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AC290r: Extreme Computing: Project-based High Performance Distributed and Parallel Systems

Numerical Simulation of Rayleigh–Bénard Convection

Yue Sun[1], Michael S. Emanuel[1], Jonathan Guillotte-Blouin[1]
[1]Harvard University School of Engineering and Applied Sciences

ABSTRACT

We present a large-scale numerical simulation of a two dimensional turbulent Rayleigh-Bénard convection using the Drekar code, a parallelized finite element fluid solver from Sandia National Labs, on Harvard's Odyssey supercomputing cluster. We postprocess the parallel outputs and visualize the temperature and velocity fields, all exhibiting the turbulent motion of the flow. We also implement Schlieren flow visualization as a bridge between painting and fluid dynamics.

INTRODUCTION

Thermal convection is a prevalent natural phenomenon in our universe. Anytime a fluid is in a gravitational field with a temperature gradient, thermal convection will happen. A flow configuration called Rayleigh-Bénard convection is used to study heat transport in a gravitational field.

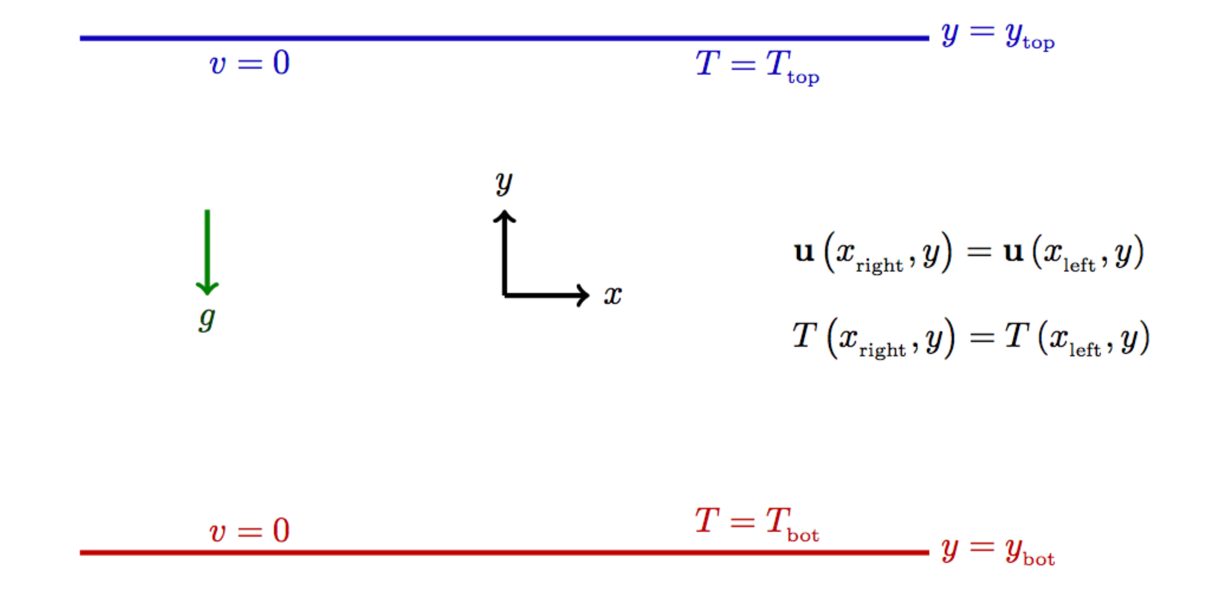

Figure 1. A schematic of two-dimensional Rayleigh-Bénard convection with no-slip and periodic boundaries. An infinite slab of fluid confined between two parallel plates with gravity acting in the downward direction. Top plate has lower temperature, and bottom plate higher temperature.

When the temperature difference between the top and bottom plates, $\Delta T = T_{bot} - T_{top}$, is large enough, fluid motion will occur bringing hot fluid up and cold fluid down. If the thermal forces become large enough, the fluid motion will become turbulent. This deceptively simple configuration requires immense computational resources to simulate accurately.

