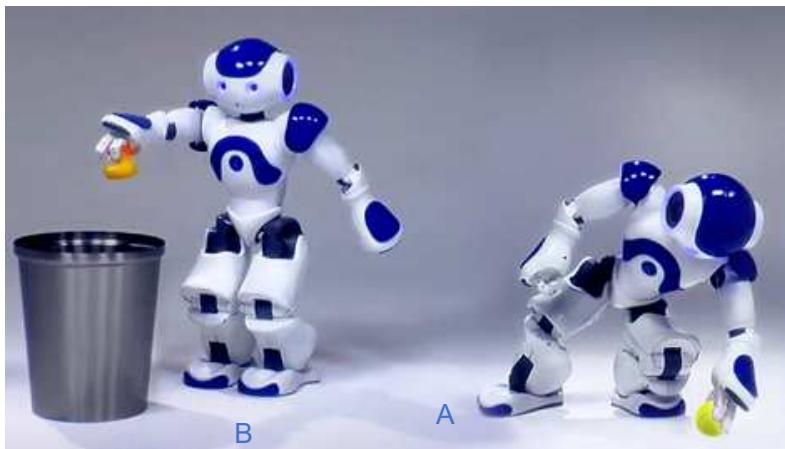
$$|S| = 100^6$$

Goal: Find shortest collision-free path from start to goal

States: configurations $q \in \mathcal{R}^6$ **Actions:** Δq **Transition:** $q' \leftarrow q + \Delta q$

5

Planning in Configuration Space



Goal: Find shortest collision-free path from

States: configurations $q \in \mathcal{R}^{20}$

Actions: Δ

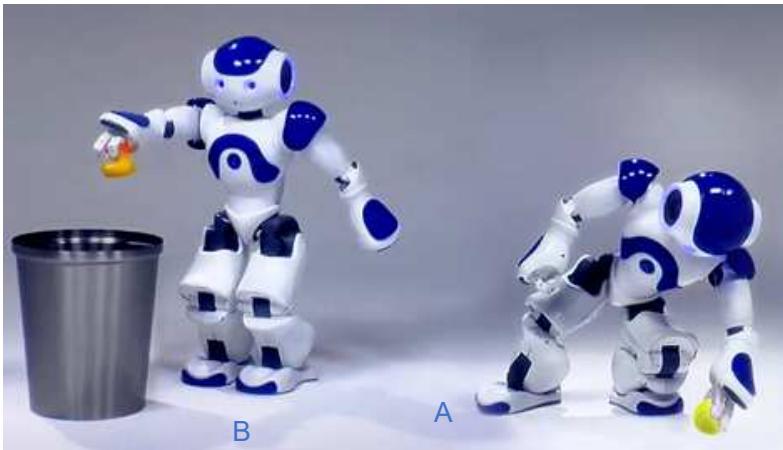
One approach is to ***discretize*** the statespace (grid it) and use graph search (think A* which is known fast)

Unfortunately if we use say 100 discrete steps in each direction we get:

(2 ankles + 2 knees + 2 hips + 2 shoulders + 2 elbows + 4 fingers + pose of com) = ~ 20 variables

5

Planning in Configuration Space



One approach is to *discretize* the statespace (grid it) and use graph search (think A* which is known fast)

Unfortunately if we use say 100 discrete steps in each direction we get:

$$|S| = 100^{20}$$

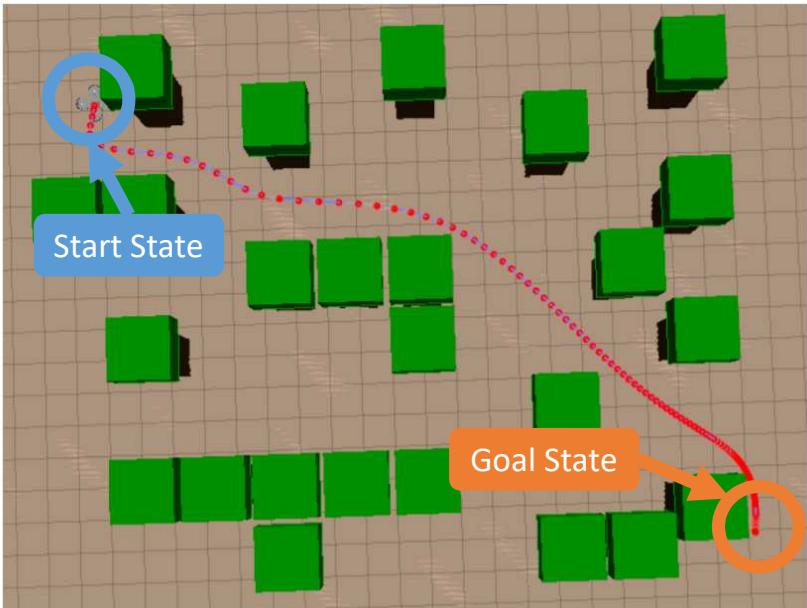
Curse of Dimensionality!

Goal: Find shortest collision-free path from start to goal

States: configurations $q \in \mathcal{R}^{20}$ **Actions:** Δq **Transition:** $q' \leftarrow q + \Delta q$

5

Planning in Configuration Space



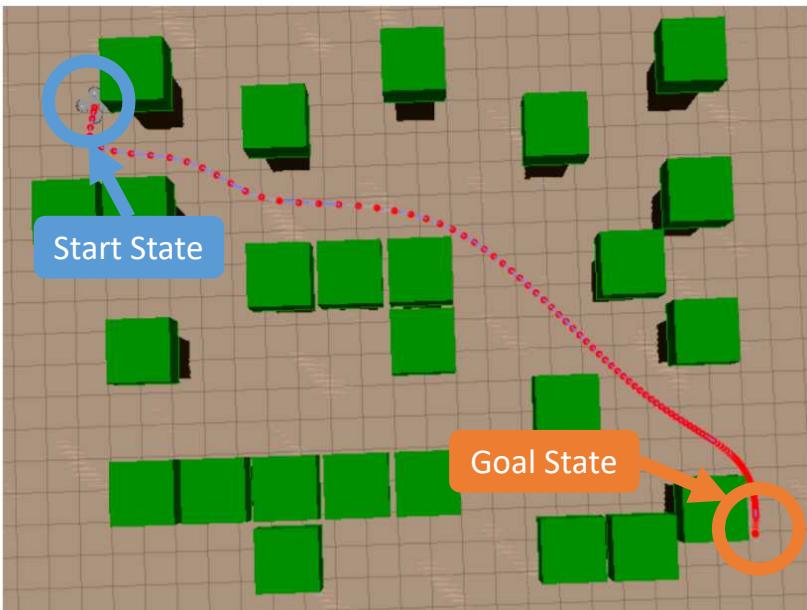
So if we can't explicitly form the graph and search the configuration space what can we do?

Goal: Find shortest collision-free path from start to goal

States: configurations $q \in \mathcal{R}^6$ **Actions:** Δq **Transition:** $q' \leftarrow q + \Delta q$

5

Planning in Configuration Space



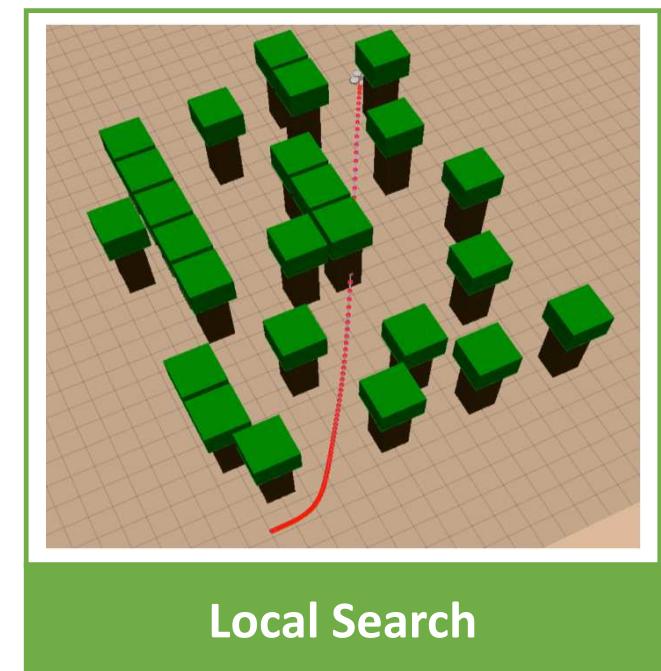
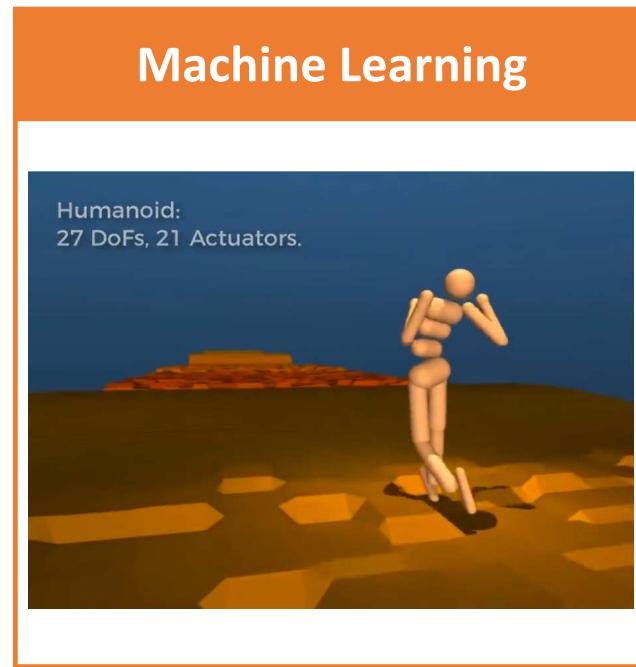
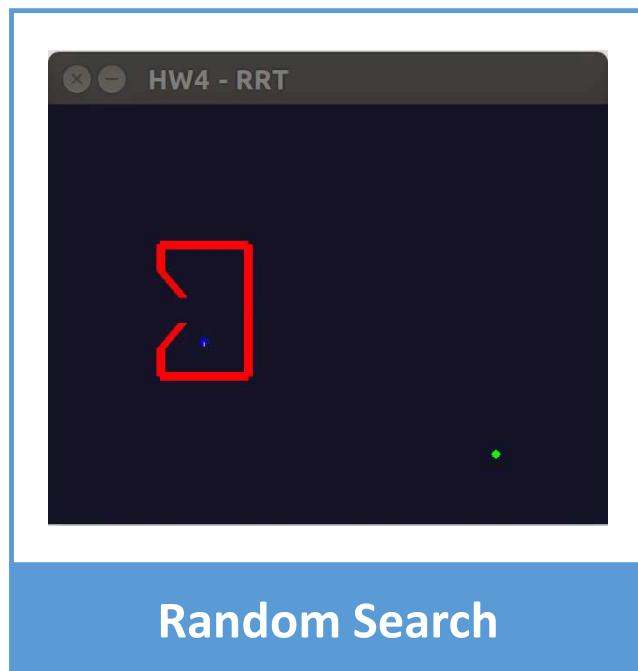
What if we **incrementally build up a path** toward the goal?

Goal: Find shortest collision-free path from start to goal

States: configurations $q \in \mathcal{R}^6$ **Actions:** Δq **Transition:** $q' \leftarrow q + \Delta q$

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Planning in Configuration Space

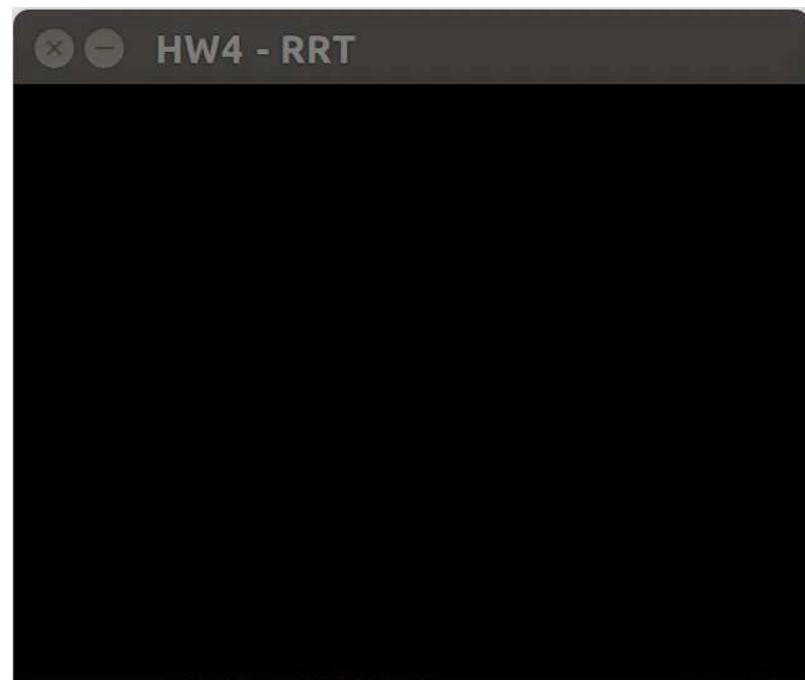


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Rapidly Exploring Random Trees (RRTs)

One of the most famous robot motion planning algorithms is **Rapidly Exploring Random Trees (RRTs)** [Lavalle & Kuffner]

The main idea is to **use randomness** to **rapidly explore** an entire state space to find a path from a given start location to the goal.



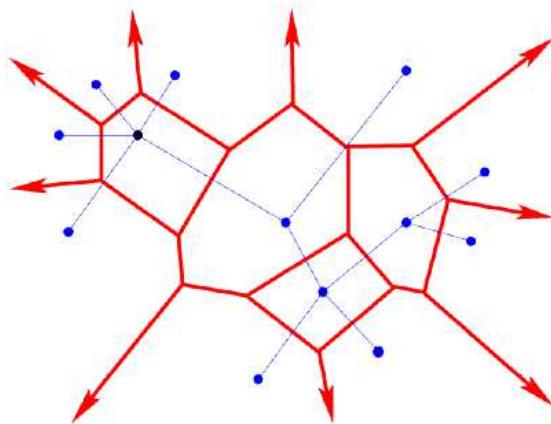
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Randomness encourages exploration

Key idea: uniform random sampling in configuration space is actually a heuristic that encourages exploration!

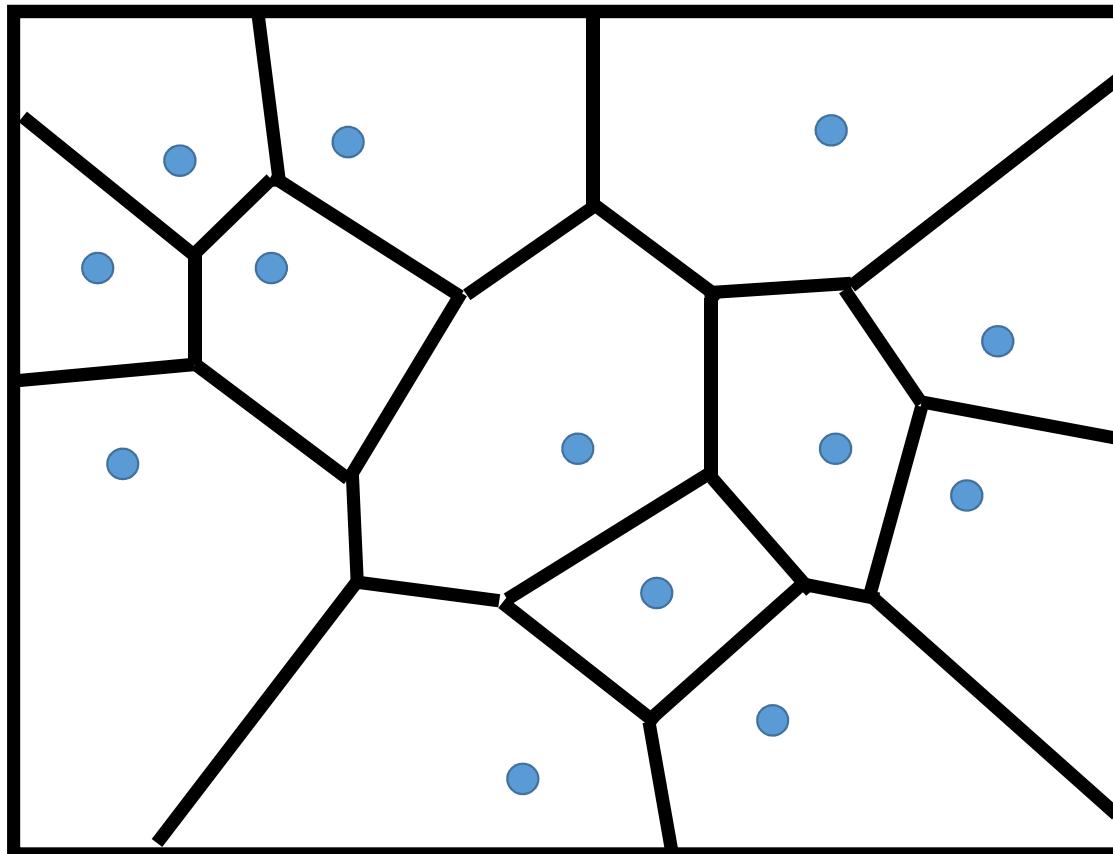
To see this we use **Voronoi regions**

Def: Voronoi region is the set of points in space that are closest to a particular node in the tree:



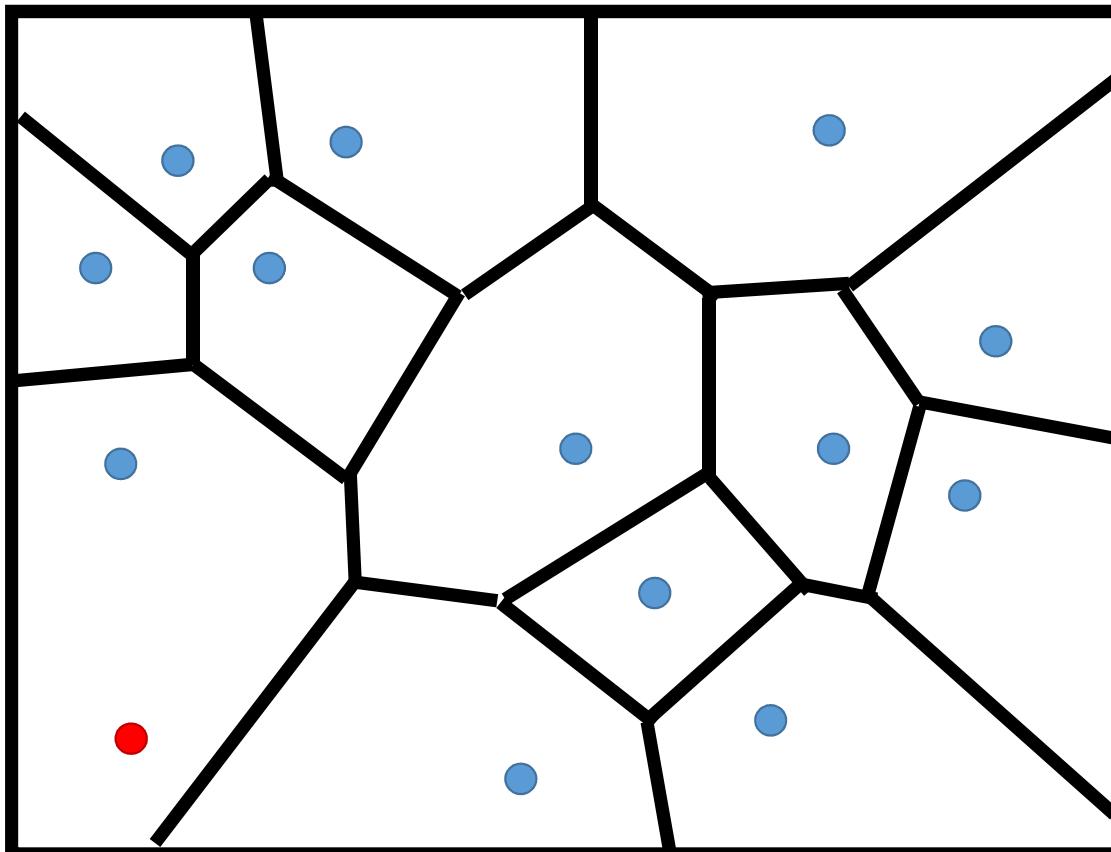
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Randomness encourages exploration



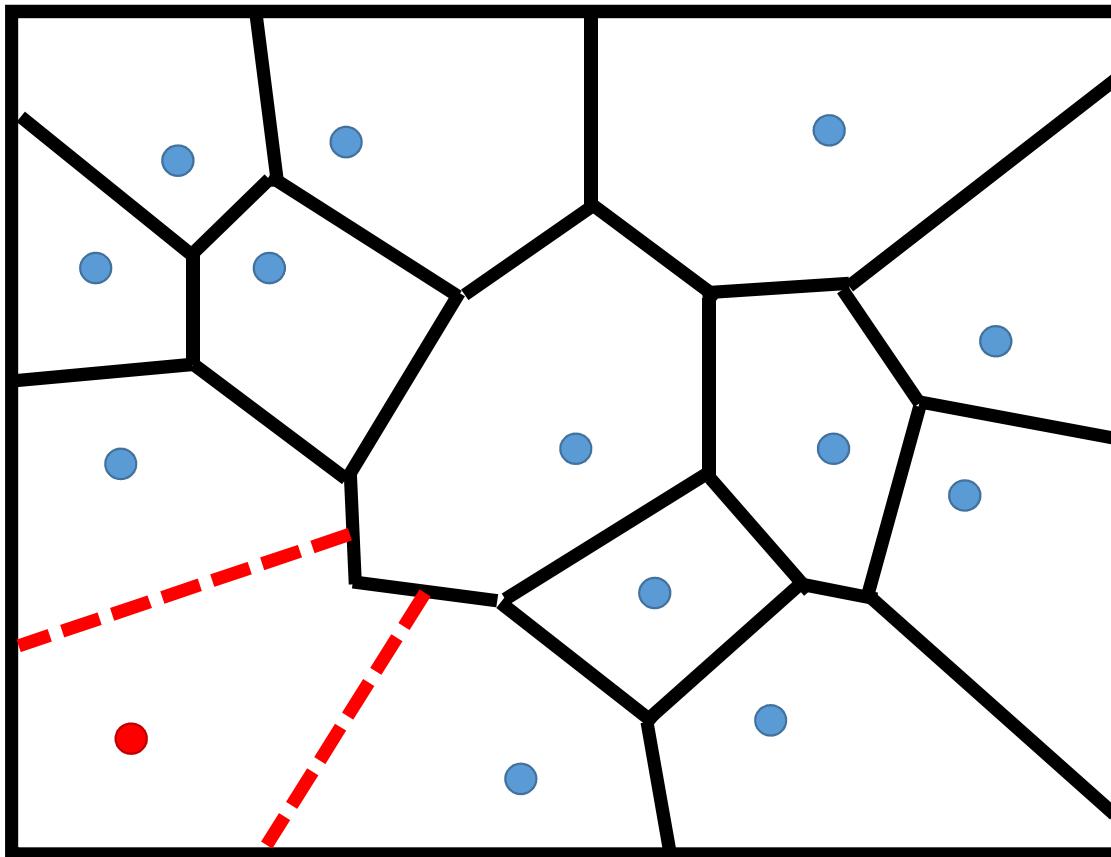
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Randomness encourages exploration



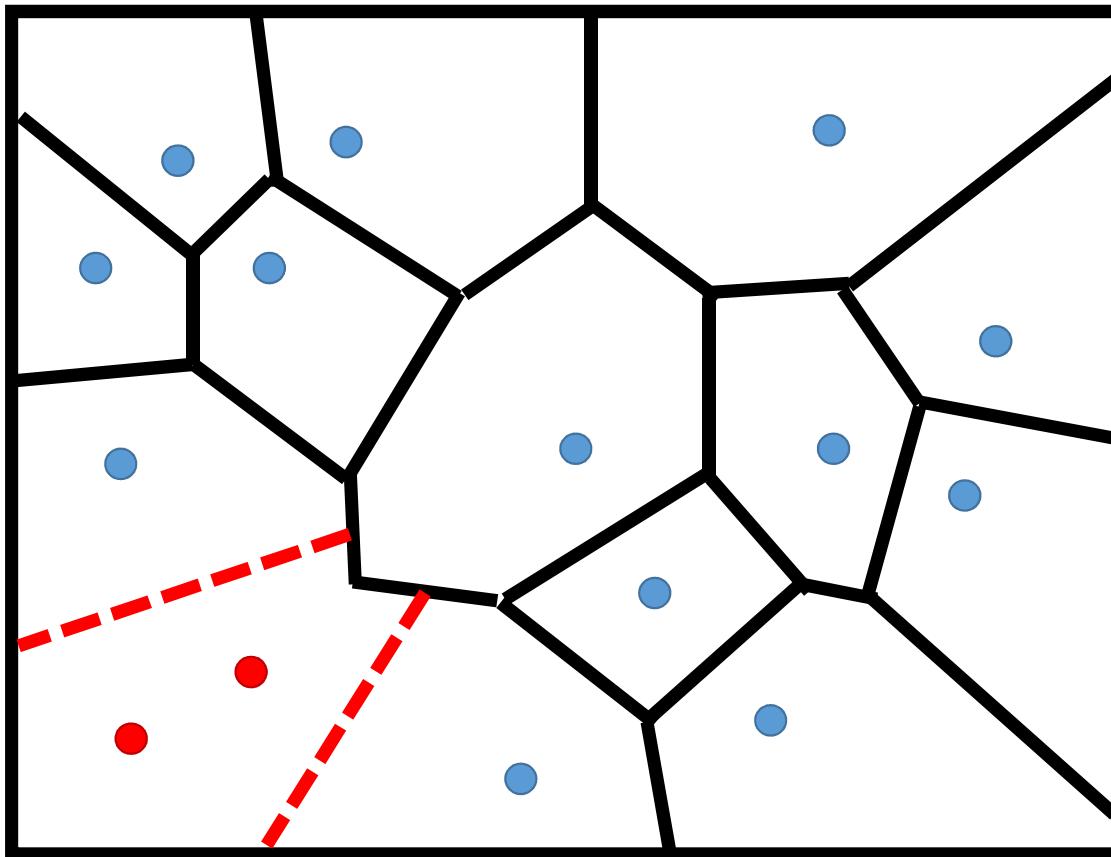
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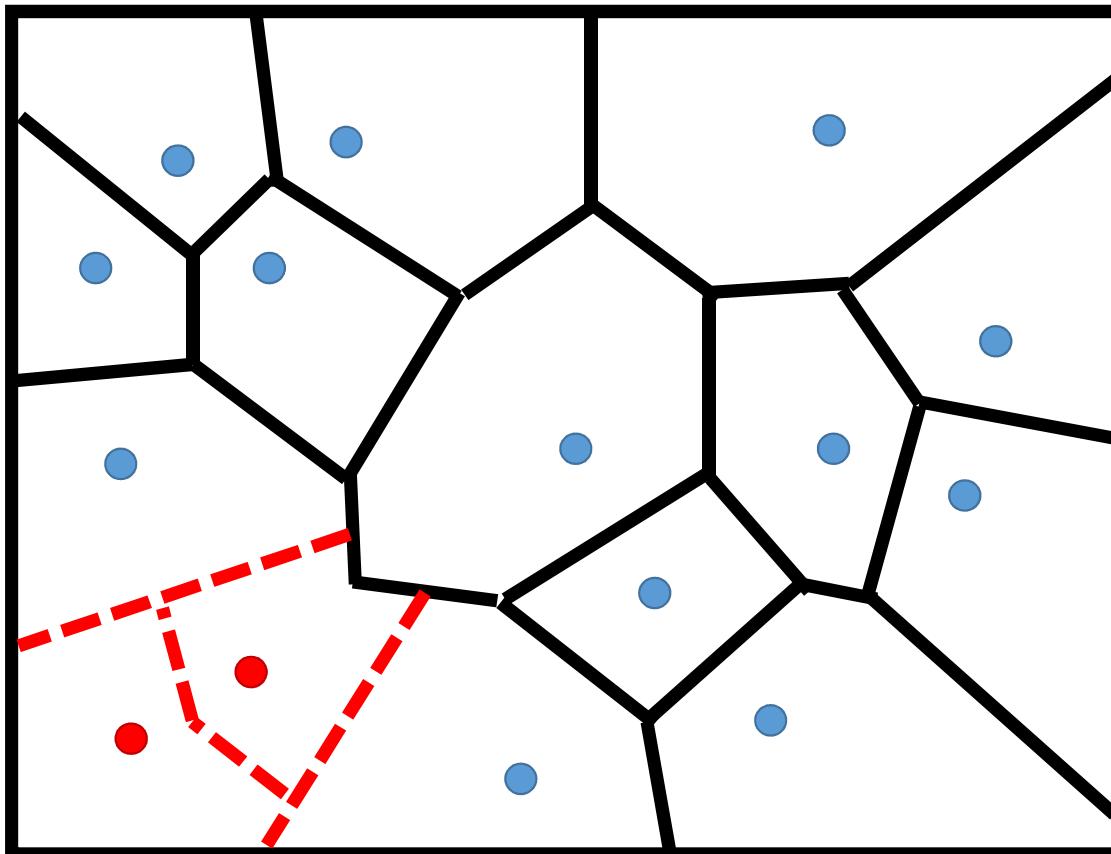
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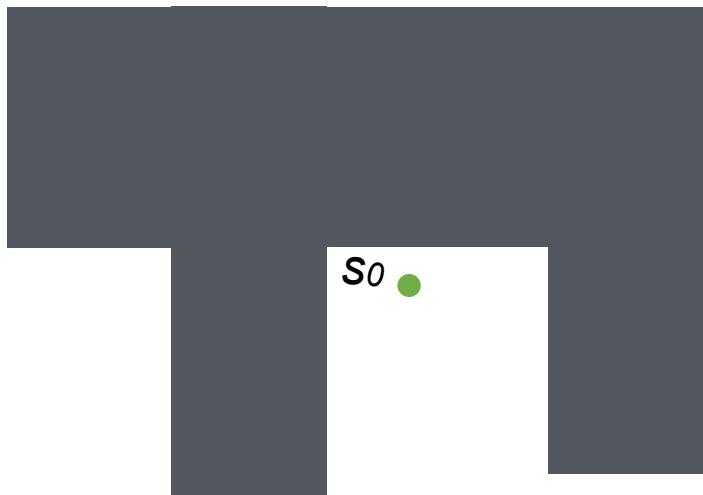
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Rapidly Exploring Random Trees (RRTs)

Algorithm (input: s_0 , s_{goal} , initial state tree T)

- Sample states $s \in S = R^n$ until s is collision-free
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- Repeat until T contains a path from s_0 to s_{goal}

● s_{goal}



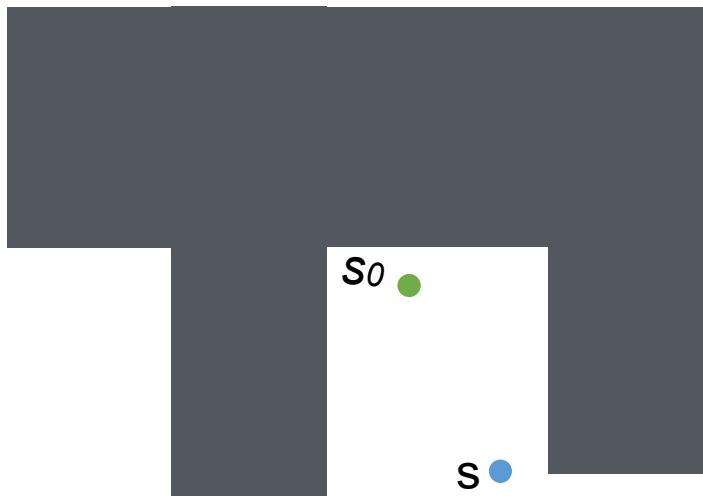
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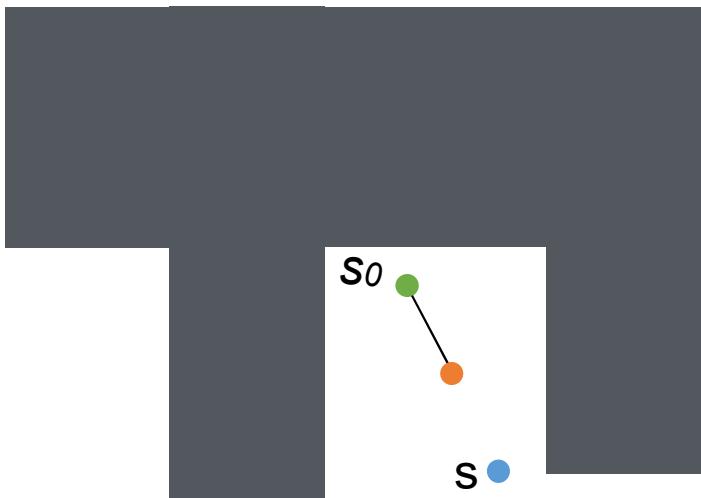
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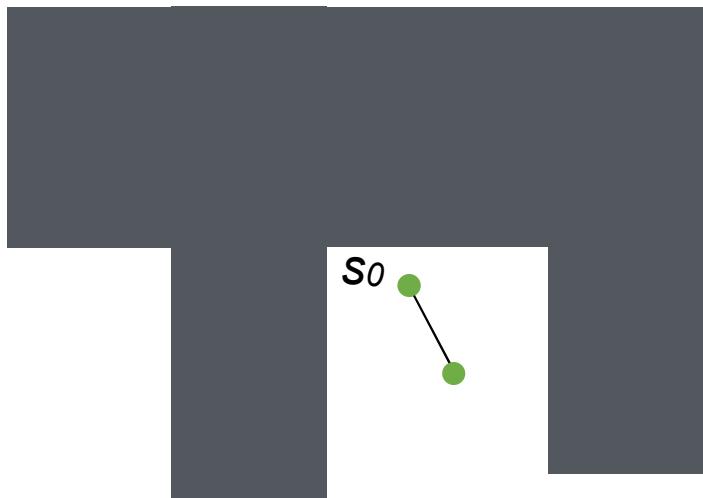
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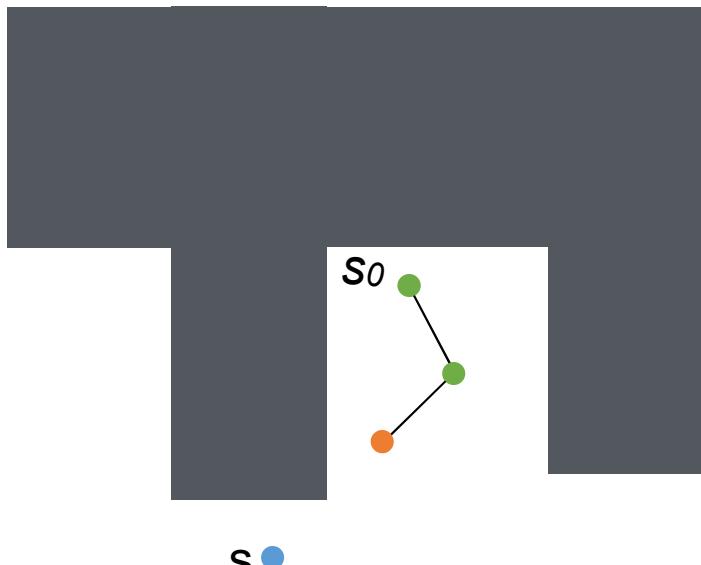
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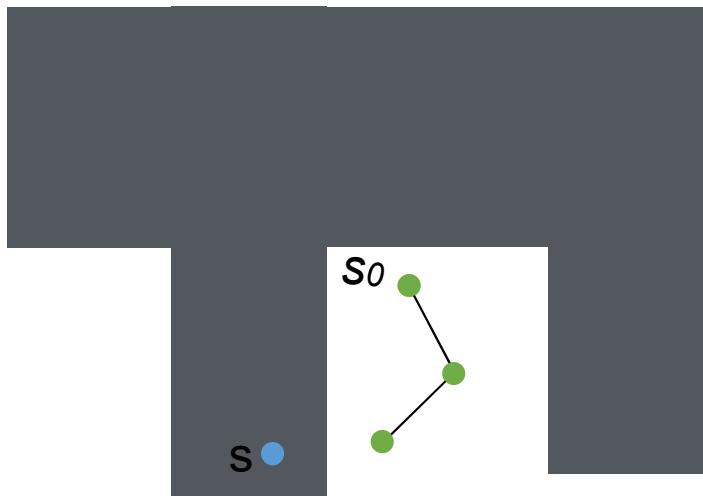
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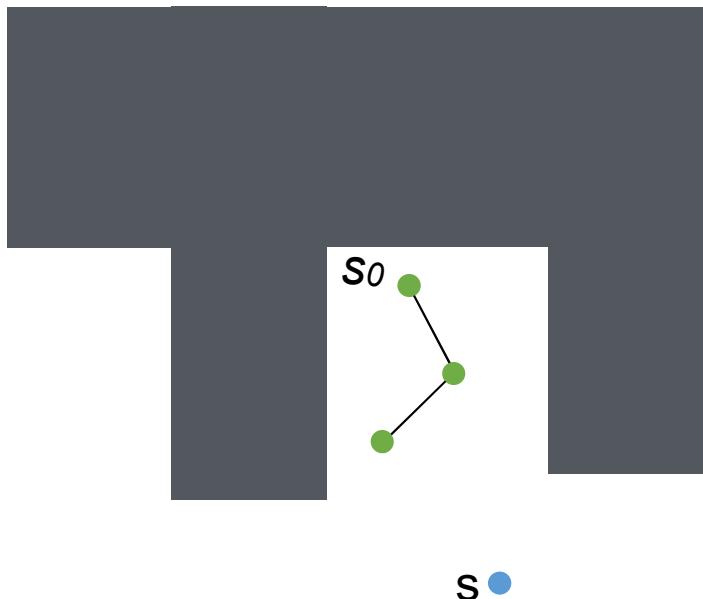
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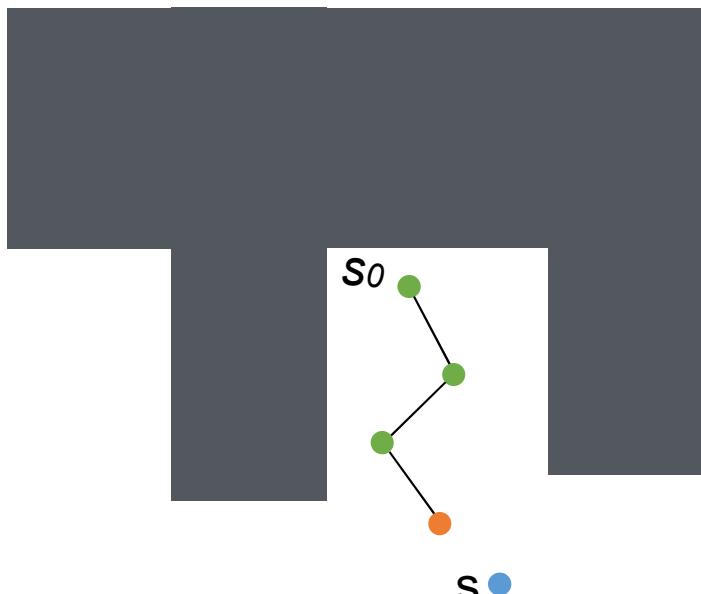
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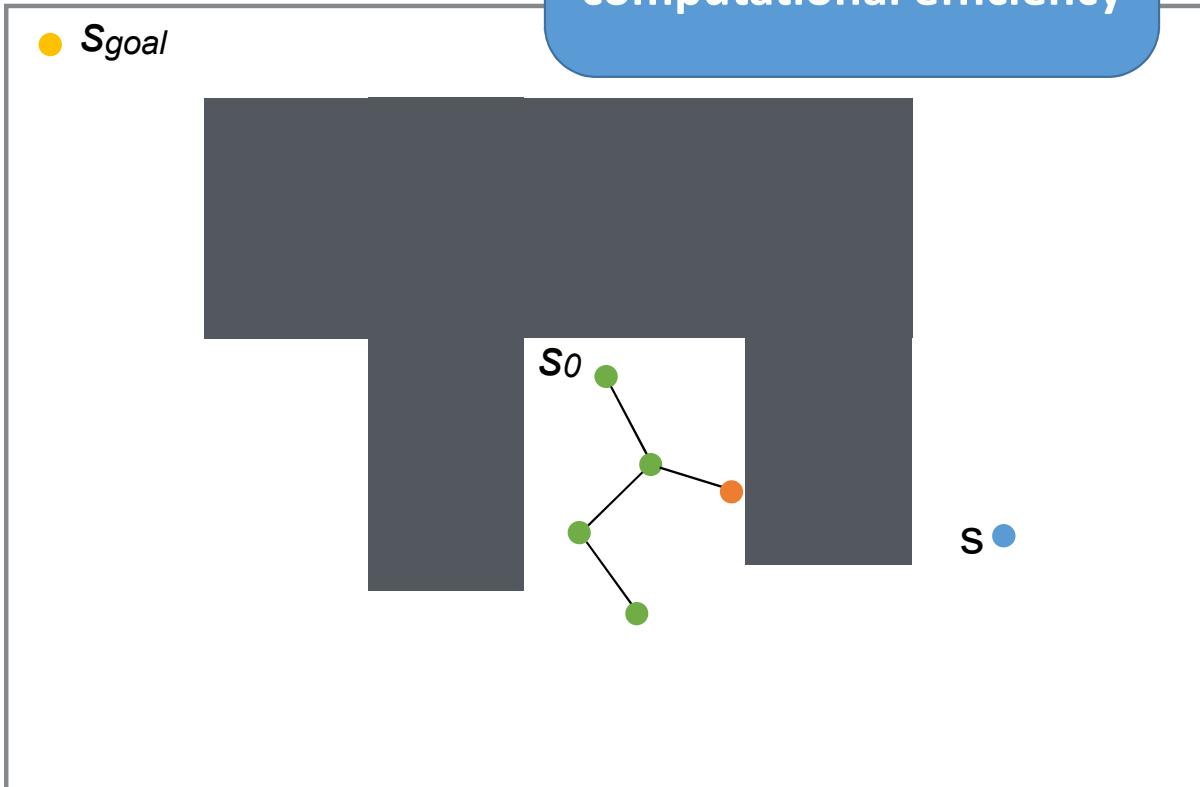
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Rapidly Exploring Random Trees (RRTs)

