Kepler’s Sieve

Learning Asteroid Orbits from Telescopic Observations

Masters of Data Science Thesis

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Acknowledgments

- Advisor: Pavlos Protopapas
- Secondary Advisor: Chris Rycroft

Introduction

The Asteroid Search Problem

- Many asteroids (about 958,000 known) in the Solar System
- We want to learn their orbits
- Biggest data source: telescope detections
- Easy once you know which detection matches which asteroid
- This is like a jigsaw puzzle with millions of pieces!

Combining Tracklets vs. Orbital Element Search

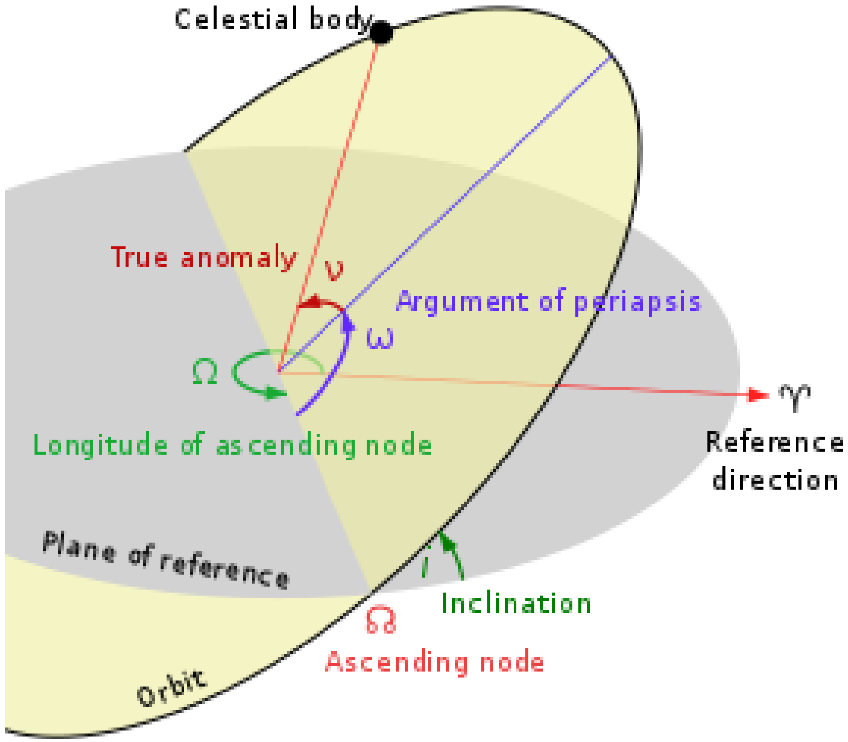
- Tracklet: two detections close to each other in time and direction
- Existing search methods: greedy search over tracklets
  - Try to extrapolate a tracklet to find additional detections
  - Attempt to fit an orbit when you have enough tracklets
- Drawbacks
  - Myopic – can only connect detections made close in time
  - Suffers from combinatorial explosion
- Proposed novel method: search Orbital Elements
  - 6D space; large, but scales well
  - Cost scales as $N_{\mathrm{ast}} \cdot N_{\mathrm{obs}}$ rather than N_{obs}^r
  - But can we make it work?

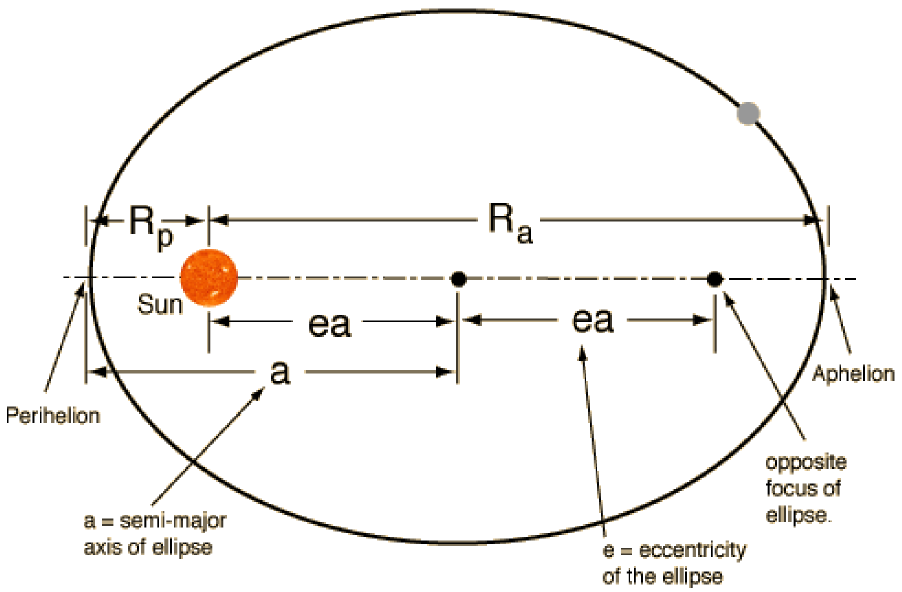
Integrating the Solar System

REBOUND Integrator for N-Body Problem

- REBOUND is a modern, open source integrator
  - github.com/hannorein/rebound
- It numerically solves the gravitational N-body problem
- Considered the “gold standard” for orbits in this thesis
- IAS15 adaptive integrator uses Gauss-Radau quadrature and a “predictor-corrector” scheme
- Horizons: API provided by NASA JPL to obtain state vectors (position and velocity) of objects in the Solar System

Keplerian Orbital Elements

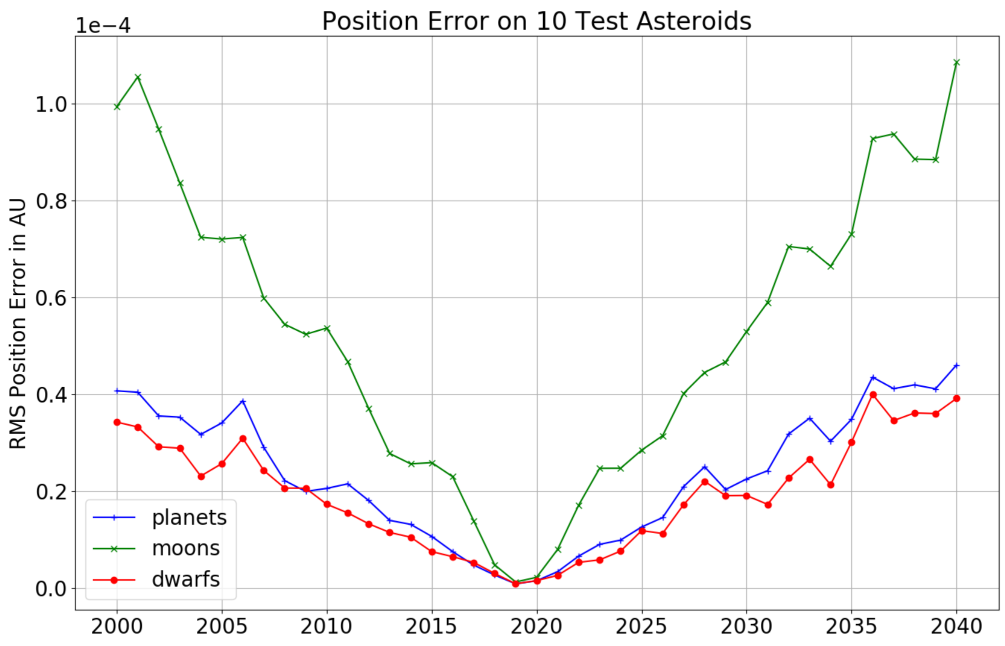


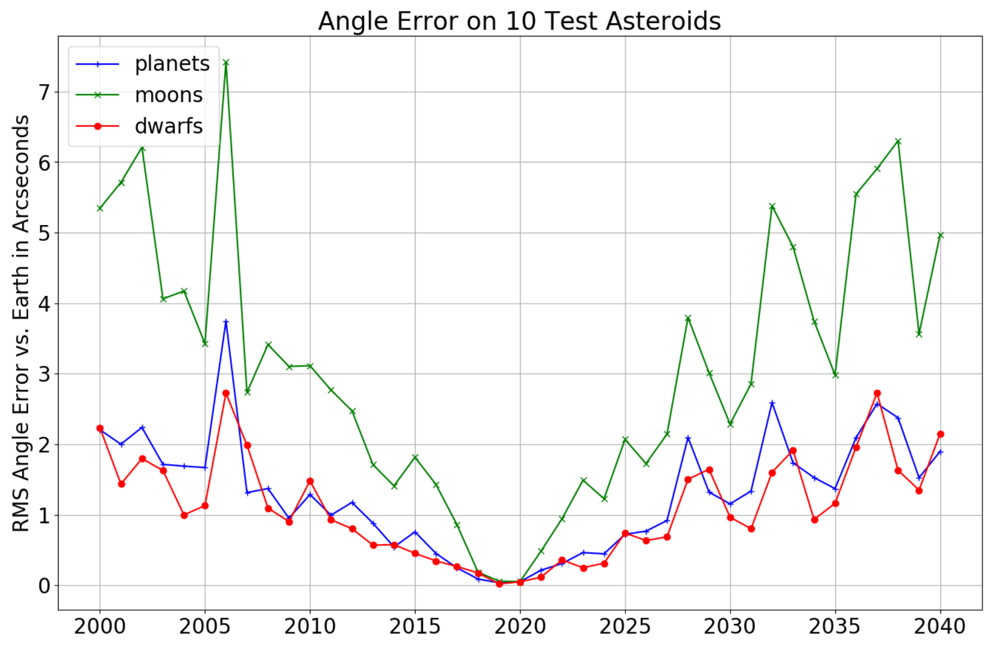


$R_a = a(1+e)$ $R_p = a(1- e)$

- Semi-major axis *a* and eccentricity *e* describe the size and shape of the orbital ellipse
- Inclination *i*, ascending node Ω, perihelion ω are angles orienting orbit in the ecliptic plane
- True anomaly *f* is location of the body on its orbital ellipse

Validating Integration vs. Horizons





- Integrate three collections of massive bodies for 40 years at daily interval
- Initial conditions from Horizons at MJD 58600 / 2019-04-27
- Test results on first 10 IAU asteroids; query their positions from Horizons
- Report error in position (AU) and instantaneous angle from asteroid to Earth (arc seconds)
- Accuracy is excellent!
  - RMS error on planets is 5.4E-6 AU
  - Angle error from asteroids to planets 0.8 arc seconds

Bulk Integration of 733,489 Asteroids

- Download asteroid orbital elements from JPL
- Data available for 733,489
- Integrate these daily for 40 years
- Save results to disk
    - REBOUND simulation archives
    - Numpy arrays
- Job takes 4:30 on 40 CPU cores
- Writes 1.37 TB output to disk

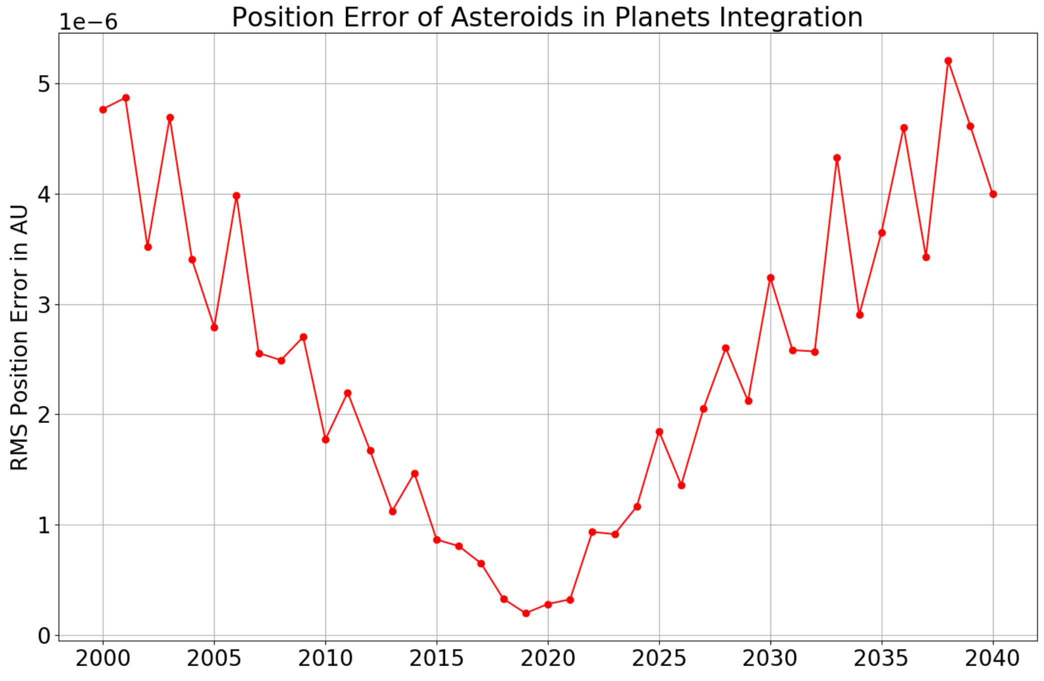
```
# Load all the asteroid elements
ast_elt = load_ast_elt()
```

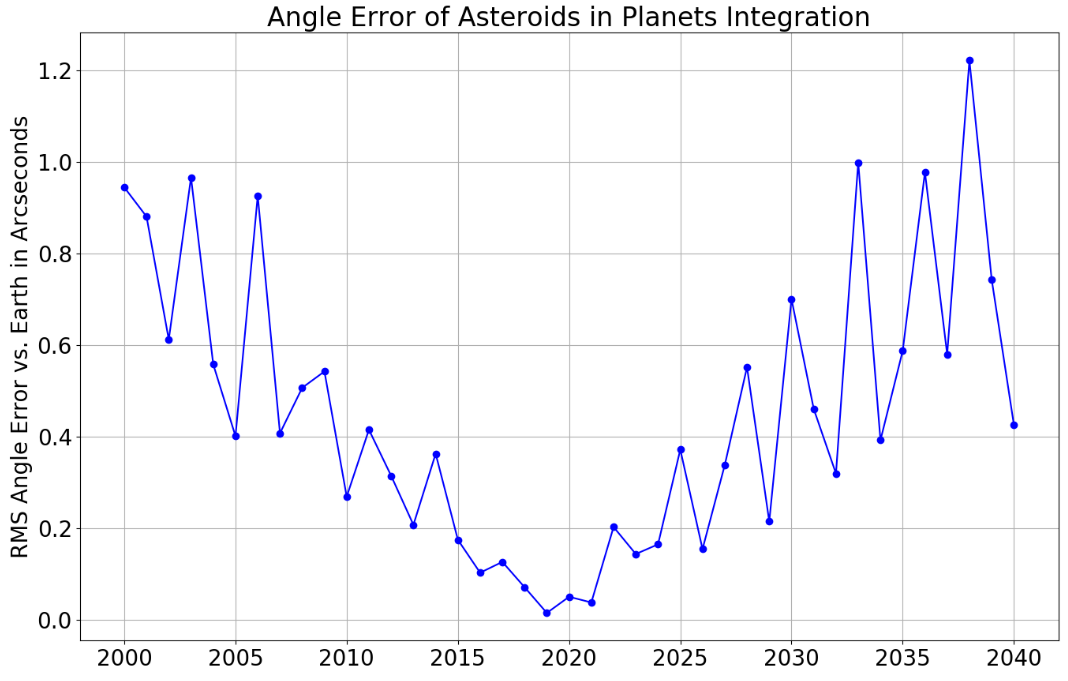
```
ast_elt
```

Num	Num	Name	epoch	a	e	inc	Omega	omega	M	H	G	Ref	f
1	1	Ceres	58600.0	2.769165	0.076009	0.184901	1.401596	1.284522	1.350398	3.34	0.12	JPL 46	1.501306
2	2	Pallas	58600.0	2.772466	0.230337	0.608007	3.020817	5.411373	1.041946	4.13	0.11	JPL 35	1.490912
3	3	Juno	58600.0	2.669150	0.256942	0.226699	2.964490	4.330836	0.609557	5.33	0.32	JPL 108	0.996719
4	4	Vesta	58600.0	2.361418	0.088721	0.124647	1.811840	2.630709	1.673106	3.20	0.32	JPL 34	-4.436417
5	5	Astraea	58600.0	2.574249	0.191095	0.093672	2.470978	6.260280	4.928221	6.85	0.15	JPL 108	-1.738676
...	...	...	...	...	...	...	...	...	...	...	...	...	...
1255499	1255499	2019 QG	58600.0	0.822197	0.237862	0.220677	5.066979	3.770460	0.503214	21.55	0.15	JPL 1	0.807024
1255501	1255501	2019 QL	58600.0	2.722045	0.530676	0.113833	4.741919	2.351059	5.297173	19.21	0.15	JPL 1	-2.082964
1255502	1255502	2019 QQ	58600.0	1.053137	0.389091	0.172121	5.648270	2.028352	3.266522	25.31	0.15	JPL 1	-3.081905
1255513	1255513	6331 P-L	58600.0	2.334803	0.282830	0.141058	6.200287	0.091869	2.609695	18.50	0.15	JPL 8	2.827595
1255514	1255514	6344 P-L	58600.0	2.812944	0.664688	0.081955	3.199363	4.094863	2.738525	20.40	0.15	JPL 17	3.032066