GOVERNING EQUATIONS

We simulate two-dimensional turbulent Rayleigh-Bénard convection based on the Boussinesq equations:

$$\frac{\partial \vec{u}}{\partial t} + \nabla \cdot (\vec{u} \otimes \vec{u}) = \nabla P + \mathrm{Pr} \nabla^2 \vec{u} + \mathrm{RaPr} T \hat{y}$$
$$\nabla \cdot \vec{u} = 0$$
$$\frac{\partial T}{\partial t} + \nabla \cdot (\vec{u} T) = \nabla^2 T$$

$\vec{u}$ is the velocity and T is the temperature. The two relevant non-dimensional numbers are:

The Prandtl number,
$$\mathrm{Pr} = \frac{\nu}{\kappa}$$
which is the ratio between the thermal viscosity and the thermal diffusivity.

The Rayleigh number,
$$\mathrm{Ra} = \frac{\alpha_V \Delta T g H^3}{\nu \kappa}$$
describes the ratio of buoyancy forces to viscous forces. Large Ra leads to turbulent flows.

APPROACH

DREKAR: CODE DESCRIPTION

- Massively parallel fluid simulation code (C++)
  - Finite element methods
  - Linear/nonlinear solvers, preconditioners, eigenvalue solvers, time integrators and automatic differentiation
- Developed at Sandia National Laboratories
- Support compressible fluids, incompressible fluids, low-mach compressible fluids, compressible and incompressible magnetohydrodynamics simulations

ODYSSEY: PLATFORM DESCRIPTION

- Harvard's largest heterogeneous clusters
  - 2,000+ nodes, 78,000+ CPU cores, 1M+ CUDA cores
  - 35PB+ storage, 250TB+ RAM
- Support by FASRC
- Core operating system is CentOS
- Utilize SLURM for the scheduling of compute jobs
- Occupy more than 10,000 square feet with 190 racks spanning three data centers separated by 100 miles

SET INPUTS

	Mesh (.pam)	Physics (.xml)
Domain size	$L_x = 2$ $L_y = 1$	$\mathrm{Ra} = 10^{10}$
Problem size	$N_x = 2049$ $N_y = 1025$	$\mathrm{Pr} = 1$
Degree of freedom: 8400900		

BUILD AND RUN

- Install first Trilinos then Drekar on Odyssey
- Create a shell script (.sh) to define SLURM flags for running batch jobs
- Submit jobs

OUTPUTS

#Core (CPU)	Run Time	Simulation Time	Data Size
512	~ 1 day	0.22 s	22.7GB
1024	~ 4 day	0.77 s	76GB
Case 2 output was corrupted. Case 1 output is used for analysis.			

RESULTS

The output data are Exodus files, we combine all parallel outputs and use `NetCDF` to extract the fields. We show velocity streamlines and temperature contours, postprocessed using `ParaView` and `matplotlib`. We also use Schlieren visualization to examine the flow structure in the temperature field,

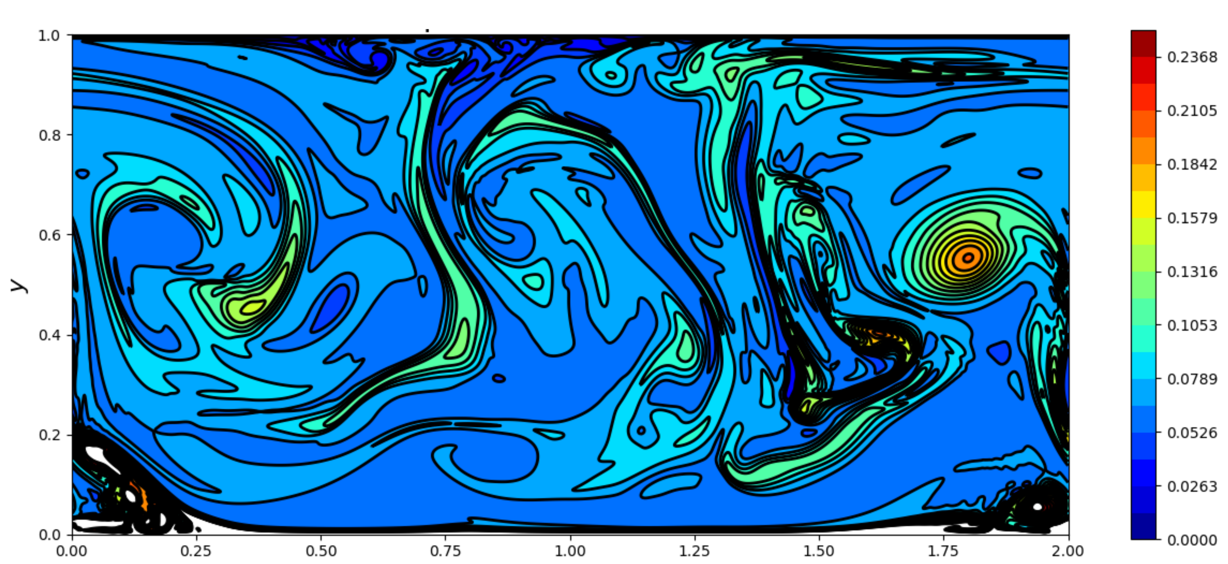

Figure 2. Temperature contour plot at $t = 0.189$, which provides direct information of the temperature distribution. Eddies and swirls structures indicate the turbulent motion of flow field.