Extend distance trades off sample vs. computational efficiency

Algorithm (input: s_0, s_{goal} , initial state tree T)

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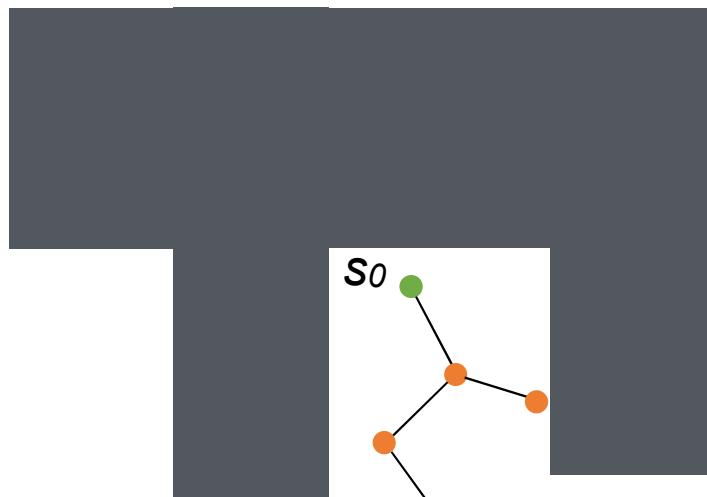
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s

It will always find a solution because it is **probabilistically complete**

5

Rapidly Exploring Random Trees (RRTs)



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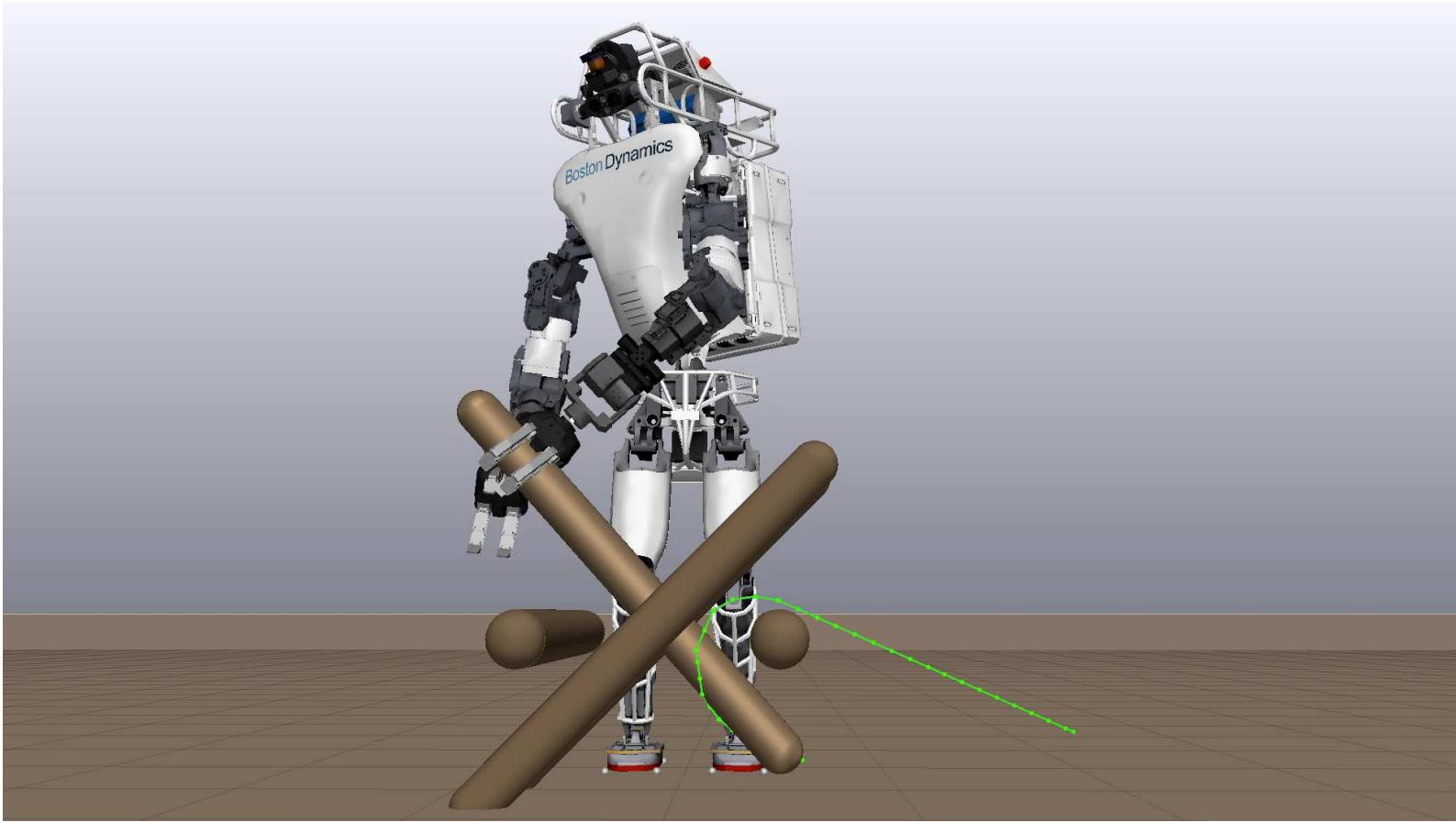
Rapidly Exploring Random Trees (RRTs)



Biased
sampling
can help!

5

RRTs often works really well in practice



5

RRTs often work really well in practice

204 J. Leonard et al.

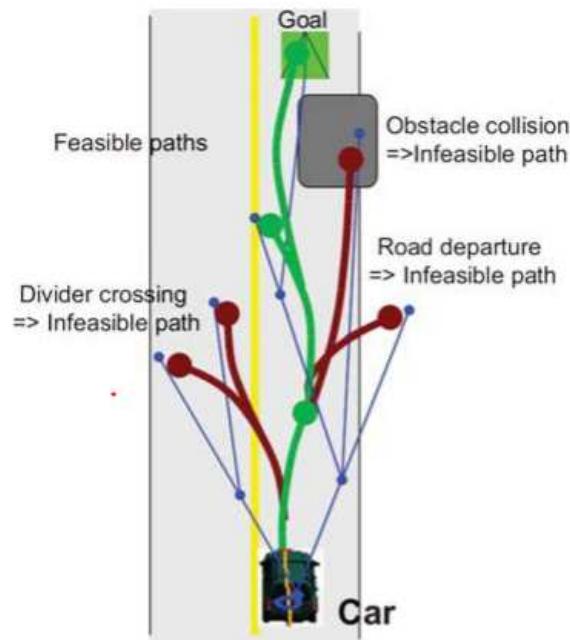


Fig. 22. Illustration of RRT Motion planning. Each leaf of the tree represents a stopping location. The motion control points (in blue) are translated into a predicted path. The predicted paths are checked for drivability (shown in green and red).

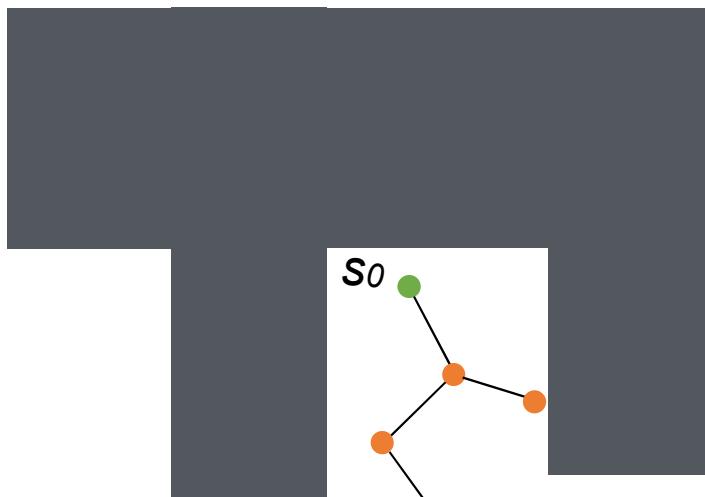
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Questions about the RRT algorithm?

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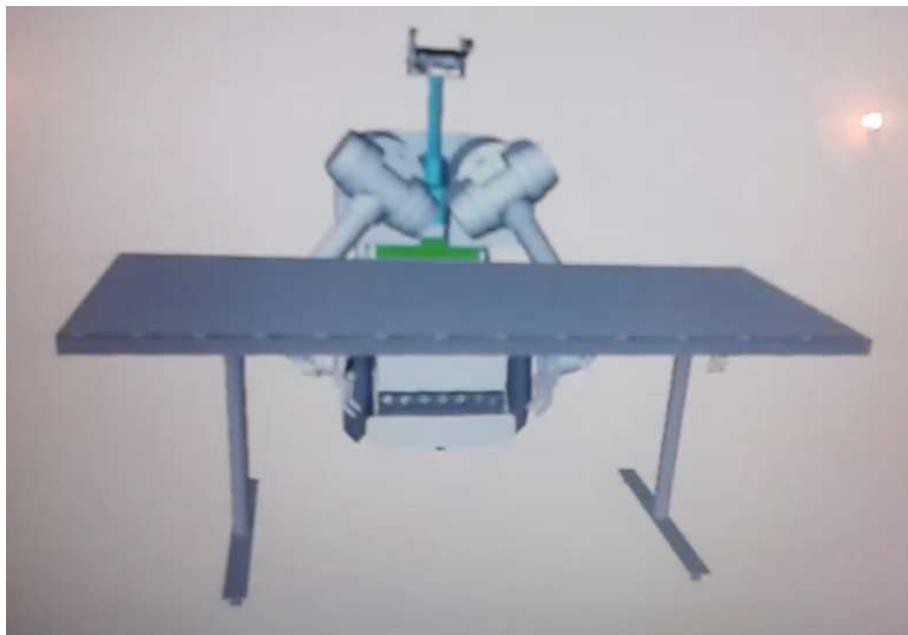
● s_{goal}



s ●

5

But we can get some WEIRD outputs...



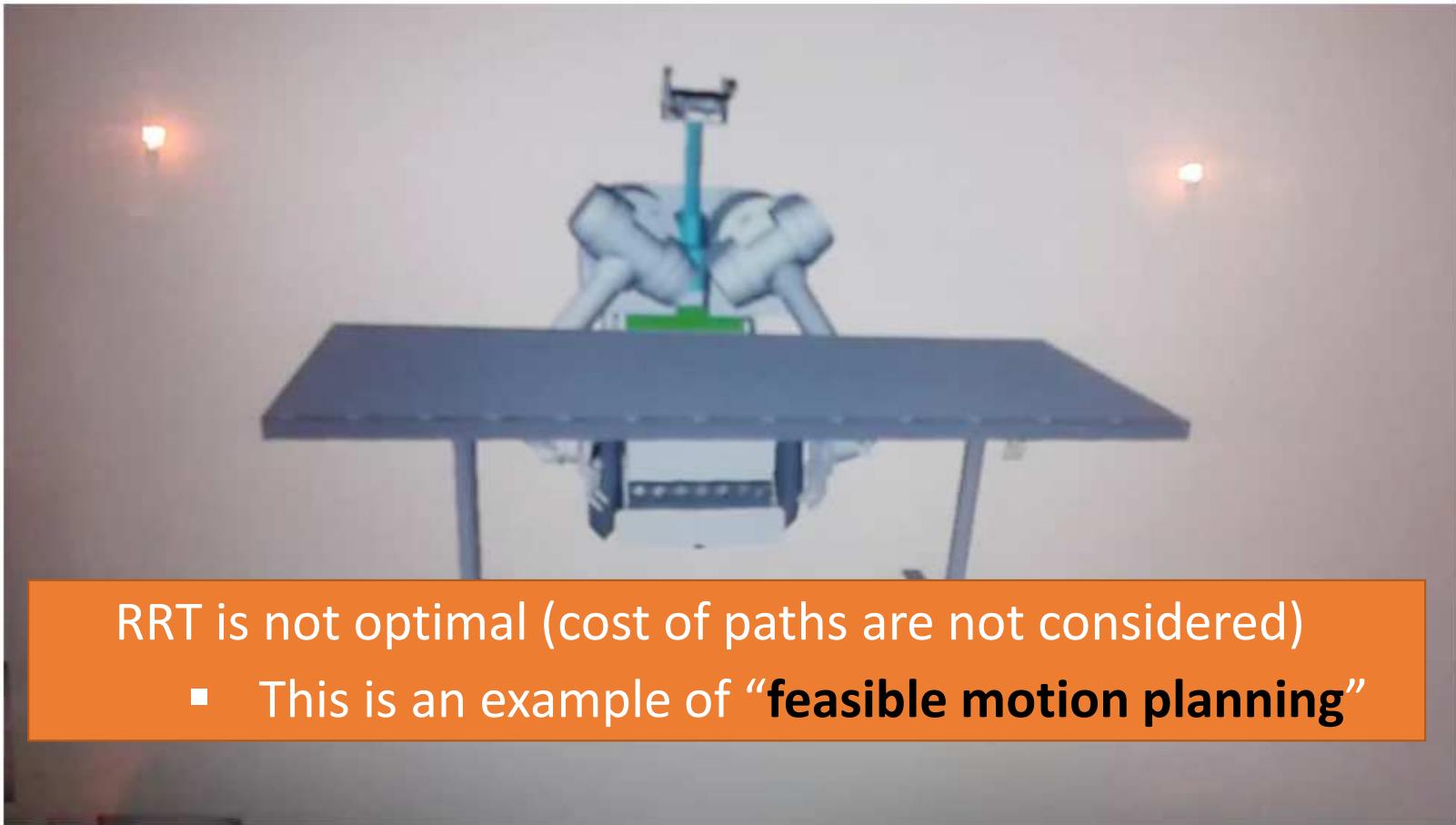
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RRT is not optimal (cost of paths are not considered)

- This is an example of “**feasible motion planning**”

5

We solve this problem with RRT*

The big trick:

- incrementally “**re-wiring**” the tree to keep locally optimal paths

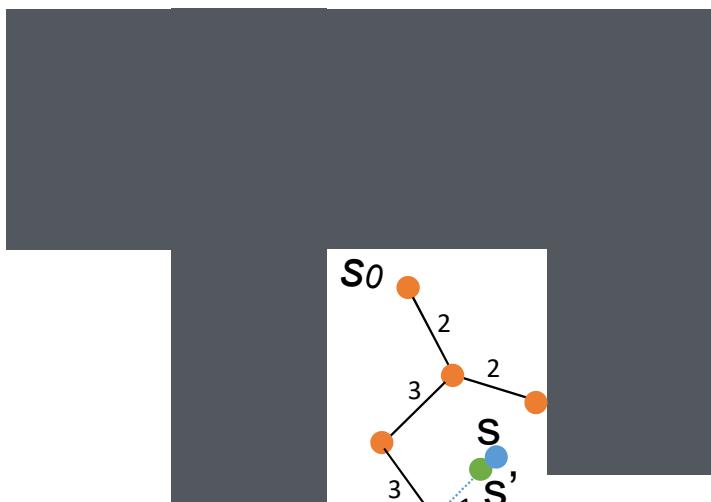
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We solve this problem with RRT*

RRT* (input: s_0, s_{goal} , initial state tree T)

- Sample states $s \in S = R^{15}$ until s is collision-free (often goal directed)
- Find closest state $s_c \in T$
- **Extend s_c toward s resulting in state s'**
- Find all $s_{near} \subseteq T$ within a distance d to s'
- Find $s_{min} \in s_{near}$, that has the lowest *path cost* to $s_0 \rightarrow s_{min} \rightarrow s'$
- Add edge $s_{min} \rightarrow s'$ to T
- Check path cost through s' to all states in $s \in s_{near}$, if any are lower than existing path cost to s , then “rewire” tree to include edge $s' \rightarrow s$
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● s_{goal}



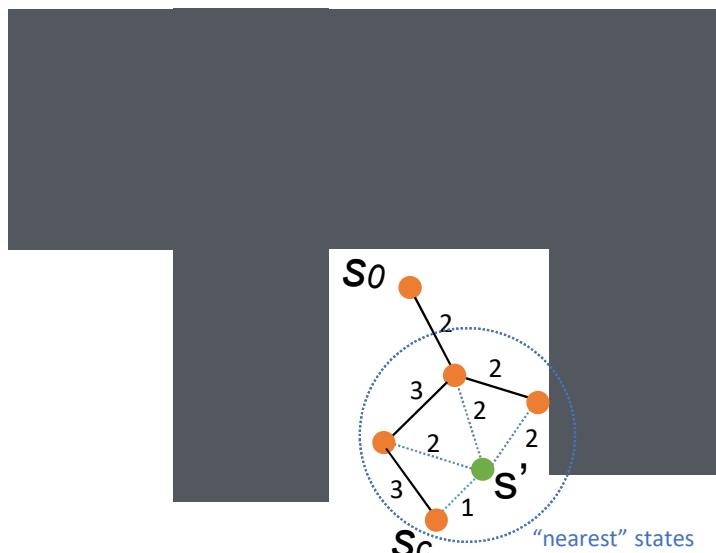
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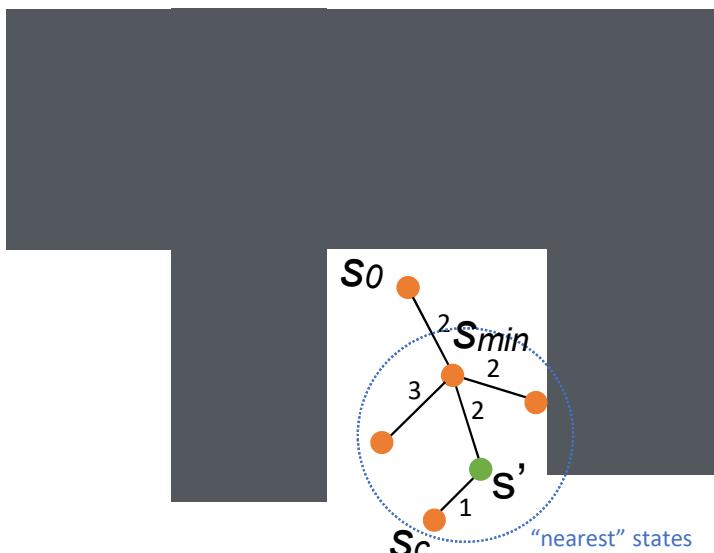
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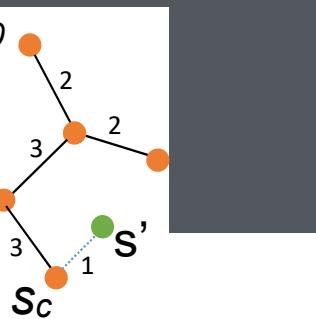
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● s_{goal}

RRT

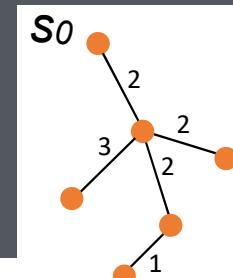
s_0



● s_{goal}

RRT*

s_0

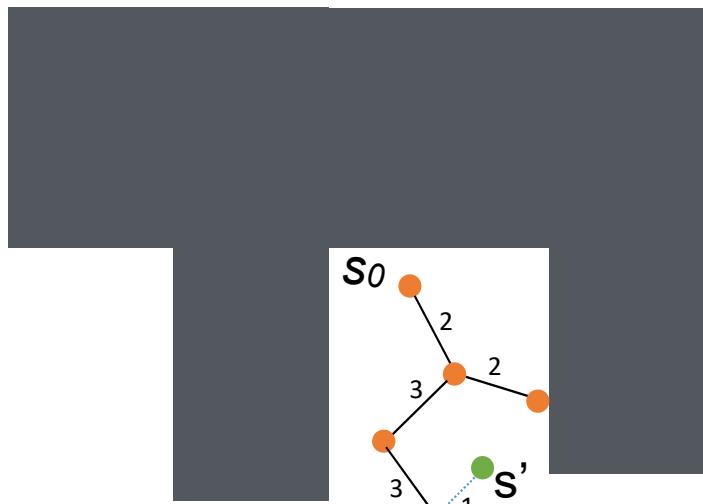


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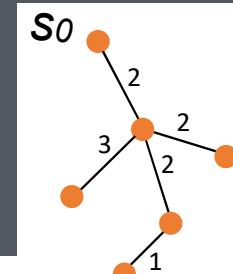
We solve this problem with RRT*

Nearest radius size is another sample vs. computational efficiency decision!

● s_{goal}



● s_{goal}



RRT*

[Karaman & Fazzoli Sampling-based Algorithms for Optimal Motion Planning]

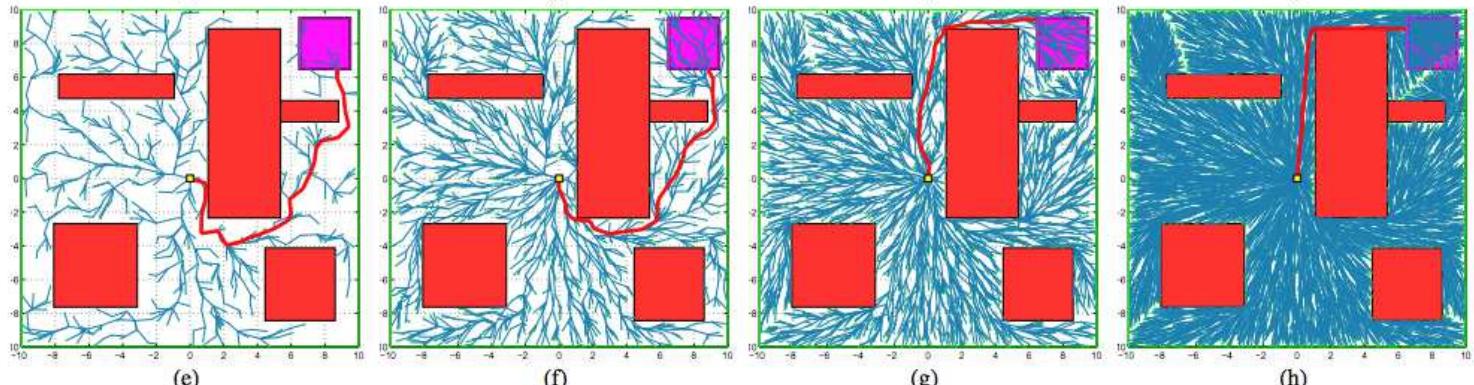
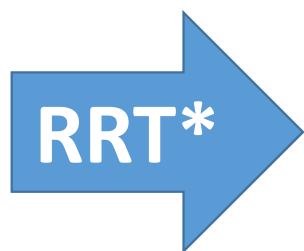
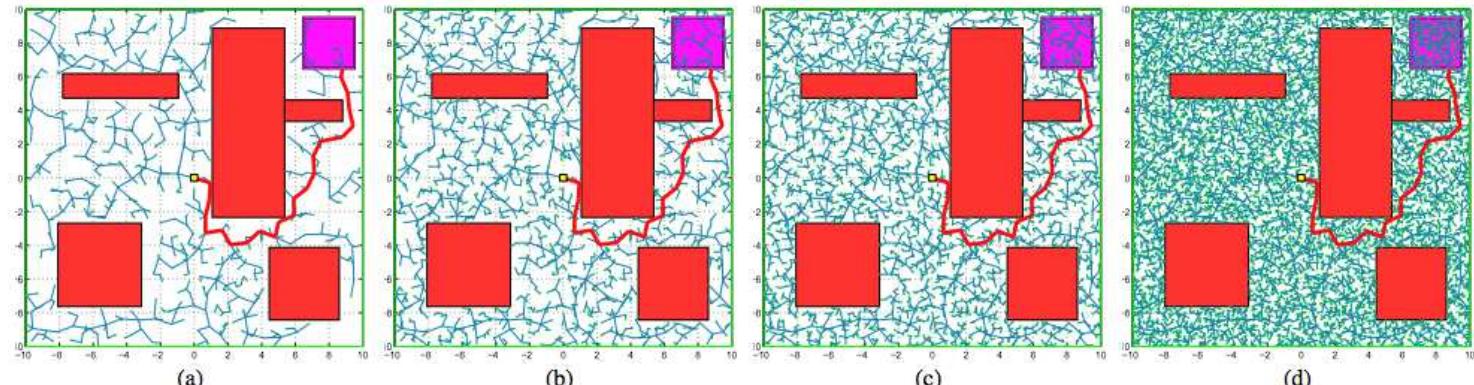
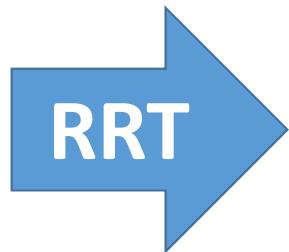


Fig. 1. A Comparison of the RRT* and RRT algorithms on a simulation example. The tree maintained by the RRT algorithm is shown in (a)-(d) in different stages, whereas that maintained by the RRT* algorithm is shown in (e)-(h). The tree snapshots (a), (e) are at 1000 iterations, (b), (f) at 2500 iterations, (c), (g) at 5000 iterations, and (d), (h) at 15,000 iterations. The goal regions are shown in magenta. The best paths that reach the target are highlighted with red.

5

So what have we learned so far?

1. Robot planning usually involves thinking about both **task and configuration spaces**
2. For many real problems, **collision checking** can be expensive
3. **RRT**: a powerful algorithm based on a very simple idea!
 - **Probabilistically complete**: If there's a solution it will find it eventually (but can still be slow for some problems)!
 - BUT RRT is not optimal (cost of paths are not considered)
 - This is an example of “**feasible motion planning**”
 - **RRT*** fixes that by incrementally rewiring the tree

5

To RRT or not to RRT that is the question!

1. Why might RRTs not be the best algorithmic choice for a robot that repeatedly does the same task?
 2. How might you adapt RRT to fix this issue?
-

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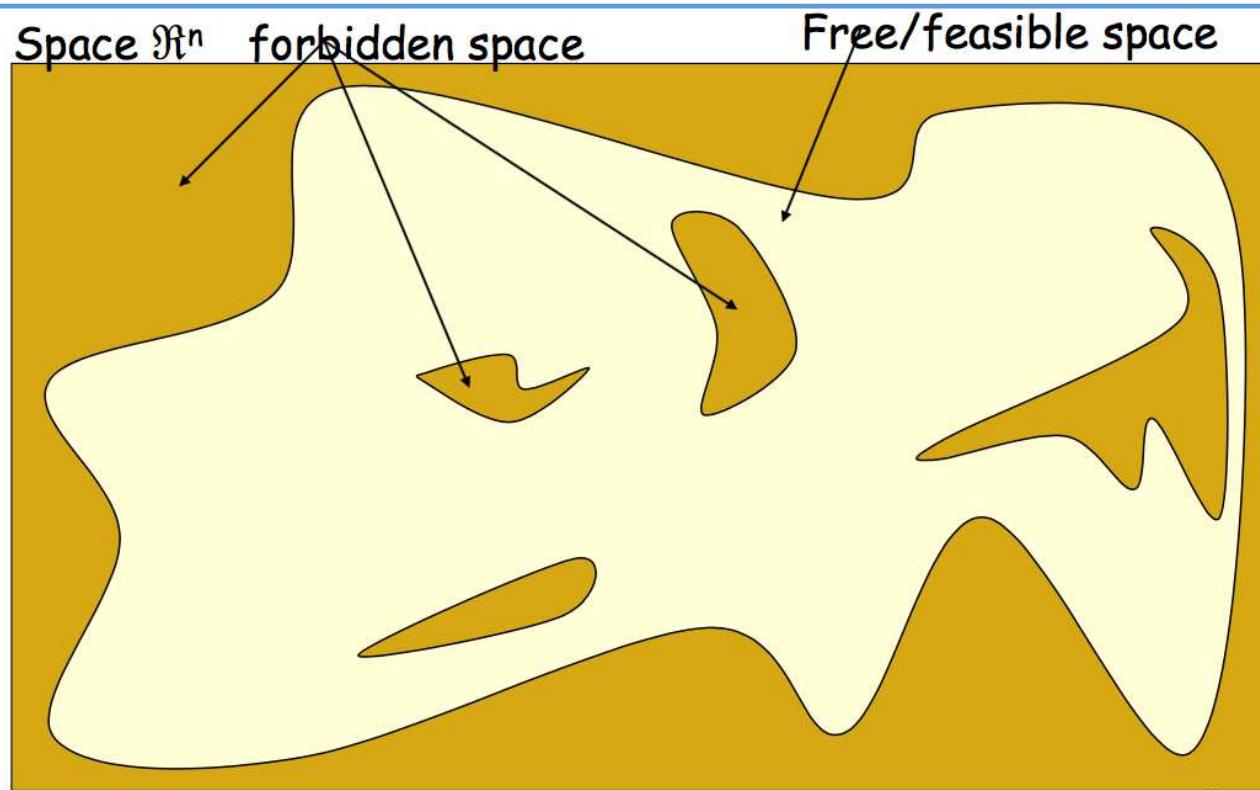
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This “**multi-query**” approach is called **Probabilistic Roadmaps (PRMs)**

5

- Probabilistic Roadmaps (PRMs) leverage an offline and an online computation phase

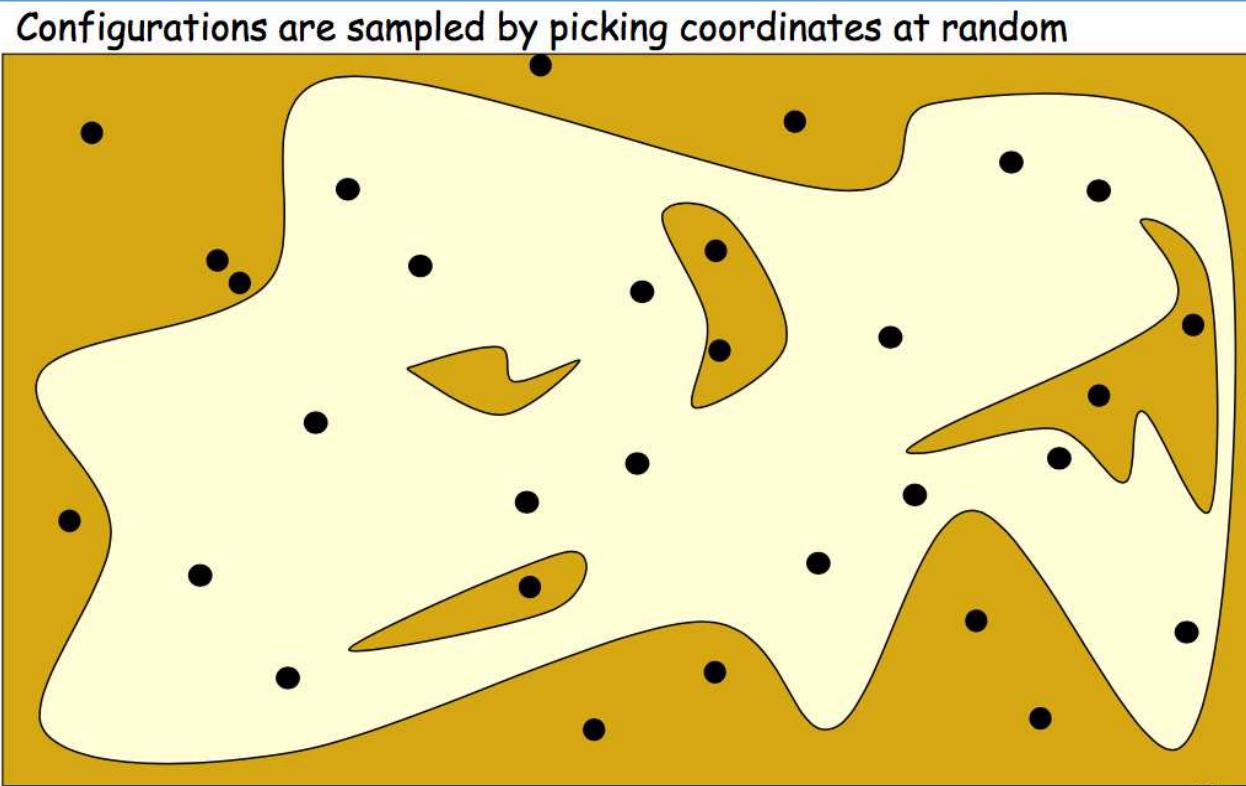
Step 1: Offline build a random graph \mathbf{G} that covers the state space