733489 rows × 19 columns

Validate Asteroid Integration vs. Horizons





- Test bulk asteroid integration on first 25 IAU asteroids
- Report position error in AU and angle error to Earth in arc seconds
- Excellent results! RMS 2.49E-6 AU and 0.45 arc seconds

Integrate Kepler Two Body Problem in TensorFlow

$$r(\theta) = \frac{a \cdot (1 - e^2)}{1 - e \cdot \cos(\theta - \theta_0)}$$

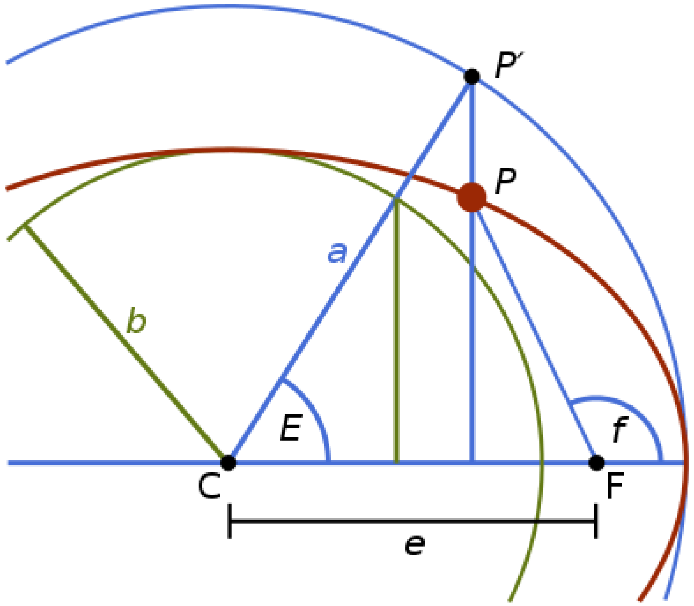
- Analytical solution to Kepler problem is an ellipse
- 5 of the 6 orbital elements *a*, *e*, *i*, Ω, ω constant
- The Mean Anomaly *M* is linear in time (2^{nd} Law)

$$M(t) = M_0 + N \cdot (t - t_0)$$

- Kepler's Equation relates orbital anomalies:

$$M = E - e\sin(E) \qquad \text{Kepler's Equation}$$

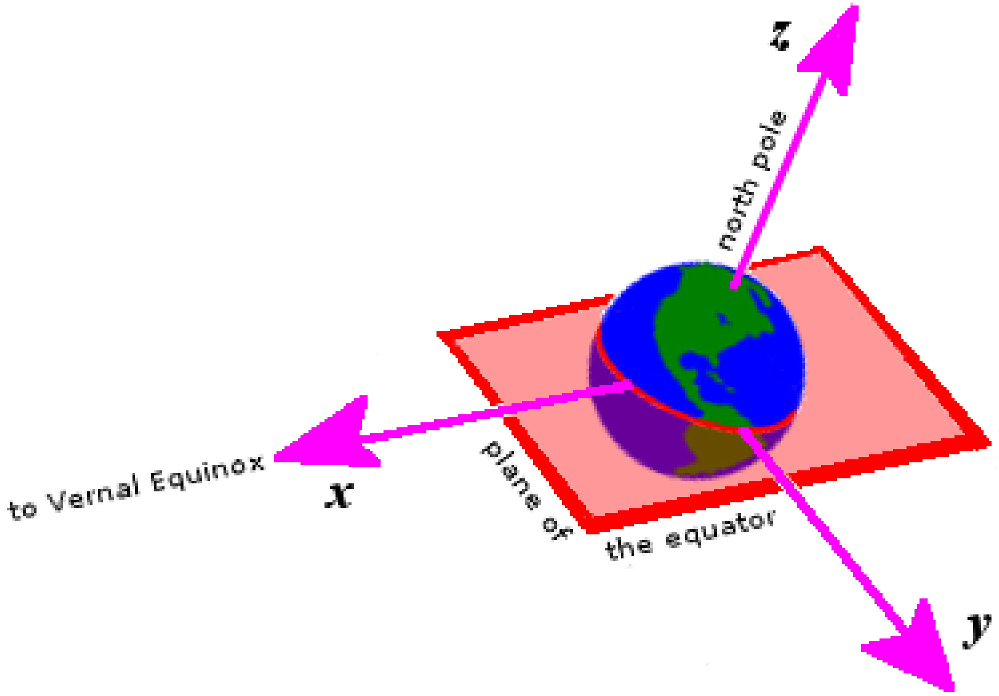
$$\tan\left(\frac{f}{2}\right) = \sqrt{\frac{1+e}{1-e}} \cdot \tan\left(\frac{E}{2}\right) \qquad \text{true to eccentric}$$

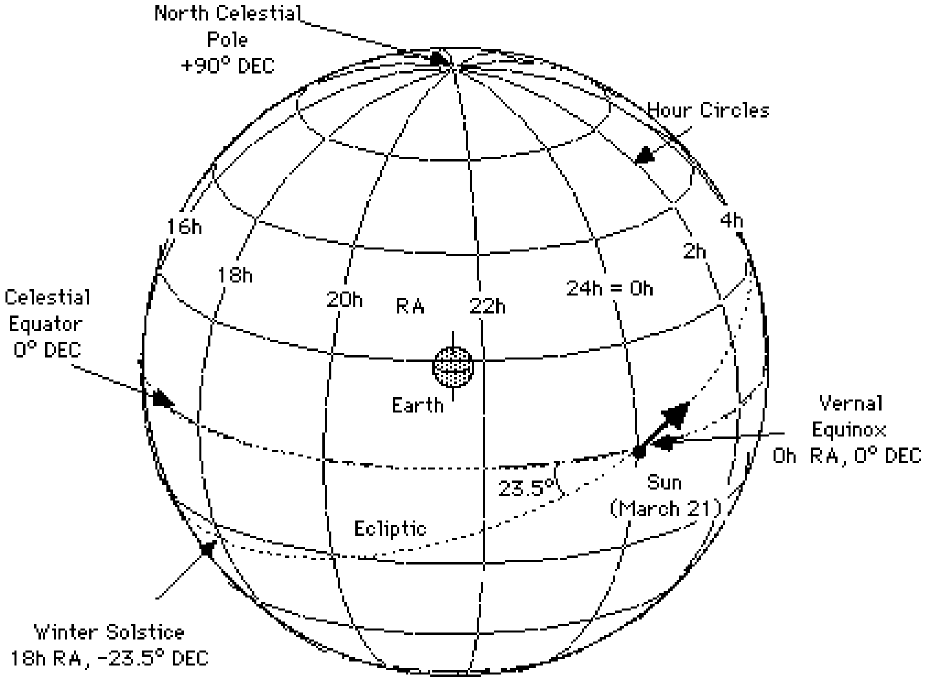


- Convert *M* to *E* to *f*, then to Cartesian coordinates
- TensorFlow is fast! 5000 time points in ~300 μ sec
- Apply calibration d**q**, d**v** to match REBOUND integration at input orbital elements

Predicting Directions from Positions

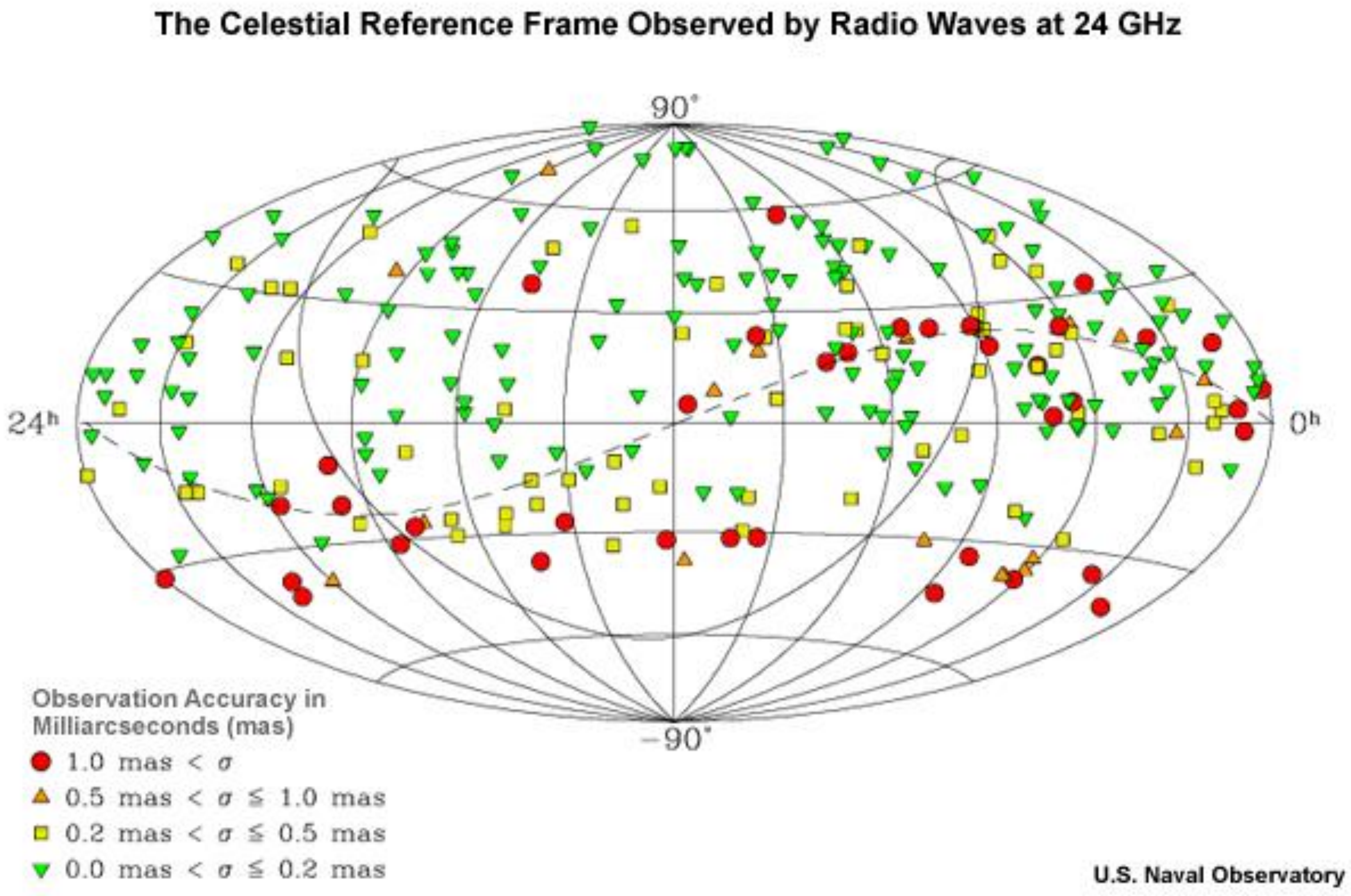
Right Ascension and Declination





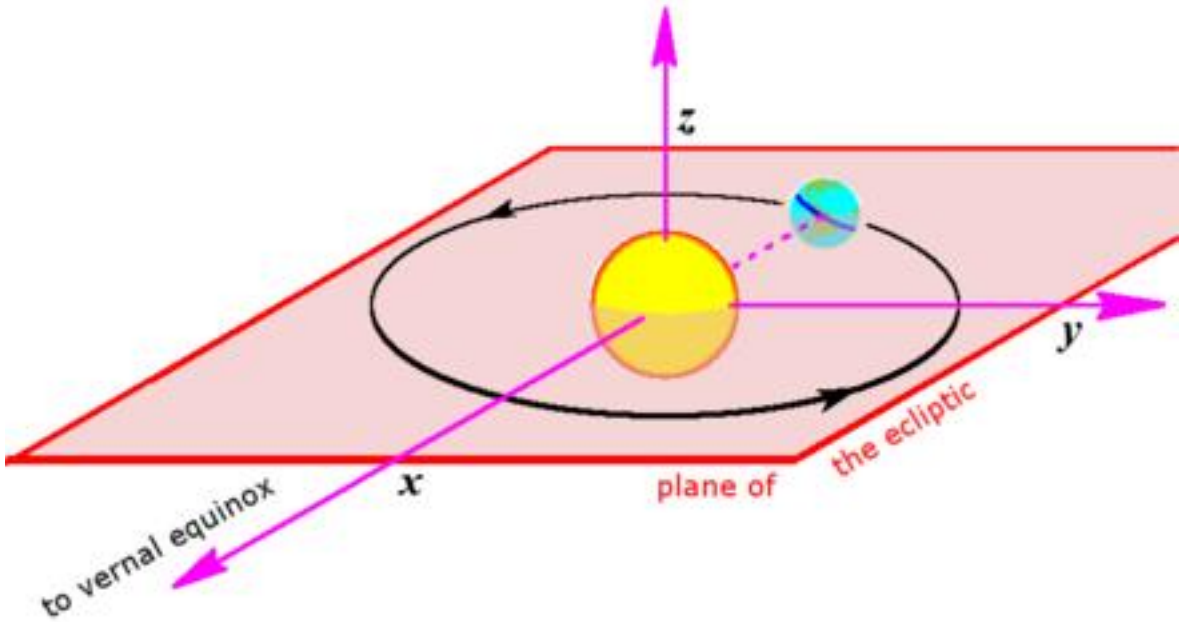
- Fundamental plane is aligned with Earth’s equator
- Intuitive, dates to ancient astronomers
- Two problems: precession (drift) and nutation (wobbles) in direction of North Pole

International Celestial Reference Frame (ICRF)



- Modern system
- Based on 232 extragalactic objects
- Addresses precession and nutation
  - Quasars don't move!
  - Not in **direction** anyway
- Amazingly accurate
  - ~2 milliarc-seconds
- Intuition: like using Polaris instead of Earth's axis for the North Pole
- Except you use 232 stars to get a highly accurate composite direction

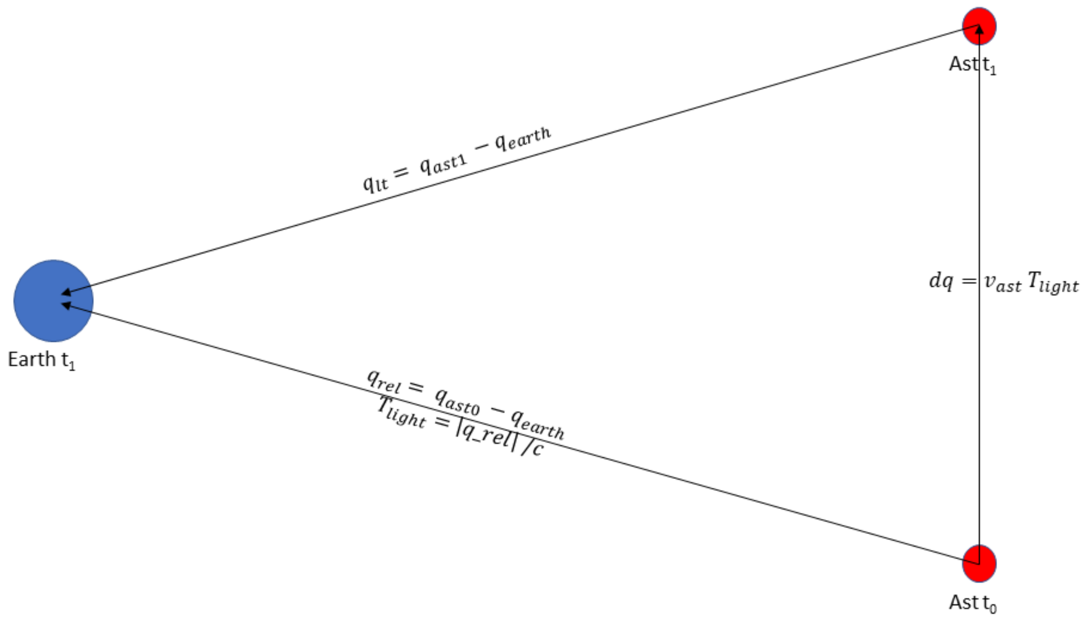
From RA/Dec to Barycentric Mean Ecliptic