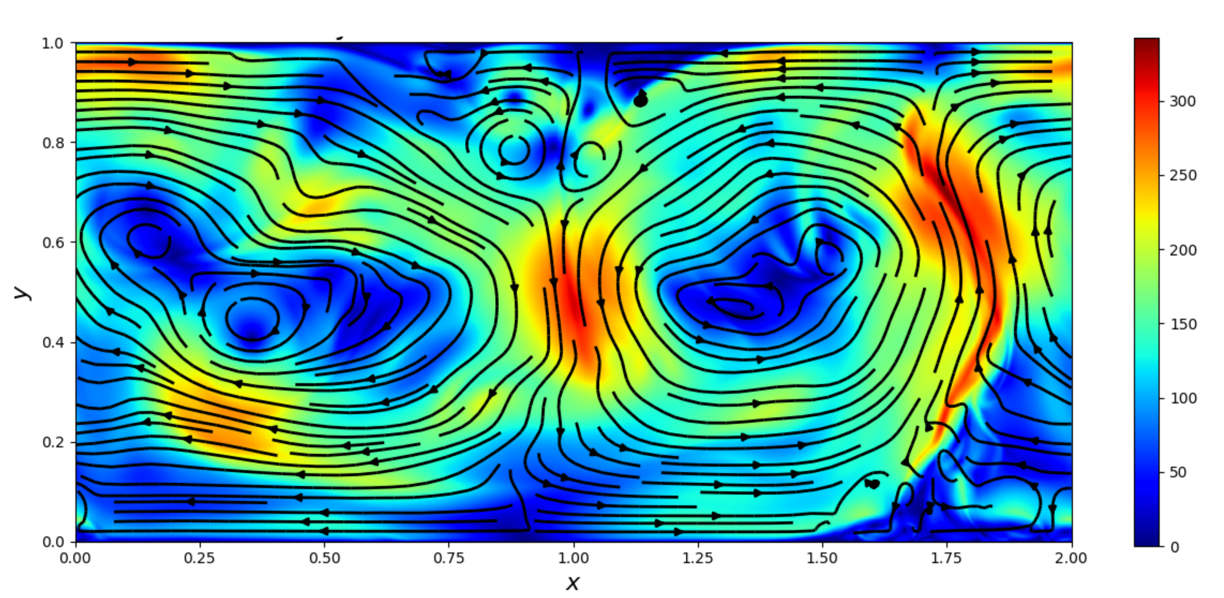

Figure 3. Velocity plot and streamlines at $t = 0.112$. The color indicates the magnitude of the fluid velocity, and the arrow on the black streamlines represents the direction of the flow.

$$Sch = \exp\left(-k \frac{|\nabla T|}{\max |\nabla T|}\right).$$

We test various color maps, and reference the color palettes of three famous painting which involve water or flowing strokes.

