2.25

1) H_0: X & Y are independent.

$$\Rightarrow \hat{\mu}_{ij} = \frac{n_{i+}\, n_{+j}}{n}$$

a) $$\sum_i \hat{\mu}_{ij} = \sum_i \frac{n_{i+}\, n_{+j}}{n} = \frac{n_{+j}}{n} \sum_i n_{i+} = \frac{n_{+j}}{n} \cdot n = n_{+j}$$

recall that $\sum_i n_{ij} = n_{+j}$, so we also have

$$\sum_i \hat{\mu}_{ij} = n_{+j}.$$

b) $$\frac{\hat{\mu}_{11} \cdot \hat{\mu}_{22}}{\hat{\mu}_{12}\, \hat{\mu}_{21}} = \frac{\frac{n_{1+}\, n_{+1}}{n} \cdot \frac{n_{2+}\, n_{+2}}{n}}{\frac{n_{1+}\, n_{+2}}{n} \cdot \frac{n_{2+}\, n_{+1}}{n}} = 1.$$

2.29.

2)

recive serum-ionized calcium

	Y	N	
treatment group	7	8	15
control group	0	15	15
			30

$$P(7) = \frac{\binom{15}{7}\binom{15}{0}}{\binom{30}{7}} = .00316$$

H_0: indep. $\Leftrightarrow$ $H_0: \theta = 1$, $H_A: \theta > 1$

P-value $= P(n_{11} \geq 7) = P(7) = .00316$

$\Rightarrow$ $\alpha >$ P-value, Reject H_0, that is, there is a strong evidence of better results by treatment than control group.

3) 2.33

a)

defendant's Race	Victim's race	death Penalty Y	death Penalty N
white	white	19	132
	Black	0	9
Black	white	11	52
	Black	6	97

b) I. white Victim

defendant's Race	death Penalty Y	death Penalty N
white	19	132
Black	11	52

$$\hat{\theta} = \frac{19.5\ (52.5)}{11.5\ (132.5)} = 0.67$$

II. Black Victim

def. Race	death Penalty Y	death Penalty N
white	0	9
Black	6	97

$$\hat{\theta} = \frac{.5\ (97.5)}{6.5\ (9.5)} = .7895$$

I. Since $\hat{\theta} < 1$, therefore a black defendant is more likely to have a death Penalty than a white defandant when the Victim is white.

II. Similarly, when the victim is black, again $\hat{\theta} < 1$, so a black defendant is more likely to have a death penalty than a white defendant.

Therefore, overall a black defendant is more likely to have a death penalty than a white defendant.

2.33

3) continue

c)

defandant's Race	Death Penalty Y	N
white	19	141
Black	17	149

$$\hat{\theta} = \frac{19 \times 149}{17 \times 141} = 1.18$$

Since $\hat{\theta} > 1$, therefore a white defendant is more likely to have a death penalty than a black defendant without considering the victim's race.

Hence, the Simpson's paradox exists, because the results and conclusions obtained from the conditional and marginal odds ratio "contradict" each other.

3.1

4) The link function specifies the function of the mean of the response that is predicted by the linear predictor in GLM.

The identity link function models the binomial probability as a linear function of the predictors. It is not a good choice, because probabilities have to be between 0 and 1, but stright lines provide predictions that can be any real number. However, the logit link would fix this issue.

3.9

5)

a) $\text{logit}(\hat{\pi}) = \hat{\alpha} + \hat{\beta} x$, $x \equiv$ income

$\log\left(\frac{\hat{\pi}}{1-\hat{\pi}}\right) = -3.556 + 0.0532x$

where π is the probability of possessing a travel credit card.

b) Because $\hat{\beta} = 0.0532 > 0$, therefore the estimated probability of having a travel credit card increases as annual income increases.

c) When, $\hat{\pi} = .5$, then $\text{logit}(\hat{\pi}) = \log\left(\frac{\hat{\pi}}{1-\hat{\pi}}\right) = \log\left(\frac{1/2}{1-1/2}\right)$
$= \log(1) = 0$

or, $\hat{\alpha} + \hat{\beta} x = 0$

$\Rightarrow x = \frac{-\hat{\alpha}}{\hat{\beta}} = -\frac{(-3.556)}{0.0532} = 66.86$.