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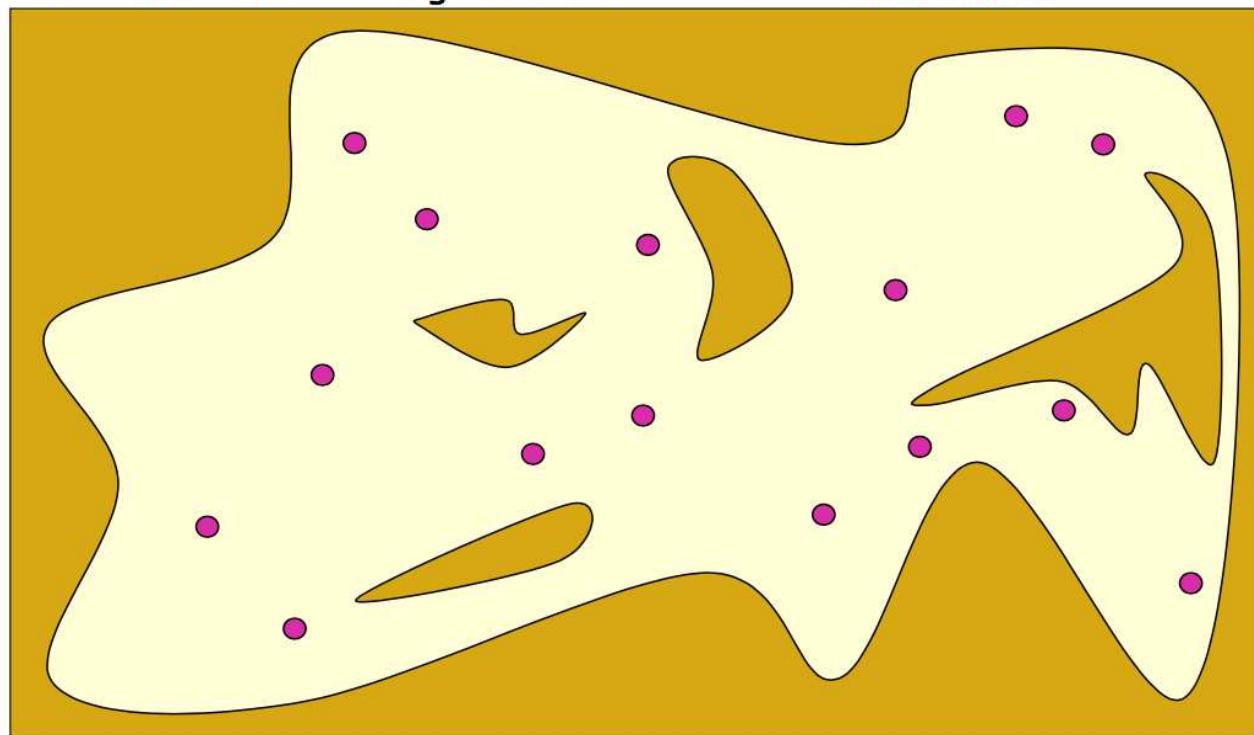


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Step 1: Offline build a random graph \mathbf{G} that covers the state space

The collision-free configurations are retained as **milestones**

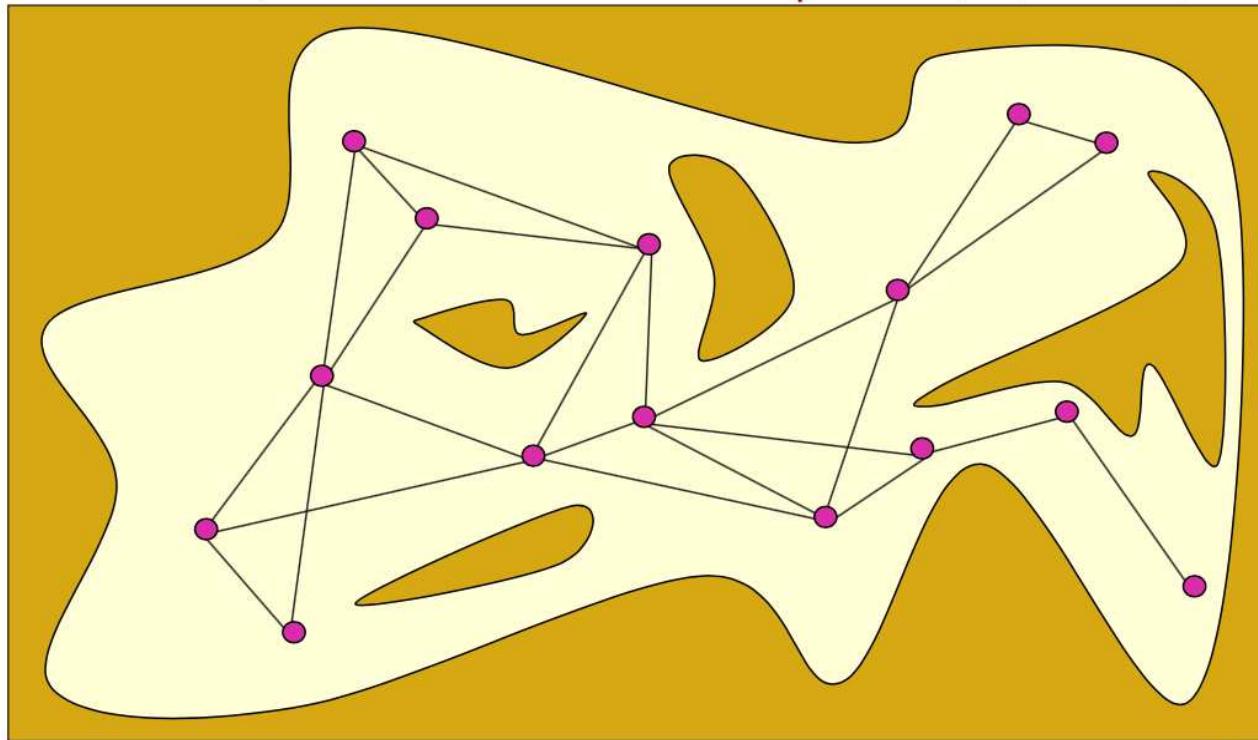


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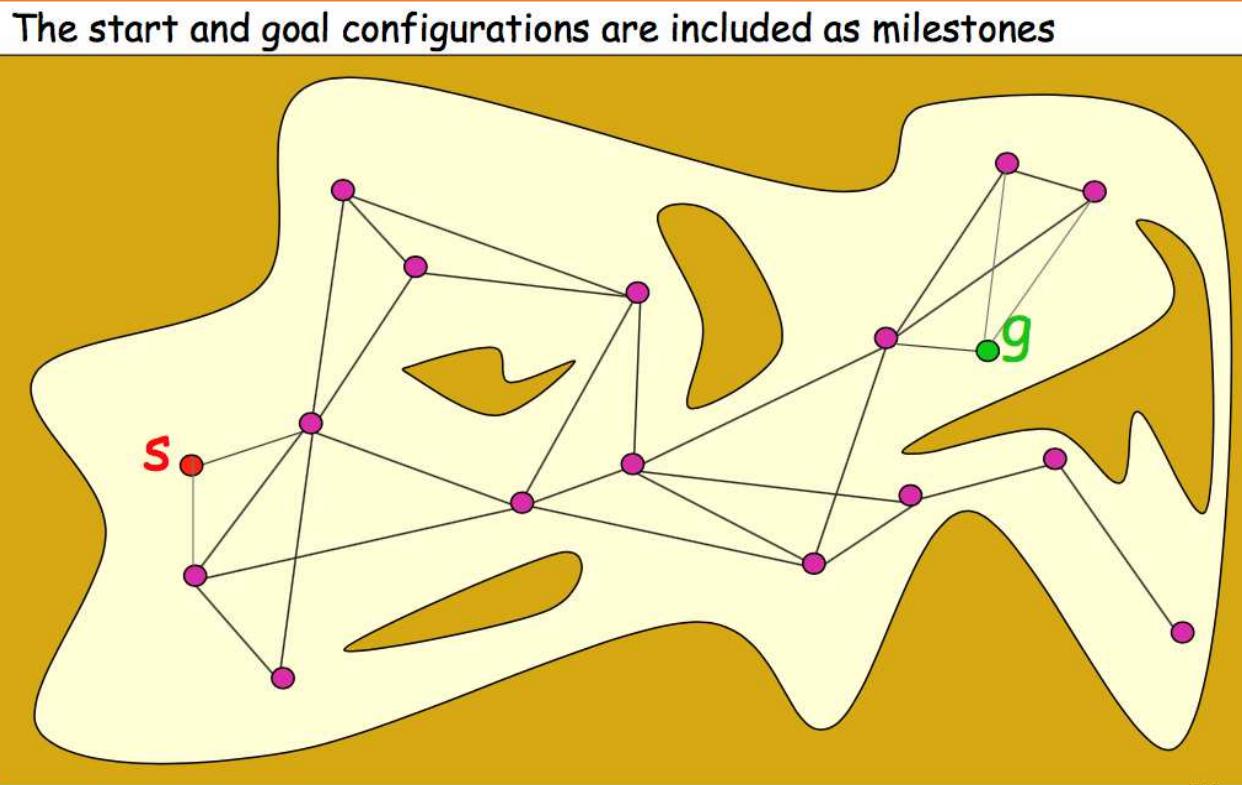
The collision-free links are retained as **local paths** to form the PRM



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- Probabilistic Roadmaps (PRMs) leverage an offline and an online computation phase
-

Step 2: Online connect the start and goal nodes and run graph search

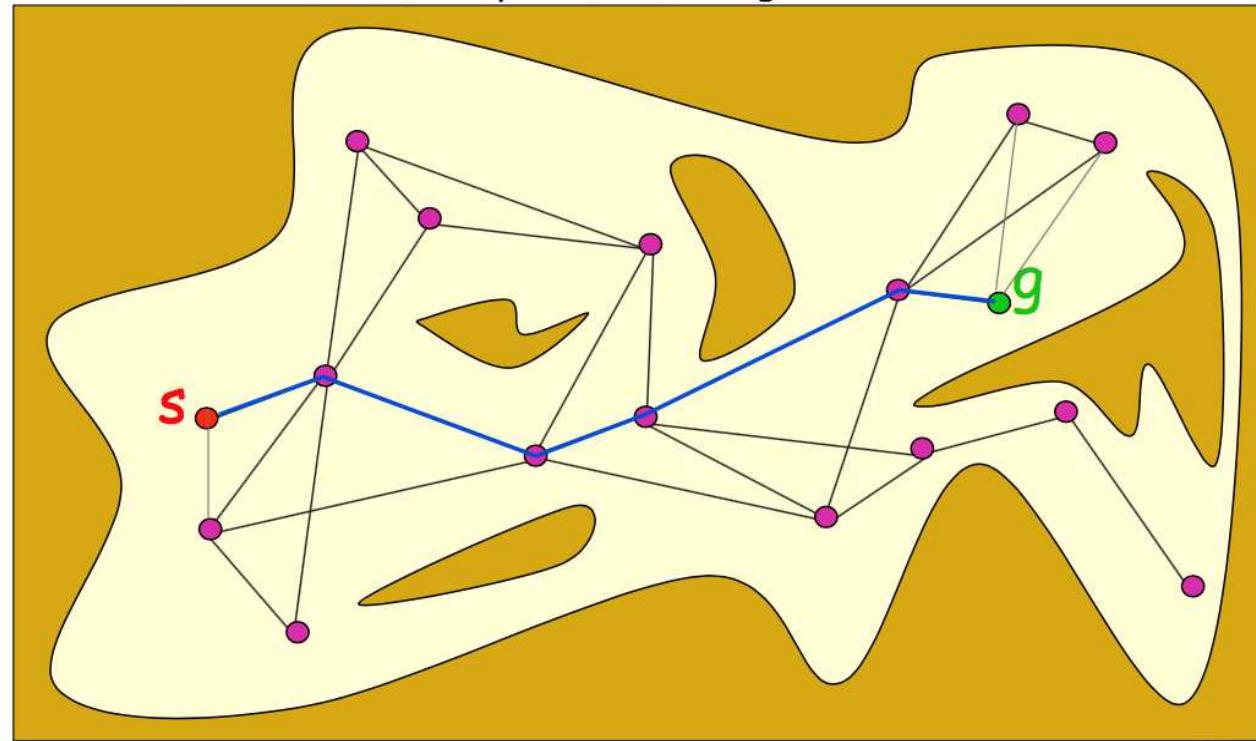


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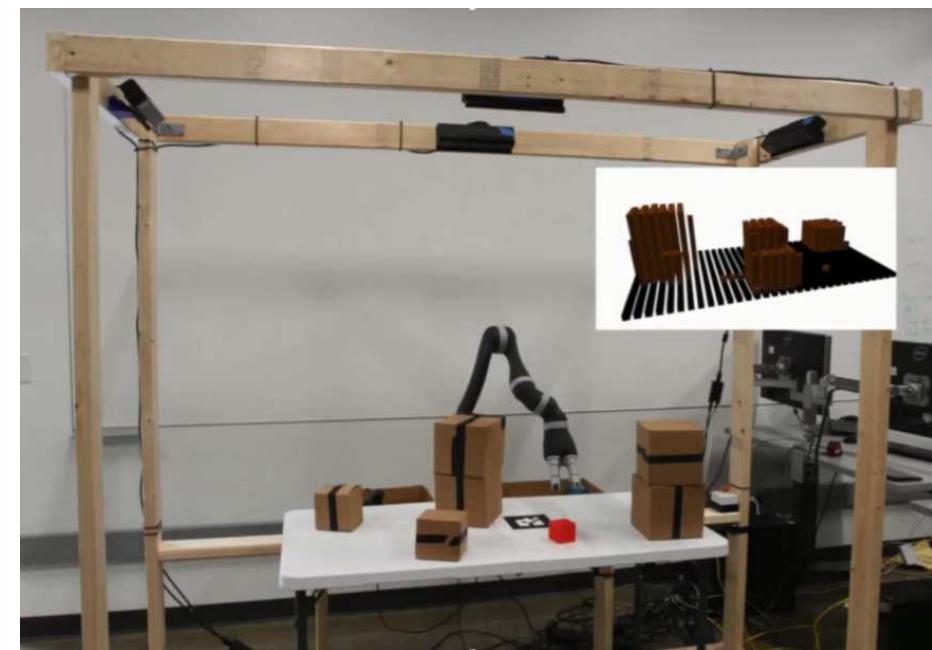
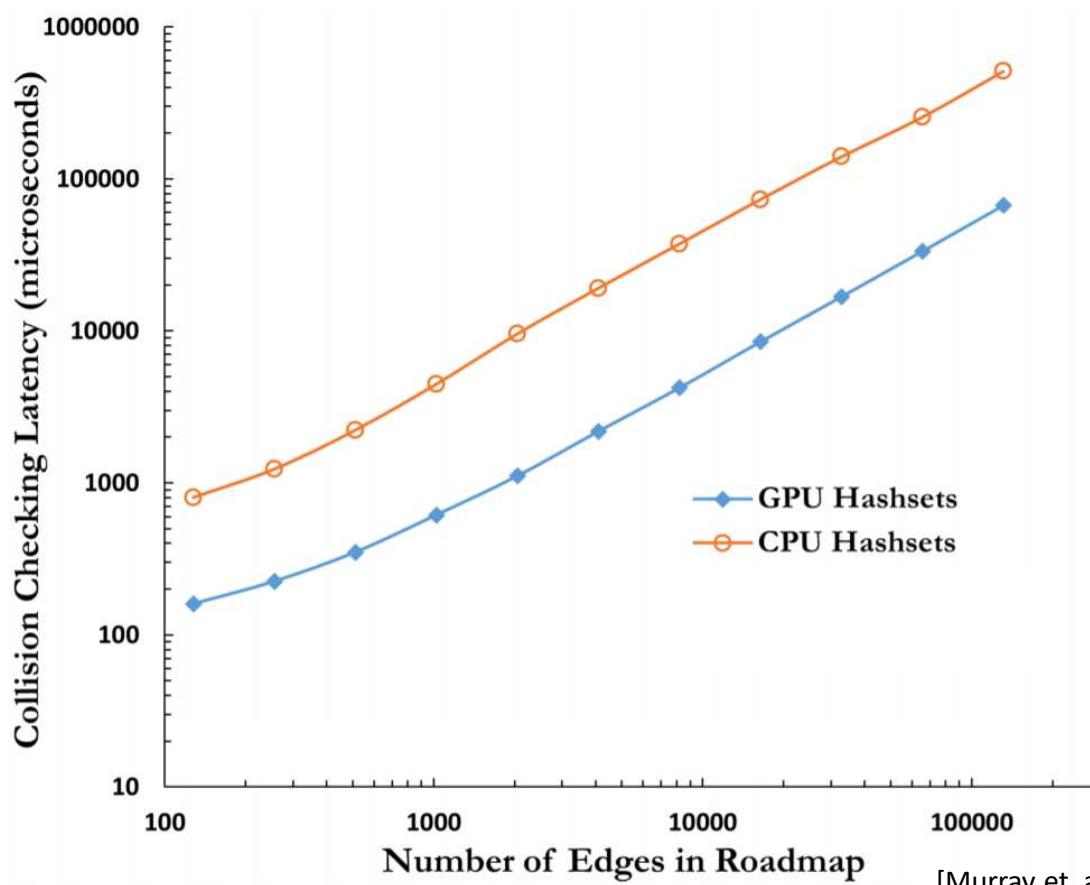
Step 2: Online connect the start and goal nodes and run graph search

The PRM is searched for a path from s to g



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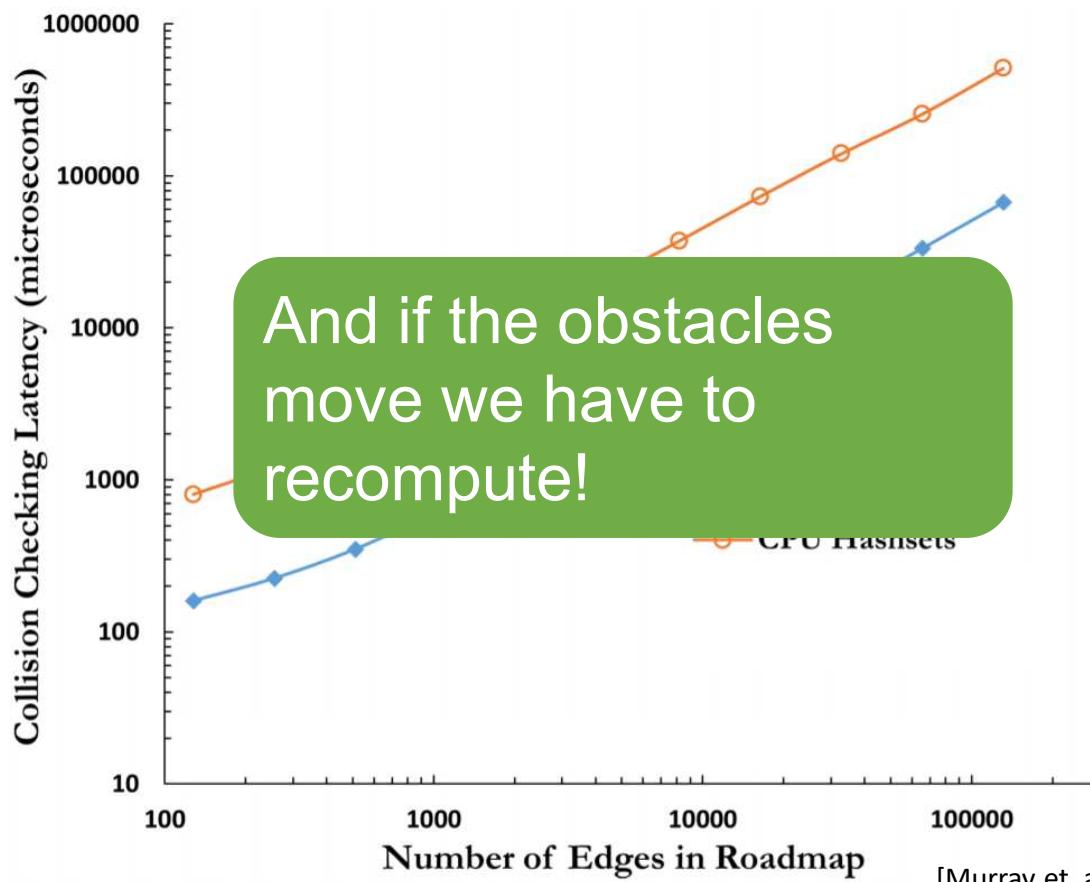
Collision detection for each connecting path in the construction of the PRM can be very expensive



[Murray et. al. The Microarchitecture of a Real-Time Robot Motion Planning Accelerator]

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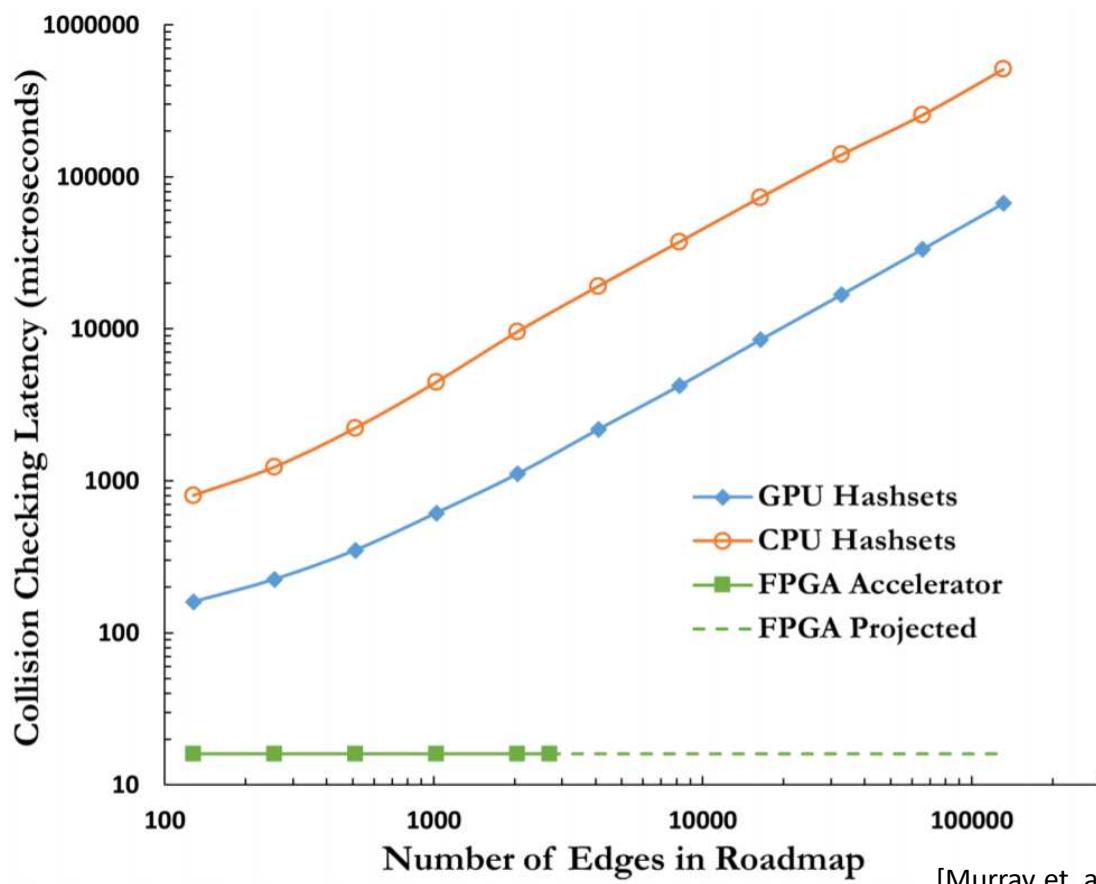
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Custom hardware can lead to near-instantaneous collision checking!



Robot Motion Planning on a Chip

Sean Murray, Will Floyd-Jones, Ying
Qi, Dan Sorin, George Konidaris



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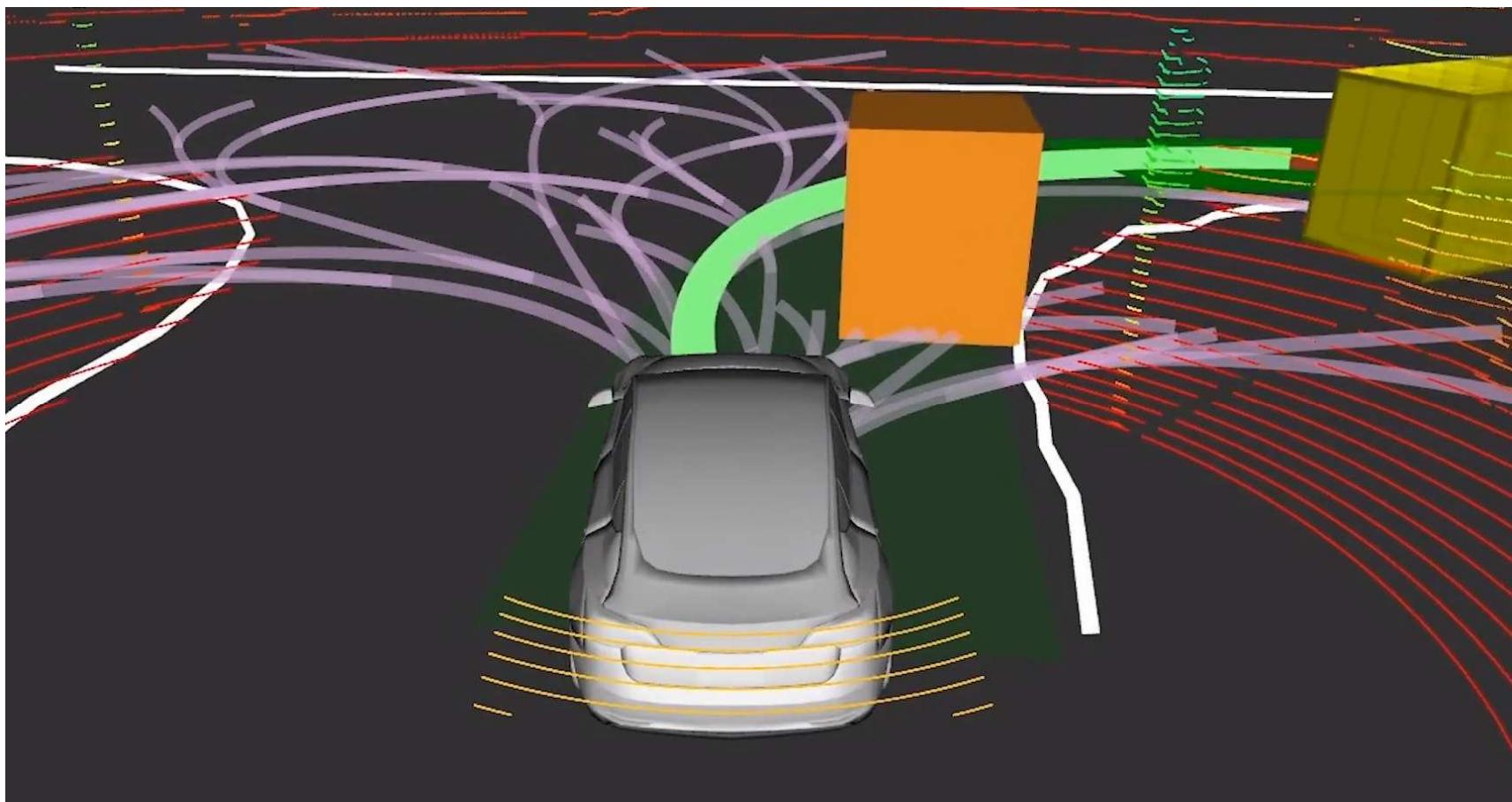
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Realtime Robotics



5

Ok so why can't robots use these awesome **kinematic** planning algorithms all the time and be better at life?!?

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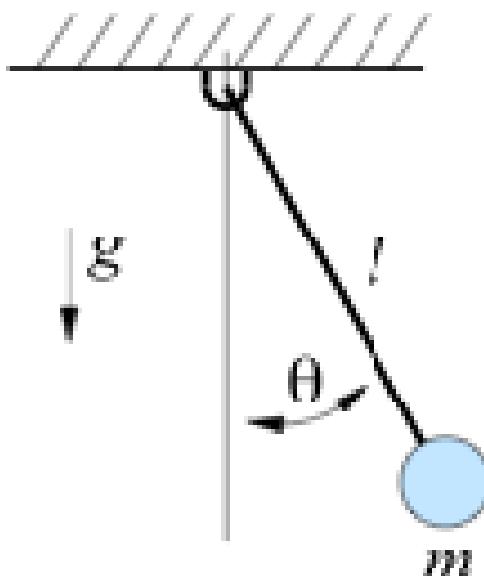
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Dynamics (aka Physics)

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The Simplest “Robot”

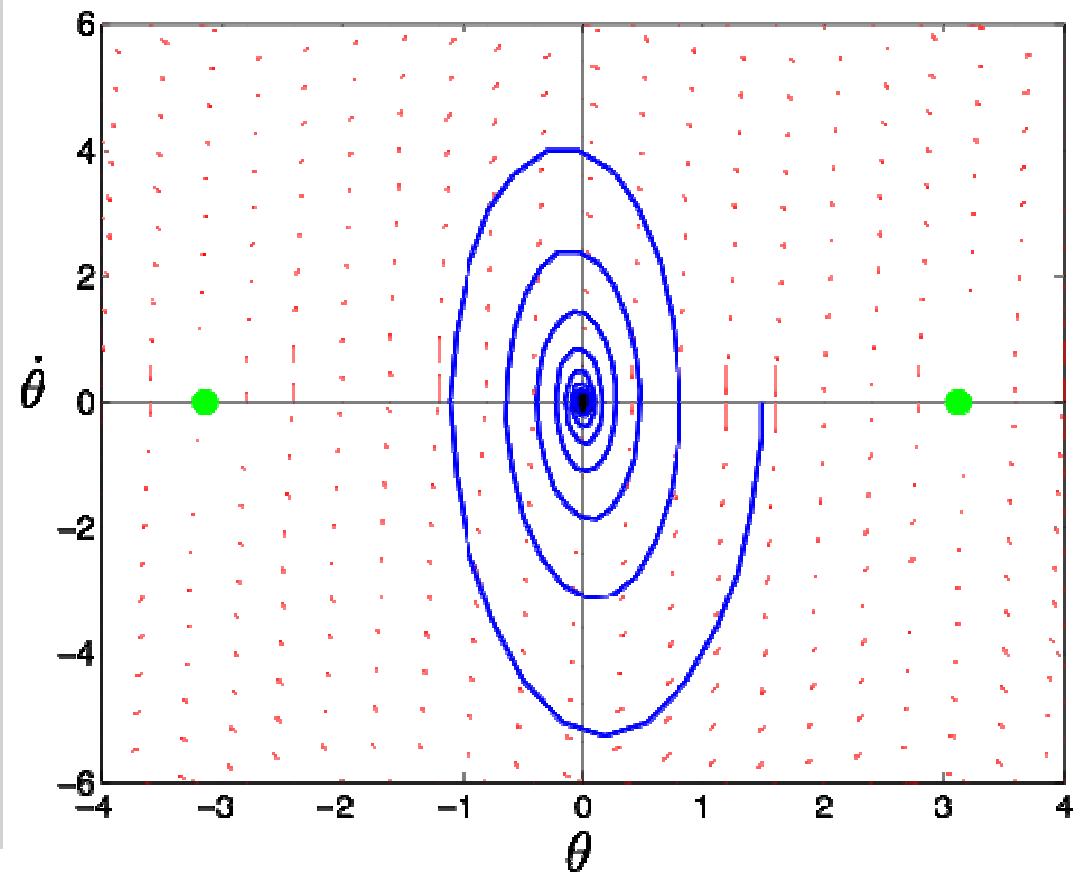
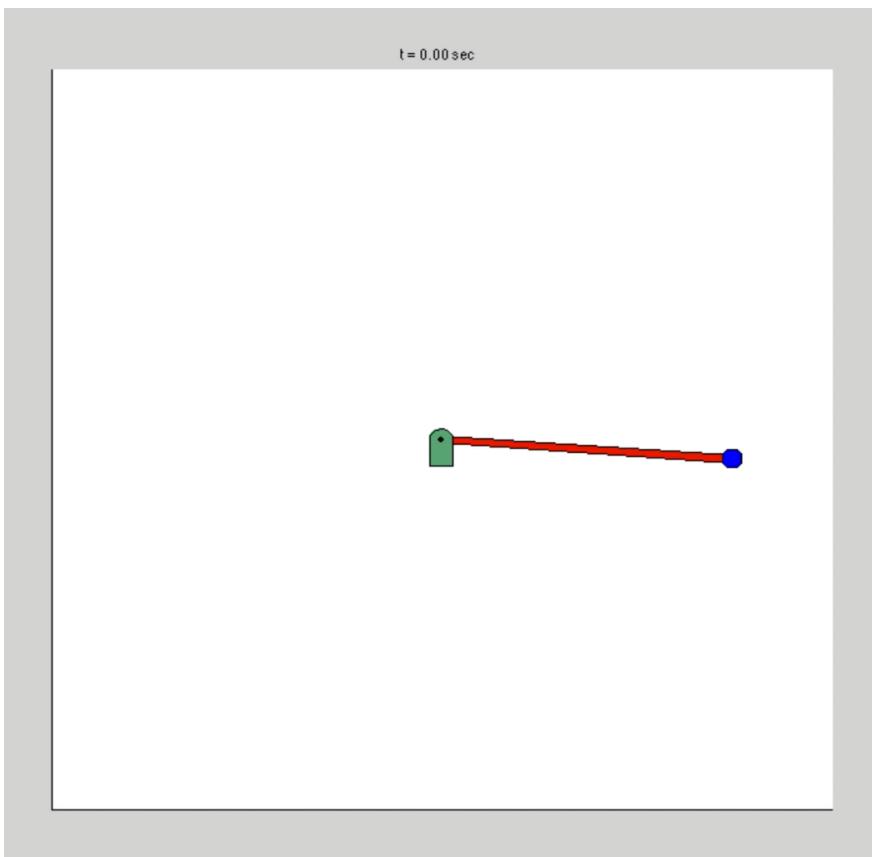


Dynamics (aka Physics)

- States: $s = \{\theta, \dot{\theta}\}$ aka angle and angular **velocity**
- Actions: $a = \tau$ aka torque at joint
- Transitions: $s' = f(s, a)$ aka physics