- RA/Dec is ideal for observing the stars
- But not for calculations involving orbits in the Solar System
- Inside the Solar System we want an inertial frame aligned with the ecliptic: BME
- Convert between ICRF and BME using `astropy` library

```
obs_icrs = astropy.SkyCoord(ra=ra, dec=dec, obstime=obstime, frame=ICRS)
obs_ecl = obs_icrs.transform_to(BarycentricMeanEcliptic)
```

Calculate Direction from Position and Velocity



- Need to remember light speed *c* is finite!
- Otherwise wrong by ~285 arc seconds

$$\mathbf{q}_{\text{rel}} = \mathbf{q}_{\text{ast}} - \mathbf{q}_{\text{earth}}$$
$$T_{\text{light}} = \|\mathbf{q}_{\text{rel}}\|/c$$
$$\Delta\mathbf{q}_{\text{ast}} = \mathbf{v}_{\text{ast}} \cdot T_{\text{light}}$$
$$\mathbf{q}_{\text{lt}} = \mathbf{q}_{\text{rel}} - \Delta\mathbf{q}_{\text{ast}}$$
$$\mathbf{u} = \mathbf{q}_{\text{lt}}/\|\mathbf{q}_{\text{lt}}\|$$

- Earth velocity doesn't matter, only asteroid velocity
- BME is an inertial frame

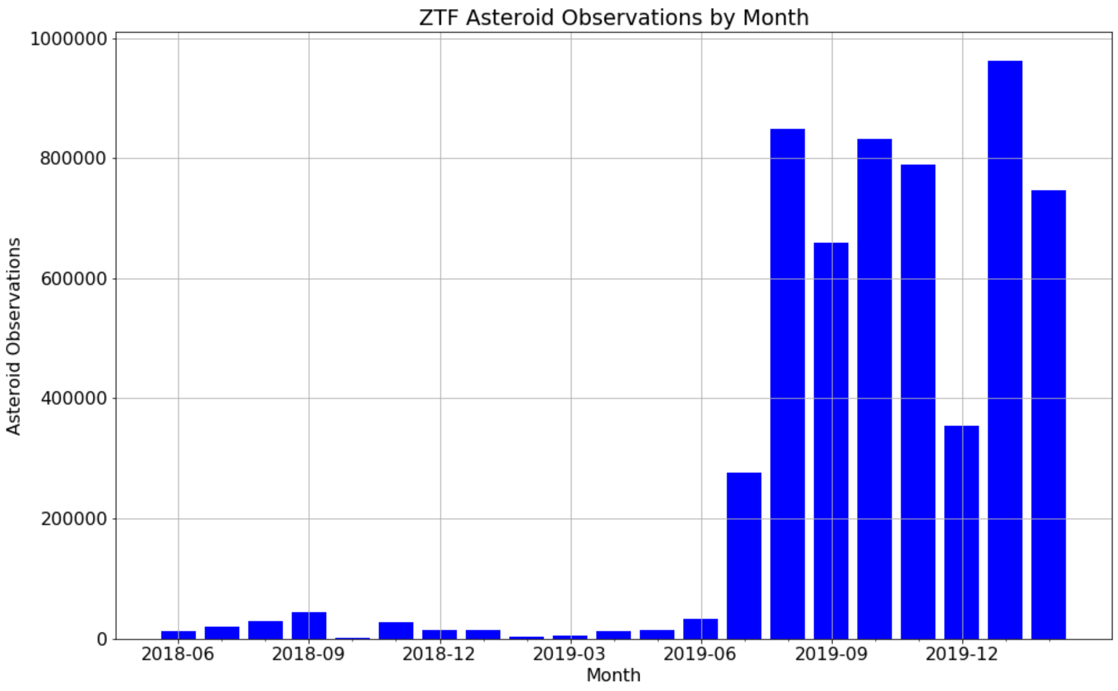
- Also need "topos adjustment" for observatory: Palomar Mountain, not geocenter!
- Topos adjustment worth 0-5 arc seconds on first 16 asteroids

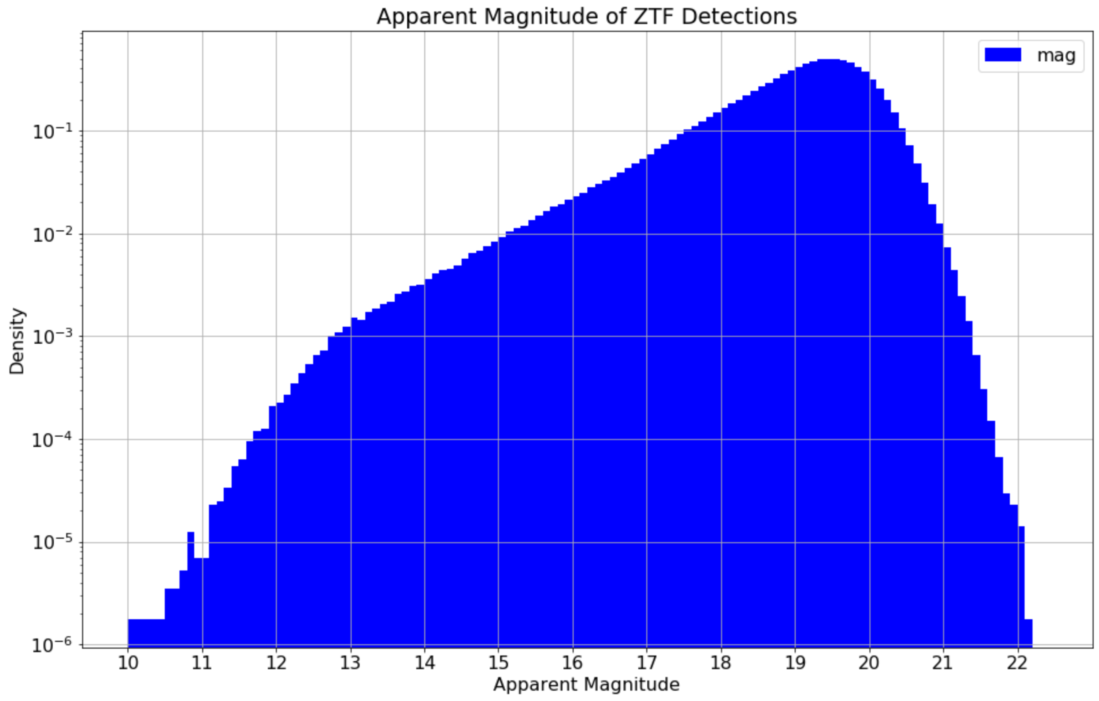
Validating Astrometric Direction

- Check these results by comparing vs. JPL, SkyField
- Downloaded Mars at 3 hour intervals over 10 years
  - Both state vectors $\boldsymbol{q}$, $\boldsymbol{v}$ and observer RA / Dec
- Computed astrometric directions from Earth to Mars
  - MSE and SkyField identical: 0.027 arc seconds
  - Both MSE and SkyField differ from JPL by 1.6 arc seconds
- Separately downloaded JPL RA/Dec on first 16 asteroids
- Compared to MSE direction calculated from integrated orbits
- RMS error: 0.873 arc seconds!

Analysis of ZTF Asteroid Detections

EDA of ZTF Detections



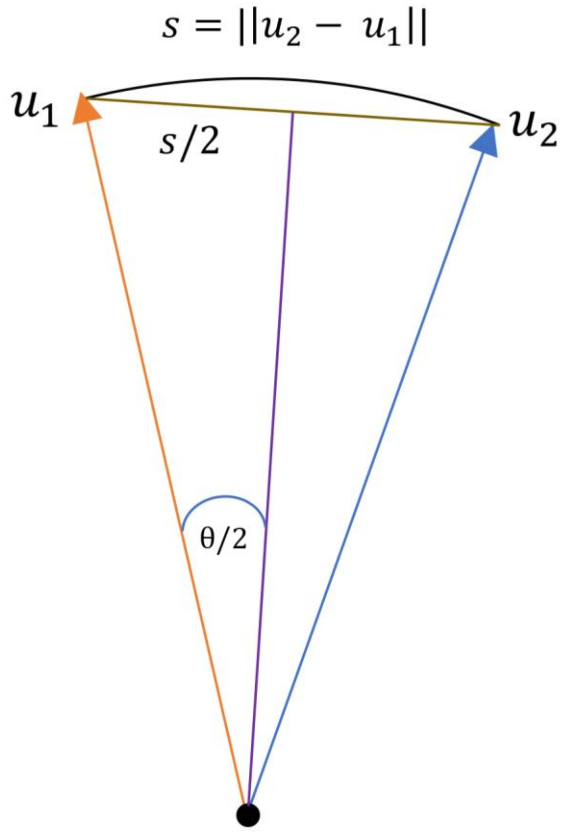


- ZTF: Zwicky Transient Facility; survey of northern sky at Palomar Mountain by Cal Tech
- Fast, deep survey: 3750 square degrees / hour to depth of 20.5 mag
- First light in 2017, but asteroid detections ramp up in July 2019; 7 months of data
- Enriched with machine learning pipeline that filters probable asteroid detections
- 5.69 million possible asteroid detections
- Data includes: MJD, RA, DEC, MAG

Converting Cartesian to Angular Distance

- How far apart are two directions in the sky?
- Convert RA/Dec to directions u_1 and u_2 in the BME
- Compute Cartesian distance s between u_1 and u_2
- Angular distance θ is geodesic (great circle distance)

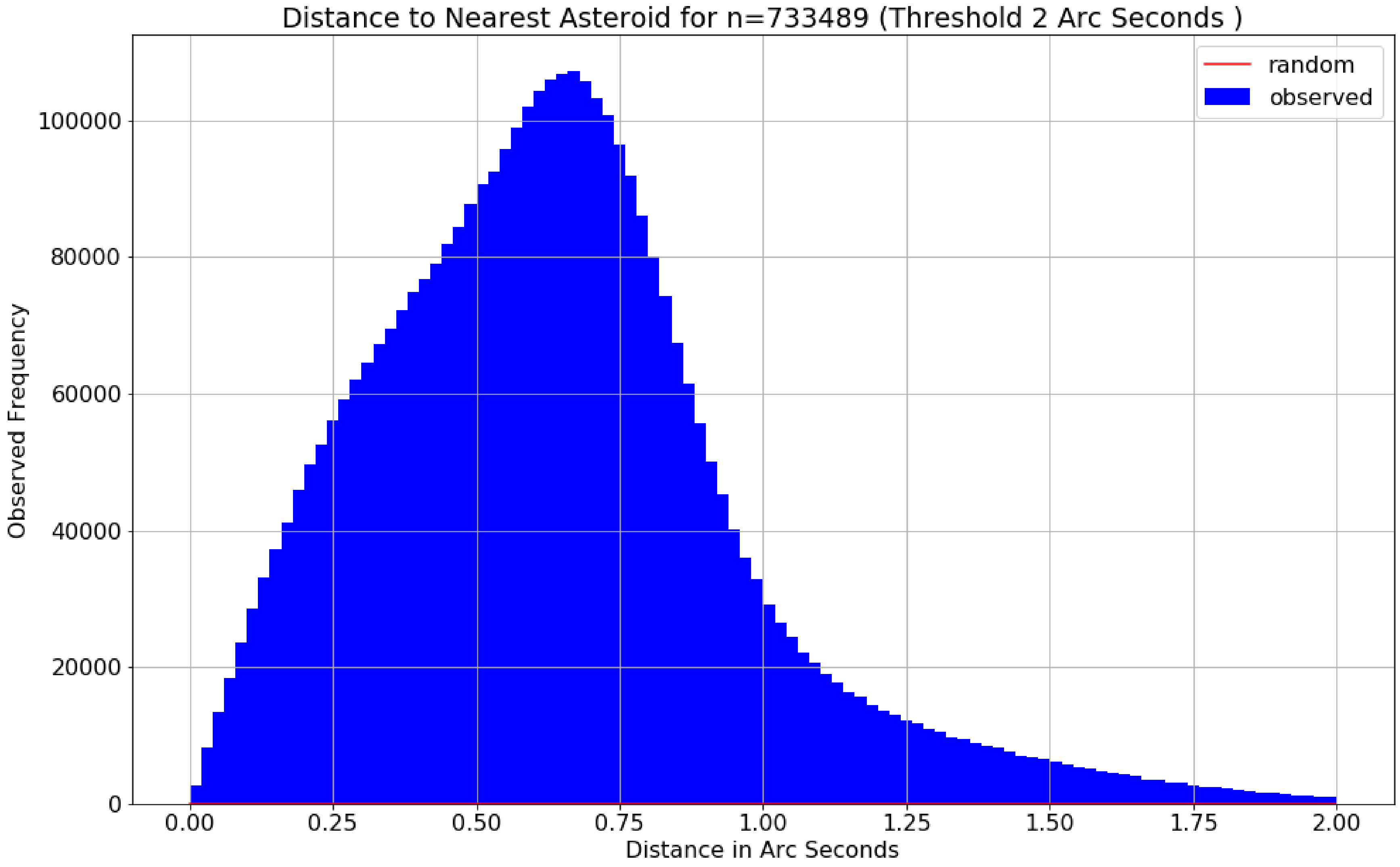
$$\sin(\theta/2) = s/2$$



Nearest Asteroid to Each ZTF Detection

- Compute direction $u_{obs} = (u_x, u_y, u_z)$ from RA/Dec for each detection
- Compute direction u_{ast} for every asteroid in the catalogue
- 5.7E6 detections x 7.3E5 asteroids = 4.2E12 (4.2 trillion) interactions!
    - Too big for naïve brute force attack
- "Only" 97,111 different MJDs with ZTF detections
- Work in chunks of 1000 asteroids at a time, find nearest to each ZTF
- Then perform reduction operation to find globally nearest asteroid
- Still large compute job: 25 hours on 40 CPUs, 256 GB RAM server

Nearest Asteroid: 65.7% Within 2.0 Arc Seconds!



