Department of Informatics, University of Zurich

BSc Thesis

Implementing an Index Structure for Streaming Time Series Data

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D B T G

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Abstract

A streaming time series is an unbounded sequence of data points that is continuously extended. The data points arrive in a predefined interval (e.g. every 5 minutes).
Such time series are relevant to applications in diverse domains. Imagine a meteorology station that sends a temperature measurement every 3 minutes or imagine a trader in the financial stock market who receives updated pricing information every 5 minutes.
We present an implementation of an index structure for streaming time series data. The system keeps a limited amount of time series data in main memory. As a result, it is able to access the information of past measurements in time window W. We introduce an implementation using two data structures, a circular array and a B^+tree, to efficiently access the data of past measurements.

Zusammenfassung

Kontinuierliche Zeitreihen werden durch neu ankommende Daten unbegrenzt erweitert. Die Daten werden in einem vordefinierten Intervall aktualisiert.
Derartige Zeitreihen sind relevant für diverse Bereiche. Beispielsweise in der Meteorologie, in welcher die Wetterinformationen kontinuierlich aktualisiert werden oder, um einen weiteren Bereich zu nennen, in Finanzmärkten, wo die Händler auf die neusten Preisinformationen angewiesen sind.
Wir präsentieren die Implementation einer Indexstruktur für kontinuierlich erweiterte Zeitreihen. Unser System behält eine limitierte Anzahl an vergangenen Daten im Arbeitsspeicher. Daraus resultiert, dass das System auf vergangenen Daten zugreifen kann. Dazu stellen wir unsere Implementation vor, welche von zwei Datenstrukturen Gebrauch macht: einem zirkulären Array und einem B^{+}baum. Die beiden Datenstrukturen erlauben den effizienten Zugriff auf alle Werte der vergangenen Daten, welche sich noch im Zeitfenster befinden.

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1. Introduction

A streaming time series is an unbounded sequence of data points that is continuously extended, potentially forever. The data points arrive in a predefined interval (e.g. every 5 minutes). Such time series are relevant to applications in diverse domains. Imagine a meteorology station that sends a temperature measurement every 3 minutes or imagine a trader in the financial stock market who depends on updated pricing information. Thus, various applications need to be fed continuously with the latest data.
Our system neither forgets about and deletes all past measurements nor keeps all of them, since a system can only keep a limited size of data in main memory. But because a portion of the data is kept, it is still possible to access past data. The system provides data structures and operations to efficiently access the time point and the value of a measurement.
The thesis proposes an implementation for an index structure for streaming time series data. In order to achieve efficient access to the time points and values of measurements, the system uses two data structures: A circular array in combination with a B^+tree. The circular array has a limited size. The measurements are stored in the array, sorted by time. The B^+tree as well contains the same measurements with time points in a predefined sliding time window. The leaves of the tree are sorted from left to right by the measurement value. Thus, range queries can be efficiently performed. For every new measurement the time window slides forward. A value drops out of the window and a new value is inserted to the circular array and the B^+tree. If a measurements time point is not in the sliding window any more, it is deleted from both data structures. The introduced system is not only implemented but also analysed in terms of runtime complexity. An experimental evaluation tries to underpin the theoretical results.

At the beginning of this thesis, in chapter 2, the TKCM algorithm designed by Wellenzohn et al.[1] is introduced. To achieve a better understanding for the system context and for the requirements our system must satisfy. Chapter 3 describes the context and introduces the operations that our system must be able to perform. In chapter 4, our approach is represented and its advantages are discussed. Further, the pseudo-code for implementing the system is outlined. After that, the runtime and space complexity of the system is described in chapter 5. Followed by an experimental evaluation of the implemented system in chapter 6. Finally, the thesis is summed up in chapter 7.

2. Background

A streaming time series is not always gapless. Due to sensor failures or transmission errors, values can get missing. To efficiently recover missing values, Wellenzohn et al.[1] present the Top-k Case Matching algorithm (TKCM). The algorithm and its connection to this thesis is introduced in Section 2.1.

2.1. TKCM

TKCM monitors a set of streaming time series and imputes missing measurement values if it detects any. It defines a two-dimensional query pattern over the most recent values of a set of time series. The idea is to derive a missing value in time series s from the *k* most similar past pattern. For each *time series* s a set of highly correlated *reference time series* are determined, such that TKCM is able to recover a missing value in streaming time series data.