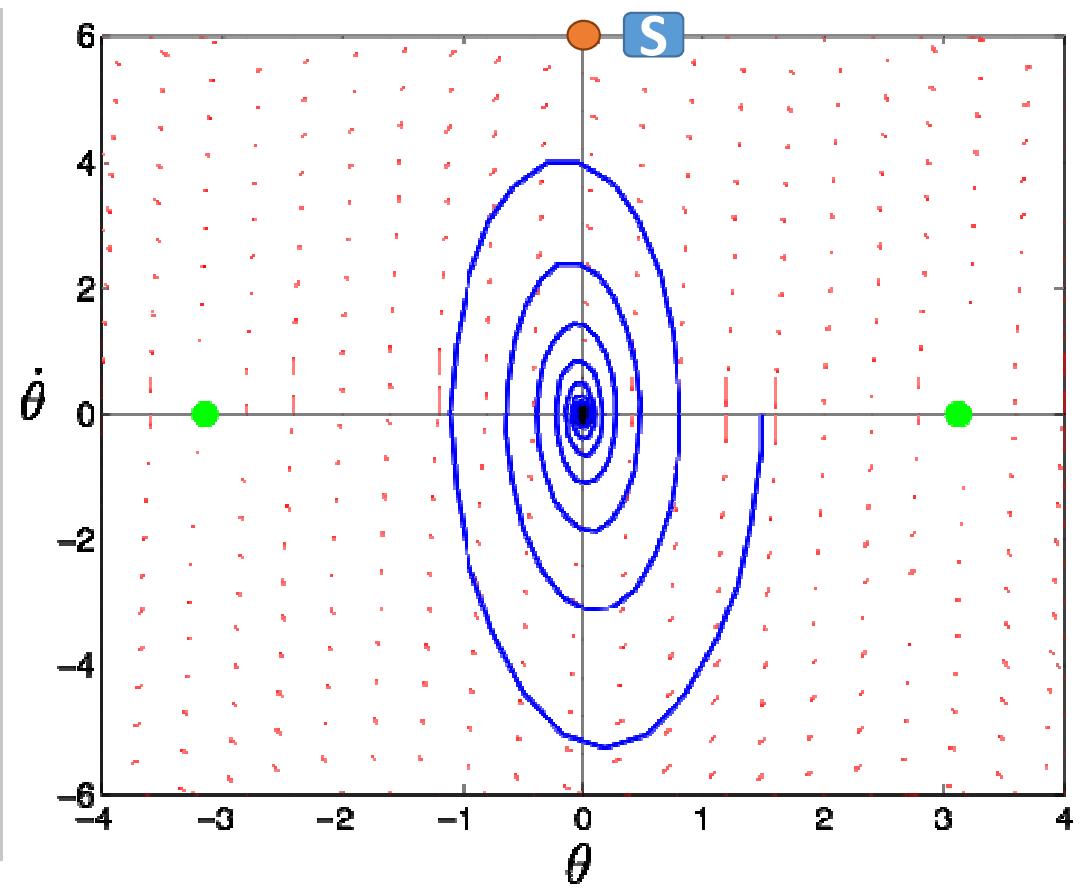
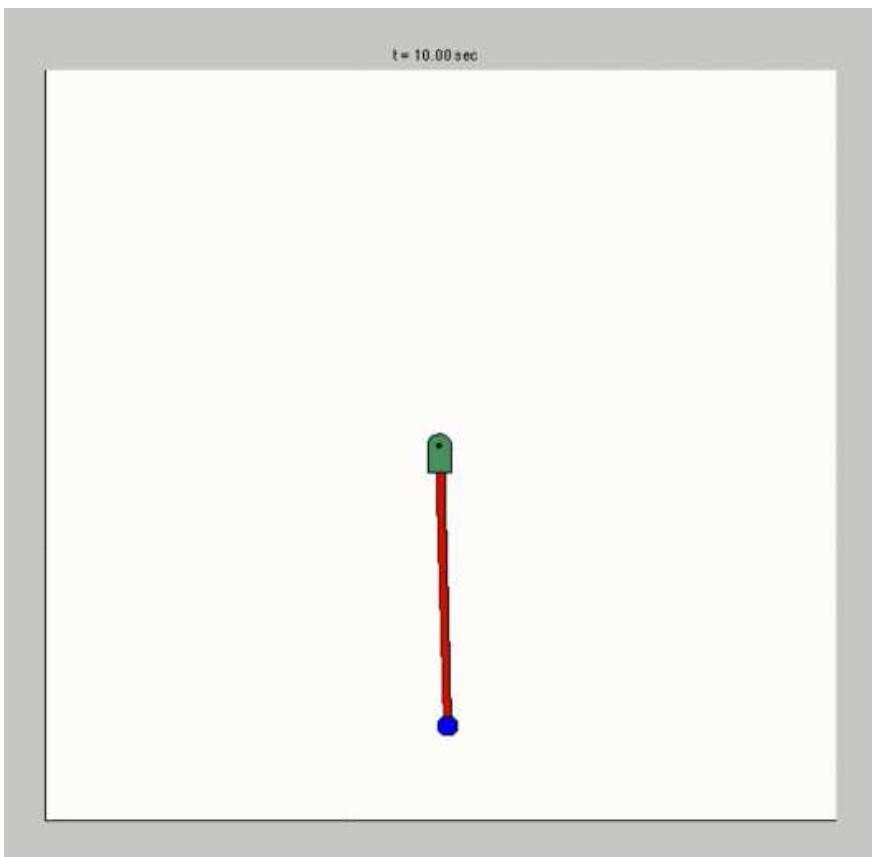
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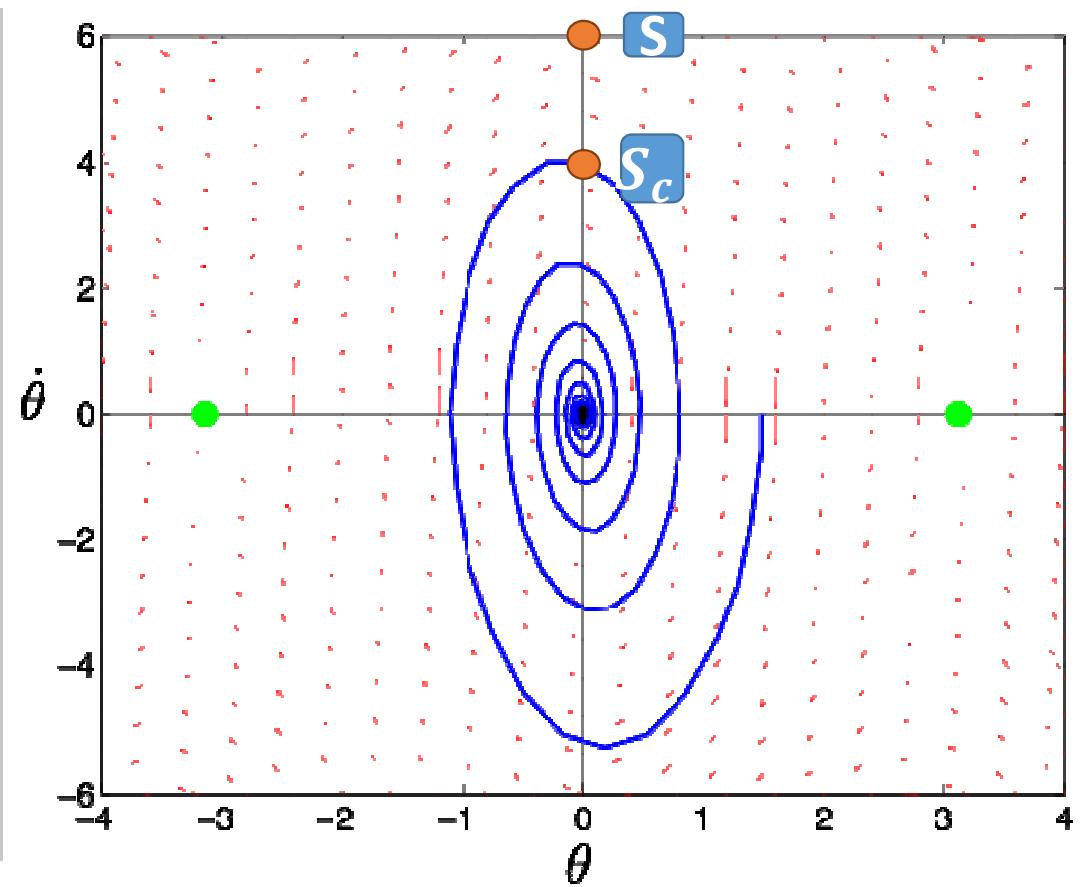
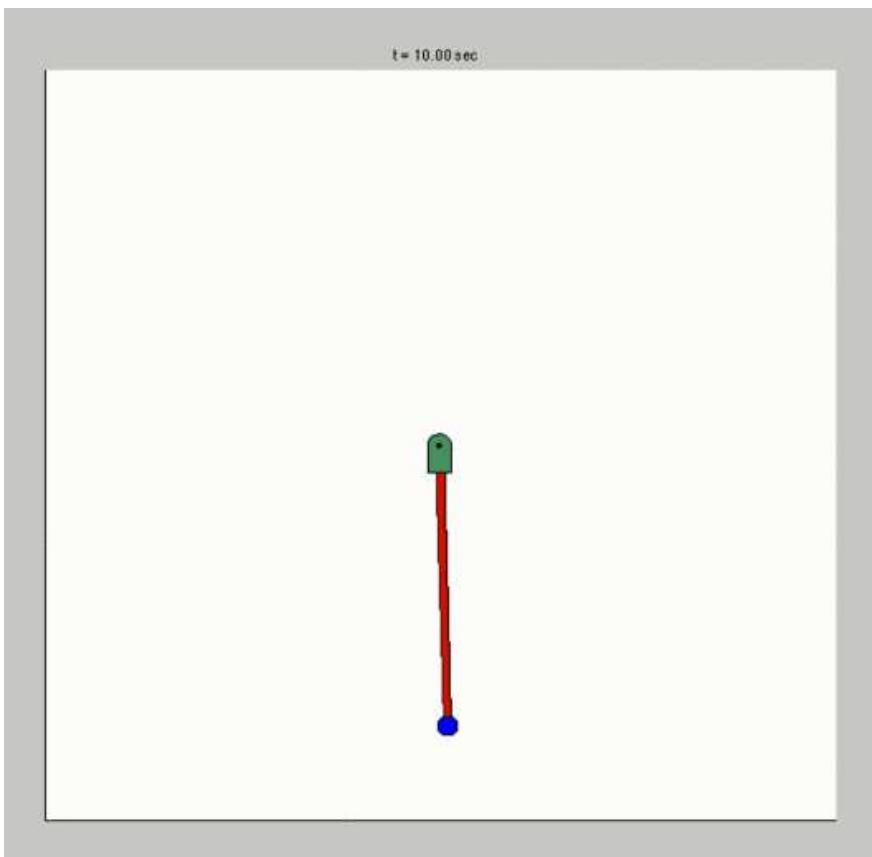
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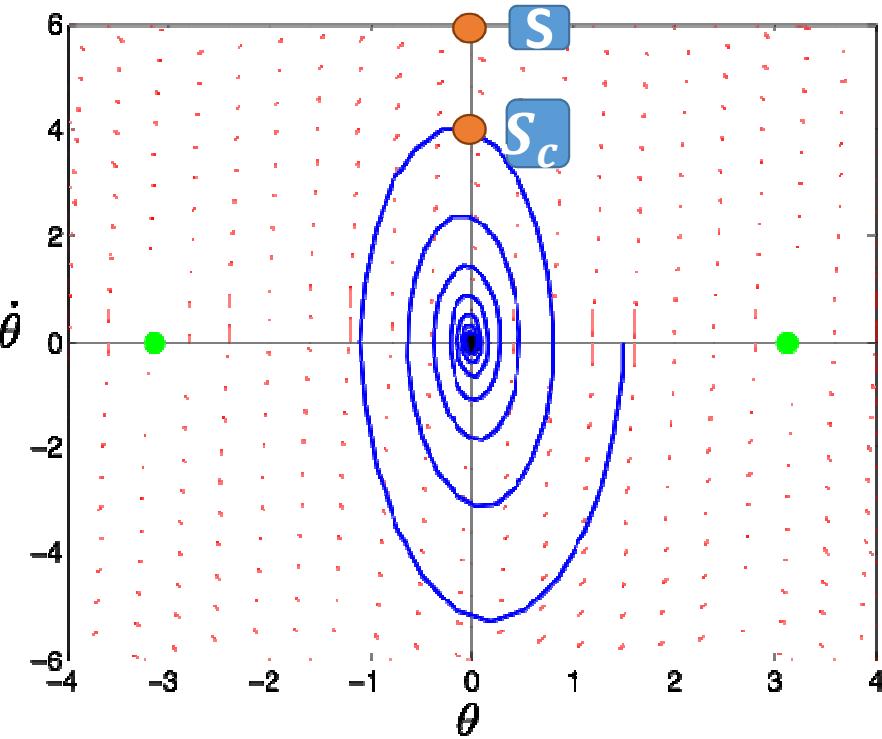
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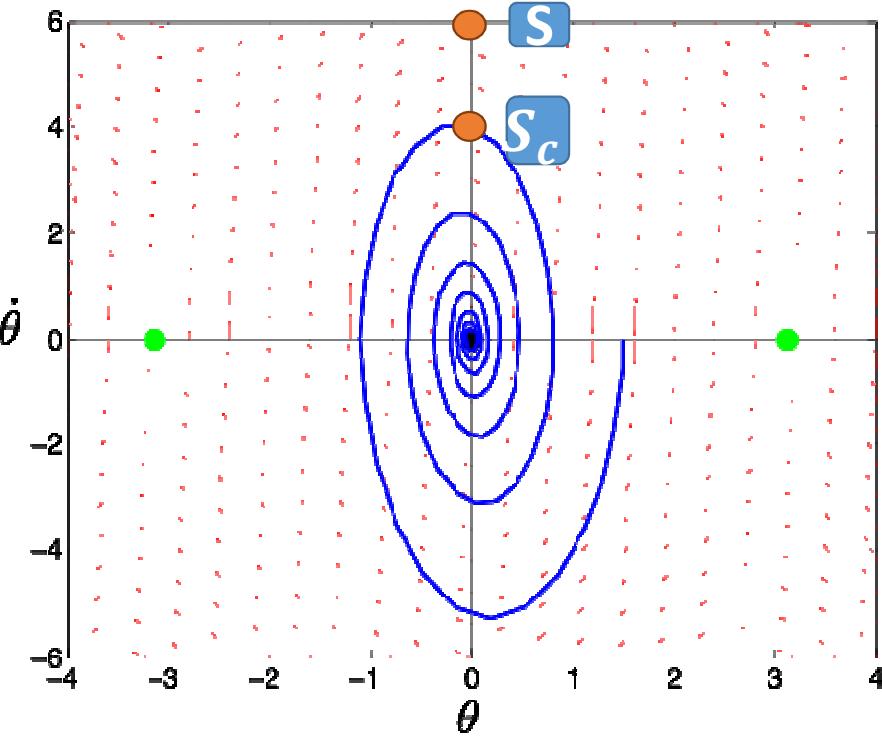
Challenges for Dynamic RRTs

The “extend” operation is complex!

- We need to solve a **boundary value problem** (find a path from s_c to s such that it follows the dynamics)
- Basically a “mini” planning problems

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Challenges for Dynamic RRTs

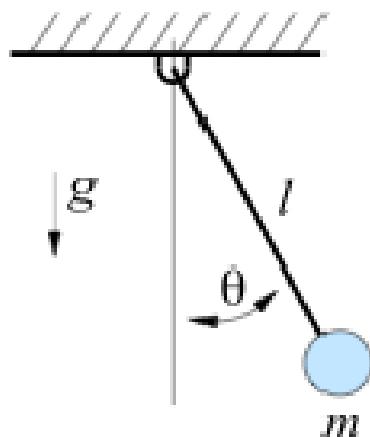
The “extend” operation is complex!

- We need to solve a **boundary value problem** (find a path from s_c to s such that it follows the dynamics)

Q: Why don't we just try a discretization of possible actions instead of solving a boundary value problem?

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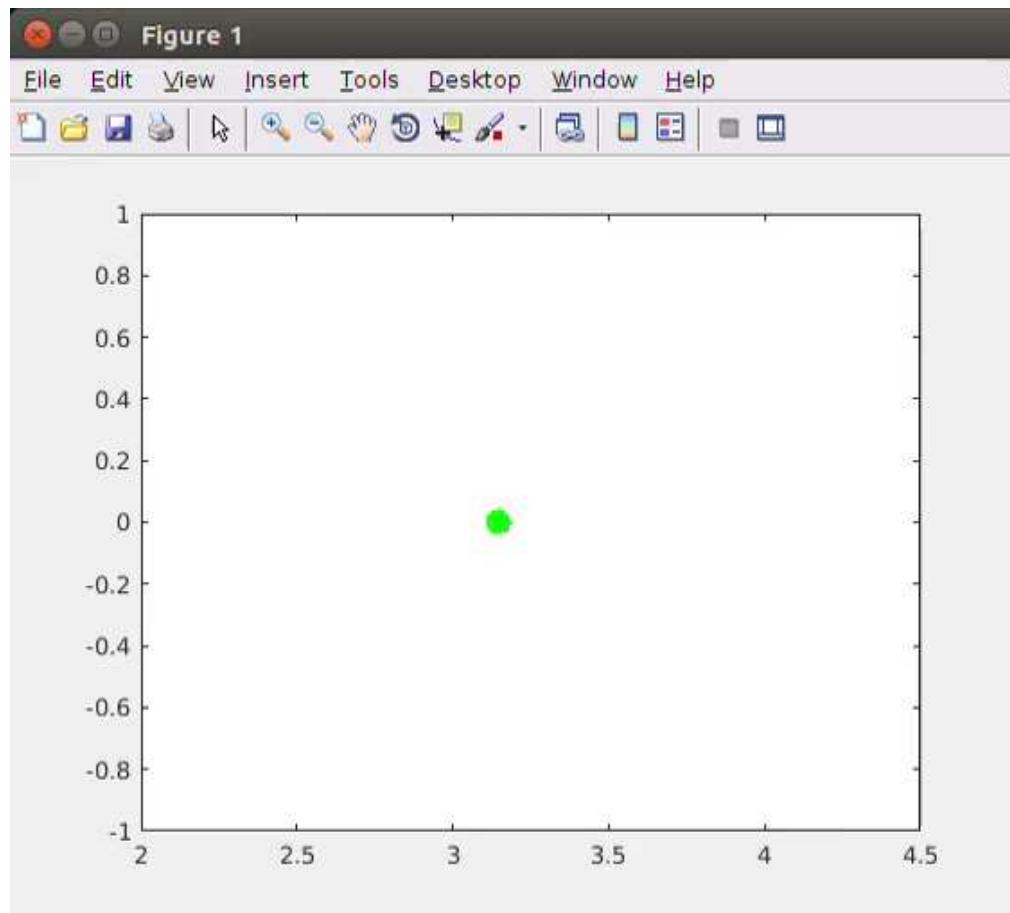


- **States:** $s = \{\theta, \dot{\theta}\}$ aka angle and angular velocity
- **Actions:** $a = \tau$ aka torque at joint
- **Transitions:** $s' = f(s, a)$ aka physics

Task: start from the **stable downward equilibrium $(0,0)$** and **swing up to the unstable upward equilibrium $(\pi,0)$**

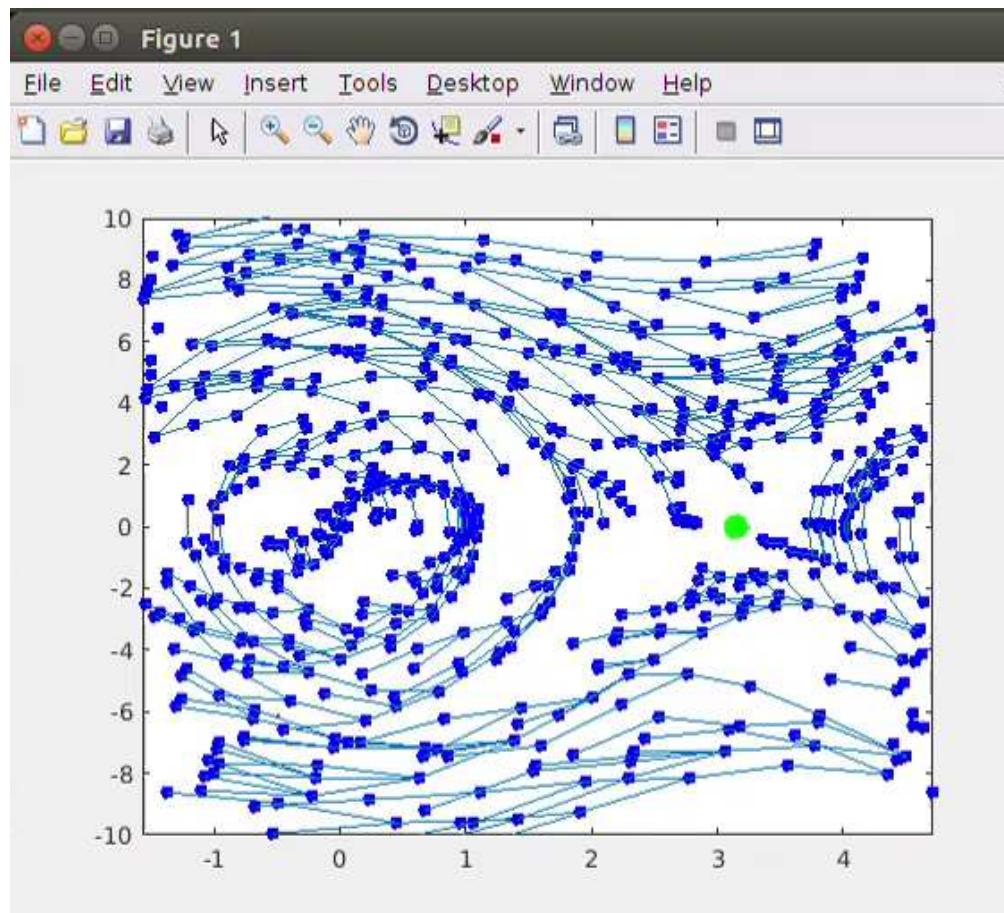
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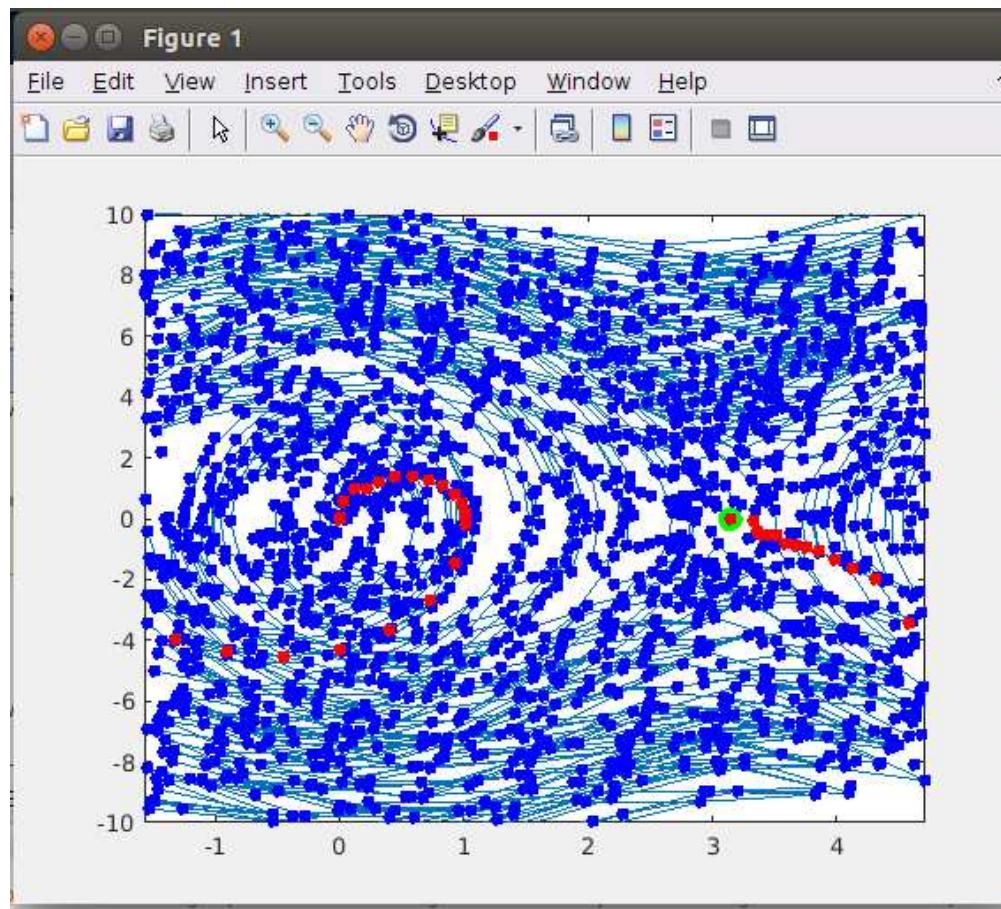
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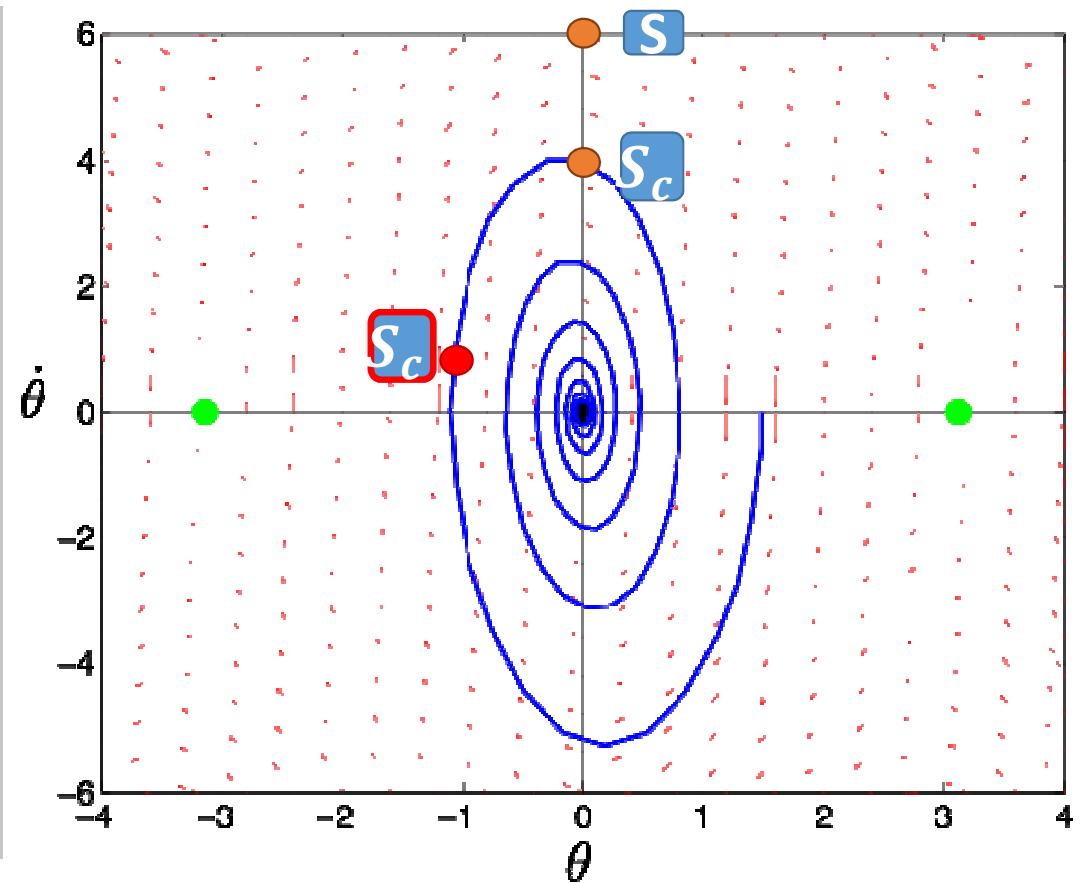
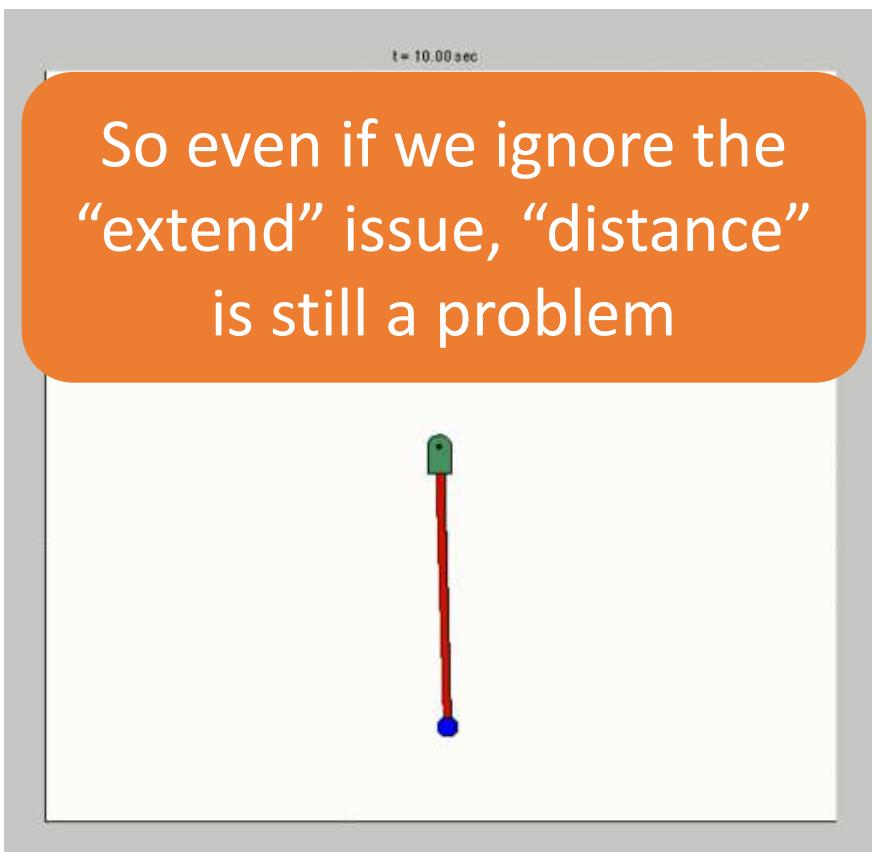
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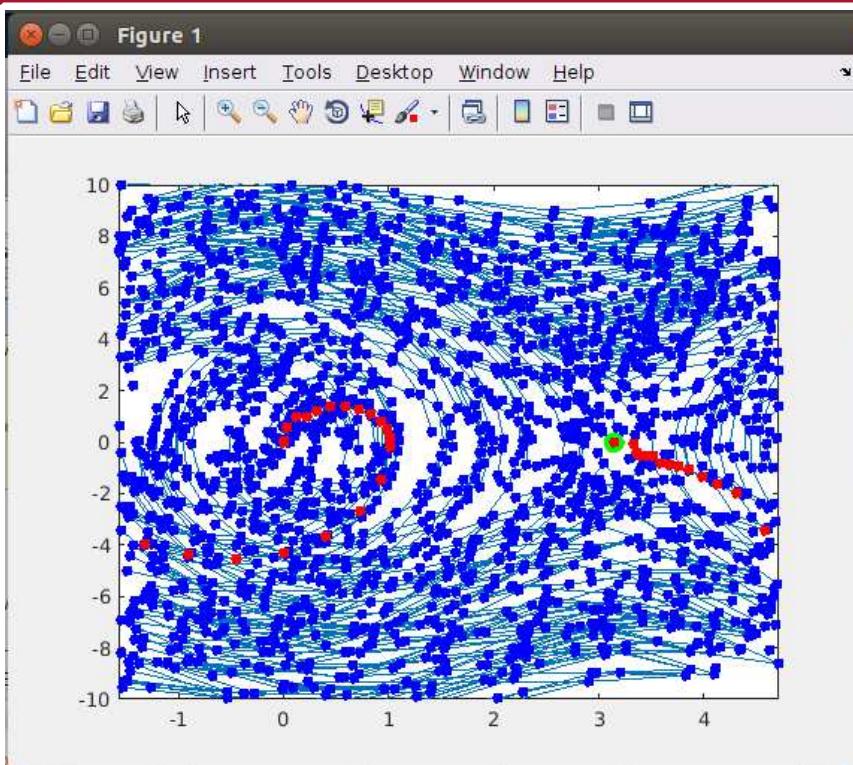
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Challenges for Dynamic RRTs

The “extend” operation is complex!

- We need to solve a **boundary value problem** (find a path from s_c to s such that it follows the dynamics)
- Basically a “mini” planning problems

What is the “closest state in the tree”

- The “**distance**” between states of dynamical systems is **not well-defined**

5

So what do we do?

5

So what do we do?

Give up and make
the computer solve
it for us?

5

So what do we do?

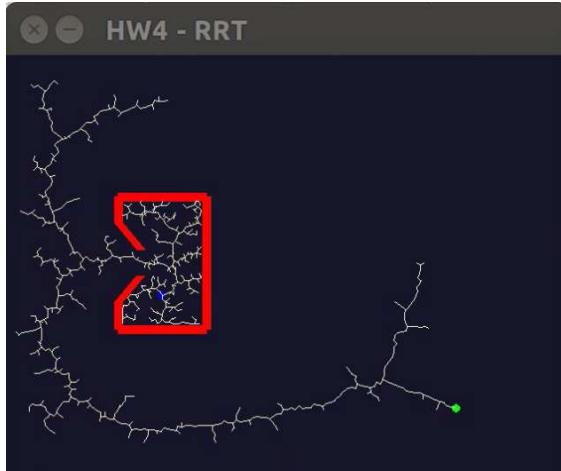
#Learning

#EfficientUseOfHumans

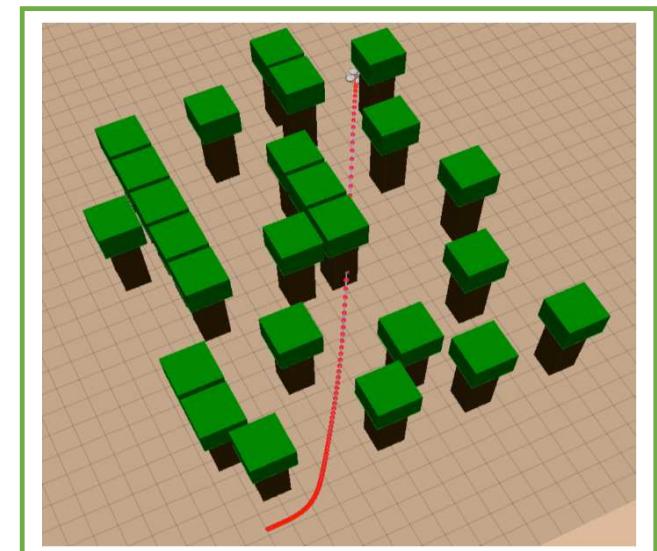
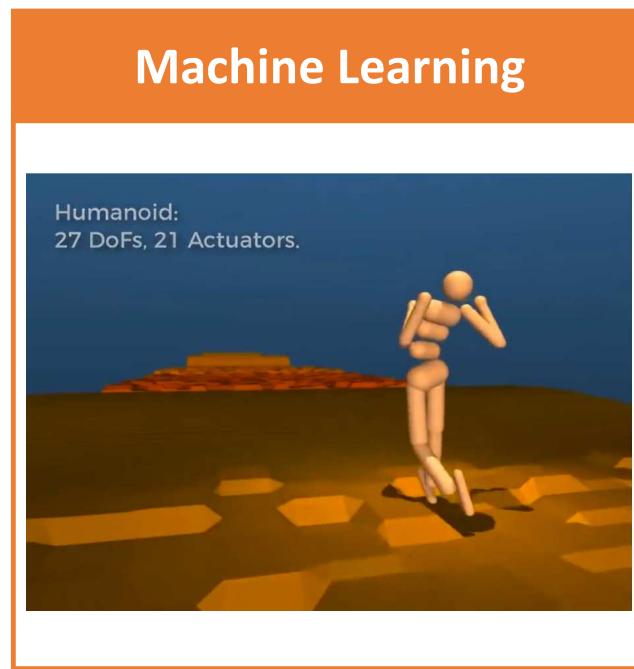
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Planning in Configuration Space



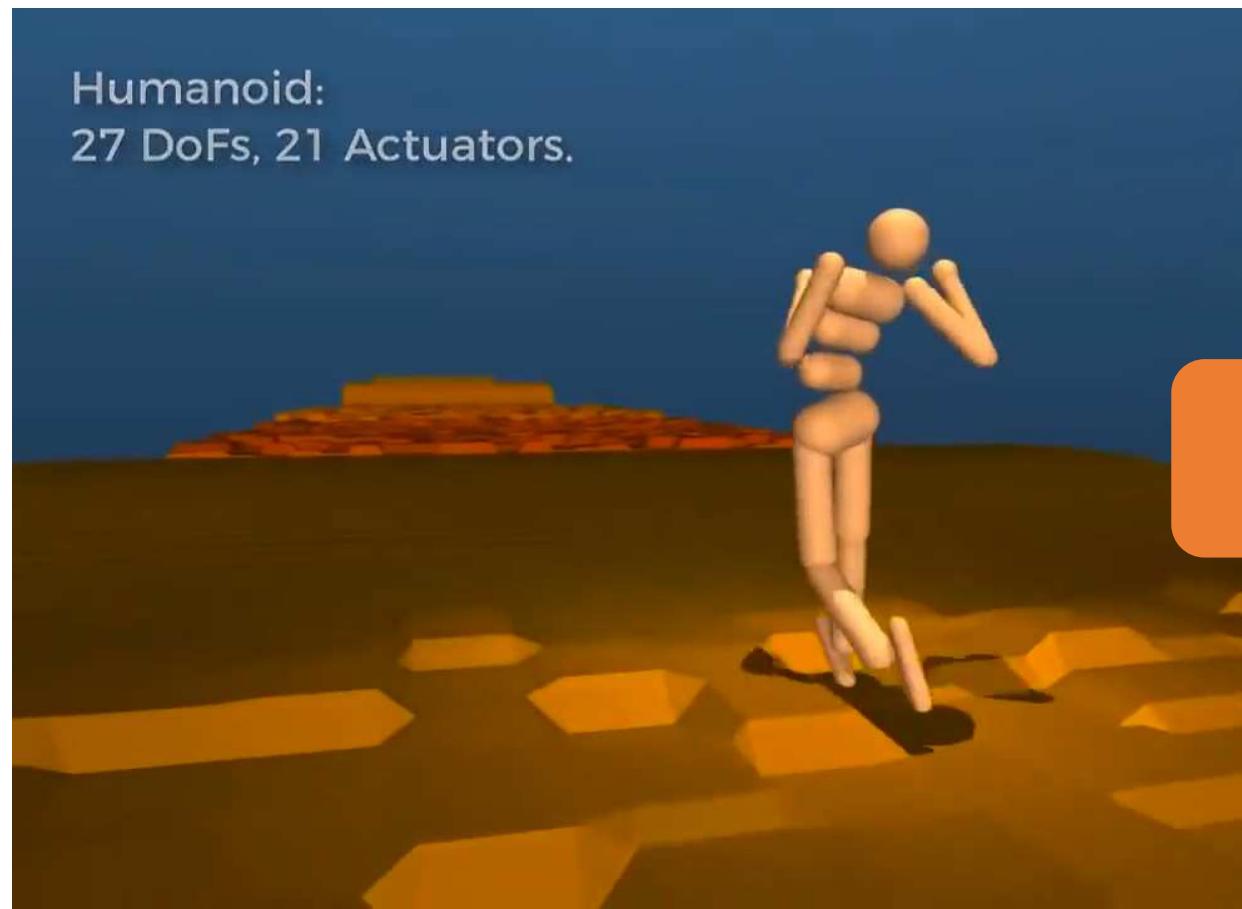
Random Search



Local Search

5

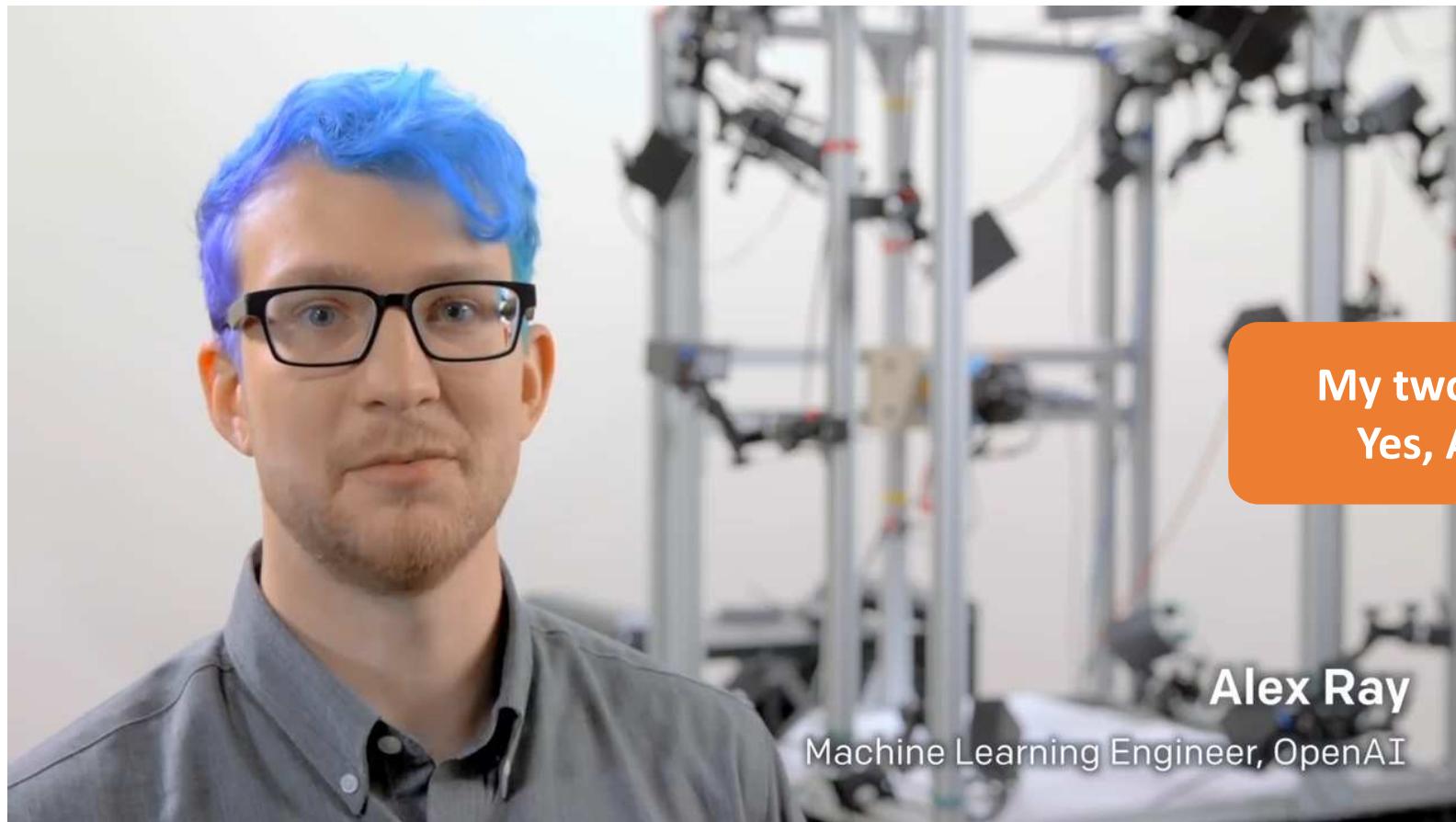
Guest Lecture in two weeks: Can I make the computer learn all of this for me automatically?