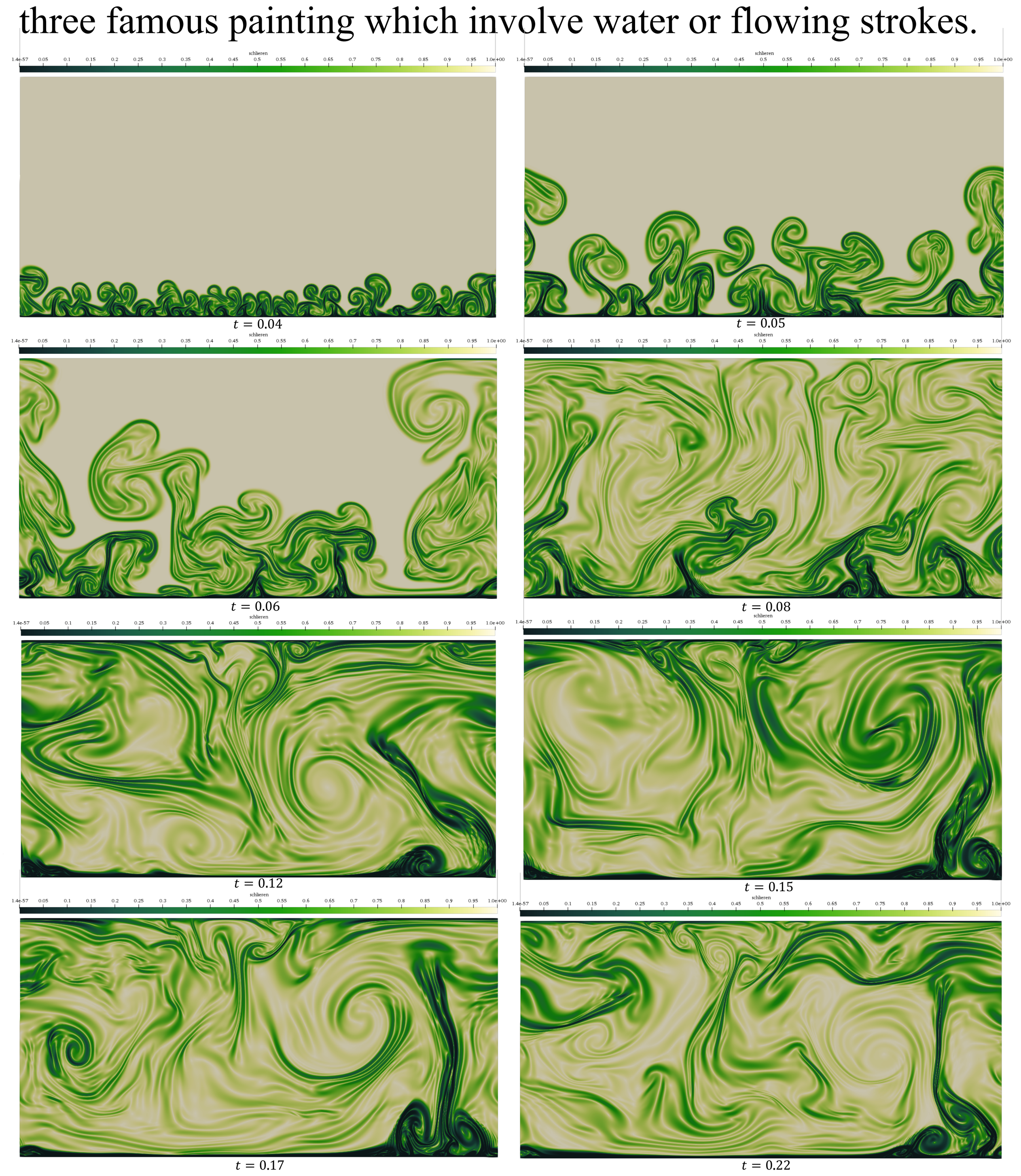

Figure 4. Schlieren flow visualizations of the temperature field show turbulence development.