Statistical Distribution of Distance on Sphere

- What is the statistical distribution of s if we guessed directions uniformly at random?

$$s^2 = 2 \cdot (1 - z) \qquad z = 1 - s^2/2$$

- This is useful parameterization because…
- "Orange Slicing Theorem" for solid angle measure:

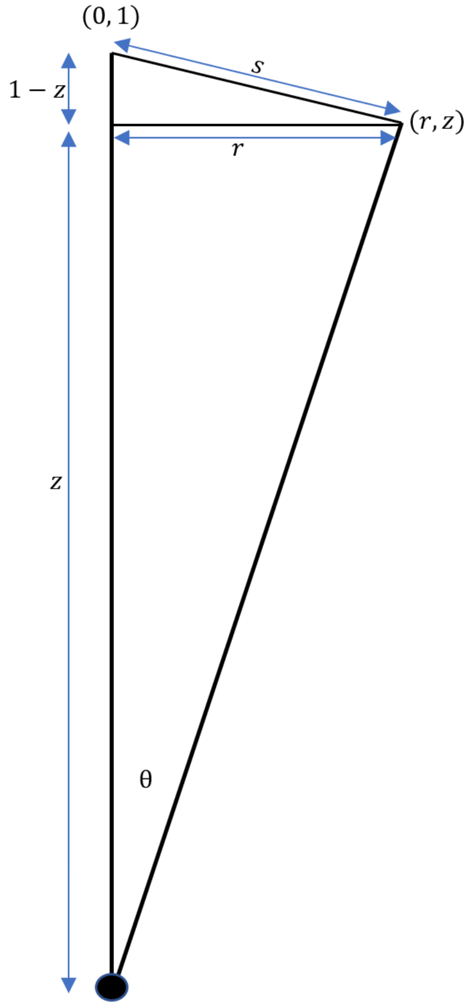
$$d\Omega = dz \cdot d\phi$$

- Think of Z and S as random variables:

$$Z \sim \mathrm{Unif}(-1, 1) \quad S^2 \sim \mathrm{Unif}(0, 4)$$

- Conditional on a max (threshold) distance τ

$$S^2 | S \leq \tau \sim \mathrm{Unif}(0, \tau^2)$$



Distribution of Nearest Asteroid Distance

- Set a threshold distance τ and define relative squared distance V

$$V = (S/\tau)^2 \qquad V \sim \mathrm{Unif}(0,1)$$

- We have n = 733,489 guesses and are picking closest

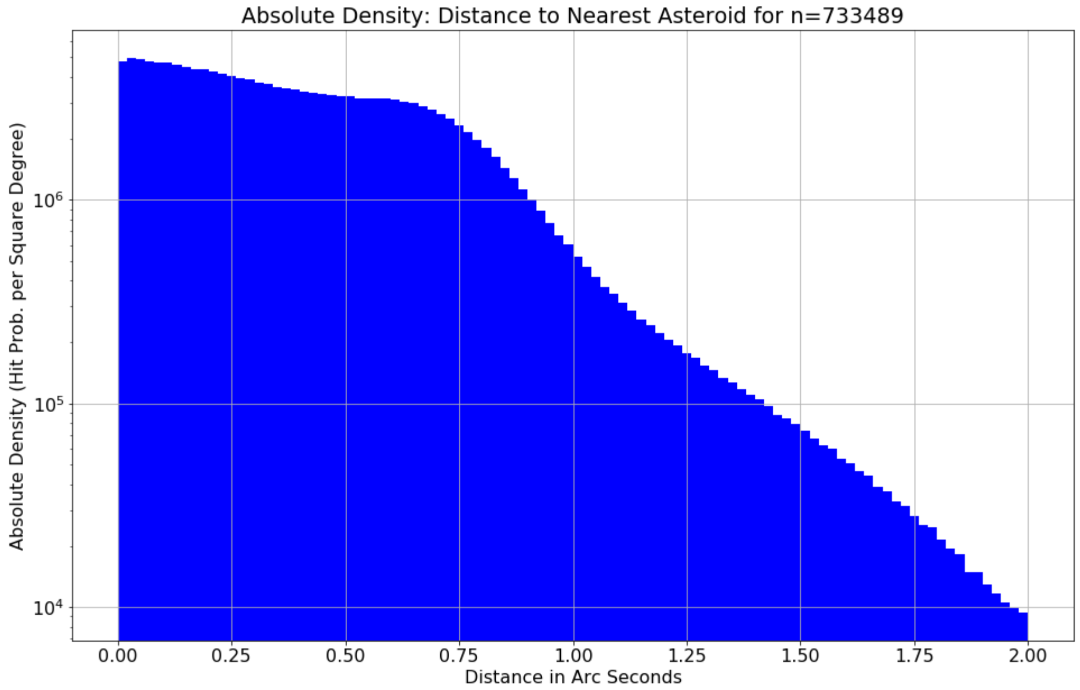
$$V_1, \ldots V_n \overset{i.i.d.}{\sim} \mathrm{Unif}(0,1)$$

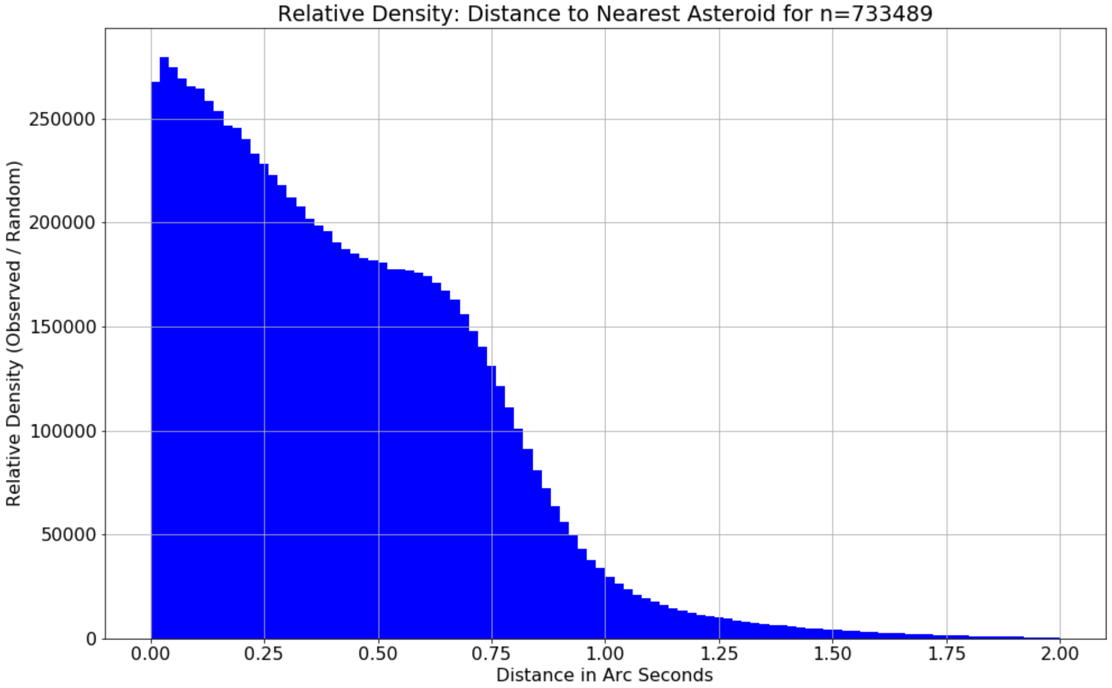
- Stat 110: The minimum of n i.i.d. uniforms has a Beta distribution

$$U_{(1)} \sim \mathrm{Beta}(1, n)$$

- How many hits at 2.0 arc seconds would we get by luck?
- Only 98. But we got 3.75 million of them!
- Conclusion: This whole apparatus works to a tolerance of 2.0 arc seconds

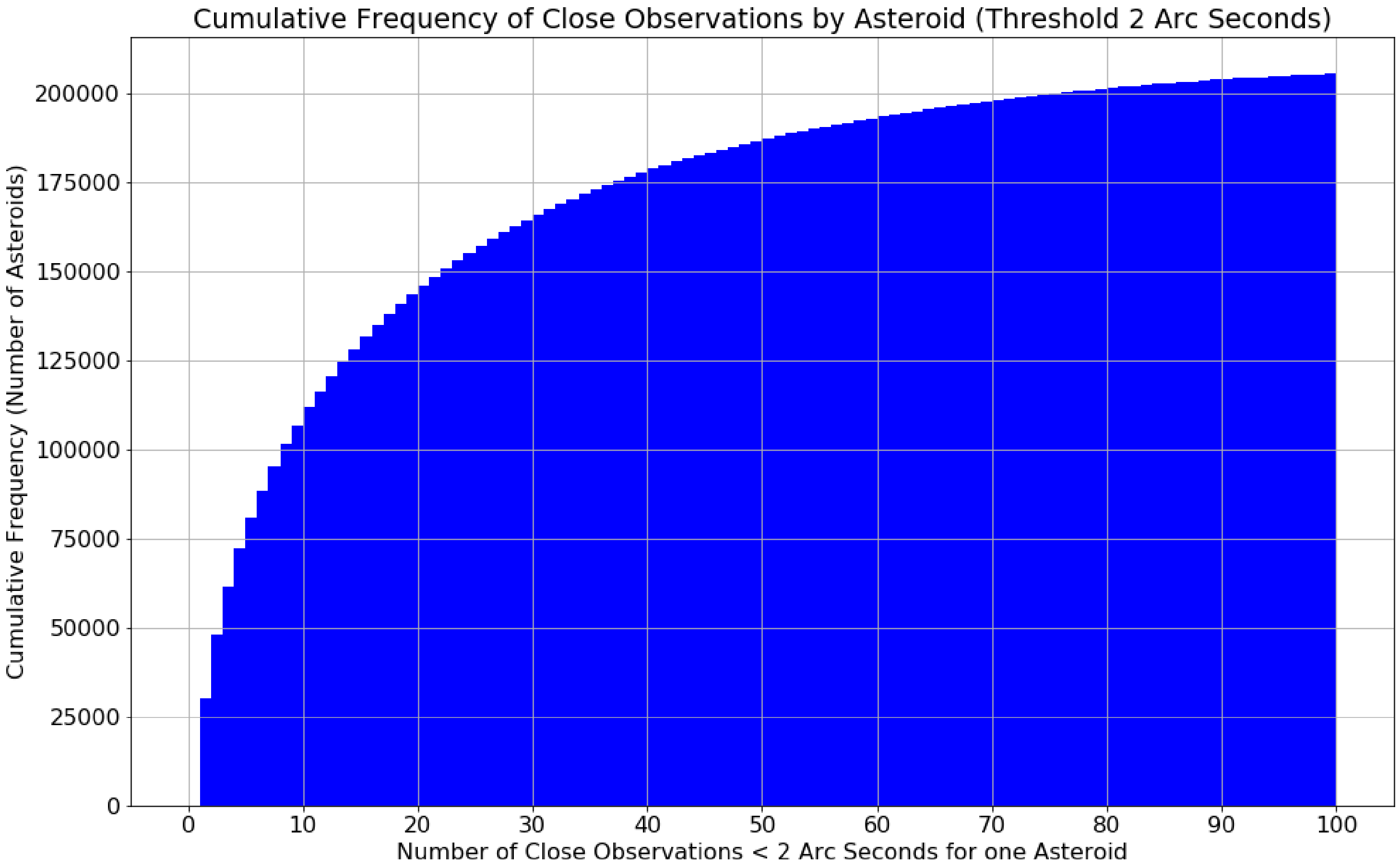
Density of Distance to Nearest Asteroid





- Plot absolute density in hits per square degree
- Plot relative density: absolute density over beta distribution PDF
- Both plots on log scale
- Right tail seems to have an exponential decay pattern

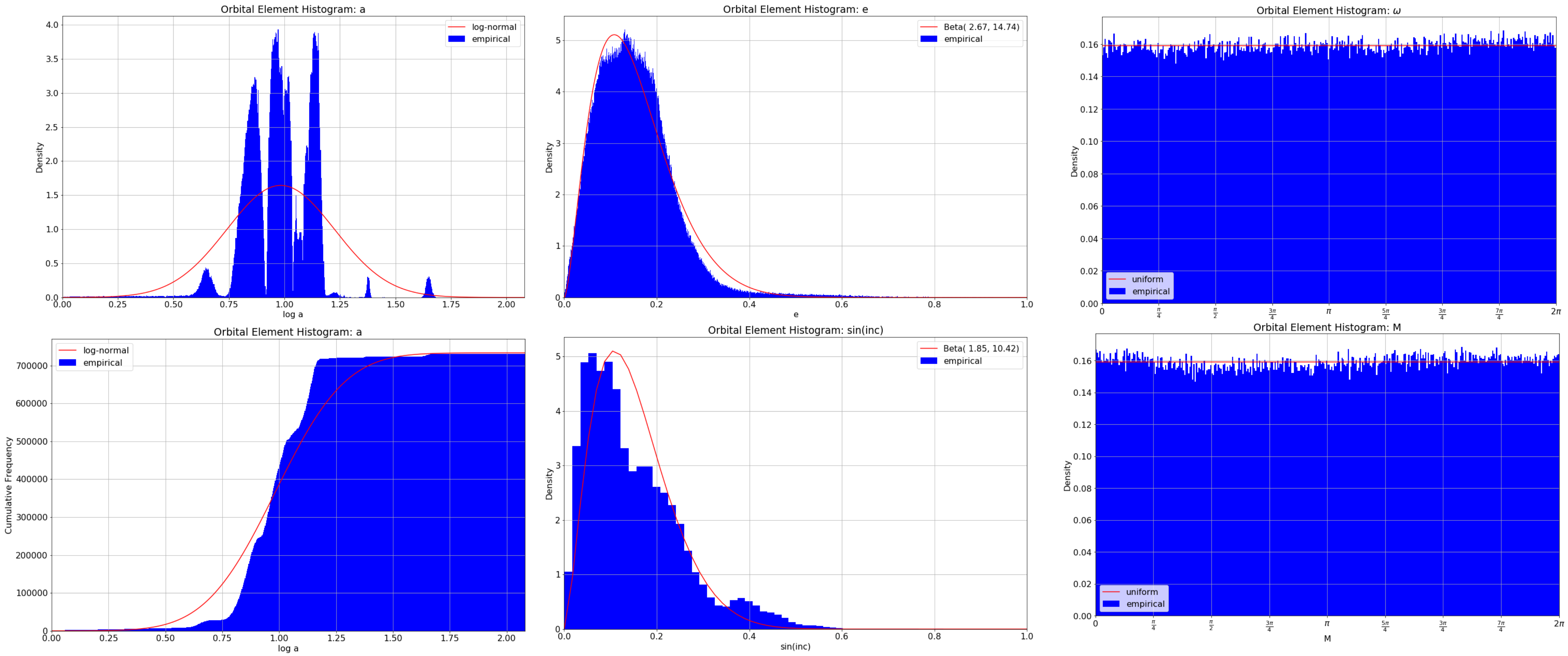
Cumulative Distribution of Hits per Asteroid



- Count hits at 2.0 arc seconds
- How many asteroids have at least
  - 20 hits? 63,746
  - 10 hits? 100,508
- We have a sporting chance to rebuild 13.6% of the catalogue if we require 10 or more hits

Asteroid Search Using Orbital Elements

Random Sampling of Candidate Elements



- Four elements sampled empirically: a, e, i, Ω
  - Randomly choose an index from 1 to 733,489 and take element of that asteroid
- Two elements sampled uniformly at random: ω, M
  - Convert mean anomaly M to true anomaly f in REBOUND

Assemble ZTF Detections Near Elements

```
# Load unperturbed element batch
ztf_elt_ast = load_ztf_batch(elts=elts_ast, thresh_deg=1.0, near_ast=False)
```