My two cents:
Yes, And...

5

Guest Lecture in two weeks: Can I make the computer learn all of this for me automatically?

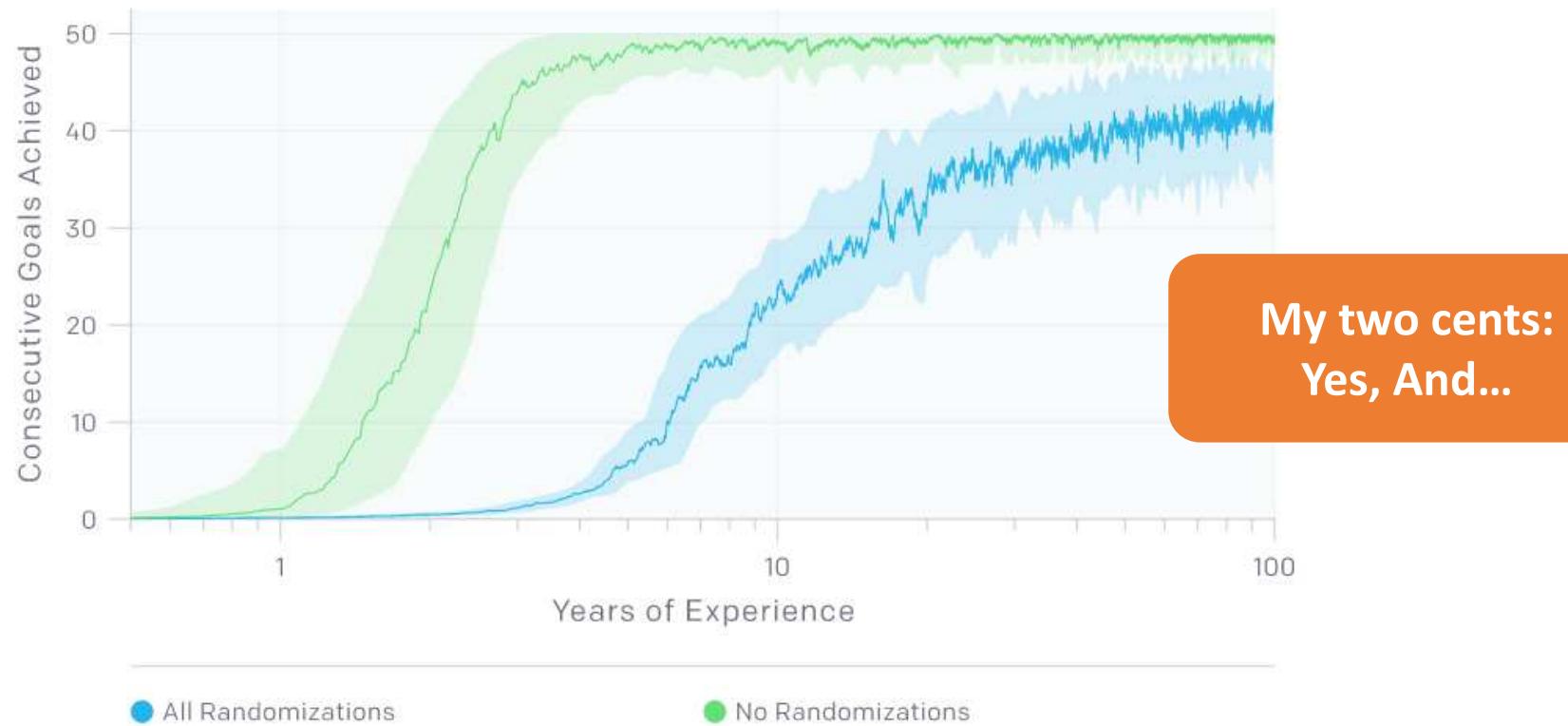


**My two cents:
Yes, And...**

Alex Ray
Machine Learning Engineer, OpenAI

5

Guest Lecture in two weeks: Can I make the computer [learn](#) all of this for me automatically?



5

So what else can we do?

•

5 So what else can we do?

Lots of math!



5 So what else can we do?

Lots of math!



5 So what else can we do?

Its actually not that
bad and the math
isn't actually that
scary I promise!



5

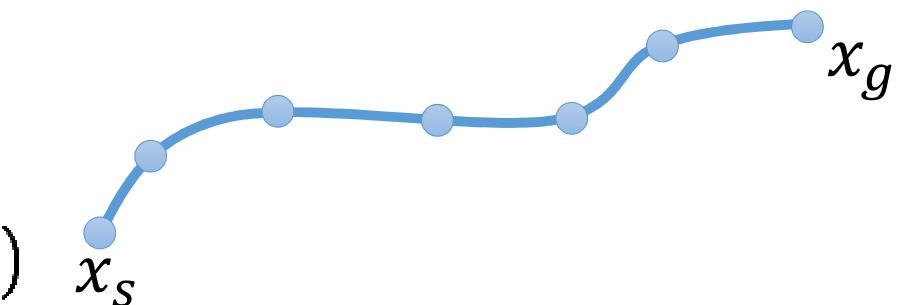
Optimization

We can write the planning problem down as an optimization problem!

$$\underset{s_0, a_0, \dots, s_N, a_N}{\text{minimize}} \sum_{k=0}^N c(s_k, a_k)$$

$$\text{subject to } s_{k+1} = f(s_k, a_k) \quad x_s$$

$$s_N = s_{\text{goal}}$$



5

Optimization

We can write the planning problem down as an optimization problem!

$$\underset{s_0, a_0, \dots, s_N, a_N}{\text{minimize}} \sum_{k=0}^N c(s_k, a_k)$$

Minimize a cost in each state
(e.g., energy used)

$$\text{subject to } s_{k+1} = f(s_k, a_k)$$

Obey physics

$$s_N = s_{\text{goal}}$$

Get to the goal

Optimization

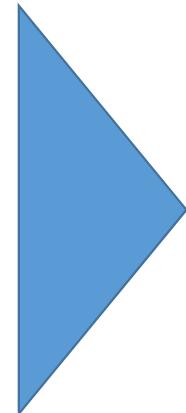
We can use Bellman updates to solve this:

- We can start at the goal state and then work backwards computing the lowest cost actions to get to all states all the way back to the start state

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$$s_N = s_{\text{goal}}$$



$$V_N(s_N) = c(s_N, a_N)$$

$$V_{N-1}(s) = \min_a c(s_{N-1}, a_{N-1}) + V_N(f(s_{N-1}, a_{N-1}))$$

This leads to the classic ***Value Iteration*** algorithm

Optimization

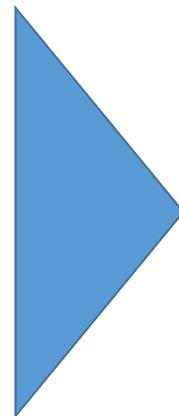
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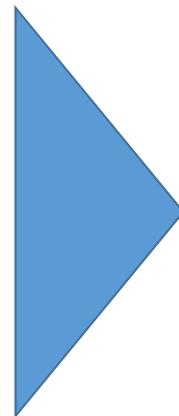
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Sadly again the complexity scales with $d^{|S| = |A|}$ and those can get **HUGE** fast! This is the “**curse of dimensionality**” again

Optimization

Lets lower our expectations!

#localOptima #efficientUseOfComputers

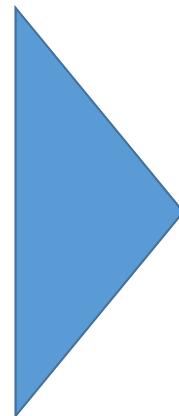
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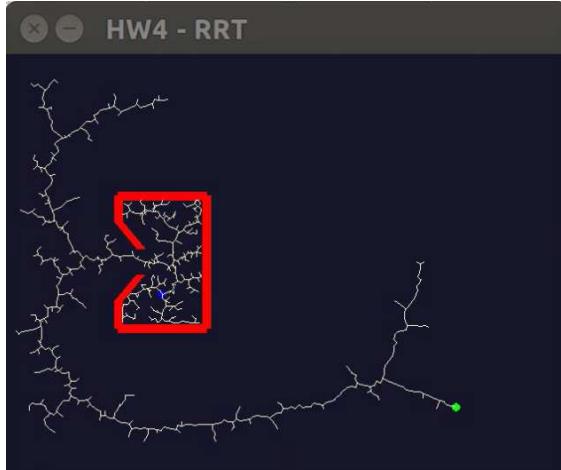
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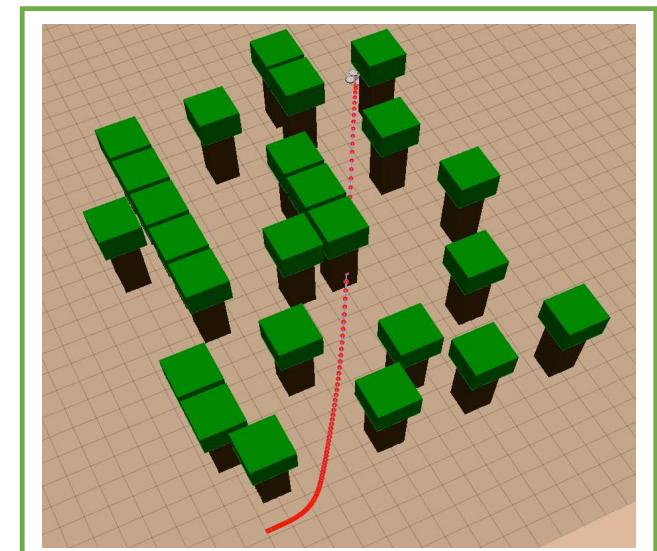
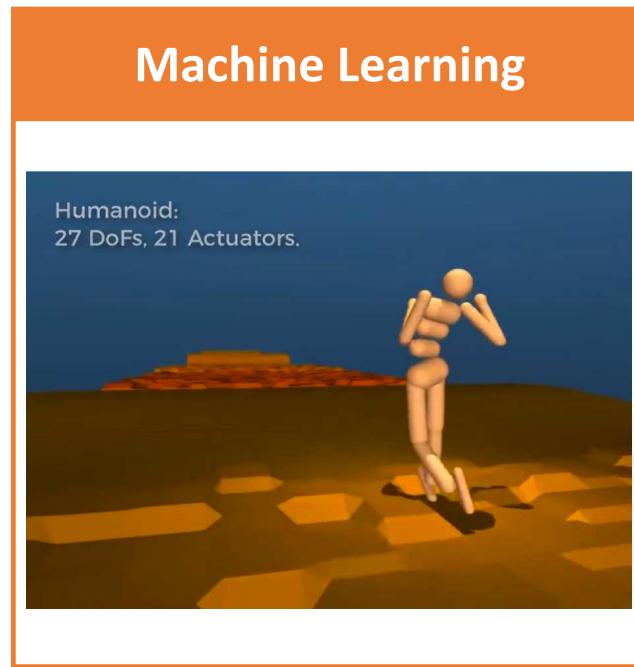
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Planning in Configuration Space



Random Search



Local Search

5 Trajectory Optimization

What if instead of finding a globally optimal path we search for a locally optimal path (off of some initial condition)?

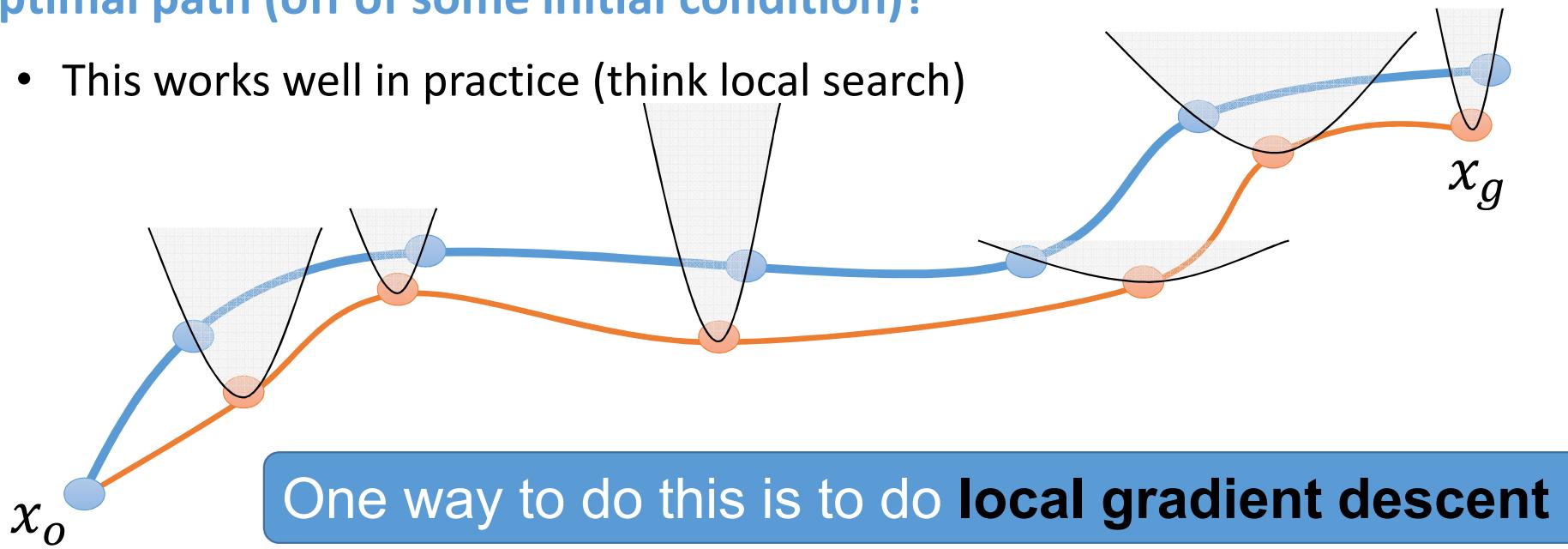
- This works well in practice (think local search)



5 Trajectory Optimization

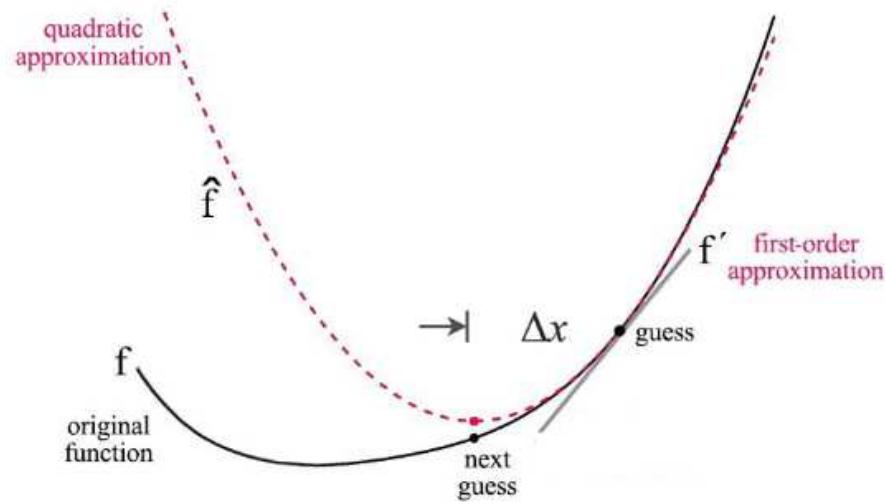
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5 Trajectory Optimization

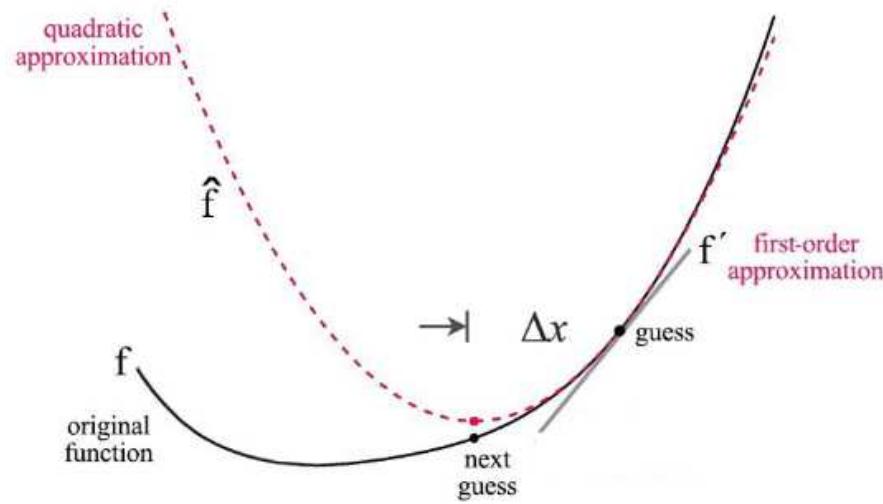
I'm drawing small quadratic bowls because most (if not all) of the practical algorithms make **linear and quadratic approximations** of the nonlinear functions allowing for efficient gradient descent



5 Trajectory Optimization

And convex optimization tells us how to descend to the minima of a quadratic function

I'm drawing small quadratic bowls because most (if not all) of the practical algorithms make **linear and quadratic approximations** of the nonlinear functions allowing for efficient gradient descent



5 Trajectory Optimization

There are also a whole host of algorithms one can use to solve these problems including:

- DDP, SQP, Interior-Point Methods, Trust-Region Methods, Stochastic Gradient Descent Methods, etc.

And you can use off-the-shelf solvers to solve these problems. Popular solvers include:

- SNOPT, IPOPT, NLOPT, fmincon (MATLAB), etc.
 - **Most people use off the shelf solvers!**
-

5 Trajectory Optimization

So trajectory optimization solves everything right?

- Can handle full robot **dynamics**
- No need for distance metrics
- Can use **off the shelf solvers** reducing the coding burden
- Finds a **locally optimal** solution – no weird paths coming out!
 - Extra motions are “optimized away”

5 Trajectory Optimization

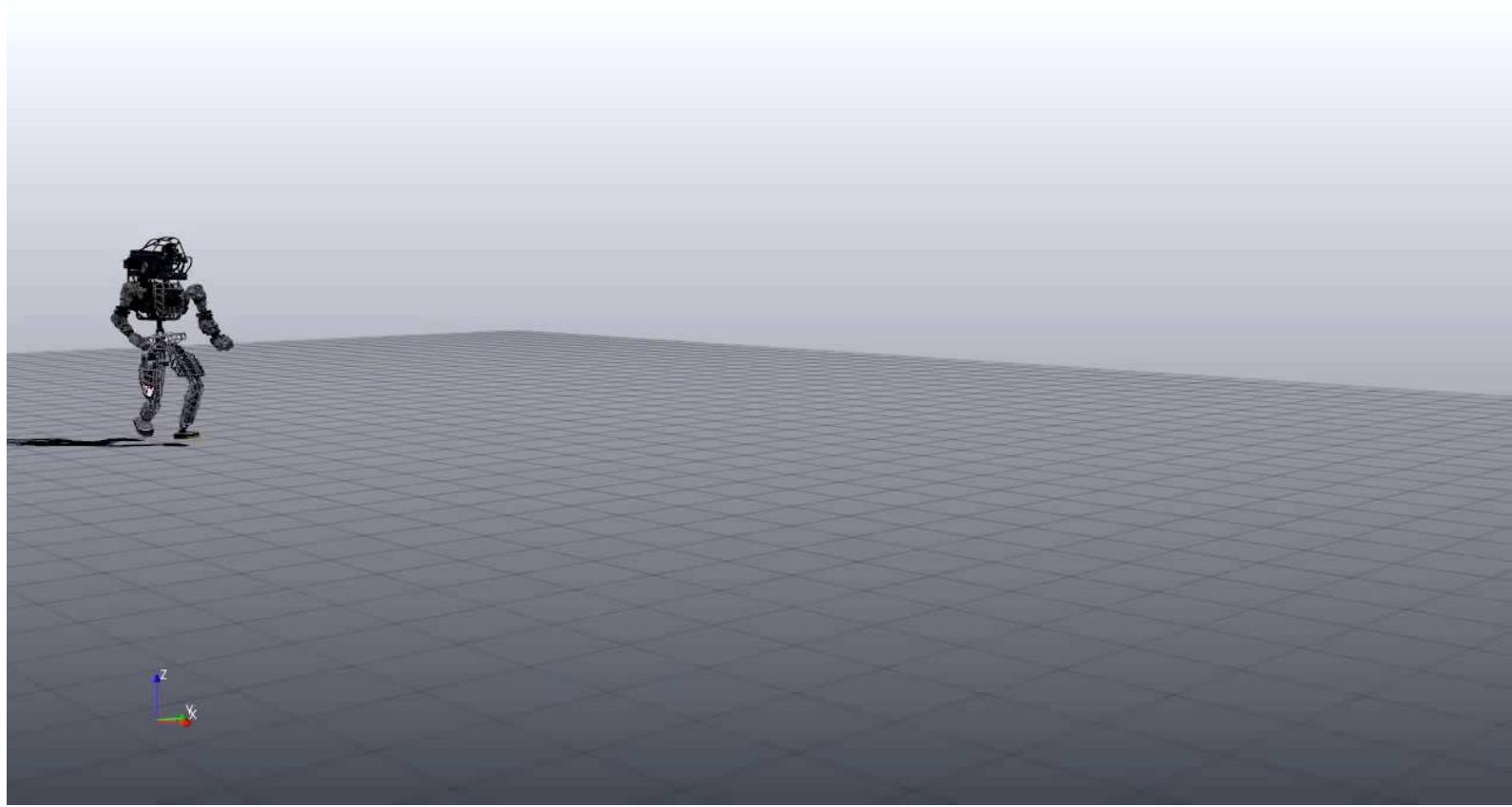
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And optimal motions often look bio-inspired as nature generally uses optimally efficient motions!

5

Atlas 1.0 Trajectory Optimization



5 Trajectory Optimization

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No free lunch strikes again!

But....

- **Not globally optimal** (will often get stuck in local minima)
- **Not even complete** (problems are often non-convex so it may not even find a feasible solution)
- **Also generally slow**

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Lets dive a little deeper into solvers!

5

There are two popular classes of solvers

Pros

Shooting Methods

(e.g., DDP, iLQR)

- Known fast

Cons

- Hard to add constraints (e.g., torque limits, obstacle avoidance)
- Generally people code it themselves

Direct Methods

(e.g., DIRTRAN using SQP or IP)

- Easy to add constraints (e.g., torque limits, obstacle avoidance)
- Easy to leverage **off the shelf solvers** (e.g., SNOPT, IPOPT)

- Considered slow

5

There are two popular classes of solvers

Pros

- K
- t
- C

Technical note: DDP reduces to a specific factorization of the **KKT matrix** solve in a direct method to exploit sparsity!

$$\begin{aligned} & \text{minimize} && (1/2)x^T Px + q^T x + r \\ & \text{subject to} && Ax = b \end{aligned}$$

$$\begin{bmatrix} P & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} x^* \\ \nu^* \end{bmatrix} = \begin{bmatrix} -q \\ b \end{bmatrix}$$

Cons

5

There are two popular classes of solvers

Pros

Shooting Methods
(e.g., DDP, iDP)

- Known fast

Cons

- Hard to add constraints (e.g., torque limits, obstacle avoidance)
- Generally people consider them slow

I'll dig a little deeper / explain this more in 2 weeks when I present my research on parallel shooting methods

Direct Methods
(e.g., DIRTRAN using SQP or IP)

easy to add constraints (e.g., torque limits, obstacle avoidance)
easy to leverage **off the shelf solvers** (e.g., SNOPT, IPOPT)

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There are two popular classes of solvers

Stephen Boyd and
Lieven Vandenberghe

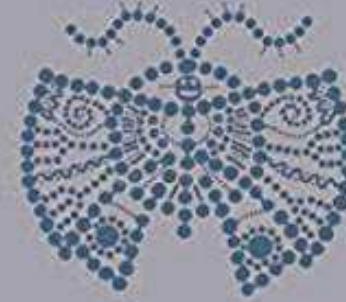
Convex Optimization

But/and these
are two great
textbooks if you
want to learn
more about the
math!

Springer Series in
Operations Research

Jorge Nocedal
Stephen J. Wright

Numerical
Optimization
Second Edition



Springer

Practical Challenges for Trajectory Optimization: Robustness

Manchester and Kuindersma 2017
Plancher and Kuindersma 2018

1. Solvers are (numerically) sensitive to:
 - Cost function designs and dynamic range
 - Regularization scheme

2. Solutions are sensitive to:
 - Initial state and input trajectories
 - Perturbations (solutions are often on constraint boundaries)

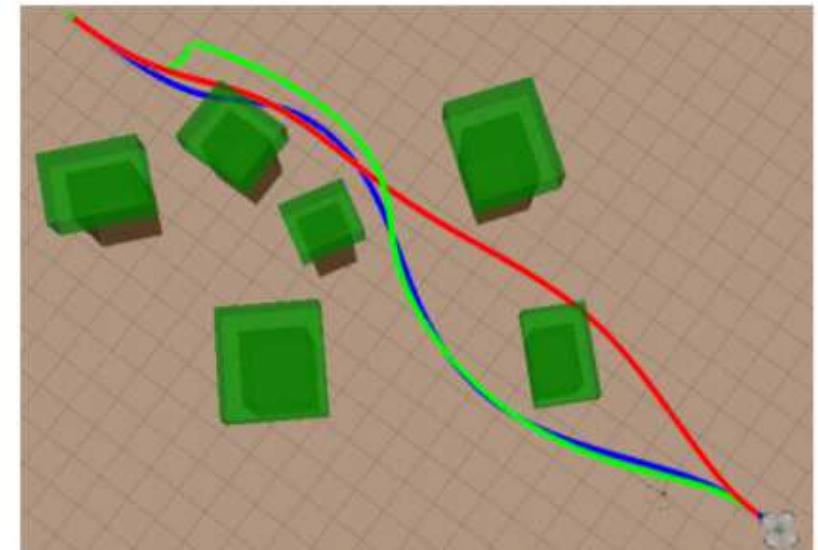
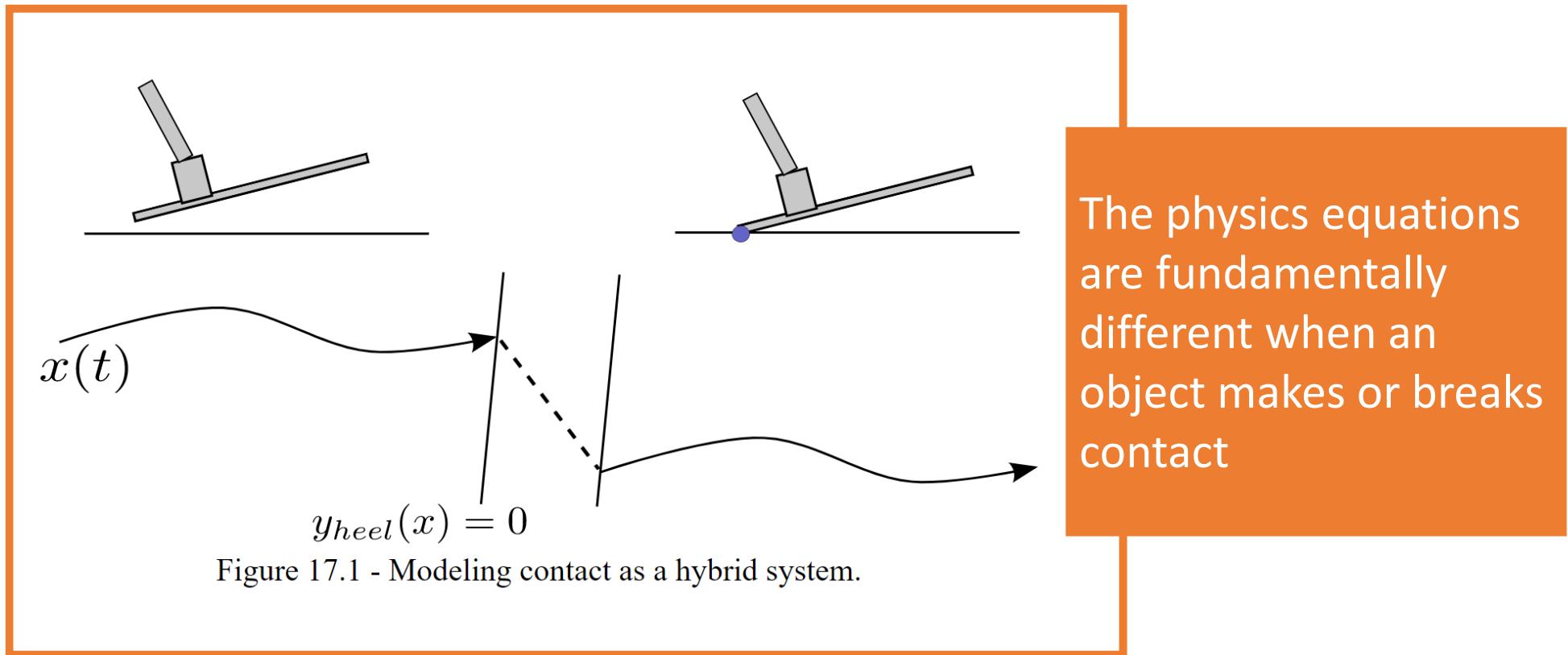


Fig. 4. DIRTRAN (red), DIRTREL-1 (blue), and DIRTREL-2 (green) quadrotor trajectories.

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Practical Challenges for Trajectory Optimization: Contact

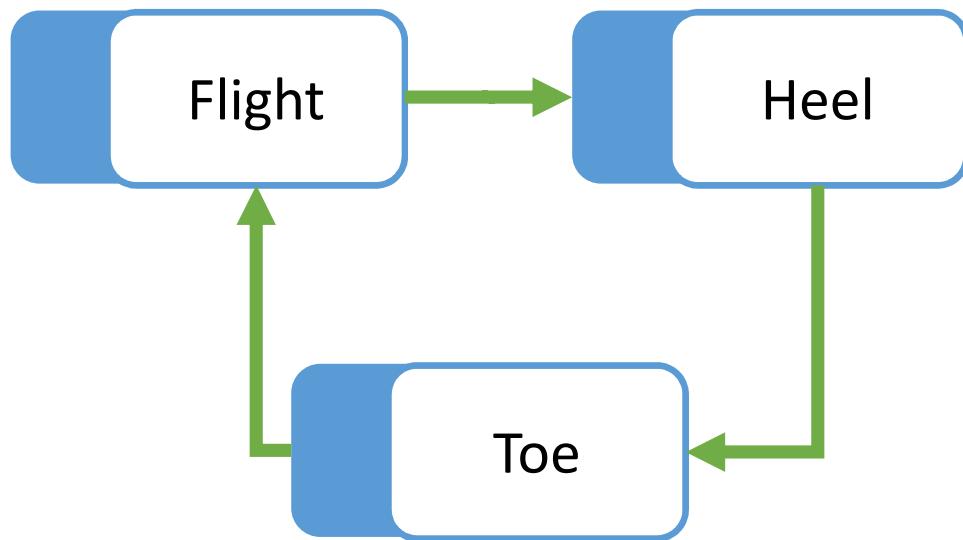
Tedrake Underactuated



5

Practical Challenges for Trajectory Optimization: Contact

Tedrake Underactuated



For walking these
hybrid modes form a
cyclic graph

If we **pre-specify** the
mode sequence and
timing we can use our
algorithms as before

5

Practical Challenges for Trajectory Optimization: Contact

Manchester and Kuindersma 2017

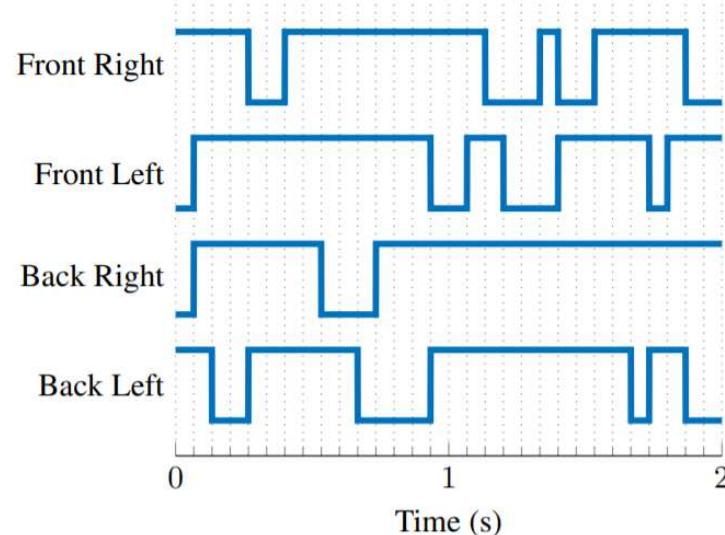
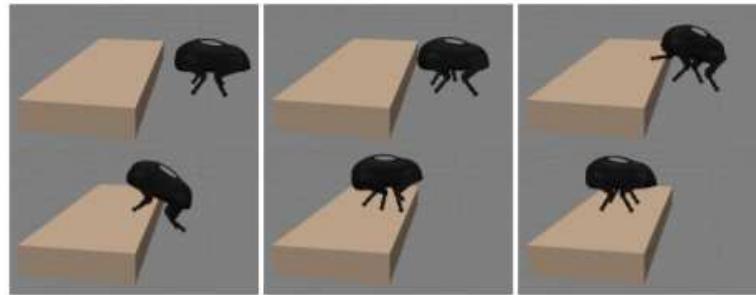


Figure 4. Contact mode sequence for each foot from LittleDog step-climbing example.



But for **complex actions** these modes start to become **hard to pre-specify**

5

Practical Challenges for Trajectory Optimization: Contact

Doshi, et. al. 2018

Contact-Implicit
Trajectory
Optimization
includes the
contact timings
and mode
transitions as
state variables

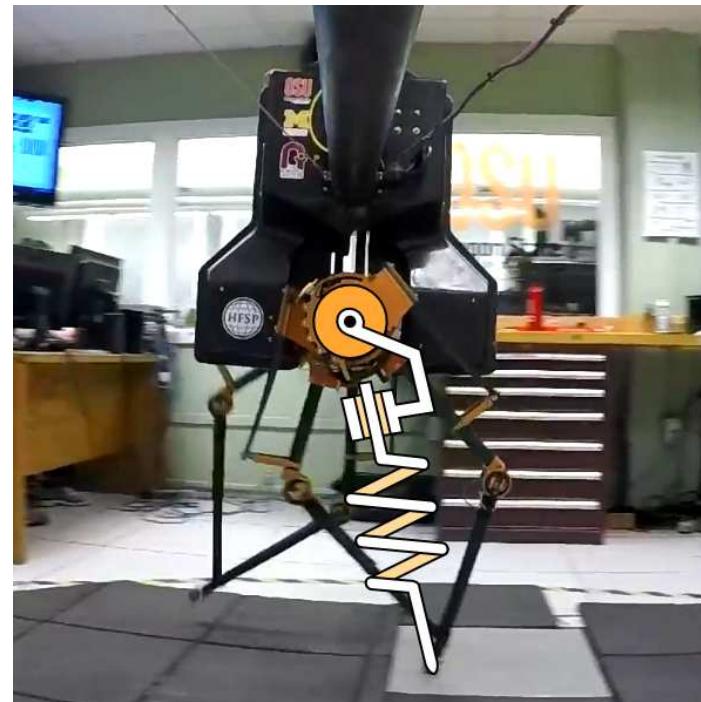
2Hz Gaits
(real-time)

But these approaches
are computationally
very expensive (read
offline) as the number
of modes explodes
combinatorically with
the number of contact
points (Mixed-Integer
Programming)!