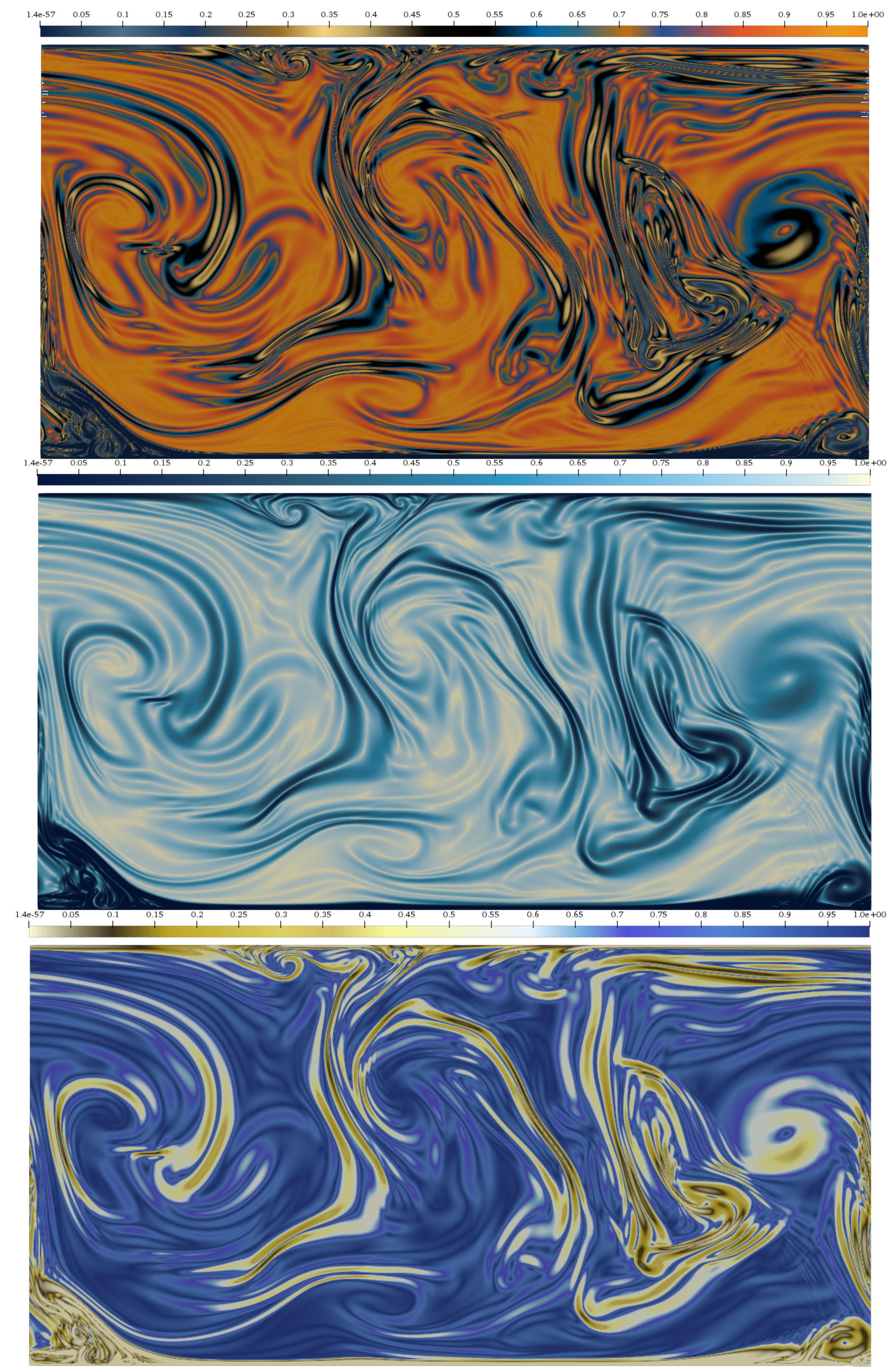

Figure 5. Schlieren flow visualizations of the temperature field at $t = 0.189$ with color palettes from *The Scream* (Munch), *The Great Wave* (Hokusai) and *Starry Night* (Van Gogh).

CONCLUSION

Extreme scale computing is a rich field with extensive capacity in driving large-scale fluid simulation. Our first brief foray allows us to examine how turbulent flow develops in Rayleigh-Bénard convection. For future work we aim to run longer simulation at a higher Rayleigh number, and implement painterly render to create more aesthetically vivid flow patterns.

CITATIONS AND LINKS

Seefeldt, Ben, Sondak, David, Hensinger, David M., Phipps, Eric T., Foucar, James G., Pawlowski, Roger P., Cyr, Eric Christopher, Shadid, John N., Smith, Thomas M., Weber, Paula D., Mabuza, Sibusiso, Bradley, Andrew Michael, Wildey, Timothy Michael, Von Winckel, Gregory John, Kramer, Richard Michael Jack, Phillips, Edward Geoffrey, Bettencourt, Matthew Tyler, Miller, Sean, Lin, Paul, Gates, Jason Matthew, Eillis, Truman Everett, and Conde, Sidafa. *Drekar v.2.0.* Computer software. *https://www.osti.gov//servlets/purl/1364765*. Vers. 00. USDOE. 5 Jun. 2017. Web.

Schoof, L. A., & Yarberry, V. R. (1994). EXODUS II: a finite element data model (No. SAND-92-2137). Sandia National Laboratories.

DRAKE, R., FOUCAR, J., HENSINGER, D., WONG, M., BUDGE, K., & GARDINER, T. (2008). Pamgen (No. PAMGEN; 002393WKSTN00). Sandia National Laboratories.

Paintings Color Palettes:
https://designshack.net/articles/inspiration/10http:/designshack.net/articles/inspiration/10-free-color-palettes-from-10-famous-paintings/