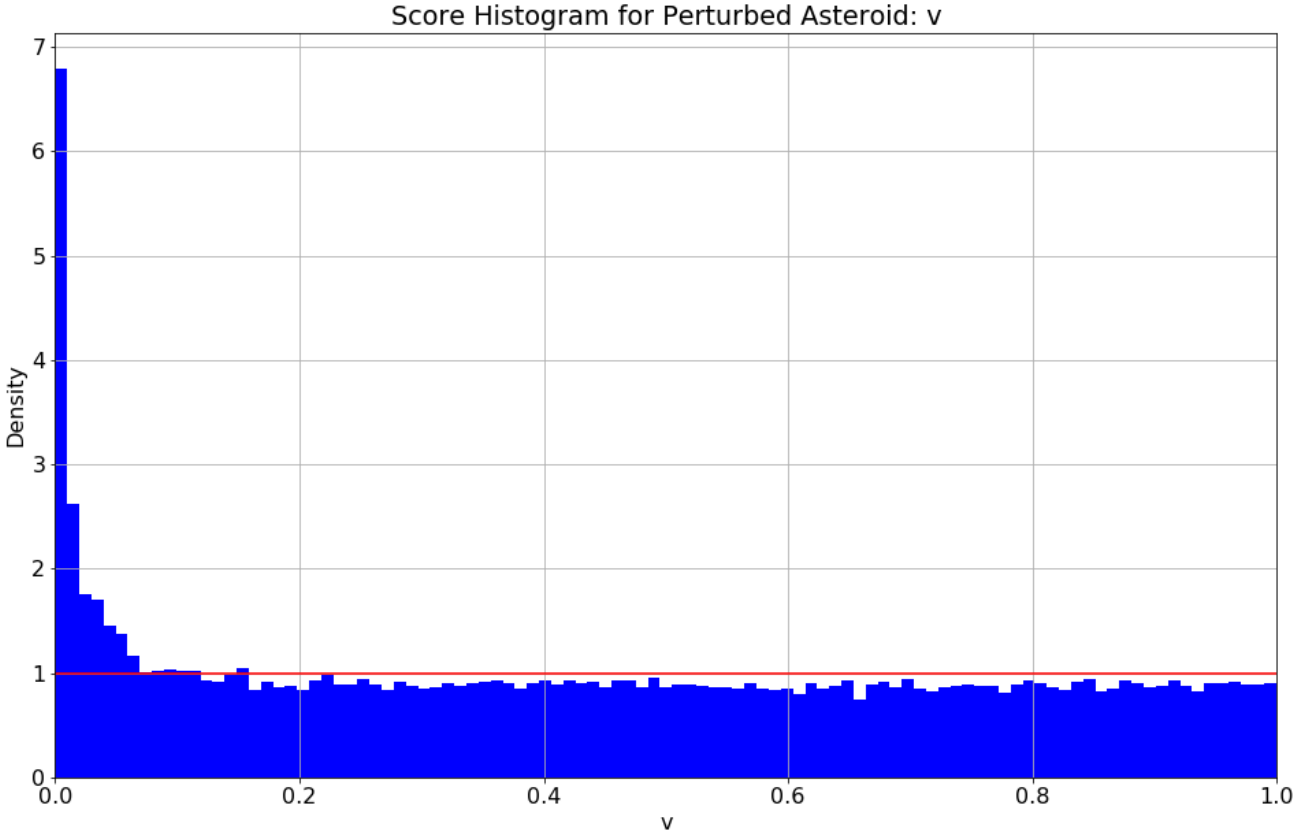
```
# Review
ztf_elt_ast[cols]
```

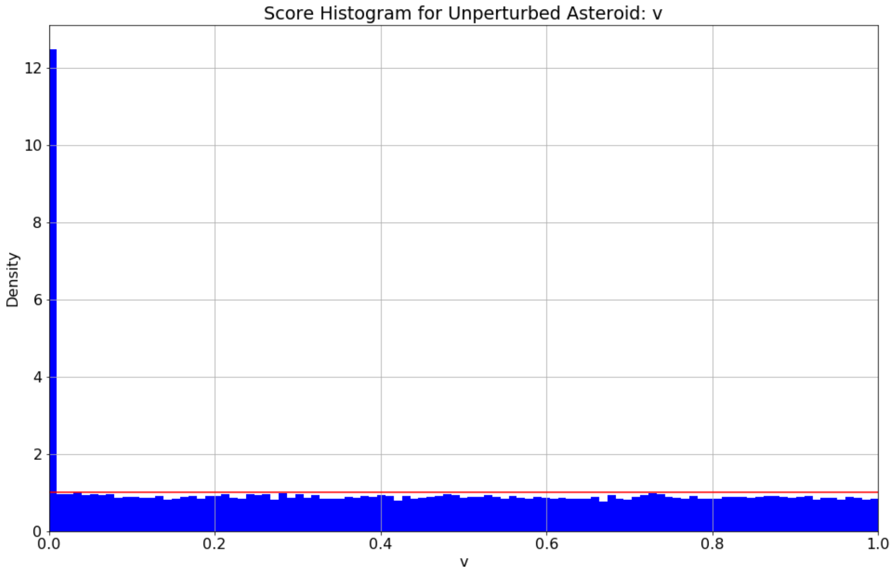
	element_id	ztf_id	mjd	ra	dec	ux	uy	uz	mag_app	elt_ux	elt_uy	elt_uz	s_sec	v	is_hit
0	733	53851	58348.197581	266.229165	-13.513802	-0.063945	-0.983101	0.171530	16.755600	-0.057300	-0.982042	0.179751	2191.408734	0.370552	False
1	733	73604	58348.197581	265.761024	-13.509148	-0.071871	-0.982578	0.171389	16.035999	-0.057300	-0.982042	0.179751	3467.151428	0.927559	False
2	733	82343	58389.193252	270.331454	-11.244934	0.005674	-0.977422	0.211222	17.196199	0.000919	-0.977996	0.208622	1124.103915	0.097503	False
3	733	257221	58685.471227	29.693832	42.180412	0.643725	0.603886	0.470042	19.289200	0.639004	0.610779	0.467571	1797.091521	0.249197	False
4	733	327000	58691.465972	33.104905	44.059131	0.601970	0.636719	0.481893	17.725201	0.606278	0.637608	0.475272	1639.539679	0.207419	False
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
90206	324582	5650588	58904.176701	44.164238	29.650540	0.623416	0.752309	0.213037	18.084700	0.627640	0.750696	0.206212	1688.638104	0.220027	False
90207	324582	5650589	58904.176250	44.164062	29.650536	0.623417	0.752307	0.213038	18.165199	0.627641	0.750695	0.206213	1688.601889	0.220018	False
90208	324582	5650665	58904.176250	44.368640	28.490480	0.628284	0.753618	0.193182	19.025200	0.627641	0.750695	0.206213	2757.856412	0.586871	False
90209	324582	5650697	58904.176250	43.296207	29.505908	0.633424	0.743491	0.214467	19.852800	0.627641	0.750695	0.206213	2555.278205	0.503822	False
90210	324582	5650705	58904.176250	44.621045	29.303550	0.620689	0.756675	0.205398	19.647400	0.627641	0.750695	0.206213	1898.912116	0.278236	False

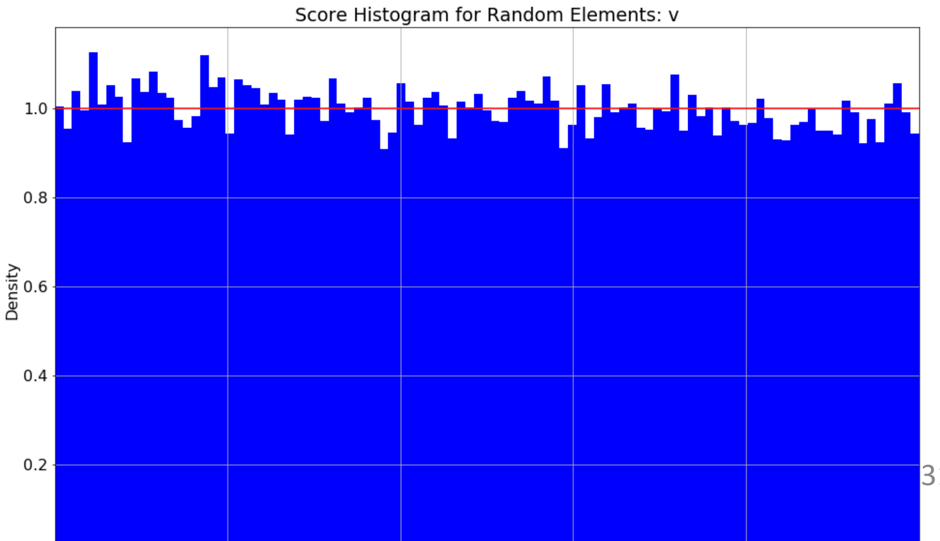
90211 rows × 15 columns

- Integrate the candidate elements on the fly in REBOUND and compute directions
- Filter the ZTF detections to those within threshold of the elements

Distribution of $V = (S/\tau)^2$ for 3 Element Batches







- Plot V for 3 batches: unperturbed, perturbed, and random
- Results match theory perfectly!
- Random elements close to uniform distribution
- Unperturbed: uniform on misses with spike in first bucket
- Perturbed: in between; hits leak out to ~250 arc seconds

Log Likelihood Objective Function

- Mixture probability model: V mixture of h hits, (1-h) misses

$$V|\text{Hit} \sim \text{Expo}(\lambda) \qquad V|\text{Miss} \sim \text{Unif}(0,1)$$

- Relate decay rate to “resolution” parameter R

$$f(v) \propto e^{-\lambda v} = e^{-\lambda s^2/\tau^2} \qquad f(v) \propto e^{-s^2/2R^2} \qquad \lambda = \frac{\tau^2}{2R^2}$$

- The resolution R controls how tightly the model focuses
- Mixture PDF:

$$h \cdot \frac{\lambda v}{1 - e^{-\lambda}} + (1 - h)$$

- Log Likelihood:

$$\mathcal{L}(\mathbf{v}, h, \lambda) = \sum_{j=1}^{n} \log\left(h_j \cdot \frac{\lambda v_j}{1 - e^{-\lambda_j}} + 1 - h_j \right)$$

Search Overview

- Six trainable orbital elements *a, e, i,* Ω, ω, *f;* epoch not trainable
- Three trainable mixture parameters: N_h, R, τ
- Compute position **q** and velocity **v** from candidate elements
- Compute direction $\mathbf{u}_{pred}$ from **q**, **v;** include light time and topos
- Compute distance s from $\mathbf{u}_{pred}$ to $\mathbf{u}_{obs}$ for ZTF observations
- Compute log likelihood $\mathscr{L}_i$ for each candidate element
- Gradient descent…

Search Techniques I: Uniform Scale, Gradient Clipping and Independent Weights

- Control variables on uniform scale in [0, 1]
  - e.g. a = a_min x exp(a_ + log(a_max / a_min); a_ trainable in [0,1]
- Clip gradients by norm; max || Grad $\mathscr{L}$ || = 1
  - would be better to do this elementwise, but requires custom class
- Weight log likelihood for each element in batch independently

$$\mathcal{L} = \sum_{i=1}^{b} w_i \cdot \mathcal{L}_i$$

  - equivalent to controlling 64 learning rates independently
  - reduce learning rate on an element when it overshoots
- Track of log likelihood and hits for each candidate element before summing them in the objective function
- Revert changes only on elements that got worse during an episode

Search Techniques II: Mixture vs. Joint Mode, Encouraging Convergence

- Mixture mode: only learn N_h, R, τ; orbital elements frozen
- Joint mode: learn all parameters jointly
  - Higher learning rate 2^{-12} in mixture mode vs. 2^{-16} in joint mode
- Modified objective function in mixture mode

$$\mathcal{L} = \sum_{i=1}^{b} w_i \cdot \frac{\mathcal{L}_i}{R_i^{\alpha} \cdot \tau_i^{\beta}}$$

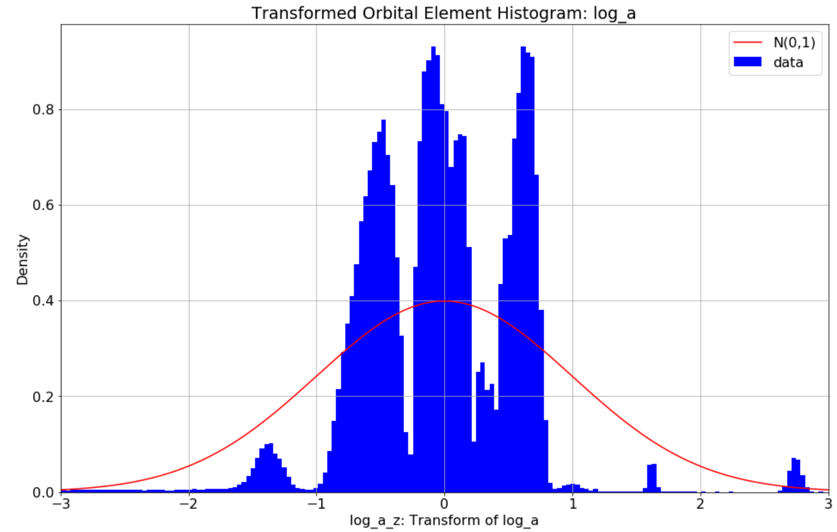
- Theoretical motivation: likelihood would always look better with a larger τ; this encourages the model to converge
- Like adjusting score for degree of difficulty in diving and gymnastics

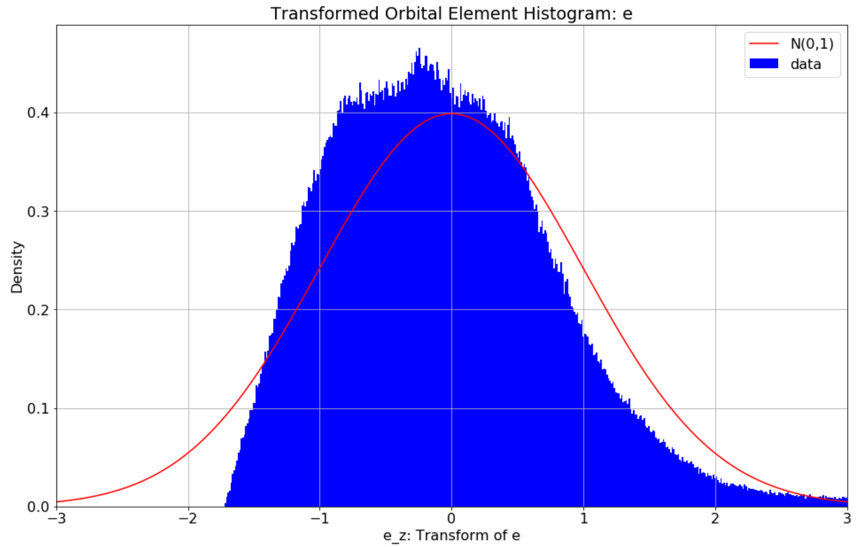
Asteroid Search Results

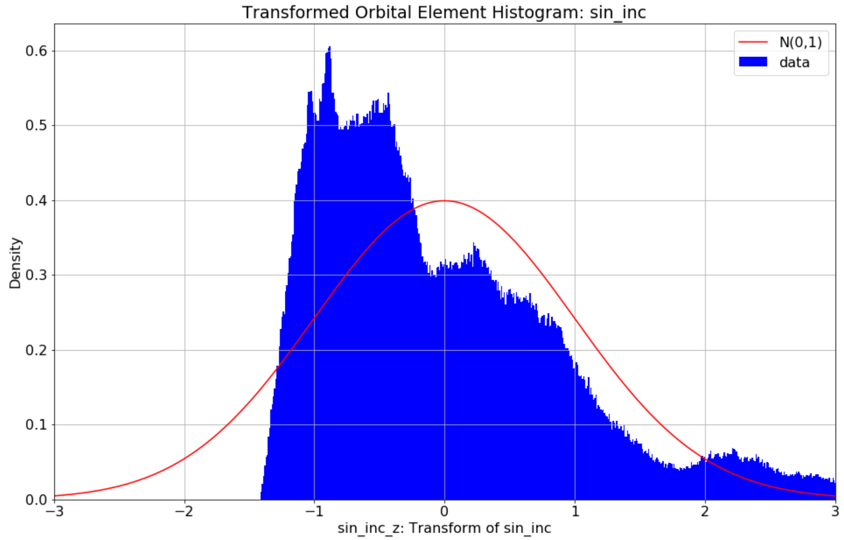
Comparing Two Orbital Elements

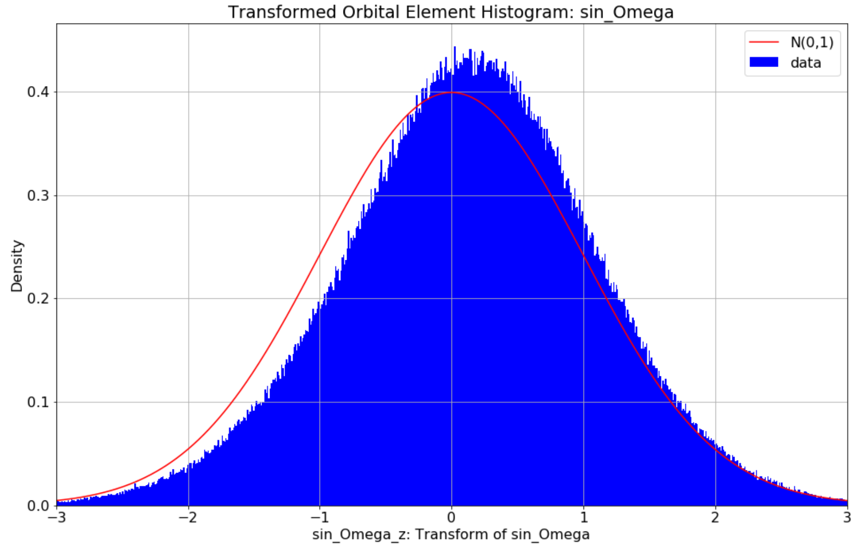
- How far apart are two 6D orbital elements ε_1 and ε_2?
- A naïve Euclidean norm makes no sense at all
- Idea 1: inject the elements into space at a set of times
  - The distance between two elements is the mean distance in AU between the orbits they describe
  - Set 240 sample time points at monthly intervals from 2010 to 2030
- Idea 2: transform elements into low dimensional Cartesian space
  - Try to make each component approximately normal
  - Try to make joint distribution approximately multivariate normal
  - Use the Mahalanobis distance on these transformed elements

Transforming Elements for Covariance Norm









- Standardize log(*a*)
- Standardize *e*
- Standardize sin(*i*)
- Same transformation for sin and cos of Ω, ω, f