5

Practical Challenges for Trajectory Optimization: Contact

One approach to avoid solving these large hard problems is to solve the problem on **simpler models** of the system



5

Practical Challenges for Trajectory Optimization: Contact

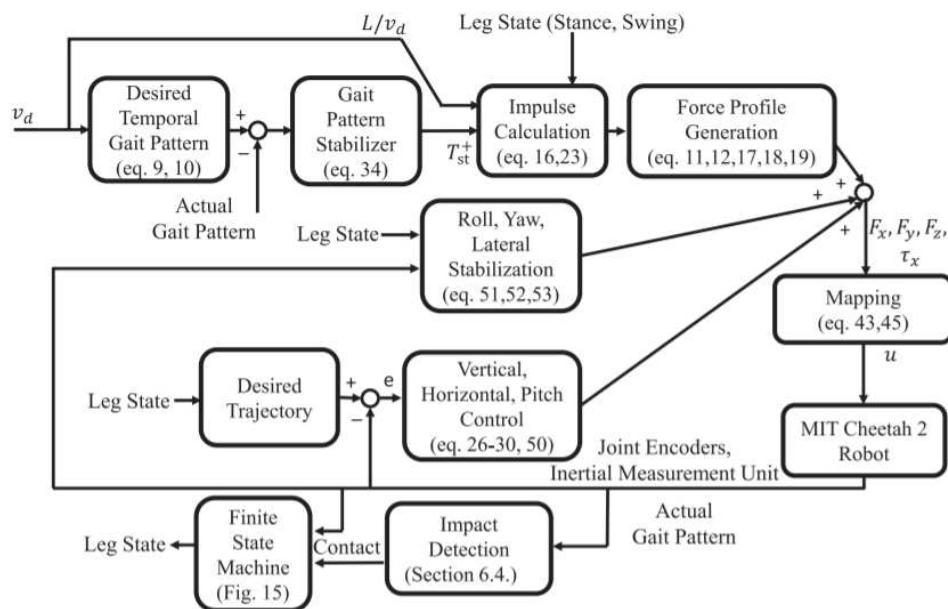
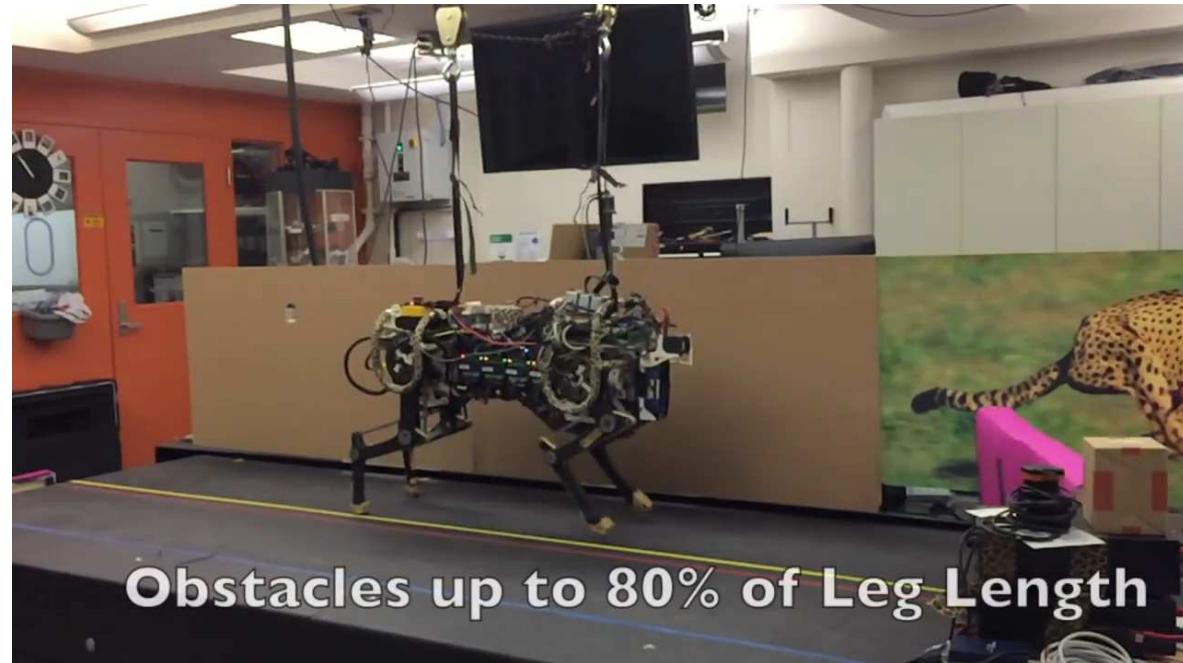


Fig. 11. Overall control system diagram.

And then **combine solutions** to these **(conservative)** simpler problems

5

Practical Challenges for Trajectory Optimization: Contact



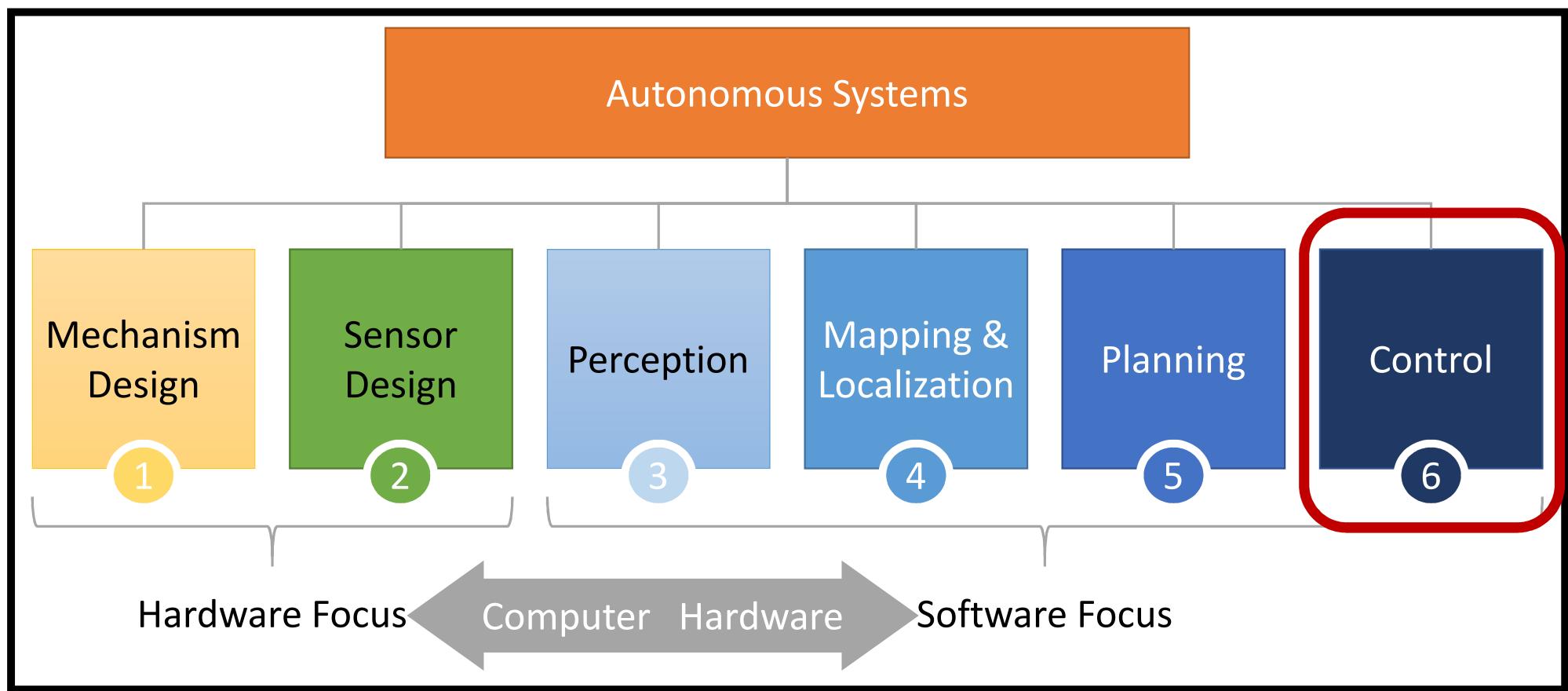
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Key Takeaways:

1. Robot planning involves both **task and configuration spaces**
 2. For many real problems, **collision checking** can be expensive
 3. **Sample Based Planners** that leverage random search (**RRT/PRM**):
 - **Probabilistically complete** (but can still be slow sometimes)
 - **Single-query (RRT) vs. Multi-query (PRM)**
 - **Probabilistically optimal (RRT*)** but generally need smoothers
 4. **Trajectory Optimization** leverages local search to find **locally optimal** (generally smooth) solutions
 - Handles dynamics well but **not complete or robust**
 - Can use **off the shelf solvers** (SQP) but **generally slower** than a solver that **exploits sparsity** in the problem (DDP/iLQR)
 - **Contact is hard** and we (sometimes) use **simpler models** for tractability
-

Autonomous Systems / Robotics is a BIG space



6

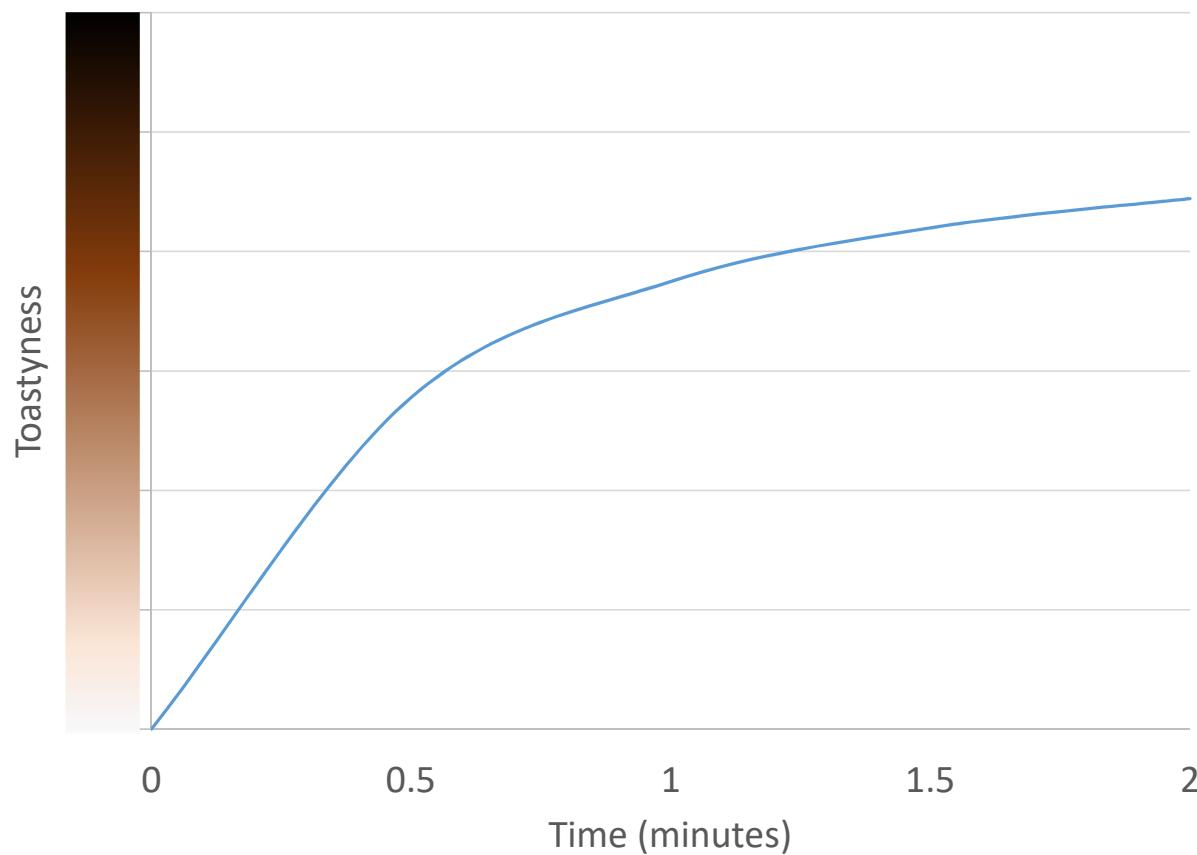
Control is the process of executing a plan in the real world

Well the simplest thing we could try would be to just execute the controls from our plan directly on the real system. This is called **Open-Loop Control!**

6

Open Loop Control

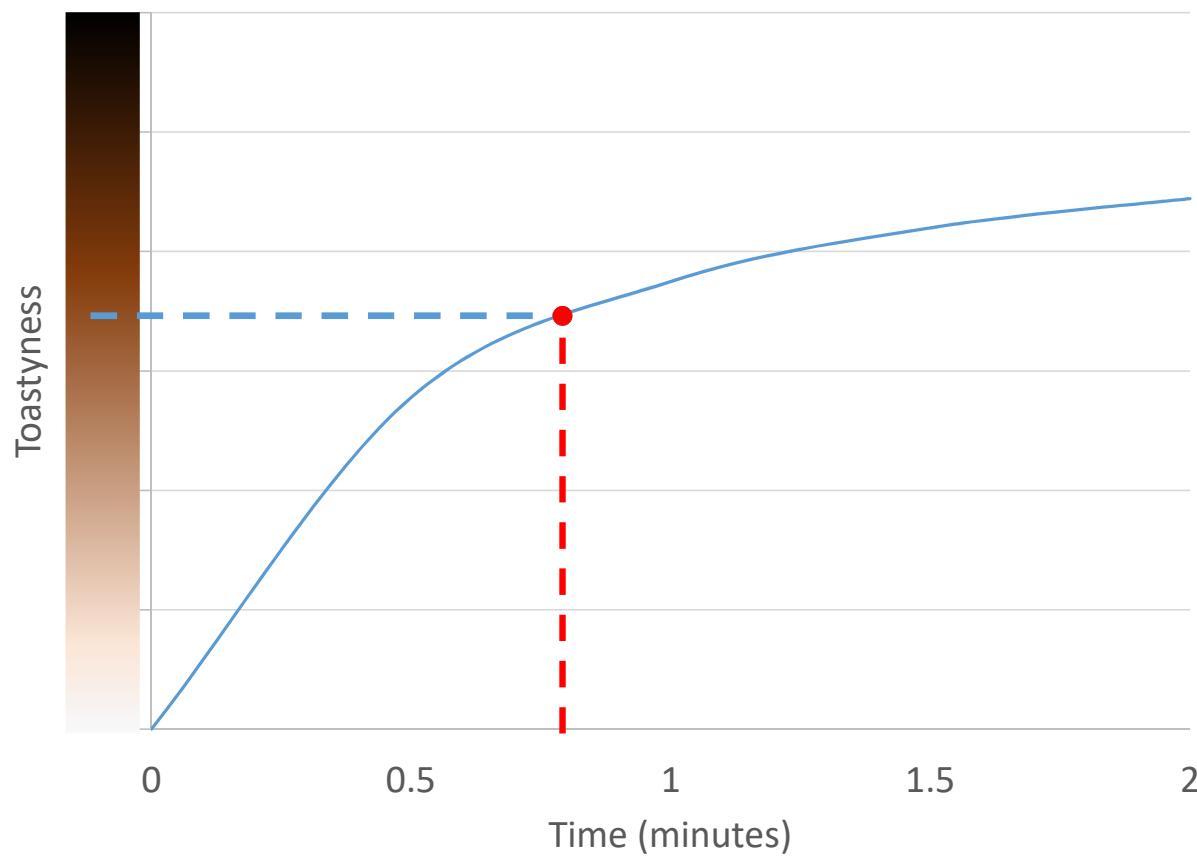
Adapted from MATLAB Control Toolbox



6

Open Loop Control

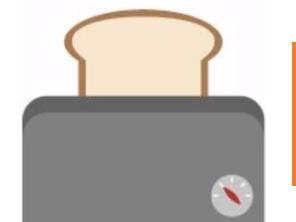
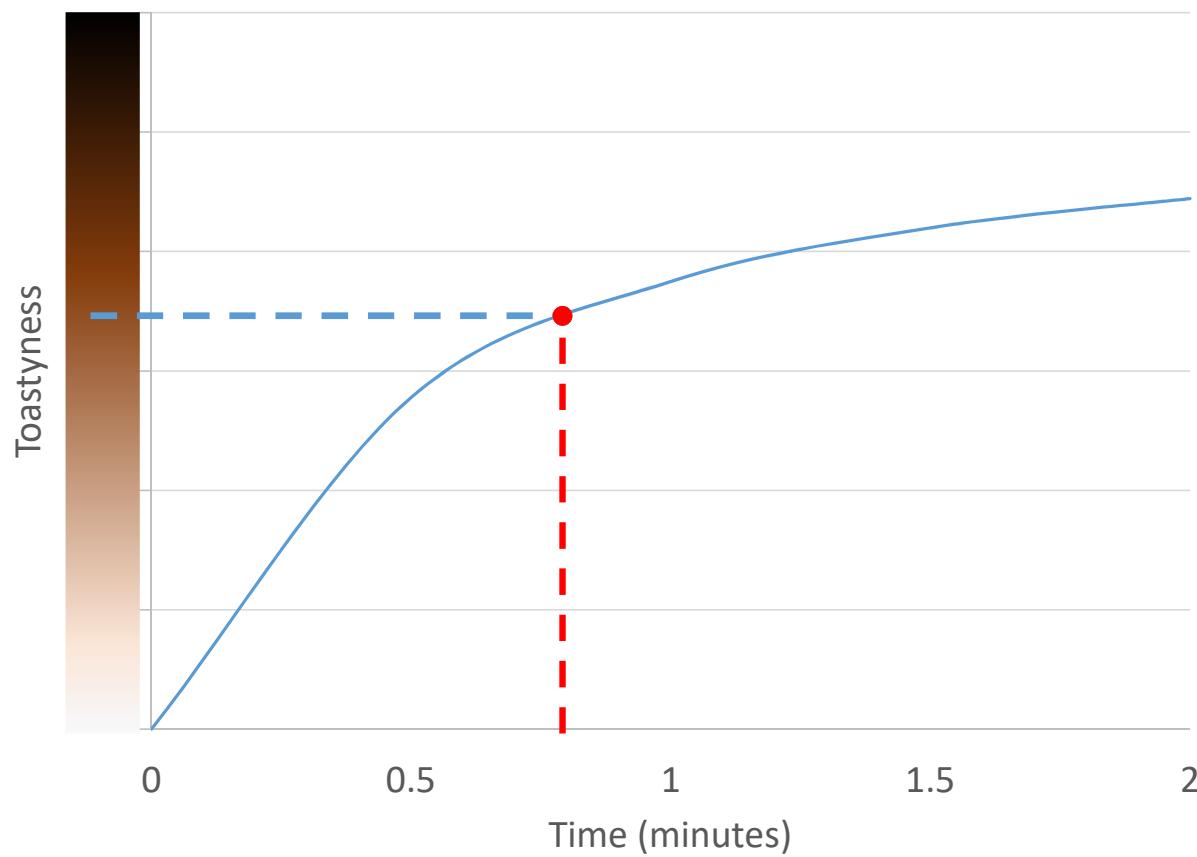
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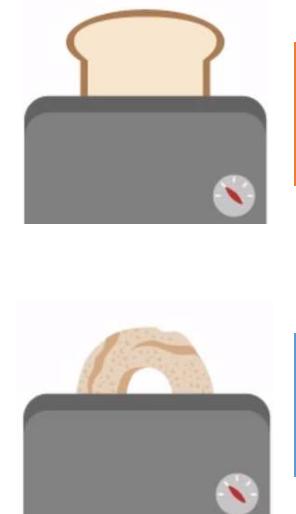
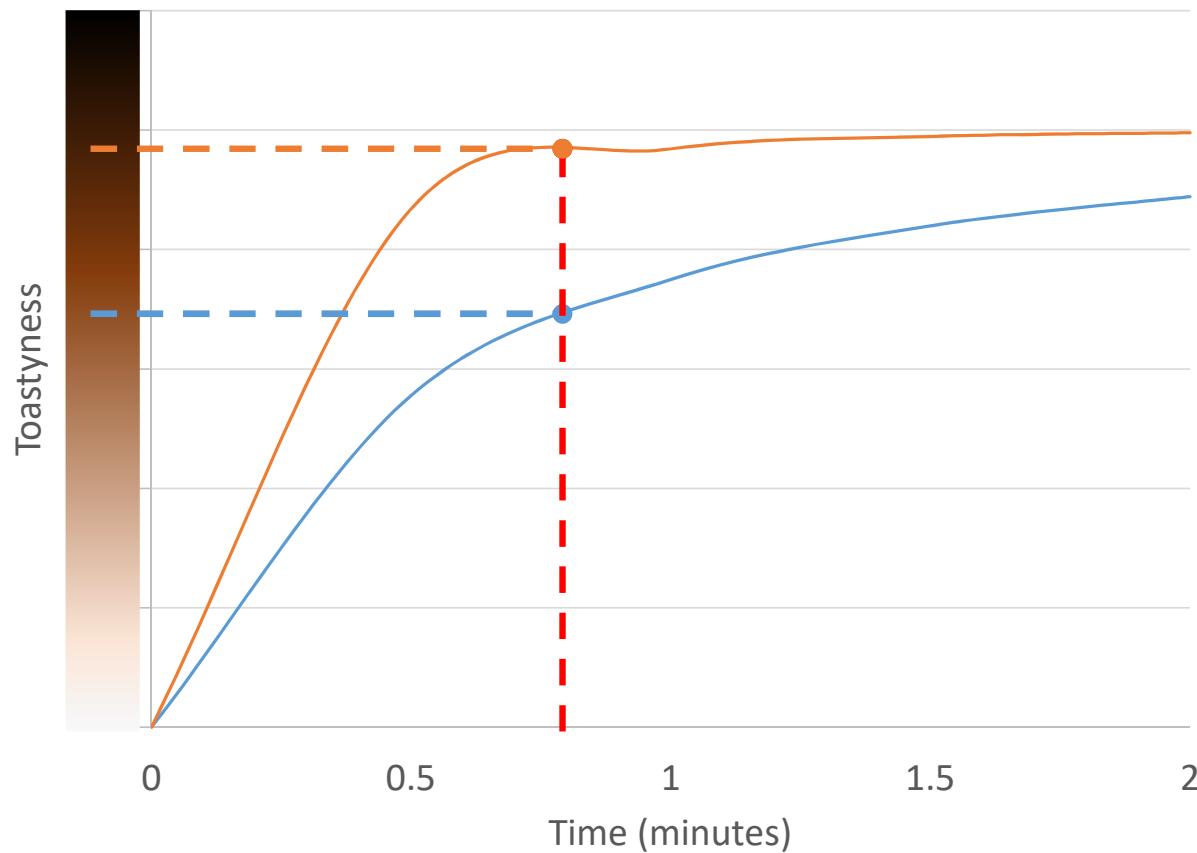
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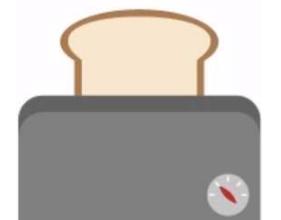
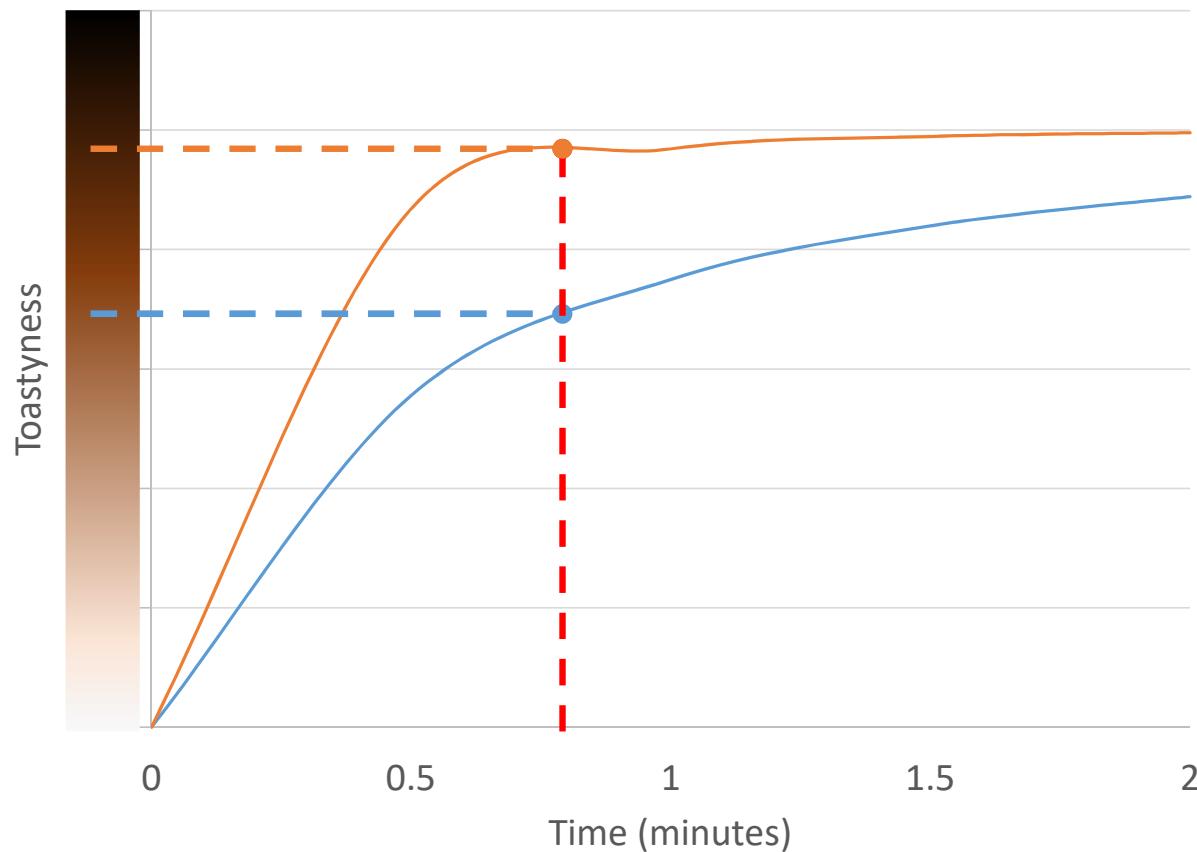
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Open Loop Control

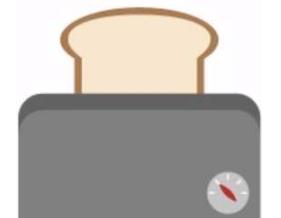
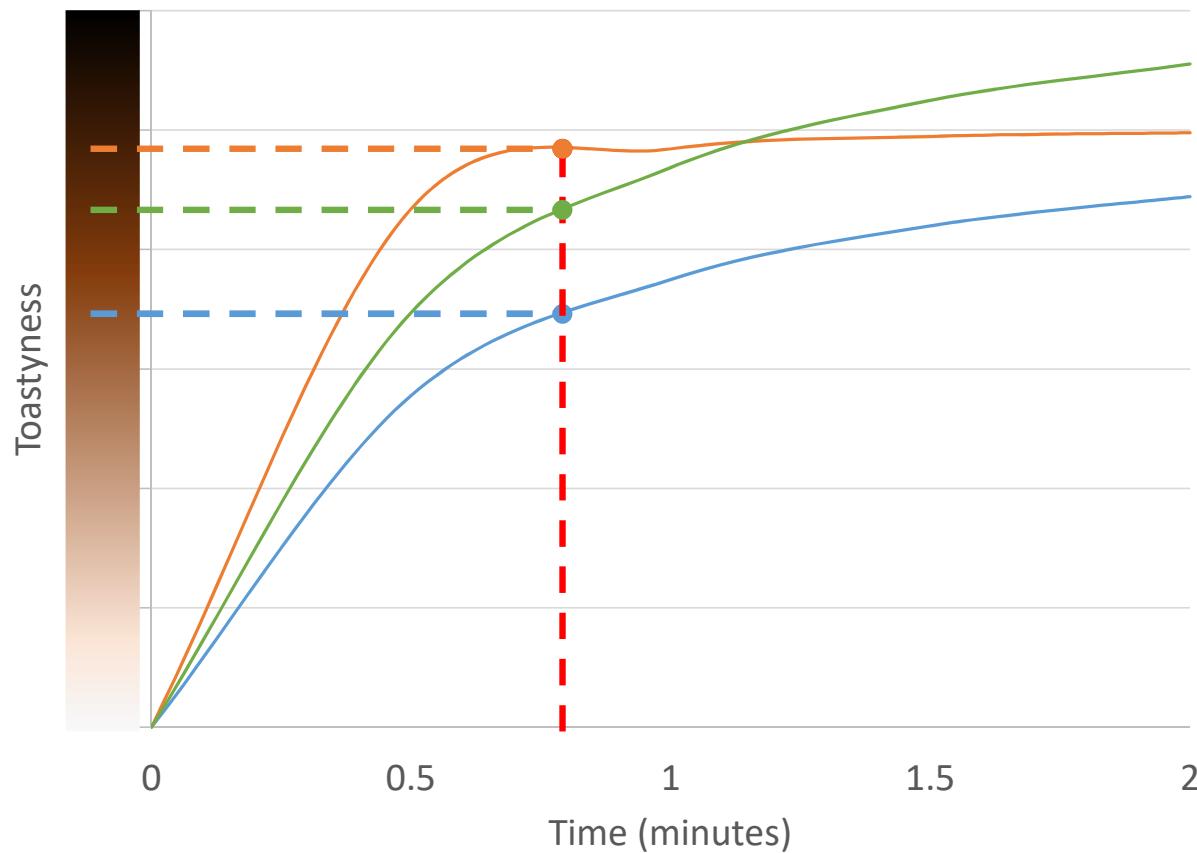
Adapted from MATLAB Control Toolbox



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Open Loop Control

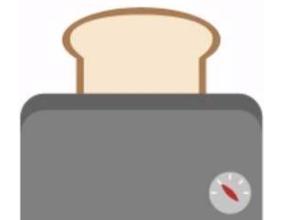
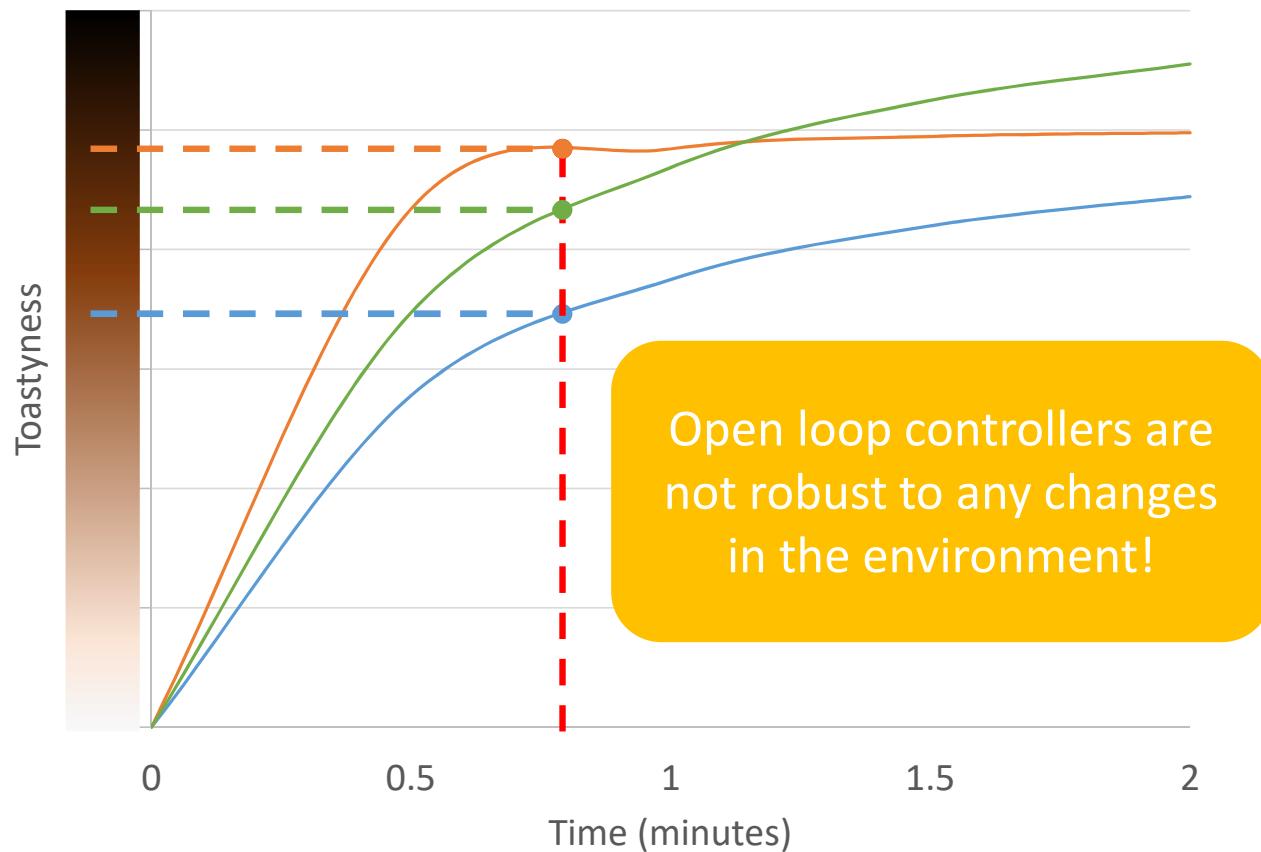
Adapted from MATLAB Control Toolbox



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Open Loop Control

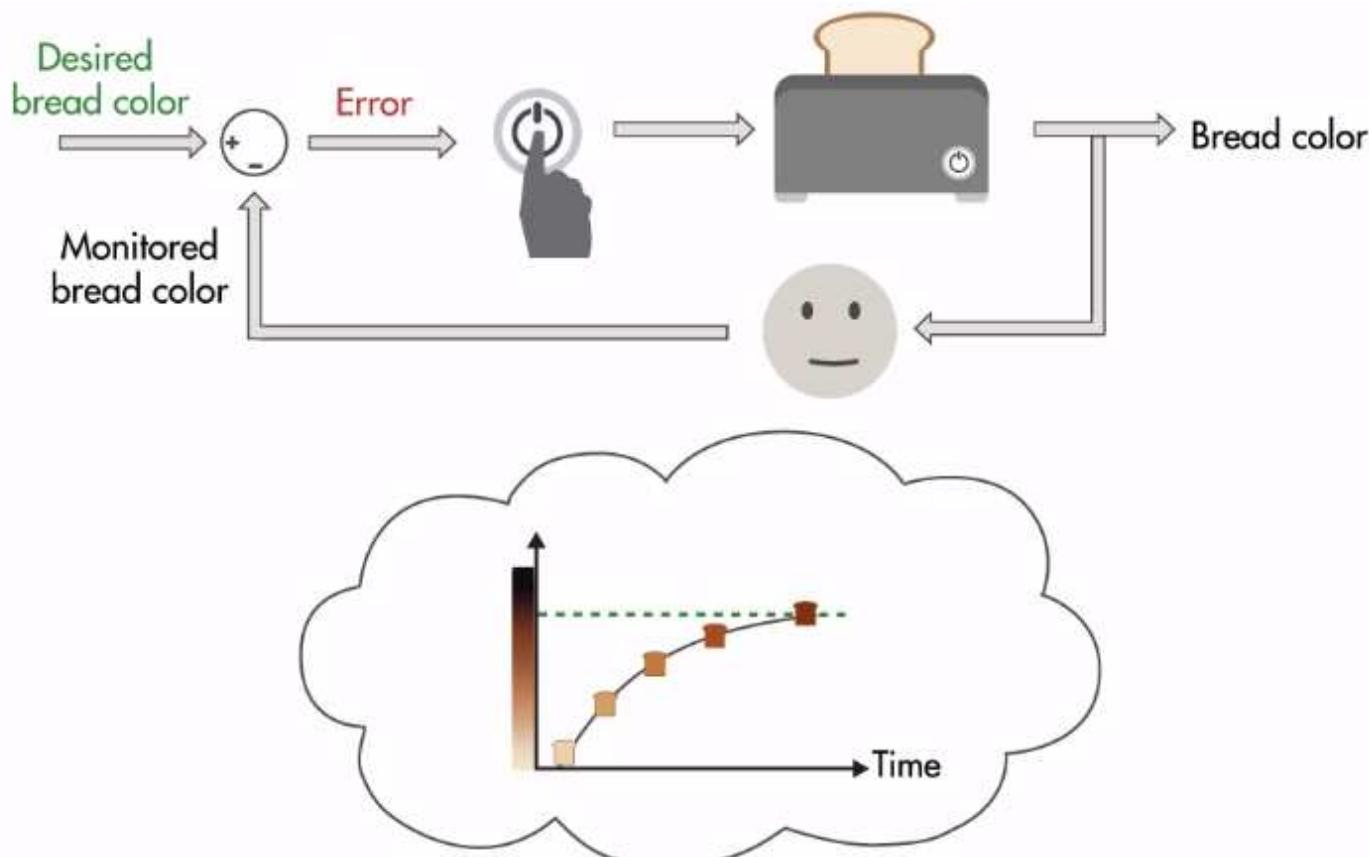
Adapted from MATLAB Control Toolbox



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Feedback (Closed Loop) Control