Definition 2.1.1 *(Time Series) Let S be a set $S = \{s_1, s_2, ...\}$ of streaming time series. The measurement value of time series $s \in S$ at time* t *is denoted as $s(t)$. For base time series s, let $R_s = \langle r_1, r_2, ...\rangle$ be an ordered sequence of the time series $r_i \in S \setminus \{s\}$. The set of reference time series for s, R_s^d, at the current time $\bar{t}$ are the first d time series in R_s for which $r(\bar{t}) \neq$ NIL. The time points in a streaming time series s are in time window W. The measurement value of a time series $s \in S$ at time point t_i is denoted as $s_i(t)$. Hence, a measurement is represented by the tupel $(t, s_i(t))$.*

To be able to recover a missing measurement value, TKCM retains a sliding window of the streaming time series in main memory.

Definition 2.1.2 *(Sliding Window) Let $W = [\underline{t}, \bar{t}]$ be a sliding window of length $|W|$. Time $\underline{t}$ represents the oldest time point that fits into the time window and time $\bar{t}$ represents the most recent time point for which the stream produced a new value.*

To impute a missing value, TKCM defines a query pattern $Q(\bar{t})$ over the most recent values of the reference time series. Afterwards, TKCM looks for the *k* most similar patterns in the sliding window.

	j=1	j=2	j=3	
i=1	16.1	16.3	16.5	r_1
i=2	17.1	17.0	17.2	r_2
	14:10	14:15	14:20	

Figure 2.1.: Example of a query pattern Q(t) of length $l = 3$ and $d = 2$ reference time series.

Definition 2.1.3 *(Pattern) Let $R_s^d = \{r_1, ..., r_d\}$ be the ordered set of reference time series for a time series s. A $pattern$ $P(t)$ of length $l > 0$ over R_s^d that is embedded at time t is defined as a $d \times l$ matrix $P(t) = [p_{i,j}]_{d \times l}$ where $p_{i,j} = S_i(t - j + l)$. $P(t)$ is a set of pattern cells $p_{i,j}$, such that $p_{i,j} \in P(t)$.*

The two-dimensional query pattern $Q(\bar{t})$ is anchored at a time point $\bar{t}$ and consists of the subsequence of length l spanning from ($\bar{t} - l + 1$ to $\bar{t}$) of each reference time series. Each row represents a subsequence of a reference time series and each column represents the values of the reference time series at a time point.
Only the values in the streaming time series with time points in time window W are kept in main memory. $\forall t : \underline{t} \leq t < \bar{t} \rightarrow s(t) \neq$ NIL since *s* contains imputed values if the real ones are missing.
TKCM initializes one neighborhood $N(q_{i,j})$ around each $q_{i,j} \in Q(\bar{t})$. The algorithm continuously looks up new candidate pattern $P(t)$ in the time window $t \in W$ by *growing* the neighborhoods $N(q_{i,j})$ until it found the k most similar patterns to $Q(\bar{t})$. It remembers the time points t in a set T to avoid considering these patterns again.

Definition 2.1.4 *(Next Pattern) The next pattern for neighborhood $N(q_{i,j})$ is pattern $P(t) = [p_{i,j}]_{d \times l}$ anchored at time point $t = \underset{t \in W \setminus T}{\operatorname{argmin}} |p_{i,j} - q_{i,j}|$*

TKCM must not only impute missing values, but also process the new measurements efficiently. Therefore, TKCM must provide an insertion method for new arriving values to insert them into a streaming time series s. Since the time window has a limited, given size $|W|$, one value has to be deleted from the time series data for each new arriving value. Provided that, the time series data in window W is already completely filled.

2.2. Access Methods

The Next Pattern (Def. 2.1.4) is built on top of two access methods.*Sorted* access is used to find the most similar value to a given query pattern cell $q_{i,j}$ (Def. 2.1.3) and *random* access retrieves the values to fill the remaining pattern cells. The two methods are defined as follows:

Definition 2.2.1 *(Sorted Access) Sorted access for a neighborhood $N(q_{i,j})$ returns the next yet unseen measurement $(t, s_i(t))$ where $t = \underset{t \in W \setminus T}{\operatorname{argmin}} |s_i(t) - q_{i,j}|$.*

Definition 2.2.2 *(Random Access) Random access returns value r(t), given time series r and time point t.*