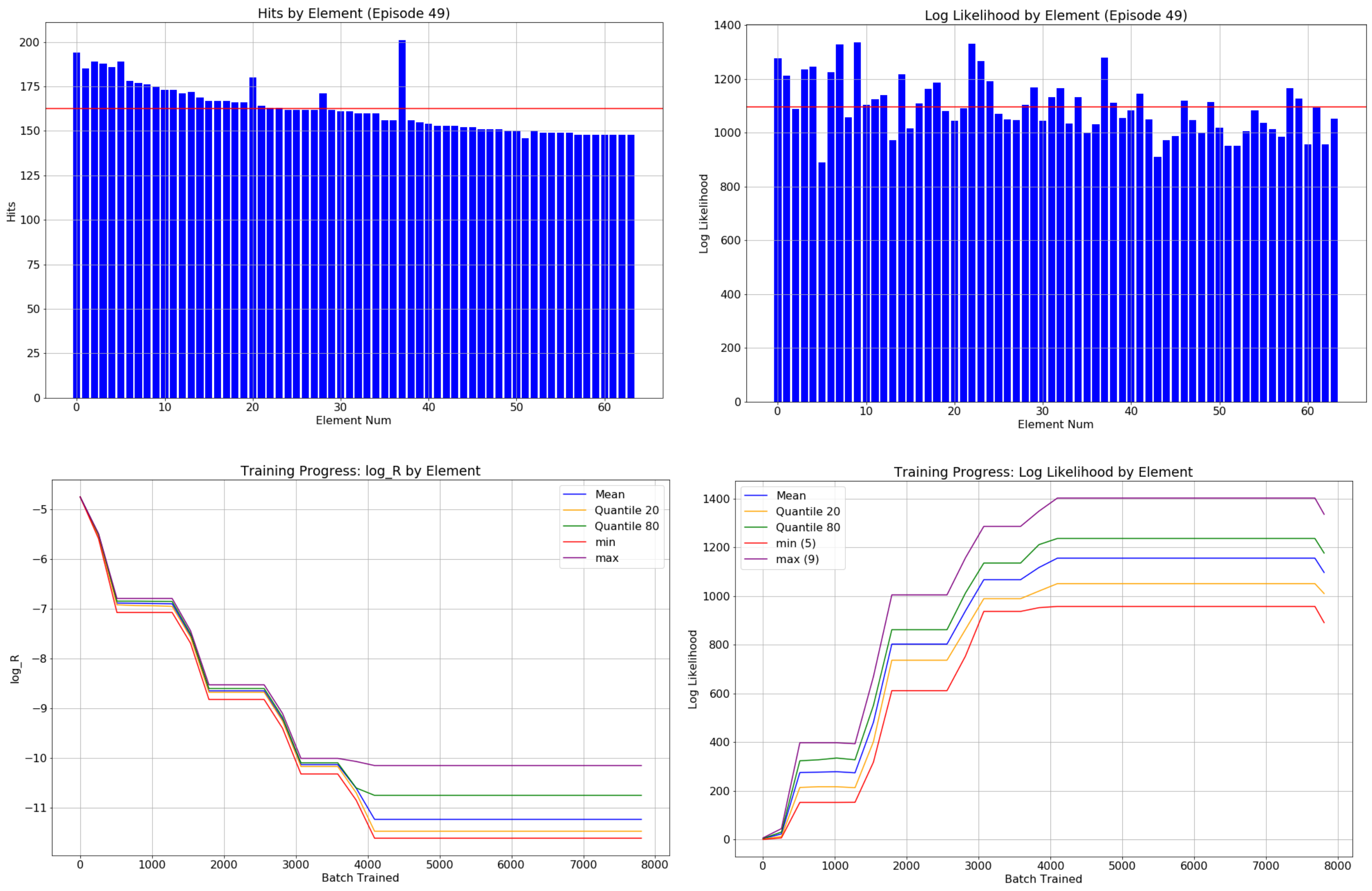
$$u = \frac{1/2 + \arcsin(x)}{\pi}$$

$$z = \Phi^{-1}(u)$$

- This injects elements into R^9
- Apply importance weights
  - 1.0 for *a*, *e*
  - 0.5 for *i*
  - 0.1 for sin, cos of Ω, ω, f
- Use PCA to find β such that X β has covariance matrix I_9

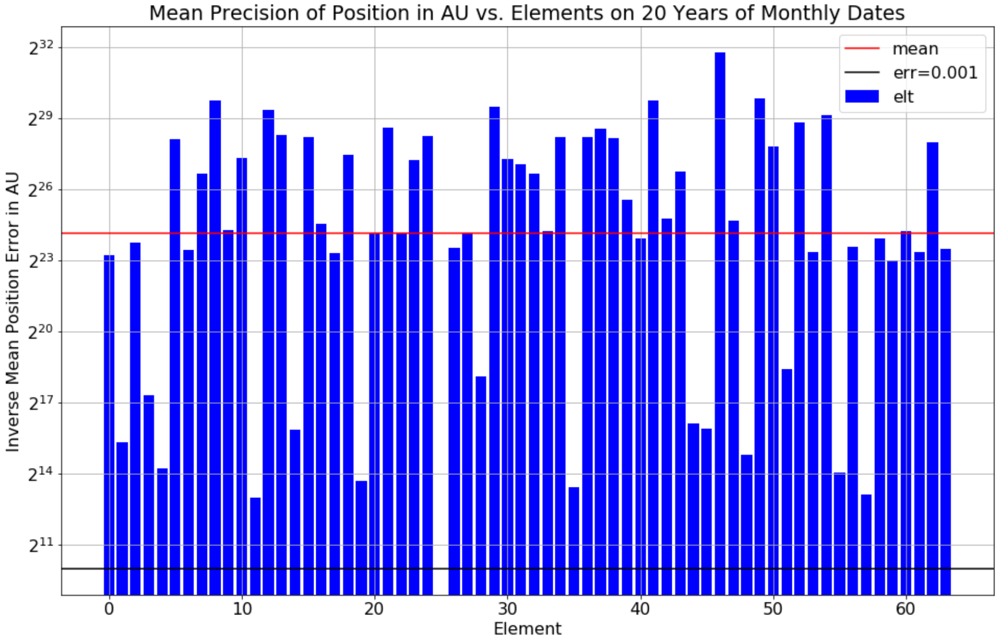
$$\|\epsilon_2 - \epsilon_2\|_{\mathrm{cov}} = \|X_2\beta - X_1\beta\|$$

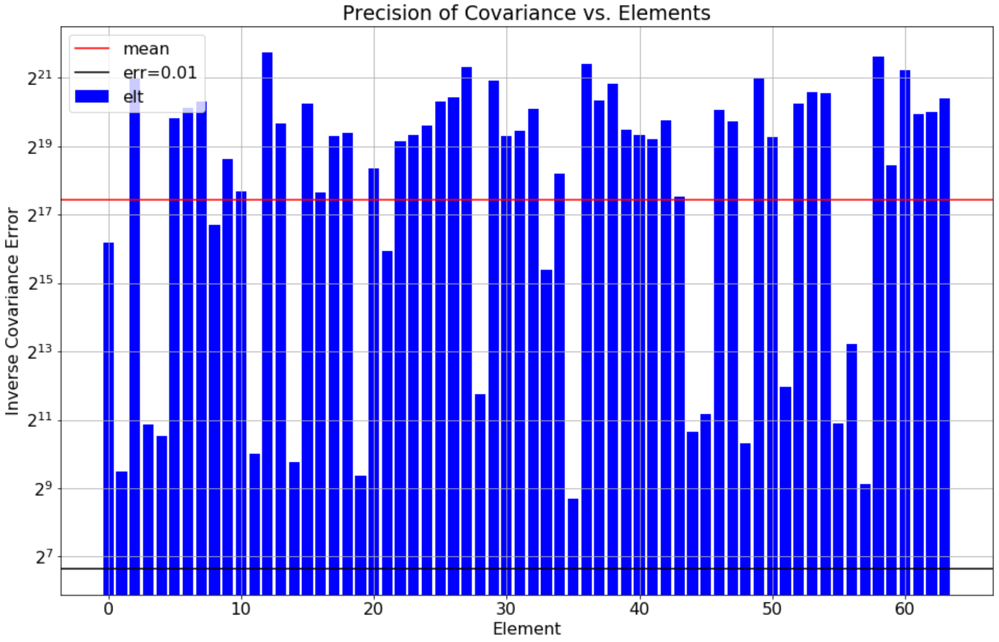
Train Known Asteroid Elements: Unperturbed



- Start with correct elements but uniformed mixture parameters
- Convergence is almost perfect
- Recovered Elements: 64 (100%)
- Hits: 162.6
- Resolution: 3.0 arc seconds
- Log Like: 1097

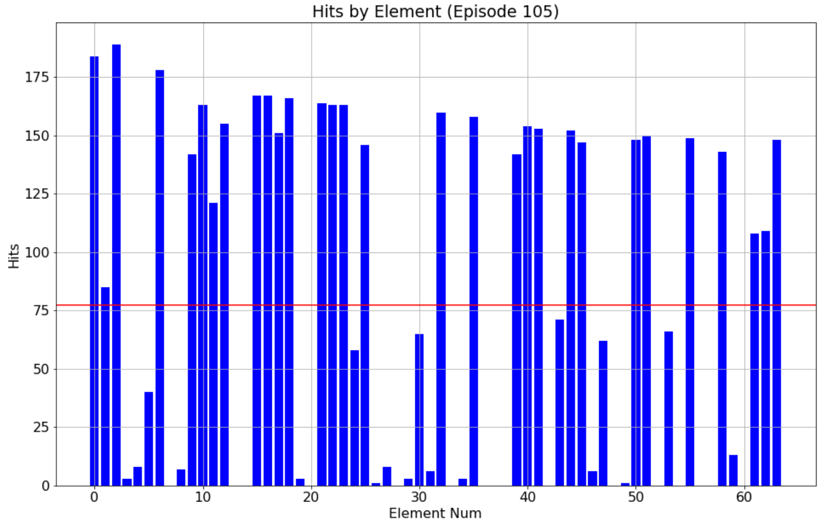
Fit Quality: Unperturbed Elements

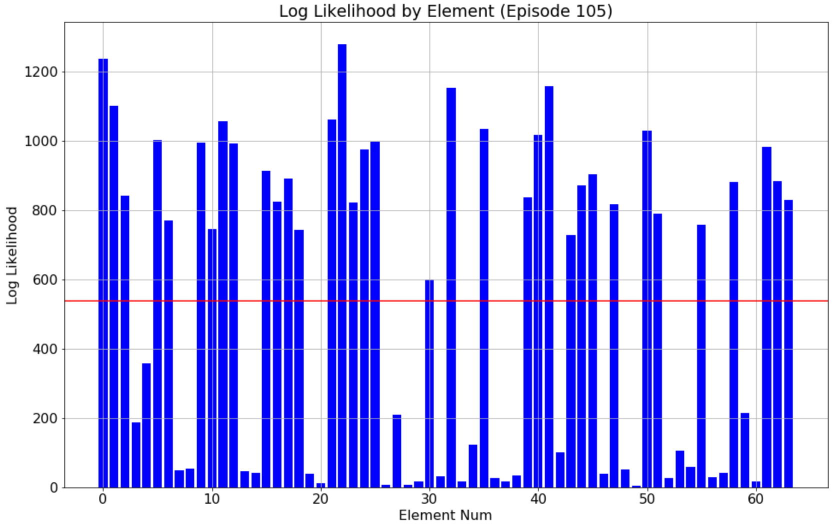


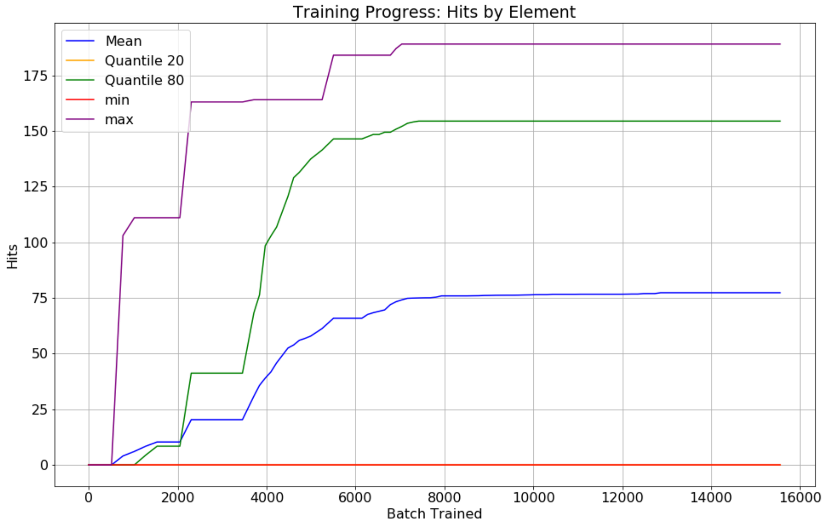


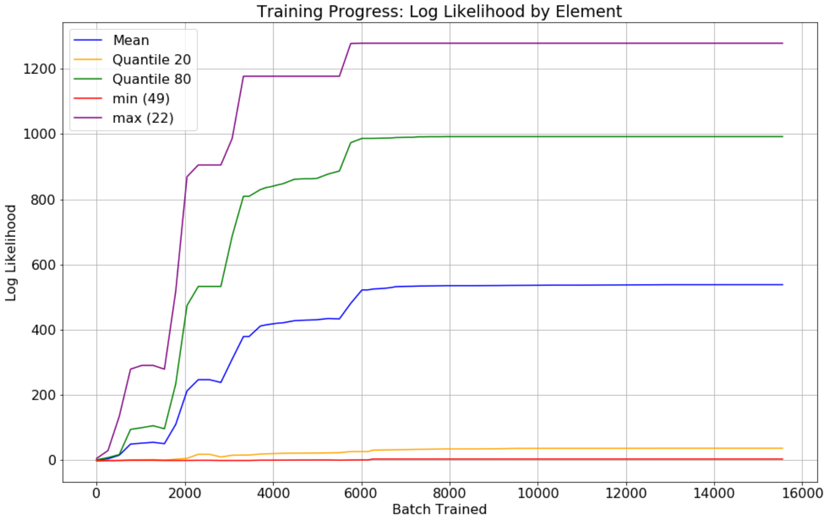
- Do recovered elements match the nearest asteroid?
- 4.6E-8 AU mean distance
- 5.7E-6 covariance norm
- The fit is almost perfect
- Big deal, this is about as hard as hitting a baseball off a tee…

Train Known Asteroid Elements: Small Perturbation



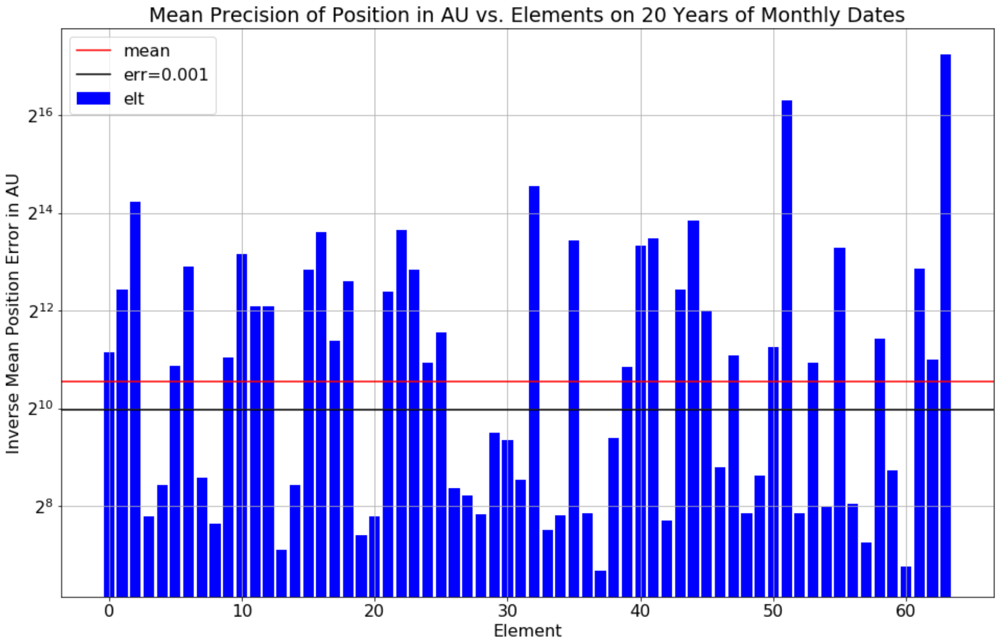


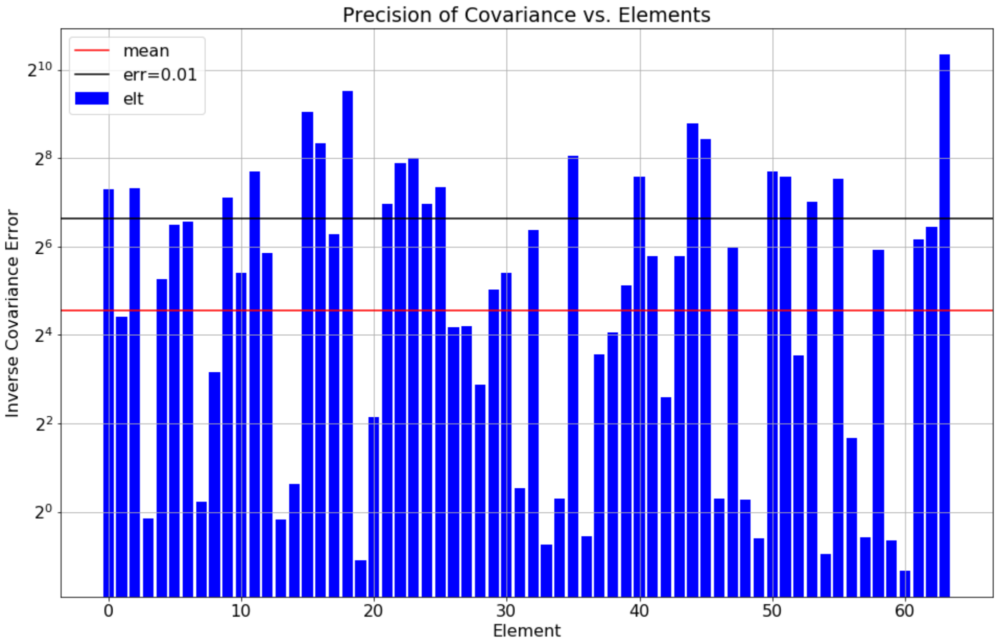




- Small perturbation:
  - 1.0% to *a*
  - 0.25% to *e*
  - 0.05 degrees to *i*
  - 0.25 degrees to Ω, ω, f
- Convergence is very good
- Recovered Elements: 42 (65.6%)
- Hits: 117.5
- Resolution: 18.2 arc seconds
- Log Like: 798