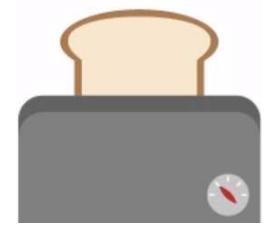
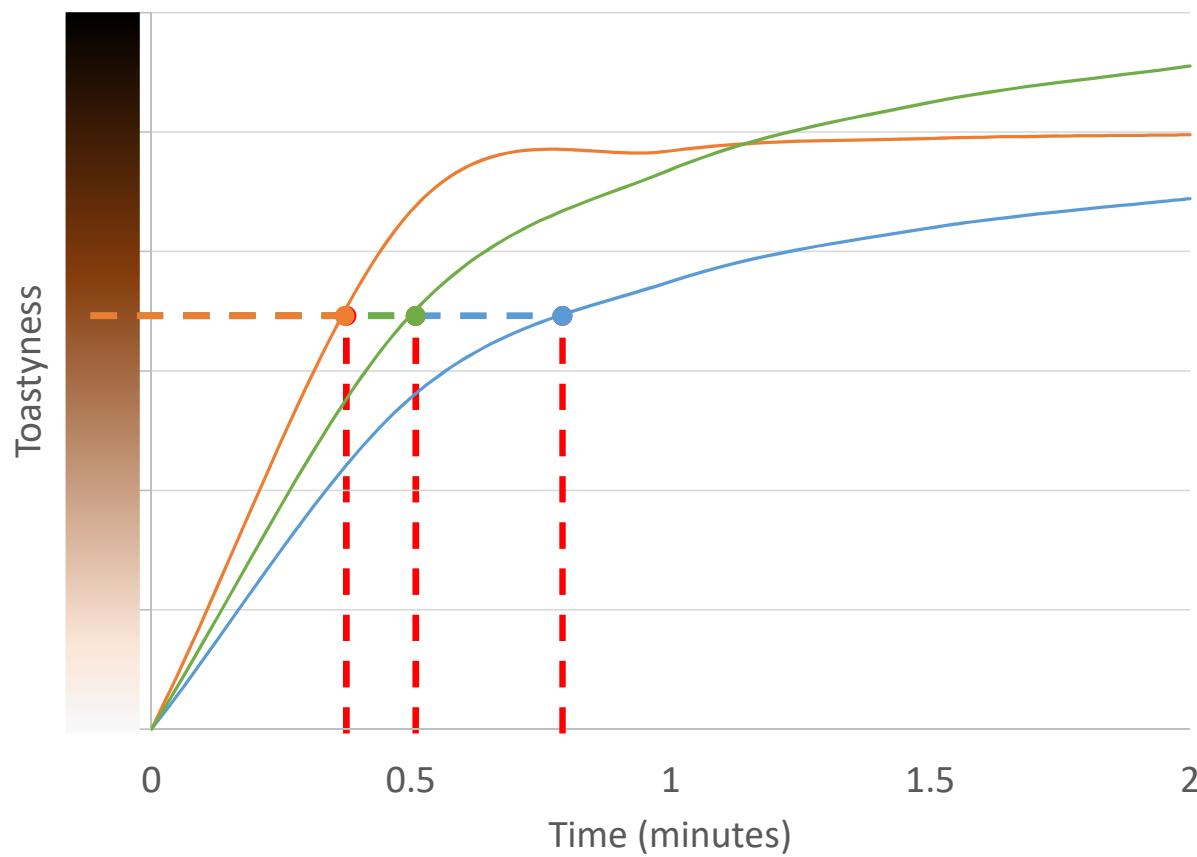
Adapted from MATLAB Control Toolbox



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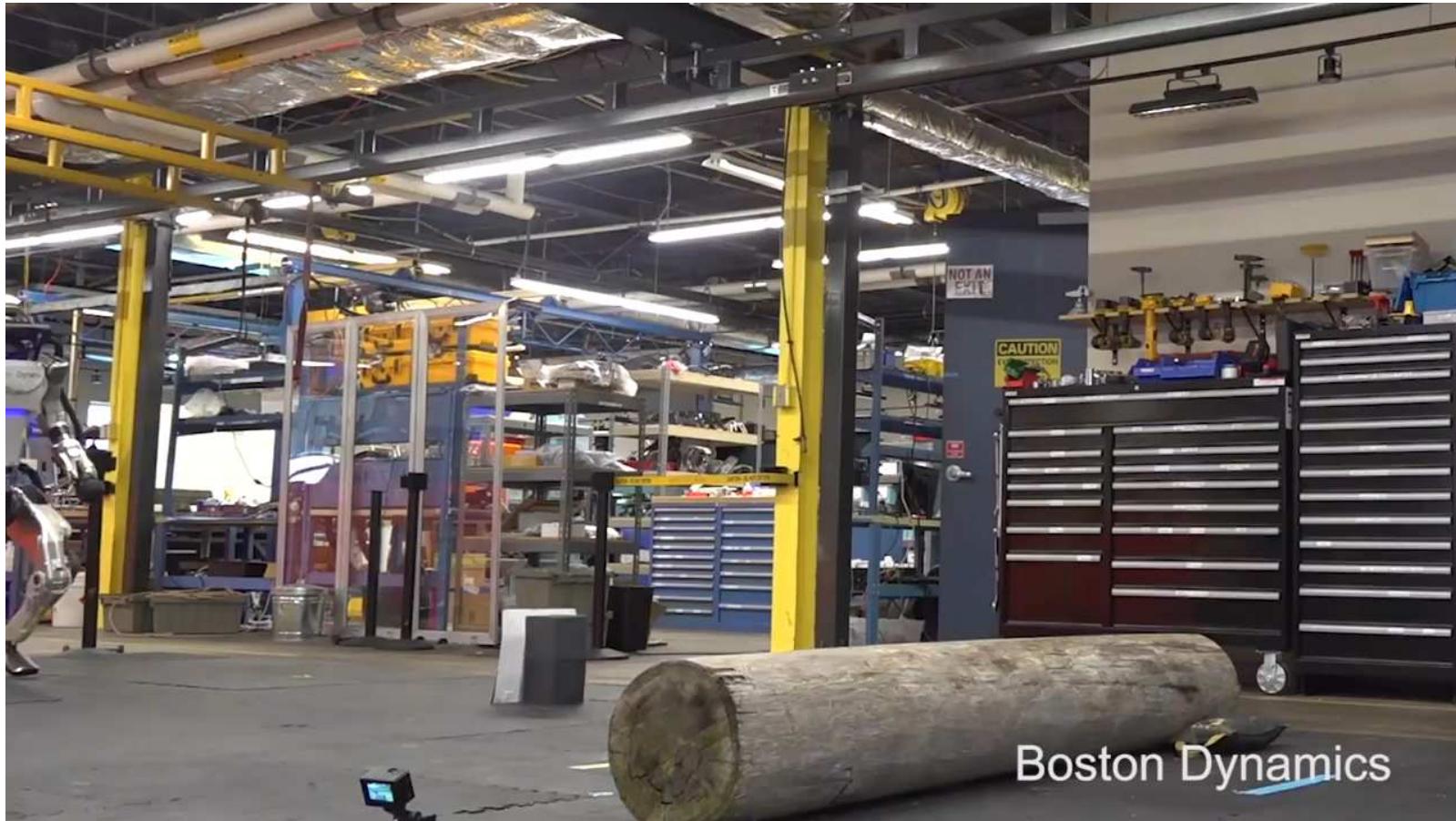
Feedback Control

Adapted from MATLAB Control Toolbox



6

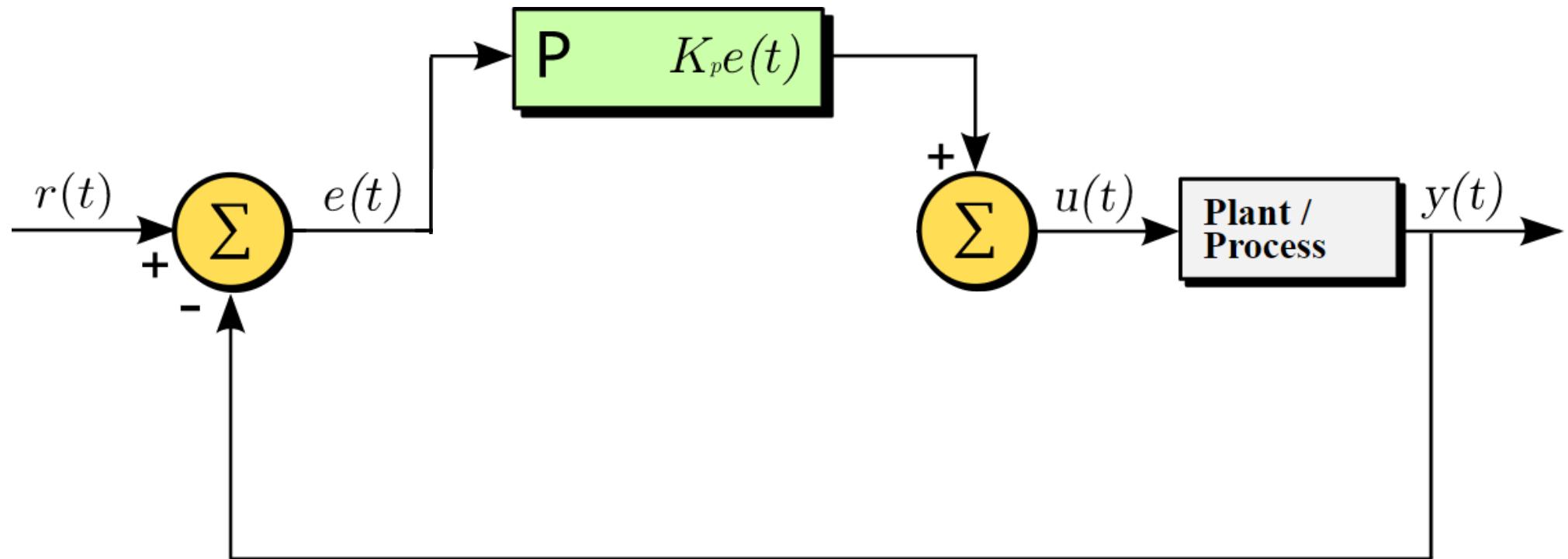
Feedback Control can lead to amazing performance!



6

So how do we do Feedback Control in practice?

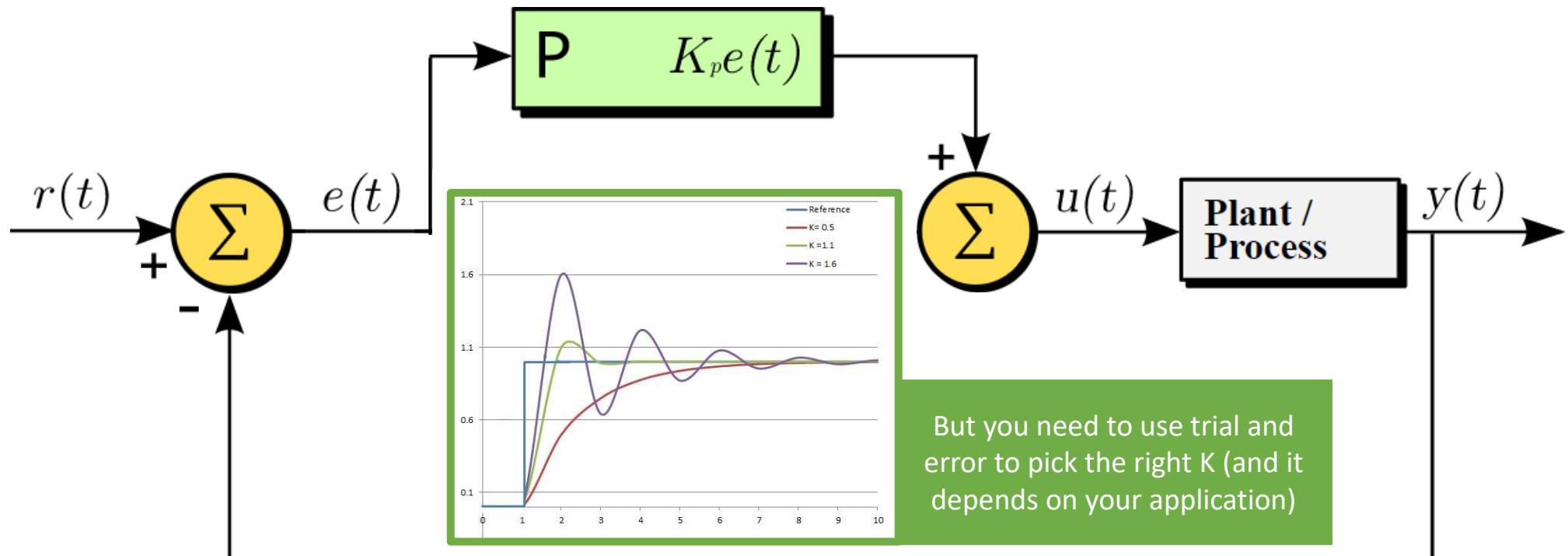
Adapted from Wikipedia



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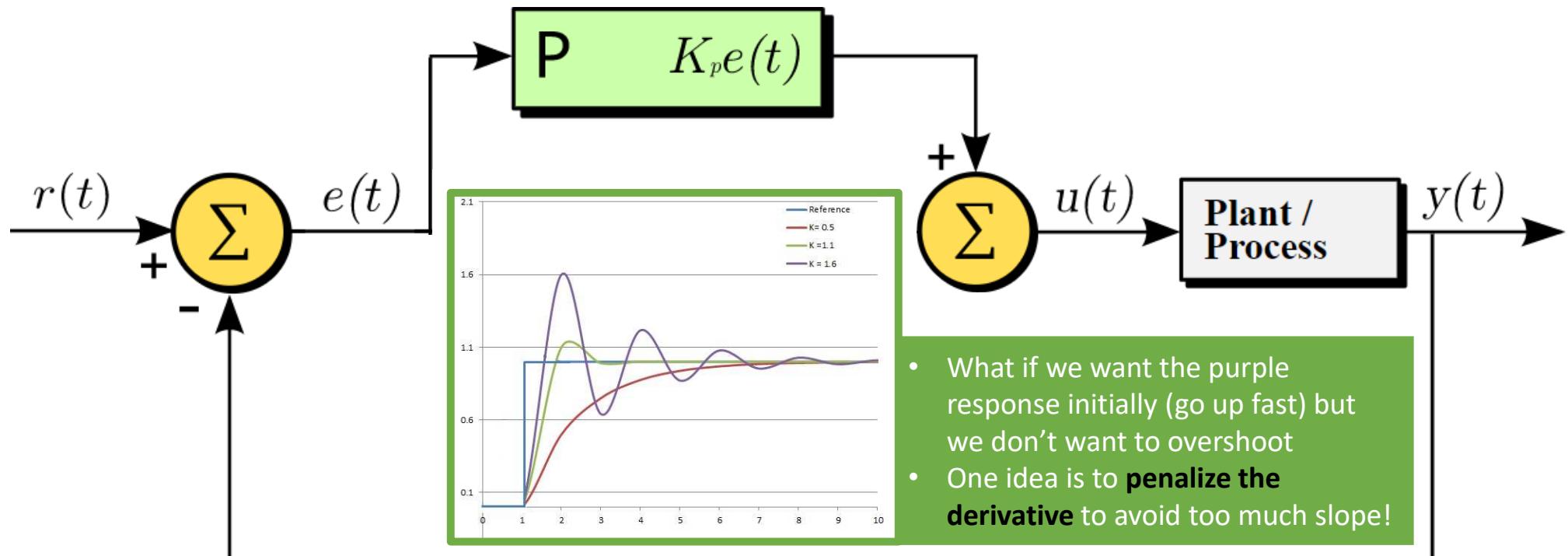
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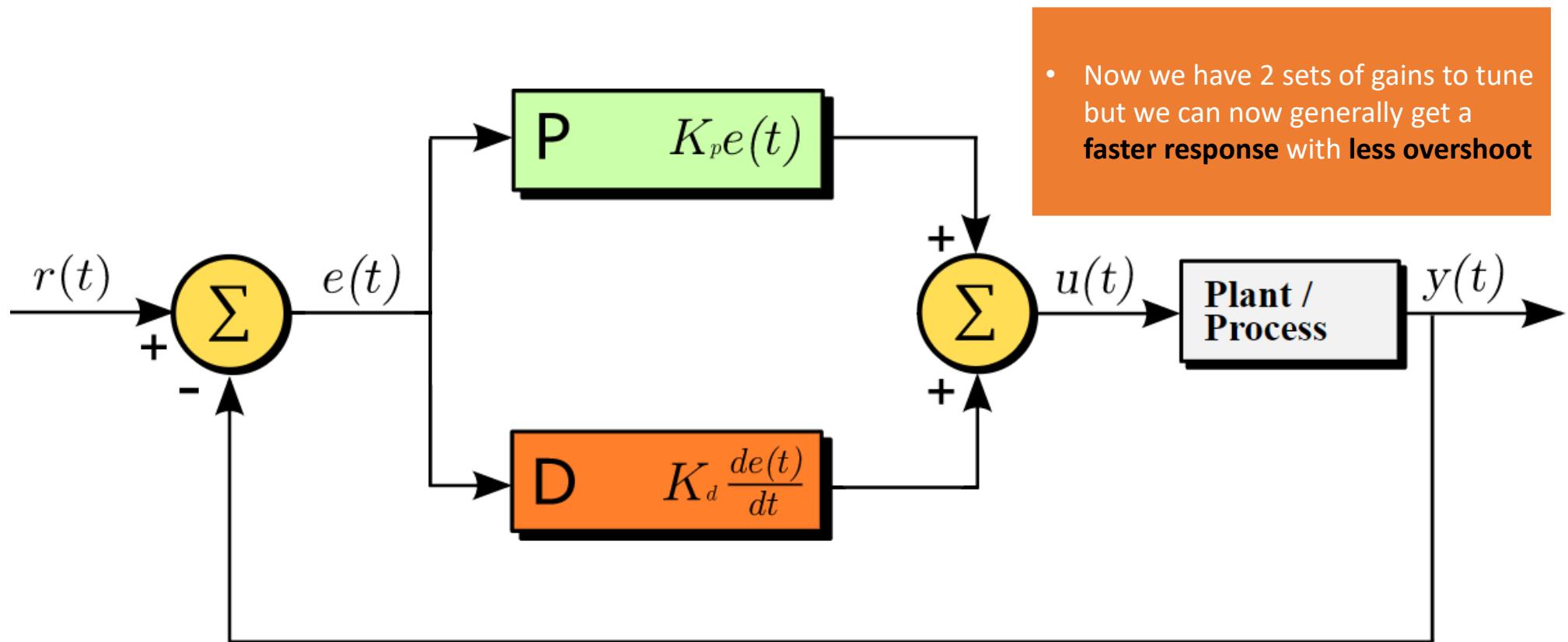
Adapted from Wikipedia



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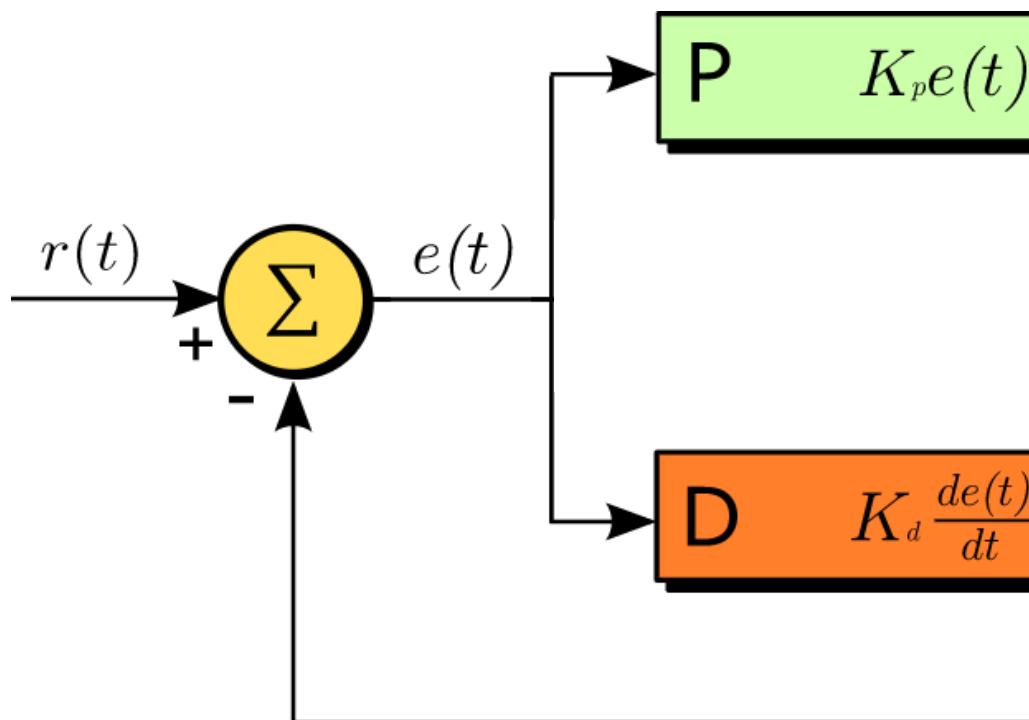
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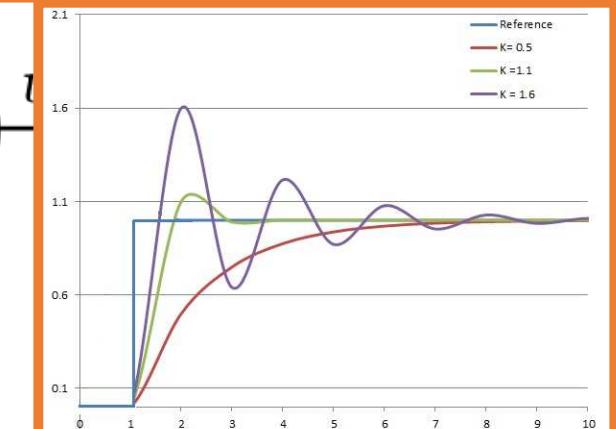
6

So how do we do Feedback Control in practice?

Adapted from Wikipedia



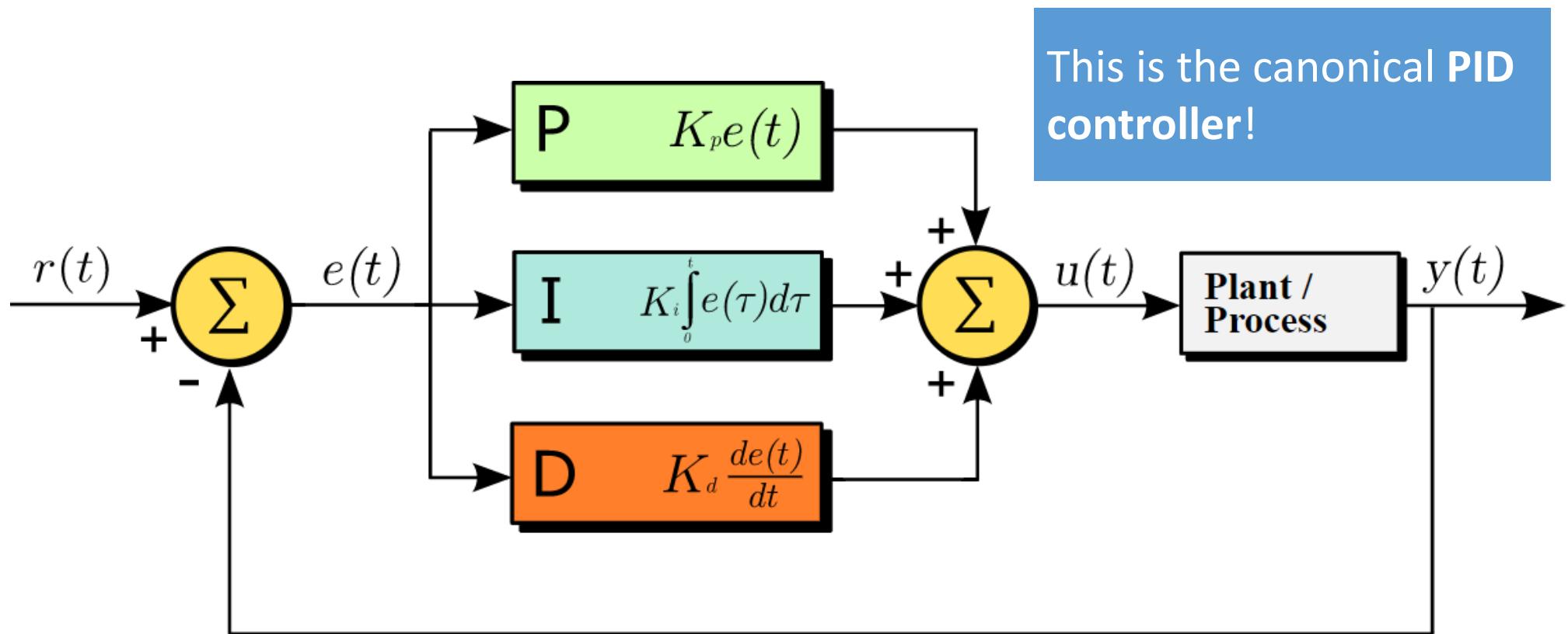
- But what if there is still an error at convergence (aka we want the graph to end at 1.1 exactly)



6

So how do we do Feedback Control in practice?

Adapted from Wikipedia



So how do we do Feedback Control in practice?

Adapted from Wikipedia

Ziegler–Nichols method

From Wikipedia, the free encyclopedia

Main article: PID controller

The **Ziegler–Nichols tuning method** is a heuristic method of tuning a PID controller. It was developed by John G. Ziegler and Nathaniel B. Nichols. It is performed by setting the I (integral) and D (derivative) gains to zero. The "P" (proportional) gain, K_p is then increased (from zero) until it reaches the **ultimate gain** K_u , at which the output of the control loop has stable and consistent oscillations. K_u and the oscillation period T_u are used to set the P, I, and D gains depending on the type of controller used:

Ziegler–Nichols method^[1]

Control Type	K_p	T_i	T_d	K_i	K_d
P	$0.5K_u$	–	–	–	–
PI	$0.45K_u$	$T_u/1.2$	–	$0.54K_u/T_u$	–
PD	$0.8K_u$	–	$T_u/8$	–	$K_u T_u/10$
classic PID ^[2]	$0.6K_u$	$T_u/2$	$T_u/8$	$1.2K_u/T_u$	$3K_u T_u/40$
Pessen Integral Rule ^[2]	$7K_u/10$	$2T_u/5$	$3T_u/20$	$1.75K_u/T_u$	$21K_u T_u/200$
some overshoot ^[2]	$K_u/3$	$T_u/2$	$T_u/3$	$0.666K_u/T_u$	$K_u T_u/9$
no overshoot ^[2]	$K_u/5$	$T_u/2$	$T_u/3$	$(2/5)K_u/T_u$	$K_u T_u/15$

Tuning PID gains is an art and there is a whole literature on a variety of methods to get particular types of response curves!

6

PID controllers work really well in practice



6

Tuning gains is hard and non-intuitive is there a better way?

6

Tuning gains is hard and non-intuitive is there a better way?

Of course there is or I wouldn't
need the transition slide!

6

The LQR Controller

What if instead of specifying gains we can specify a **cost function** we want to achieve...

6

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Maybe something like track the desired state but don't use too much energy to do it?

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The LQR Controller

What if instead of specifying gains we can specify a **cost function** we want to achieve...

Maybe something like track the desired state but don't use too much energy to do it?

Instead of tuning gains we can tune cost weights (Q, R) which are often more intuitive

$$L(x, u) = \underbrace{(x - x_g)^T Q (x - x_g)}_{\text{Deviation of the state from some goal state}} + \underbrace{u^T R u}_{\text{Effort (torque)}}$$

Deviation of the state from some goal state

Effort (torque)

6

The LQR Controller

It turns out if we minimize this quadratic cost over time with a linear model of the dynamics

$$\begin{aligned} \min_{x,u} \sum_{k=0}^N & (x_k - x_g)^T Q (x_k - x_g) + u_k^T R u_k \\ \text{s.t. } & x_{k+1} = Ax_k + Bu_k \end{aligned}$$



There is a closed form solution to the optimal feedback controller!
(Riccati Equation)

$$u_k = -K_k x_k$$

6

The LQR Controller

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$$\min_{x,u} \sum_{k=0}^N (x_k - x_g)^T Q (x_k - x_g) + u_k^T R u_k$$

s.t. $x_{k+1} = Ax_k + Bu_k$

This is used widely in practice!

There is a closed form solution to the optimal feedback controller!
(Riccati Equation)

$$u_k = -K_k x_k$$

6

We can also use LQR in RRT as a better metric of “distance” and the feedback controller as the best “extend”

Finite-horizon, discrete-time LQR [edit]

For a discrete-time linear system described by:^[1]

$$x_{k+1} = Ax_k + Bu_k$$

with a performance index defined as:

$$J = x_N^T Q x_N + \sum_{k=0}^{N-1} (x_k^T Q x_k + u_k^T R u_k + 2x_k^T N u_k)$$

the optimal control sequence minimizing the performance index is given by:

$$u_k = -F_k x_k$$

where:

$$F_k = (R + B^T P_{k+1} B)^{-1} (B^T P_{k+1} A + N^T)$$

Feedback Controller for “Extend”

and P_k is found iteratively backwards in time by the dynamic Riccati equation:

$$P_{k-1} = A^T P_k A - (A^T P_k B + N) (R + B^T P_k B)^{-1} (B^T P_k A + N^T) + Q$$

Cost-to-Go as “Distance Metric”

from terminal condition $P_N = Q$. Note that u_N is not defined, since x is driven to its final state x_N by $Ax_{N-1} + Bu_{N-1}$.

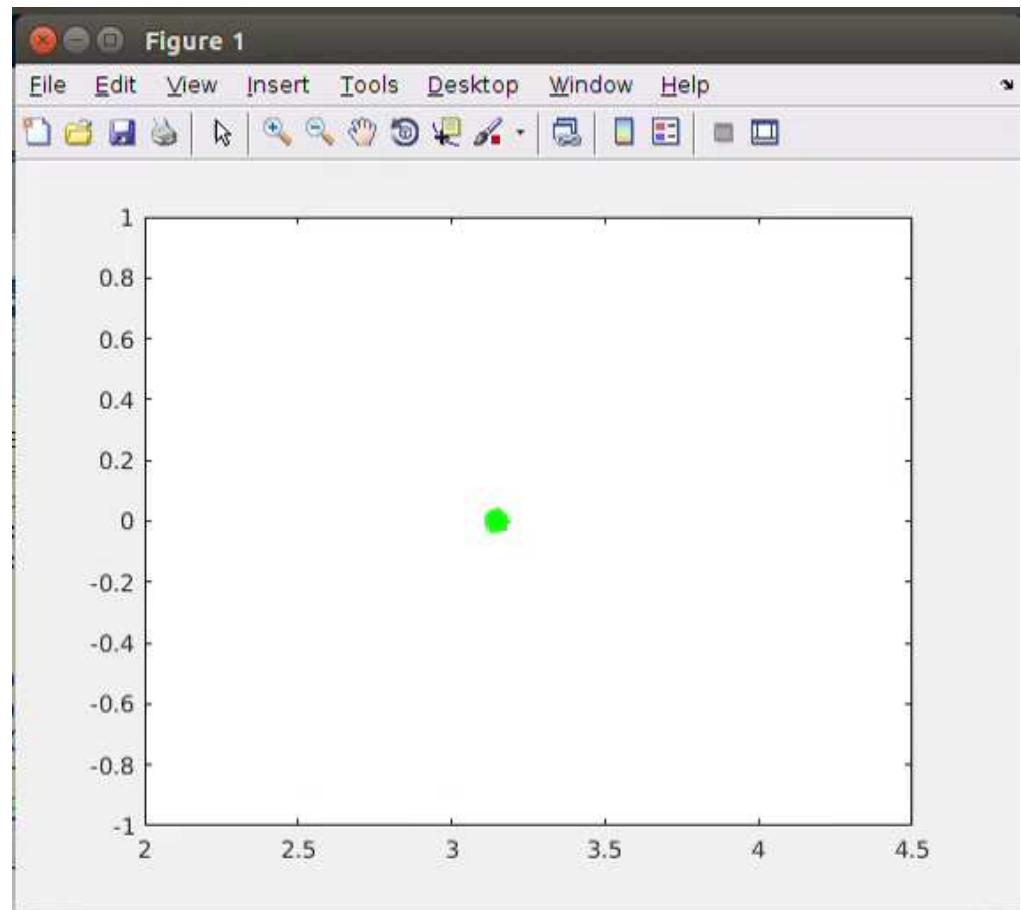
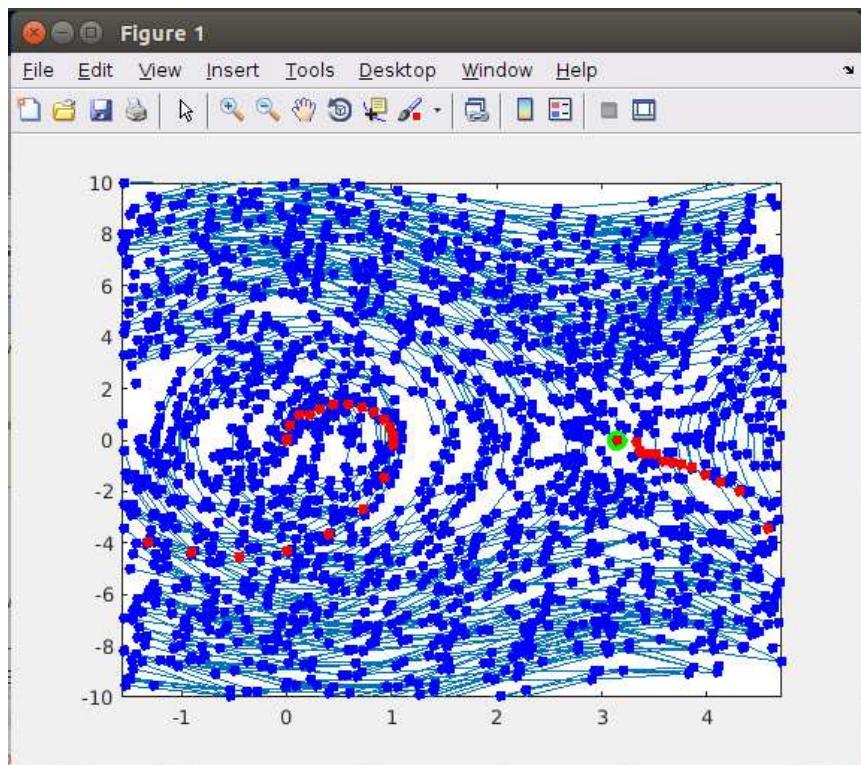
Bellman Updates

$$V_N(x_N) = c(x_N, u_N)$$

$$V_{k+1}(x) = \min_a c(x, u) + V_k(f(x, u))$$

6

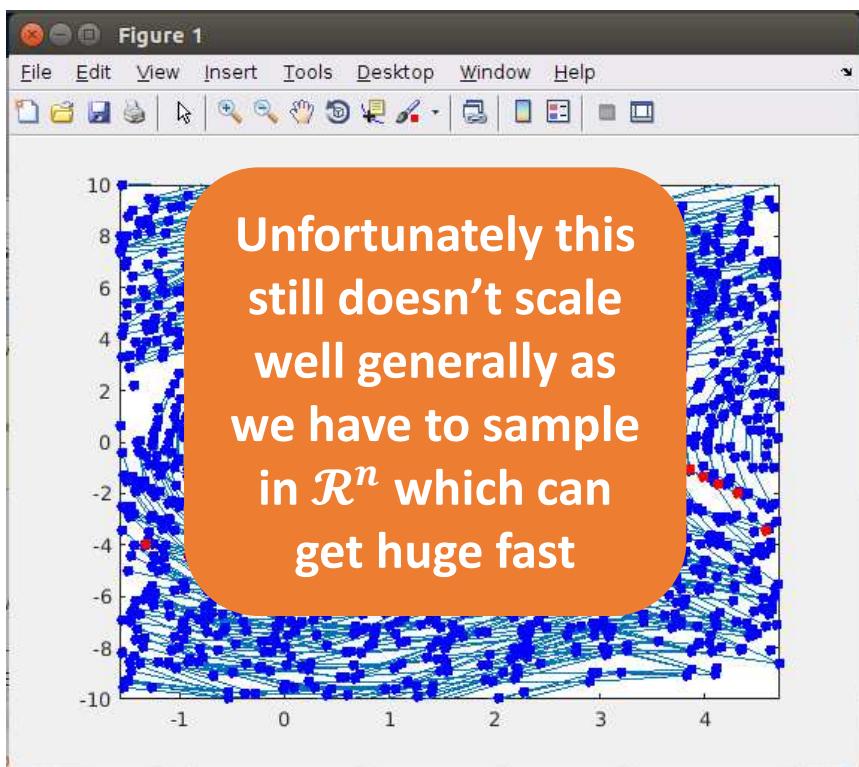
We can also use LQR in RRT as a better metric of “distance” and the feedback controller as the best “extend”



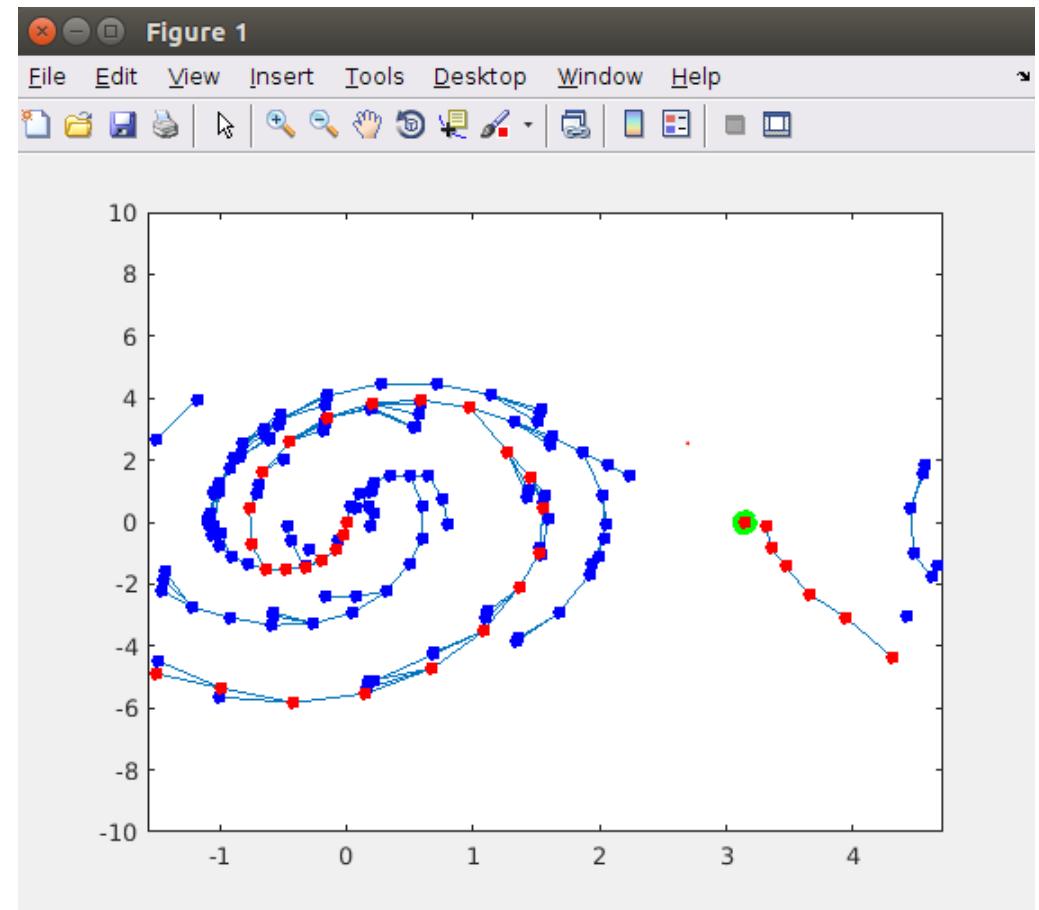
[Perez et. al. LQR-RRT*]

6

We can use LQR in RRT as a better metric of “distance” and the feedback controller as the best “extend”



[Perez et. al. LQR-RRT*]



6

So what have we learned so far?