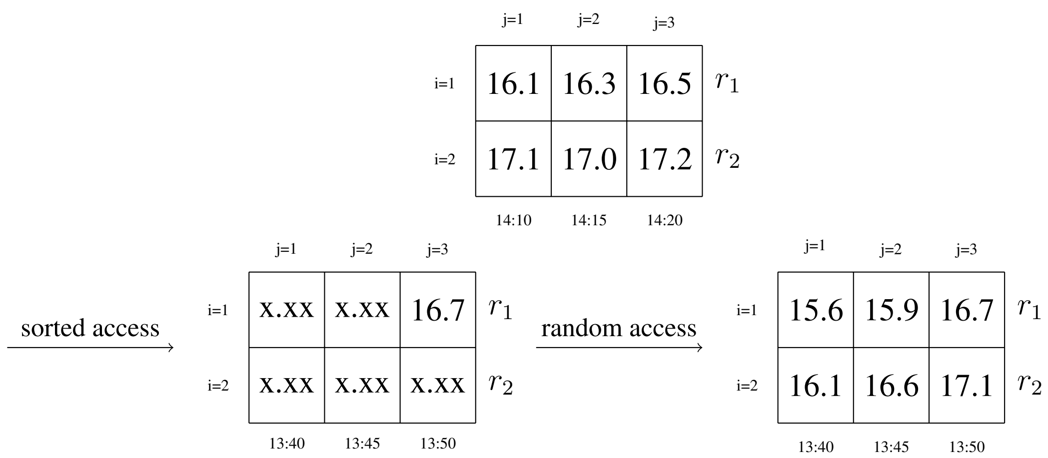


Figure 2.2.: Pattern for query pattern cell $q_{1,3}$.

The next pattern search is presented in algorithm 1. TKCM initializes a set $T = \{\}$. The set is filled during execution with all time points *t* for which a pattern P(t) has been processed. Using the sorted access mode, the algorithm finds the next yet unseen time point $t \notin T$ for which the value is most similar to a given value in a query pattern cell $q_{i,j}$. The random access mode is used to look up the values that pattern $P(t)$ is composed of.

Algorithm 1: NextPattern

Input: Neighborhood $N(q_{i,j})$ and seen time points T
Output: Next pattern $P(t')$

```
P ← matrix of size d × l
(t, s_i(t)) ← sortedAccess(N(q_{i,j}),T)
P[i,j] ← s_i(t)
foreach q_{x,y} ∈ Q(t̄) \ q_{i,j} do
    P[x,y] ← randomAccess(s_x, t − y + j)
end
return P anchored at time t − j + l
```

3. Problem Definition

The present thesis introduces an implementation of the $random$ and $sorted$ access method for a streaming time series s. The access modes are described in the previous Section 2.2. The required operations and the context for our system are presented in the following.

3.1. Context

We make the following assumptions for our system:

- The measurements arrive in a fixed interval (E.g. every five minutes).
- There are no gaps between the measurements since the gaps are filled before values are inserted in the data structure.
- There are no measurements that arrive out-of-order.

3.2. Operations

The system needs to efficiently perform on the streaming time series s in a sliding window W:

- shift($\bar{t}, v$): add value *v* for the new current time point $\bar{t}$ and remove value *v'* for the time point $\underline{t} - 1$ that just dropped out of time window W.
- sortedAccess($N(q_{i,j}), T$): given a value *v* and a set of time points T, return the time point $t \notin T$ such that $|v - s(t)|$ is minimal (Def. 2.2.1).
- randomAccess(t): return the value of time series *s* at time *t*, denoted by $s(t)$ (Def. 2.2.2).
- newNeighborhood($q_{i,j}, t, j, l$): given a value *v* for the time point t, the pattern length l and the index j, return the new neighborhood N at t.

Wellenzohn et al.[1] suggest a combination of two data structures: a B^+tree and a circular array. The random access operation can be performed by the circular array, while the sorted access operation is executed on the leaves of a B^+tree.
Additional reasons for the data structures are described in Chapter 4. Further, the implementation of the random and sorted access modes using the suggested data structures is presented and a solution for handling duplicate values is proposed.