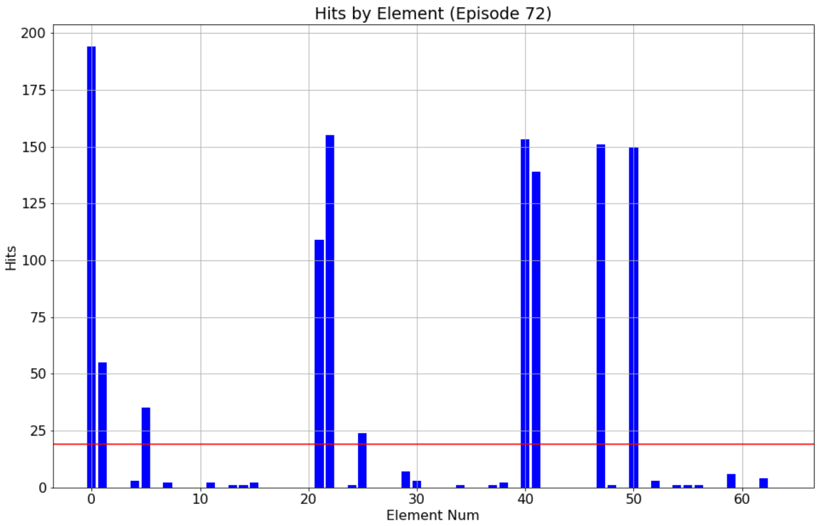
Fit Quality: Small Perturbation

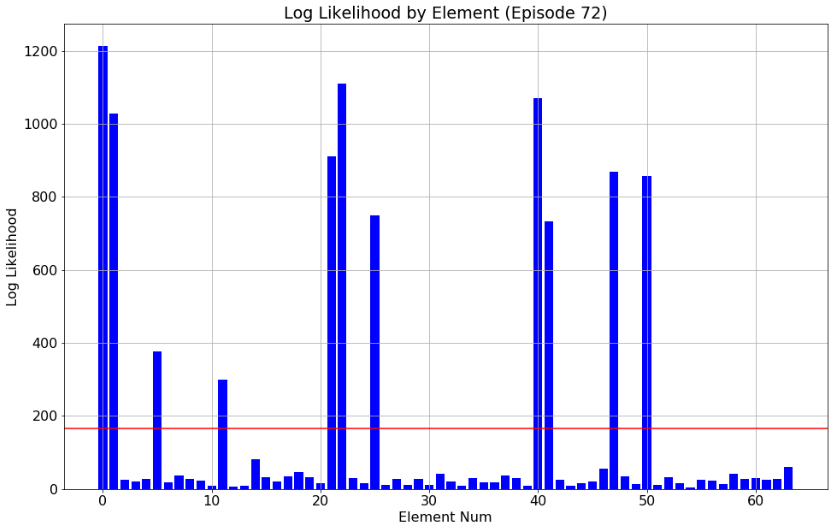


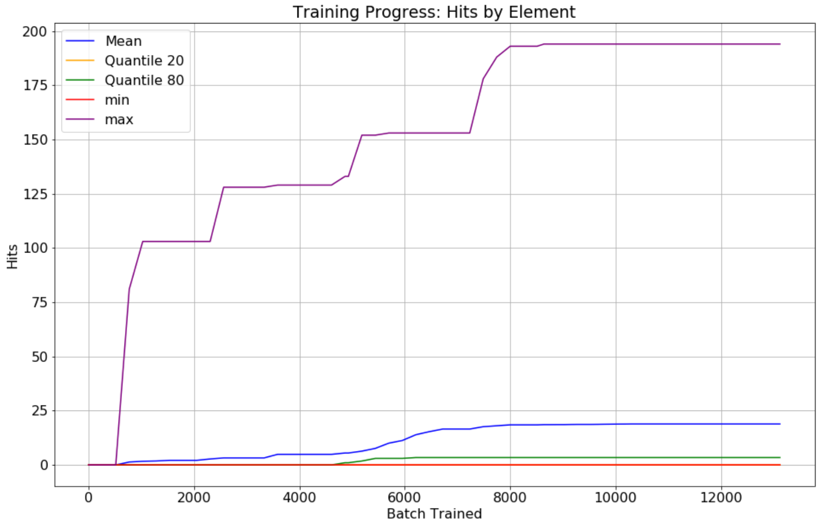


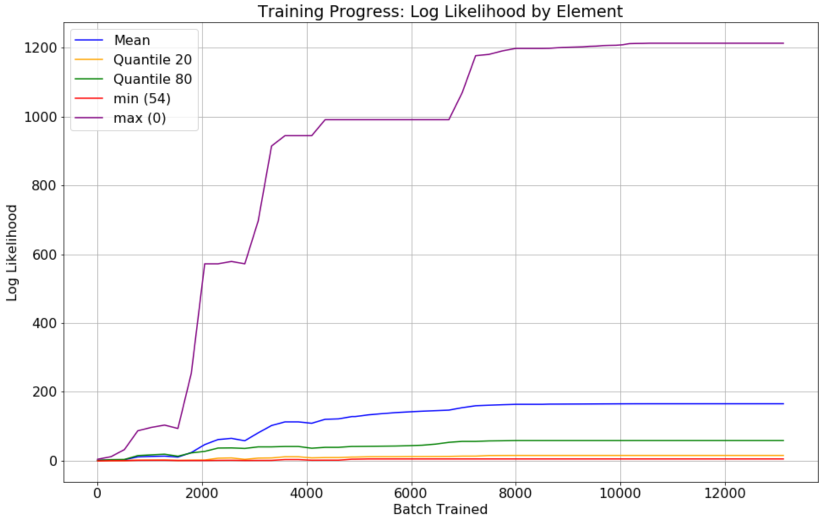
- Do recovered elements match the nearest asteroid?
- 2.6E-4 AU mean distance
- 0.012 covariance norm
- This is still a very good fit on the 42 elements that have been recovered
- This is like your little league coach lobbing the ball over the plate in batting practice…

Train Known Asteroid Elements: Large Perturbation



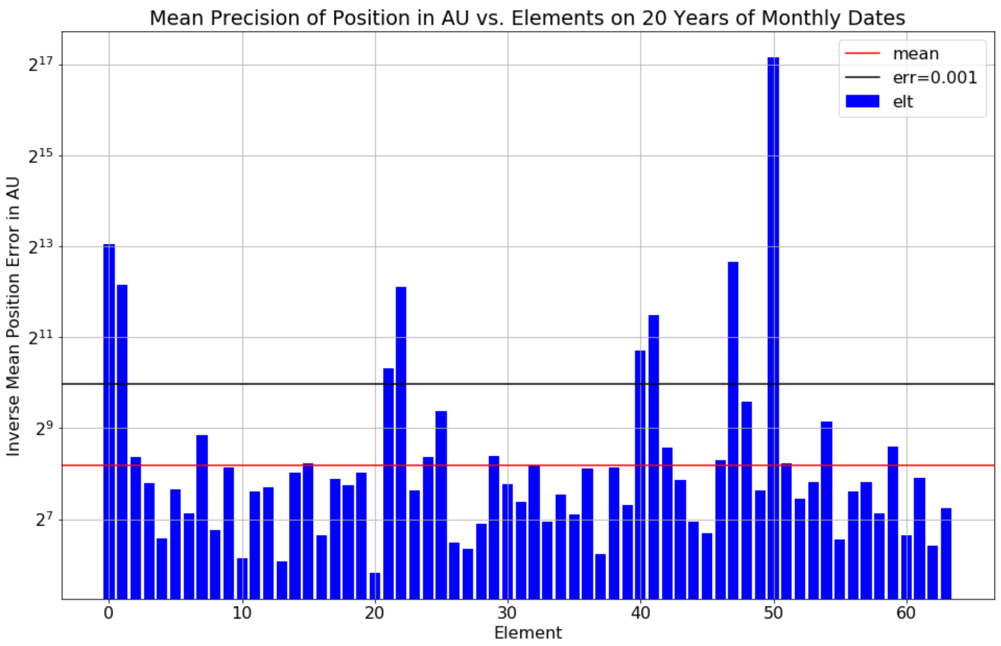


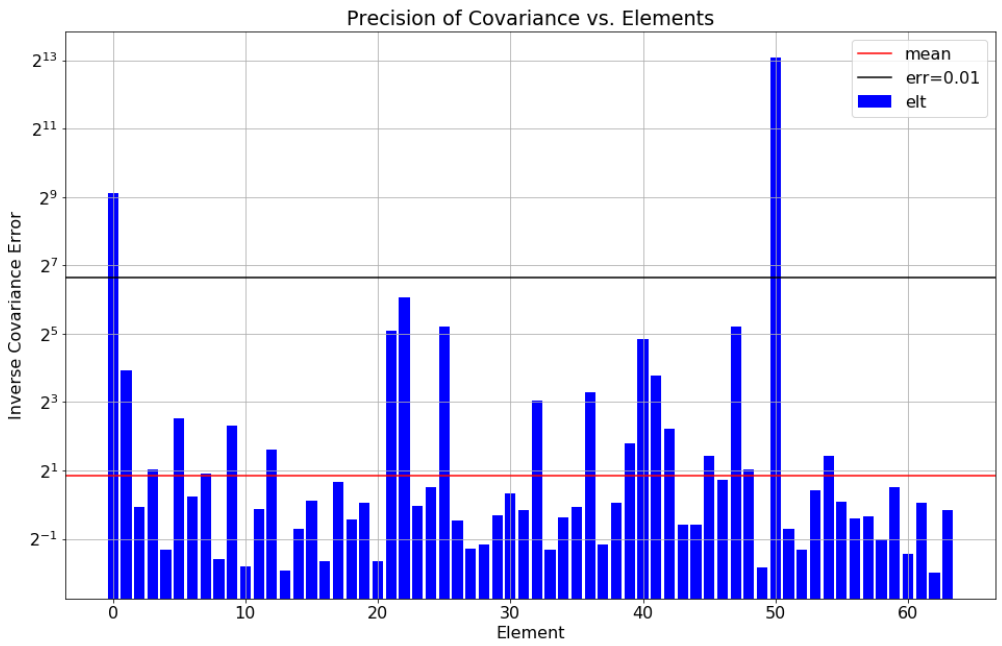




- Small perturbation:
  - 5.0% to *a*
  - 1.0% to *e*
  - 0.25 degrees to *i*
  - 1.0 degrees to Ω, ω, f
- Convergence is decent
- Recovered Elements: 12 (18.8%)
- Hits: 98.2
- Resolution: 32.4 arc seconds
- Log Like: 748
- Many of these elements were perturbed so far the original is no longer even the nearest asteroid!

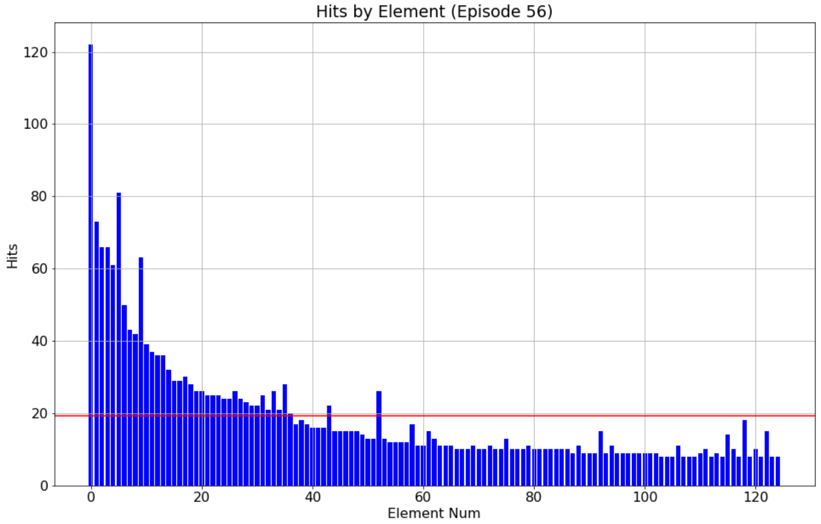
Fit Quality: Large Perturbation

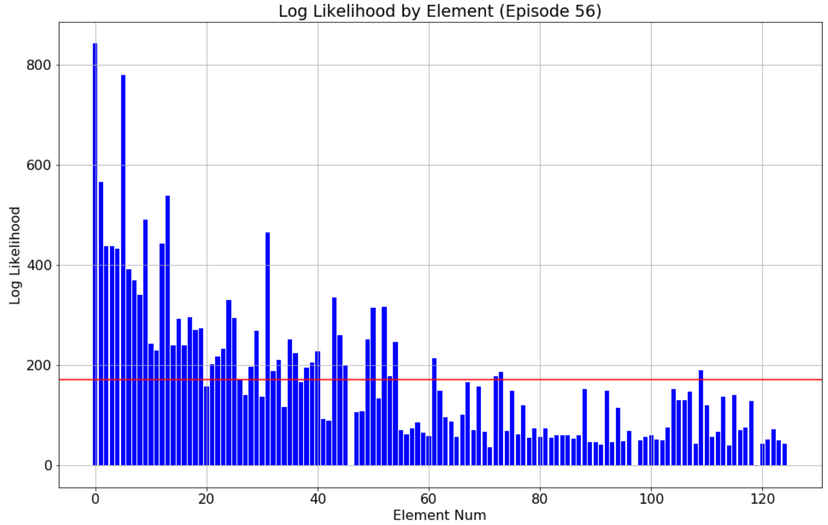


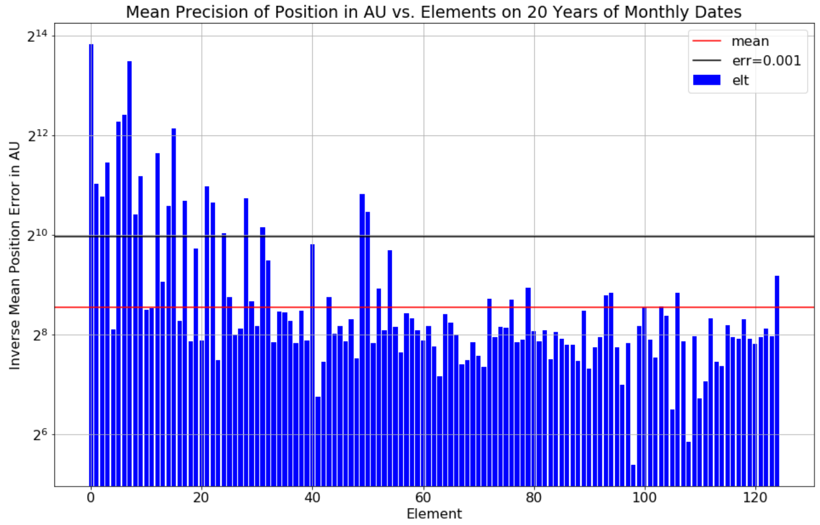


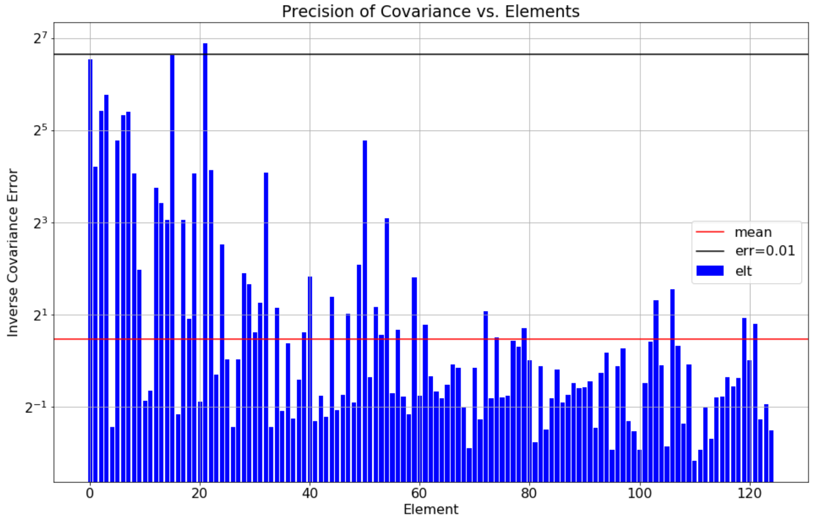
- Do recovered elements match the nearest asteroid? Quite well.
- 4.5E-4 AU mean distance
- 0.032 covariance norm
- This is a decent fit on the 12 elements that have been recovered
- This is a lot harder than the last task-like facing a high school pitcher…