1. Real world autonomous systems need to use **Feedback Control**
2. **PID** controllers are simple and effective but require **gain tuning**
3. **LQR** controllers allow for **cost function design** instead
4. PID and LQR require a plan to already exist and are simply **tracking controllers**

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- But what happens if we **deviate** so much from our original plan that it is **no longer valid**? How do we initiate **re-plans**?

6

So what have we learned so far?

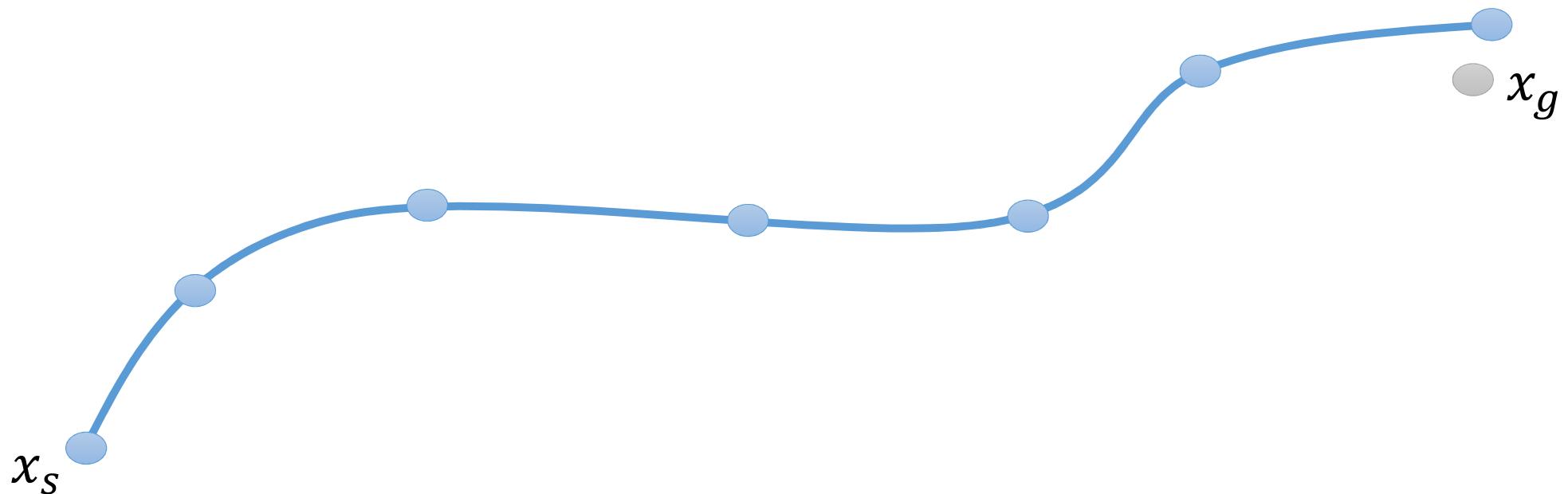
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- But what happens if we **deviate** so much from our original plan that it is **no longer valid**? How do we initiate **re-plans**?

This is an open unsolved problem!

6

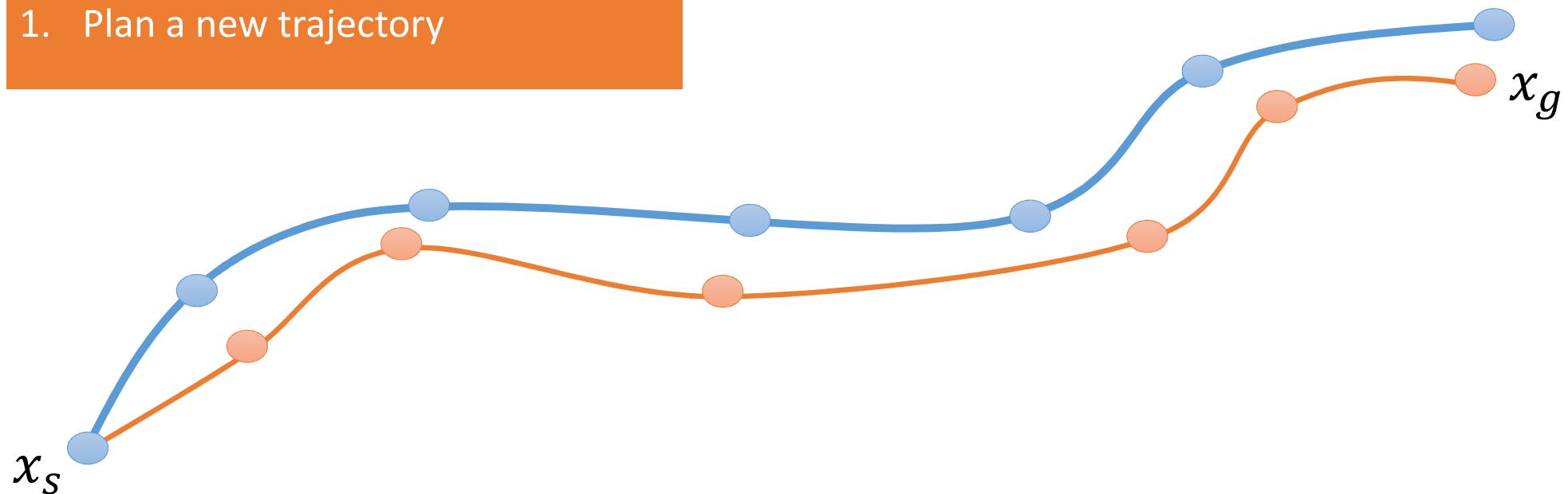
Model Predictive Control: re-planning fast enough that the plan becomes the controller!



6

Model Predictive Control: re-planning fast enough that the plan becomes the controller!

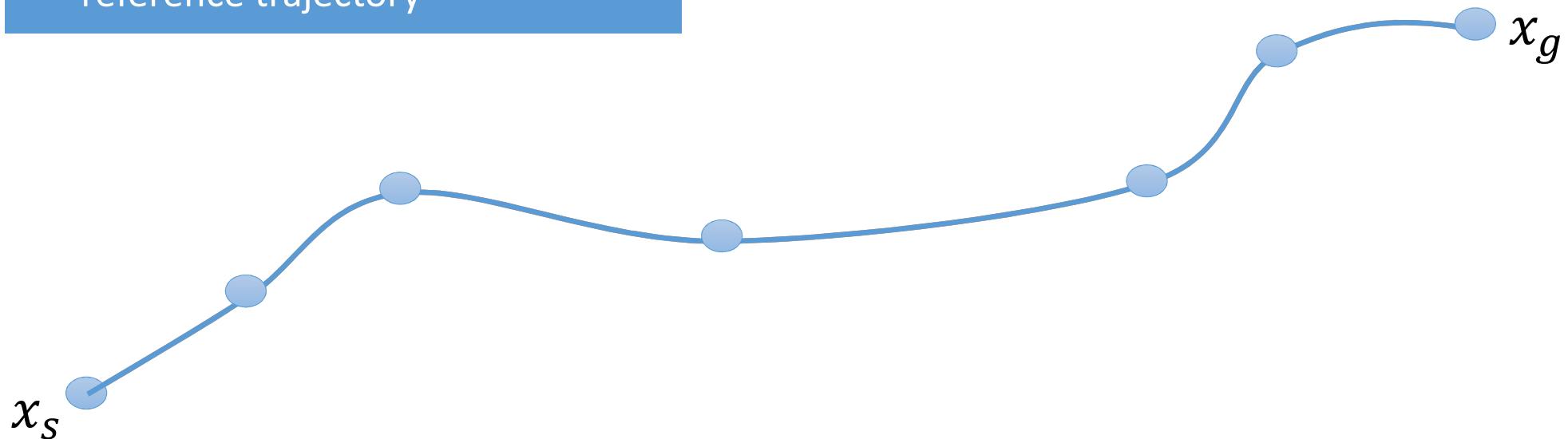
1. Plan a new trajectory



6

Model Predictive Control (MPC): re-planning fast enough that the plan becomes the controller!

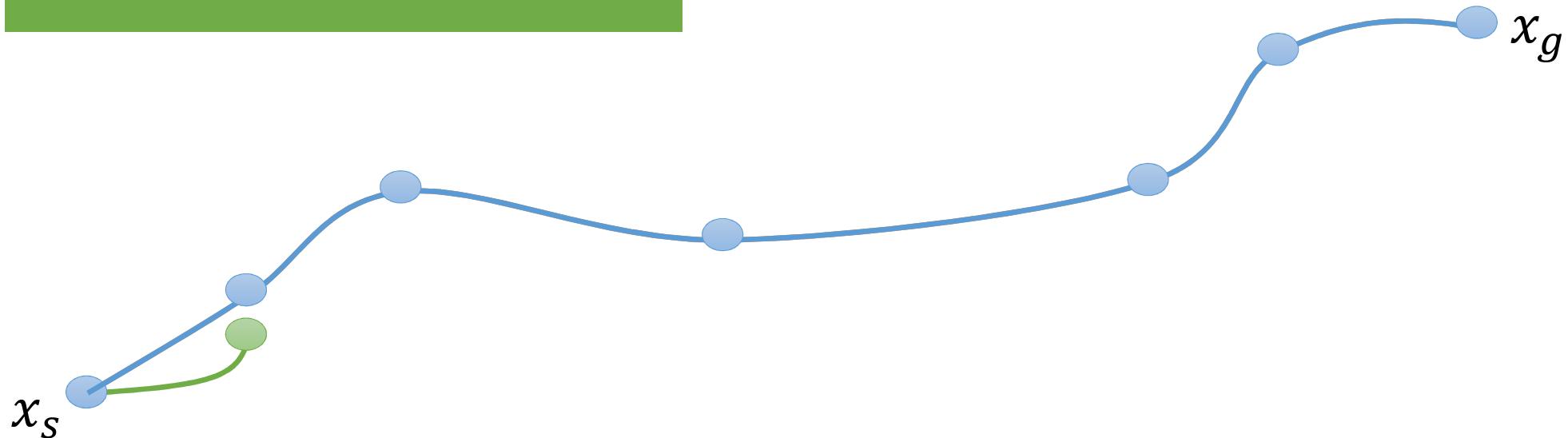
2. The new plan becomes the reference trajectory



6

Model Predictive Control (MPC): re-planning fast enough that the plan becomes the controller!

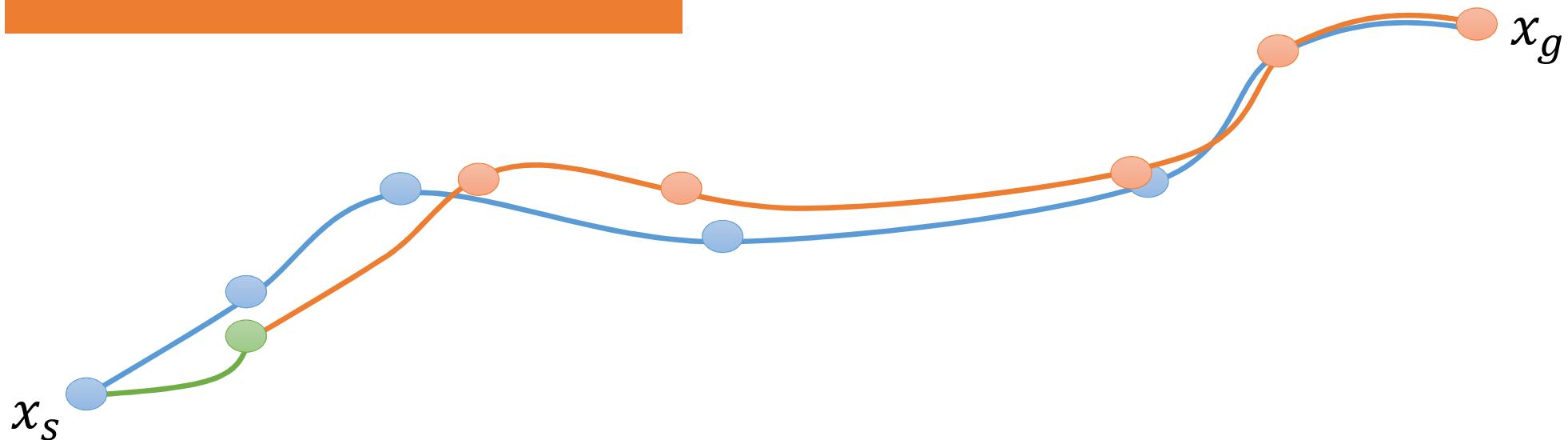
3. Execute the first step of the plan



6

Model Predictive Control (MPC): re-planning fast enough that the plan becomes the controller!

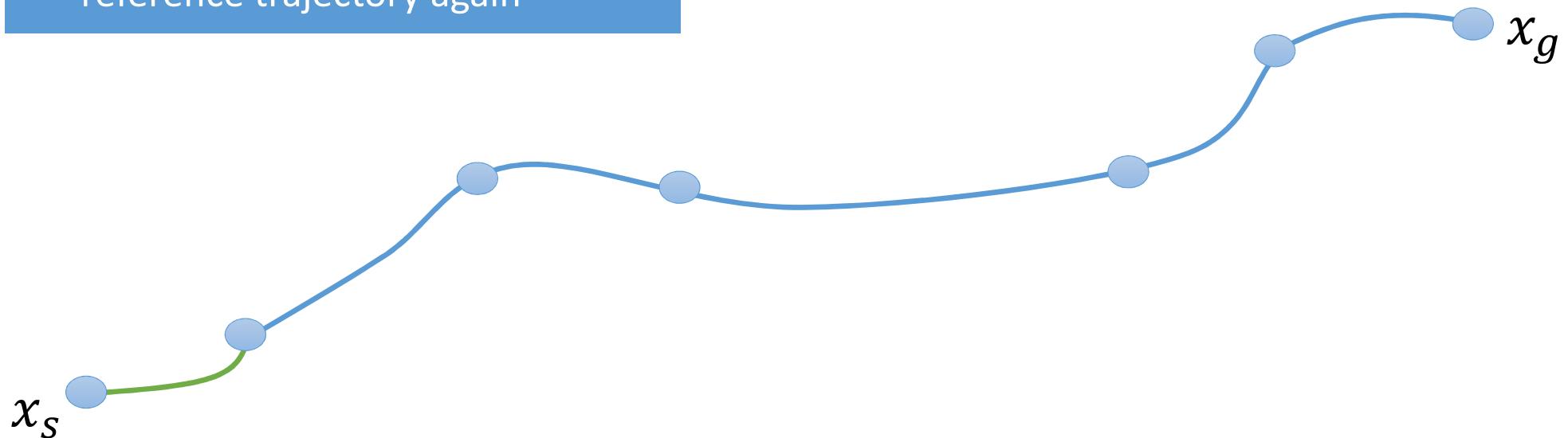
1. Re-plan based on that step



6

Model Predictive Control (MPC): re-planning fast enough that the plan becomes the controller!

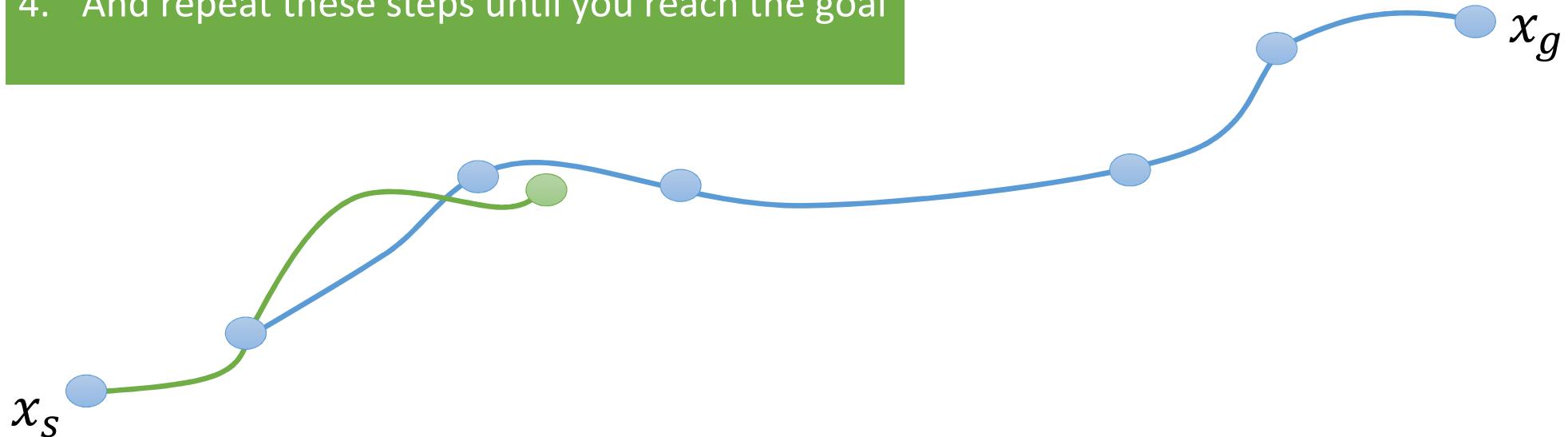
2. The new plan becomes the reference trajectory again



6

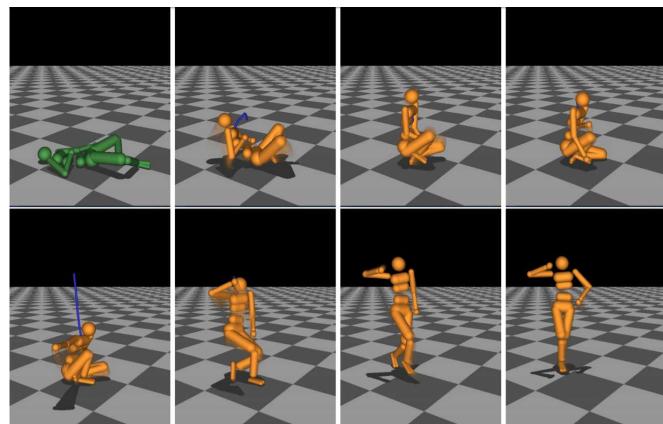
Model Predictive Control (MPC): re-planning fast enough that the plan becomes the controller!

- 3. Execute the first step of the new plan again
- 4. And repeat these steps until you reach the goal

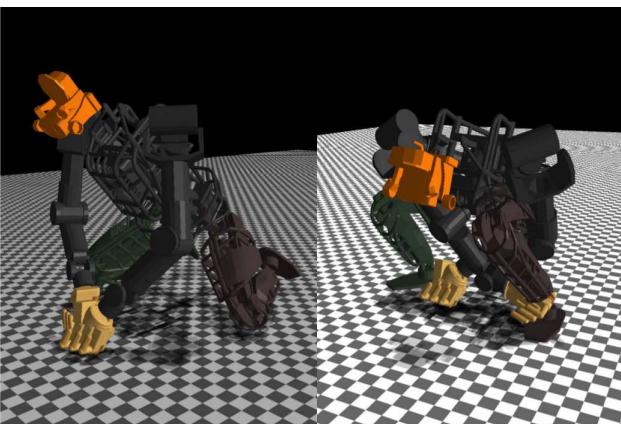


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Recently MPC has been used in a variety of complex autonomous systems in simulation and on physical robots



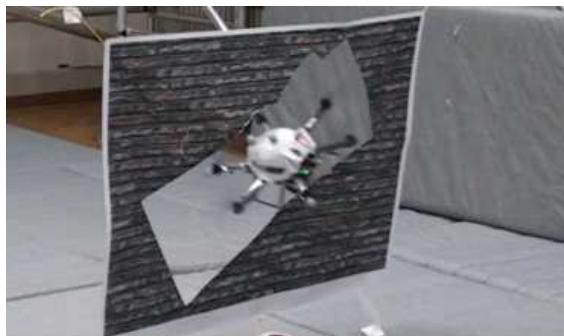
[Tassa et. al. IROS 2012]



[Erez et. al. Humanoids 2013]



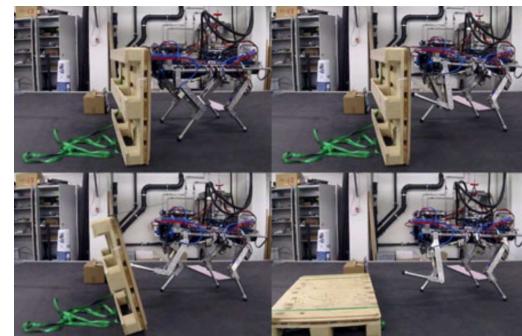
[Koenemann et. al. IROS 2015]



[Neunert et. al. ICRA 2016]



[Neunert et. al. Humanoids 2017]

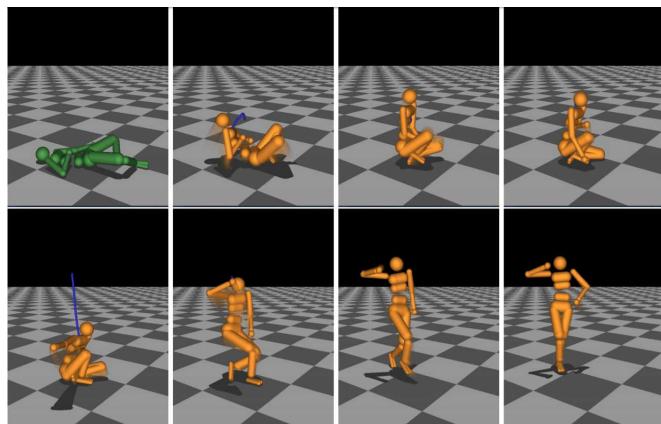


[Farshidian et. al. IEEE RAL 2017]

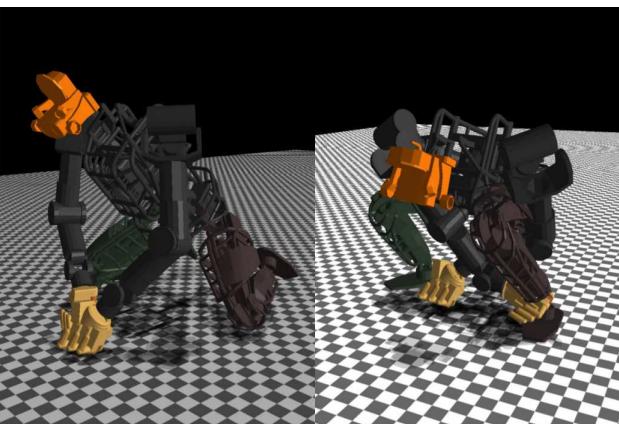


[Plancher et. al. WAFR 2018]
[Plancher et. al. ICRA 2019]

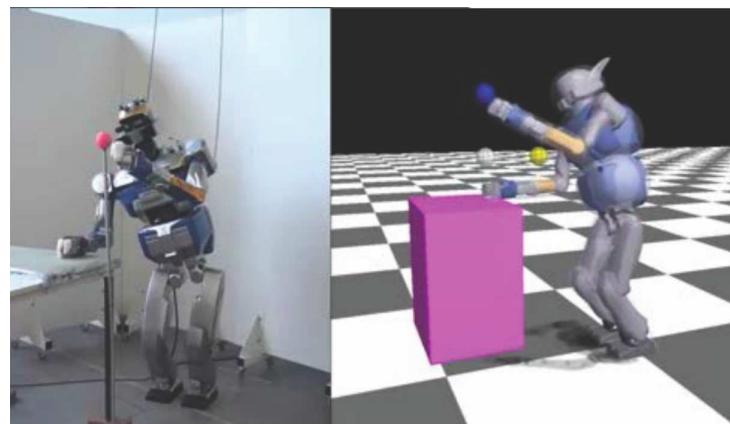
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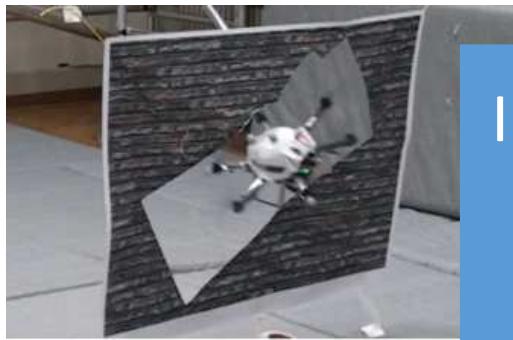
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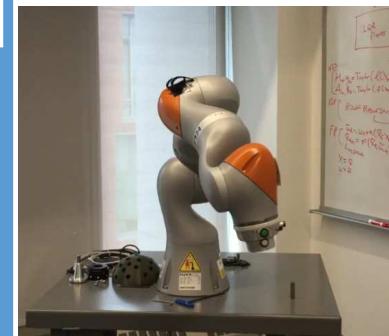


[Neunert et. al. ICRA 2016]

I will go into far more detail on this when
I present my recent work during the
sample paper presentations!

[Neunert et. al. Humanoids 2017]

[Farshidian et. al. IEEE RAL 2017]



[Plancher et. al. WAFR 2018]

[Plancher et. al. ICRA 2019]

6

Practical Challenges for Control: Contact

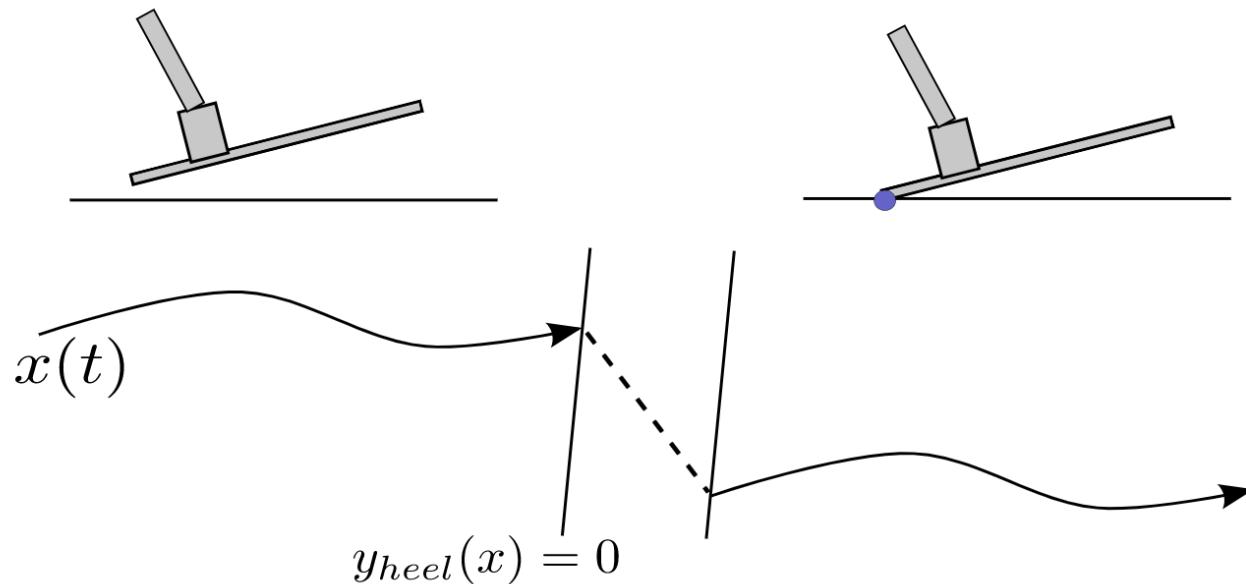


Figure 17.1 - Modeling contact as a hybrid system.

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Practical Challenges for Control: Contact

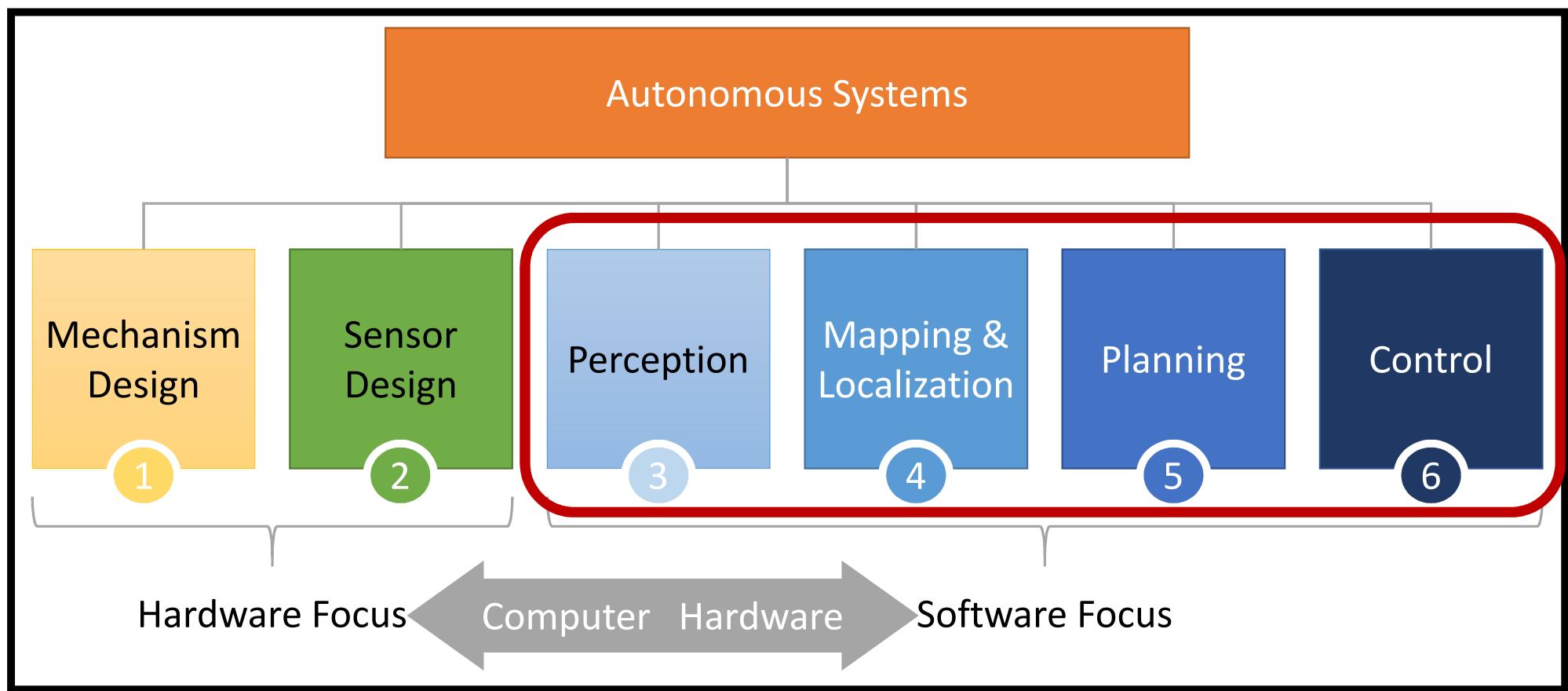


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Key Takeaways:

1. Real world autonomous systems need to use **Feedback Control**
 2. **Tracking controllers** allow for simple control design and are quite effective in practice. Two common controllers are:
 1. PID with gain tuning
 2. LQR with cost function design
 3. Using **MPC** allows for the planner to be the controller which enables more **sophisticated control strategies**
 4. Contact is really hard!
-

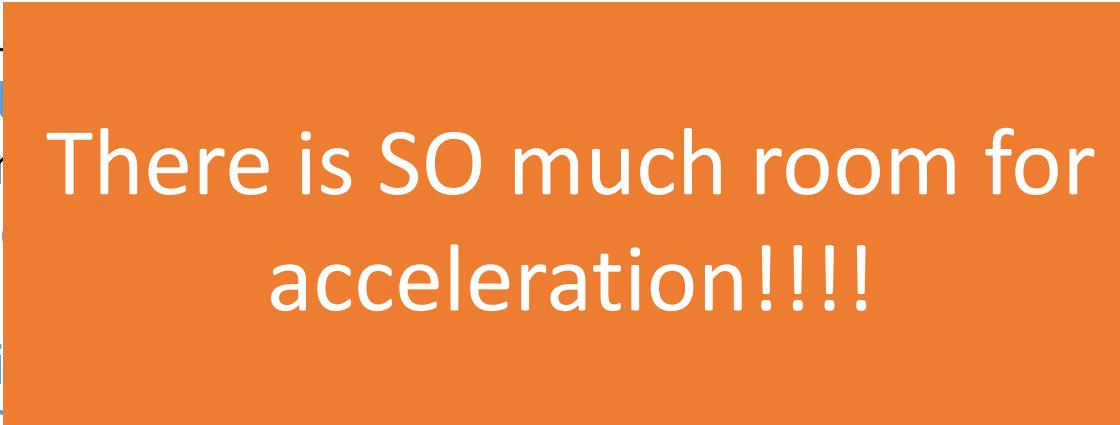
Autonomous Systems / Robotics is a BIG space



Key Takeaways:

1. NNs running on **accelerator chips** solve most perception problems
 2. The **Kalman/Particle Filter** uses probability to solve the localization problem but **modeling and/or approximations** are needed to run online
 3. Mapping quickly becomes a **memory storage problem**
 4. **Stereo Depth** and **Visual Odometry** also need acceleration to run online
 5. Robot planning involves both **task and configuration spaces**
 6. **Collision checking** can be expensive
 7. Sample Based Planners (**PRM, RRT, RRT***) leverage random search and are **probabilistically complete** but do not scale well to high dimensions
 8. Trajectory Optimization finds **locally optimal** paths but is **not complete or robust** and (often) solved with (slow) **off the shelf solvers**
 9. Tracking controllers (**PID, LQR**) work well in practice but **MPC** is a much more powerful (and computationally expensive) approach
 10. **Contact is hard** and we (sometimes) use **simpler models** for tractability
-

Key Takeaways:

1. NNs running on **accelerator chips** solve most perception problems
 2. The **Kalman/Particle Filter** uses probability to solve the localization problem but **modeling and/or approximations** are needed to run online
 3. Mapping qu...
 4. **Stereo Depth** is used to run online
 5. Robot planning is used to run online
 6. **Collision checks** are used to run online
 7. Sample Based Planning is used to search and plan in high dimensions
 8. Trajectory Optimization finds **locally optimal** paths but is **not complete or robust** and (often) solved with (slow) **off the shelf solvers**
 9. Tracking controllers (**PID, LQR**) work well in practice but **MPC** is a much more powerful (and computationally expensive) approach
 10. Contact is hard and we (sometimes) use **simpler models** for tractability
- 
- There is SO much room for acceleration!!!!

And that's everything!

<http://bit.ly/CS249-Feedback-L2>