3.11

6)

a) $\log \mu = \alpha + \beta x \Rightarrow$ $x = 1 \leftarrow$ treatment B : μ_B
$x = 0 \leftarrow$ treatment A : μ_A

so

$\log(\mu_B) = \alpha + \beta(1) = \alpha + \beta$
$\log(\mu_A) = \alpha + \beta(0) = \alpha$

$\log(\mu_B) - \log(\mu_A) = \log\left(\frac{\mu_B}{\mu_A}\right)$
$= \alpha + \beta - \alpha = \beta$.

$\Rightarrow e^{\beta} = e^{\log\left(\frac{\mu_B}{\mu_A}\right)} = \frac{\mu_B}{\mu_A}$

6) continue see the R codes of output in the slides.

b)
$$\log(\hat{\mu}) = 1.609 + 0.588\, x$$

From part (a) we have $\beta = \log\left(\frac{\mu_B}{\mu_A}\right)$

or: $e^{\beta} = \frac{\mu_B}{\mu_A}$, so $e^{.588} = 1.8 = \frac{\mu_B}{\mu_A}$

That is, the mean of treatment B 80% is higher than the mean of treatment A.

c) $H_0: \mu_B = \mu_A \iff H_0: \frac{\mu_B}{\mu_A} = 1 \iff e^{\beta} = 1$

$\iff H_0: \beta = 0.$ v.s. $H_A: \beta > 0$

Wald test $\quad Z = \frac{\hat{\beta}}{S.E(\hat{\beta})} = \frac{.588}{.176} = 3.33$

p-value $= P(Z > 3.33) < .001$

LRT $\quad G^2 = 26.86 - 16.27 = 11.6 \quad df = 1.$

P-value $< .001$.

$\alpha >$ P-value $\Rightarrow$ reject H_0 $\Rightarrow$ treatment B has higher defect rate.

3.11

6) continue

d) 95% CI for β: $\hat{\beta} \pm Z_{\alpha/2}\, S.E(\hat{\beta})$

$$.588 \pm 1.96\,(0.176)$$

$$\Rightarrow (.2416,\ .9343)$$

Note that $\beta = \log \frac{\mu_B}{\mu_A}$

$\Leftrightarrow e^{\beta} = \frac{\mu_B}{\mu_A}$, so the C.I. for $\frac{\mu_B}{\mu_A}$ is given by

$$(e^{.2416},\ e^{.9343}) \Leftrightarrow (1.27,\ 2.54).$$

3.12.

7)

a) $\log(\hat{\mu}) = \alpha + \beta_1 x + \beta_2 Z$

$$= 1.72 + 0.588X - .229Z$$

This model is fitted with no intraction. see the R code.

The conditional effect of treatment is: $e^{.5878} = 1.8$

That is, controling for thickness, the estimated mean number of defects in B is 80% higher than in A. whenease, $e^{-.2296} = .79$, i.e., controlling for treatment, the estimated mean number of defects is 21% lower with thick coating than with thin coating. However, this effect is not significant. (See the P-value)

8)

a) See the R code & out put. We have

$$\log(\hat{\mu}) = \hat{\alpha} + \hat{\beta} x$$
$$= -0.4284 + .5893\, x$$

b) $\hat{\mu} = e^{\log(\hat{\mu})} = e^{-.4284 + .5893(2.44)} = 2.74$

c) C.I for β: $\hat{\beta} \pm Z_{\alpha/2} \cdot SE(\hat{\beta})$

$$.5893 \pm 1.96\,(0.065)$$
$$\Rightarrow (0.4619,\ 0.7167)$$

Note that $\mu = e^{\alpha + \beta x} = e^{\alpha} (e^{\beta})^{x}$

So, the C.I. for e^{β} is : $(e^{+.4619}, e^{.7167}) \equiv (1.59, 2.05)$

i.e., the estimated mean number of satellites increase between 59% and 105% for each aditional kg increase in weight

d) $Z = \dfrac{\hat{\beta}}{S.E(\hat{\beta})} = \dfrac{.5893}{0.06502} = 9.064$ with P-value $\simeq 0$.

So the effect of the weight is strongly significant.

8) continue

e) The LR test statistics can be found from the null deviance and the residual deviance as follows

$632.79 - 560.87 = 71$, $df = 1$

P-value $= P(\chi^2_{(1)} > 71) \cong 0$

which supports the results of part d.

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9) 3.16

a) No, because for both groups Male & female the sample variance is way higher than the sample means. that is, ther is a clear evidence of overdispersion.

b) Since the poisson model doesn't take into account the overdispersion, therefore giving an unreasonable small SE.

c) The negative binomial C.I. is more appropriate. The Poisson C.I is too optimastic as it does not take into account the overdispersion.

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10). a) True b) False