Search Known Asteroids with Random Initializations



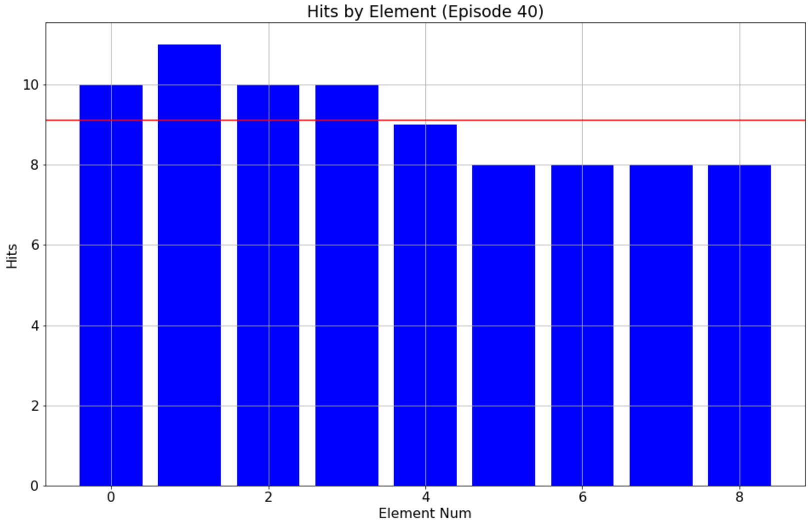


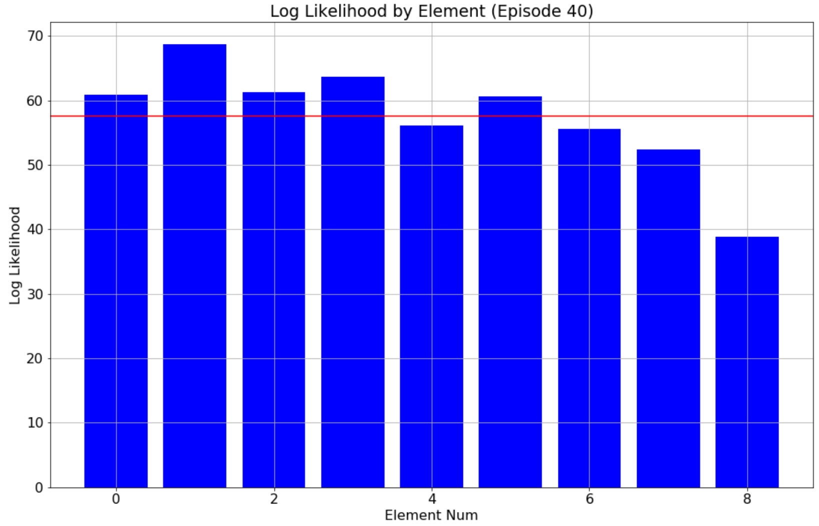


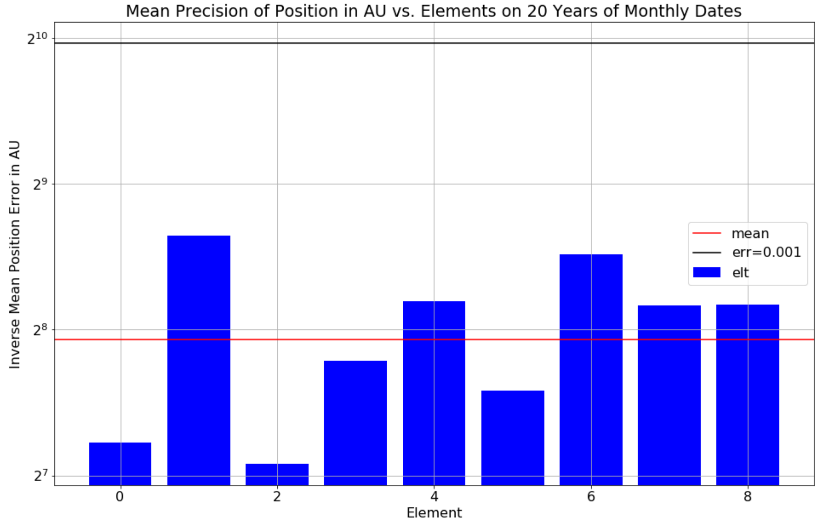


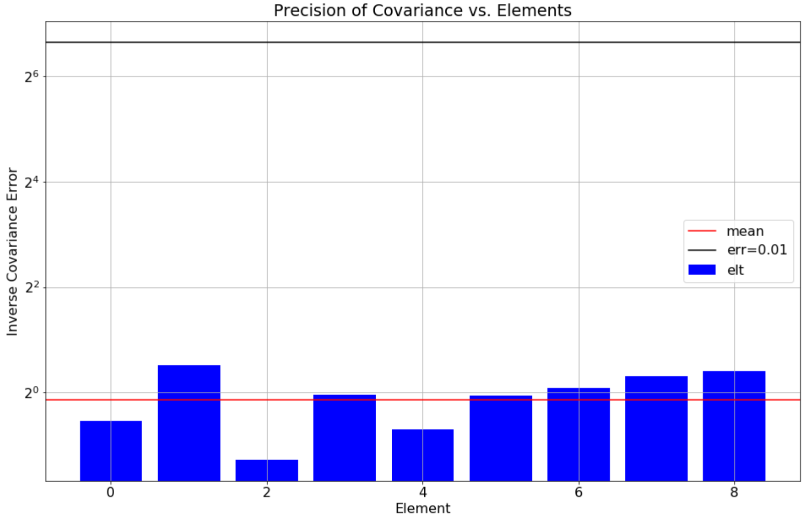
- ~4096 random seeds
- Each batch started with 1024 random elements
- Selected best 64 by mean log(v)
- Trained for ~5 days on 4 GPUs
- Reported fits with ≥ 8 hits and resolution < 20 arc seconds
- Recovered Elements: 125
- Hits: 19.2
- Resolution: 9.3 arc seconds
- Nearest Asteroid Distance: 2.66E-3 AU
- Nearest Asteroid Cov. Norm: 0.73
- Comments on fit quality:
  - Decent fit on some
  - Probably spurious on others
  - Overall shows that this can work
  - But not ready for production
- Baseball analogy continued: facing Roger Clemens, but trying to get hit by the pitch to get on base cheaply

Search Unknown Asteroids with Random Initializations









- 4096 random seeds
- Each batch started with 1024 random elements
- Selected best 64 by mean log(v)
- Trained for ~5 days on 4 GPUs
- Reported fits with ≥ 8 hits and resolution < 20 arc seconds
- Recovered Elements: 9
- Hits: 9.1
- Resolution: 5.3 arc seconds
- Nearest Asteroid Distance: 4.10E-3 AU
- Nearest Asteroid Cov. Norm: 1.10
- Comments on fit quality:
  - Pretty good resolution
  - But not many hits
  - Not surprising: searching for new asteroids!
- Would prefer to see greater differentiation in distance to nearest known asteroid vs. previous run

Presenting 9 New Asteroid Candidates

element_id	a	e	inc	Omega	omega	f	epoch	num_hits	R_sec	thresh_sec	num_rows_close	log_like
178421	3.160327	0.089064	0.153620	2.668766	4.773995	-0.463213	58600.0	10.996569	3.773662	350.229950	15.0	68.716019
3308	3.026962	0.119945	0.129409	3.903006	4.525979	4.819063	58600.0	9.996422	3.821569	351.971649	14.0	63.716438
44117	2.935863	0.187419	0.124516	3.166528	1.230836	-3.122138	58600.0	9.994445	5.225097	357.466431	16.0	61.298466
170789	2.735335	0.152867	0.403704	6.038029	3.016815	-3.443415	58600.0	9.996418	7.083936	347.111053	13.0	60.860794
113970	2.897024	0.068932	0.209250	5.663728	3.868474	4.450756	58600.0	7.999145	3.958026	348.778046	9.0	60.584415
45801	2.754677	0.047293	0.118126	3.139070	5.767782	-1.949476	58600.0	8.996888	4.321019	326.387482	14.0	56.090981
50775	2.374712	0.100280	0.165483	4.280114	5.955170	2.995417	58600.0	9.948814	9.705722	809.804260	29.0	55.641151
96507	2.820351	0.068964	0.080233	2.222034	0.960931	-2.630966	58600.0	8.821539	3.905472	262.245087	12.0	52.416988
191915	2.315446	0.192885	0.057156	2.130249	2.865086	-4.122024	58600.0	7.982198	9.668445	388.469055	21.0	38.837540

ZTF Hits for Selected Asteroid Candidates

element_id	ObjectID	mjd	ra	dec	mag_app	s_sec
178421	b'ZTF18aboluox'	58430.166620	346.704046	-10.675142	15.787200	1.260372
178421	b'ZTF18aboluox'	58430.170313	346.704079	-10.675160	15.800000	0.982009
178421	b'ZTF18aboluox'	58430.166620	346.704001	-10.675028	15.587700	1.699850
178421	b'ZTF18aboluox'	58430.170313	346.704073	-10.675099	15.654600	1.175734
178421	b'ZTF18acewaex'	58863.138472	67.229666	17.295098	19.328100	6.219038
178421	b'ZTF18acewaex'	58863.138472	67.229789	17.295055	19.343500	5.789846
178421	b'ZTF18acewaex'	58863.152465	67.229835	17.295083	19.283501	2.389730
178421	b'ZTF18acewaex'	58863.152465	67.229783	17.295077	19.263100	2.392794

element_id	ObjectID	mjd	ra	dec	mag_app	s_sec
3308	b'ZTF18abtpdzg'	58670.439884	354.565806	-8.964600	17.605200	3.286346
3308	b'ZTF18abtpdzg'	58670.440336	354.565888	-8.964426	17.601101	3.888707
3308	b'ZTF18abtpdzg'	58670.462604	354.565787	-8.964429	16.858500	0.697156
3308	b'ZTF18abtpdzg'	58670.463056	354.565909	-8.964536	17.128700	1.329765
3308	b'ZTF18abtpdzg'	58670.462604	354.565824	-8.964445	17.092899	0.837276
3308	b'ZTF18abtpdzg'	58670.463056	354.565809	-8.964412	16.658600	0.769468
3308	b'ZTF18abspkzw'	58863.110058	346.642277	1.047679	16.541800	8.282505
3308	b'ZTF18abspkzw'	58863.109606	346.642261	1.047839	16.946899	7.604068

element_id	ObjectID	mjd	ra	dec	mag_app	s_sec
191915	b'ZTF18abtxtgd'	58430.170313	341.181109	-12.234676	19.693501	0.036218
191915	b'ZTF18abtxtgd'	58430.166620	341.181092	-12.234595	19.260099	2.621655
191915	b'ZTF19abtsqmn'	58899.139884	94.436593	22.583914	18.914301	6.521414
191915	b'ZTF19abtsqmn'	58899.140336	94.436579	22.583982	18.900700	6.623162
191915	b'ZTF19abtsqmn'	58899.192569	94.436720	22.583899	19.819700	0.946245
191915	b'ZTF19abtsqmn'	58899.222442	94.436672	22.583855	19.873501	4.829147
191915	b'ZTF19abtsqmn'	58899.220162	94.436545	22.583886	20.234699	4.129297
191915	b'ZTF19abtsqmn'	58899.220613	94.436587	22.583902	20.670900	4.263810

element_id	ObjectID	mjd	ra	dec	mag_app	s_sec
170789	b'ZTF17aaaqwwg'	58903.113588	84.725827	15.284655	19.437901	7.917067
170789	b'ZTF17aaaqwwg'	58903.116806	84.725856	15.284678	19.699699	7.043546
170789	b'ZTF17aaaqwwg'	58903.129097	84.725897	15.284685	20.146000	3.737848
170789	b'ZTF17aaaqwwg'	58903.126840	84.725899	15.284678	19.689501	4.333303
170789	b'ZTF17aaaqwwg'	58903.128194	84.725824	15.284668	19.916201	3.767049
170789	b'ZTF17aaaqwwg'	58903.149248	84.726023	15.284693	19.143600	3.802050
170789	b'ZTF17aaaqwwg'	58903.149699	84.725870	15.284728	19.707399	4.209481
170789	b'ZTF17aaaqwwg'	58903.151053	84.725889	15.284665	19.904301	4.672150

- Should we believe these new asteroid candidates? Look at ZTF hits to decide.
- Element `178421` has 4 hits on `ZTF18aboluox` and 4 hits on `ZTF18acewaex`, made 433 days apart
  - Magnitudes are too different (4), spurious connection of 2 different objects
- Element `3308` has 6 hits on `ZTF18abtpdzg` and 2 hits on `ZTF18abspkzw`, made 193 days apart
- Element `191915` has 6 hits on `ZTF19abtsqmn` and 2 hits on `ZTF18abtxtgd`, made 469 days apart
  - Magnitudes are compatible: The model has made a non-obvious connection on compatible tracks!
- Element `170789` has 8 hits on `ZTF17aaaqwwg` made in a 55 minute interval
  - Model agrees with ZTF that this is one track for the same object

Conclusions

- Prove that asteroid search over orbital elements works
  - Need an adequate initialization and representation in data set
- Built a working prototype in TensorFlow
  - First demonstration of efficient astrometric computations on GPU?
- High quality integration of the Solar System and astrometric directions
  - Associated each of 5.7E6 ZTF observations to nearest asteroid
  - 4.2E12 interactions, possibly novel and useful data set
- Proposed candidate orbital elements for 9 new asteroids
- Proof of concept for an automated pipeline to search for new asteroids

Future Work I: Initialization, More Data

- Intelligent initialization of candidate elements
  - Random initialization was just a quick and dirty placeholder; ran out of time
  - ZTF ObjectID is a great starting point for initializations
  - Provisionally assume that all the detections belong to same object
  - Build least squares fit for candidate element
  - Code mostly there now; modify AsteroidSearch with new loss function
- Add a second data set
  - ZTF is great, but it only dates back to July 2019
  - Want to add a second data source
  - Ideally this should have an ML pipeline to classify probable asteroid hits
  - Failing that, can use any data set with a real-bogus classifier, then subtract known stars and galaxies
  - Is Pan-STARRS a good choice?
  - Advice from astronomers on my Committee would be welcome here!

Future Work II: Magnitude, Automated Pipeline

- Incorporate magnitude into log likelihood
  - Have prototype to predict magnitude and incorporate it
  - It was too finicky, needed to turn it off to get first version working on time
- Develop an automated pipeline
  - Initial goal: Accurately rebuild a large fraction of the asteroid catalogue
  - ZTF alone has over 100,000 asteroids with 10 or more hits in the data
  - Plausible that with intelligent initialization, we can recover many of these
  - With a second data set, we could really go far
  - An automated process that can accurately recreate the known catalogue…
  - …is also an automated process that can provisionally classify new asteroids!

Mille Grazie: Thank you for Your Attention!

- Questions?
- Comments?
- Suggestions?