4. Approach

Each time series $s \in S$ is represented by a circular array. The circular array is kept in main memory. It uses random access to look up value $s(t)$ for a given time t. Further, for each time series s a B^+tree is maintained that is also kept in main memory. The B^+tree is ideal for sorted access by value and therefore for range queries. Both data structures are described in detail in Section 4.1 and Section 4.2, respectively. Figure 4.1 illustrates the data structures. All measurements in the circular array are represented in the B^+tree as well.

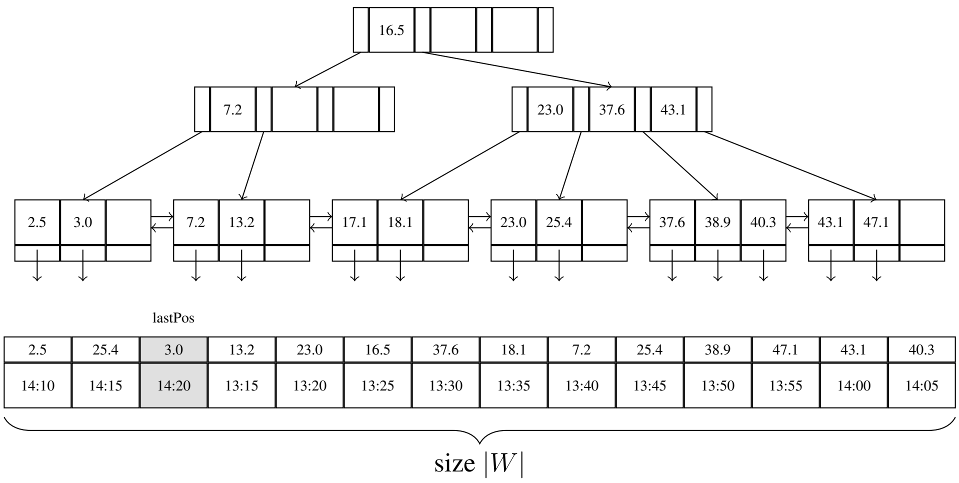


Figure 4.1.: A B^+tree with the related circular array.

4.1. Circular Array

A circular array is used to store the time series data sorted by time and the time interval is predefined. The value and time of a measurement is directly stored in the circular array. The size of the array is determined by $|W|$ and represents the capacity of the number of measurements that the array may hold. The last update position $lastPos$ is stored in a variable and updated with every insertion. In detail, a circular array contains the following attributes: a counter, which counts the number of measurements in the array, a $lastPos$ to store the position that was last updated with a measurement and the data, which actually holds the time points and values of all measurements.

```
struct Measurement{
  timeStamp time;
```

```
  double value;
};

struct CircularArray{
  Measurement * data;
  int lastPos;
  int count;
};
```

4.1.1. States

The circular array can be in two different states:

1. First, the array is filled with every new arriving measurement until every position in the circular array is occupied as illustrated in Figure 4.2, array 1.

2. Afterwards, if all spaces in the circular array are occupied, the number of measurements stays constant because a value drops out if a new value is inserted in Figure 4.2, array 2.

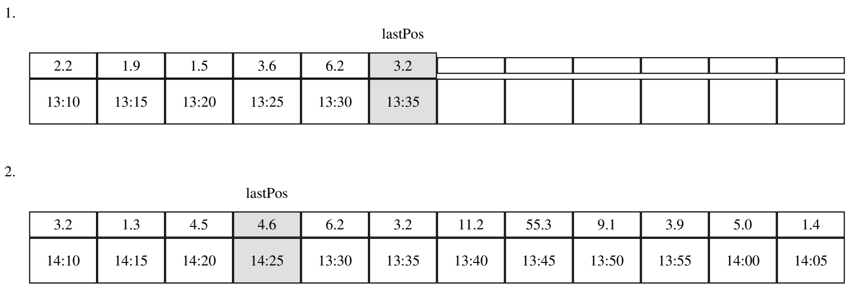


Figure 4.2.: Shifted circular array.

Example 1 *The value at time point 14:25 in array 2 in Figure 4.2 is the newest measurement. Hence, the last update position is at time point 14:25. A new measurement will be inserted at the next position in the circular array. So at the position of the oldest time point 13:30. In order to lookup the value at time point 14:00 we can take advantage of the fixed interval. If the last update position is at time point 14.25, we can directly calculate the position for time point 14:00, using the last update position and the fixed interval.*

4.1.2. Shift$(\bar{t}, v)$

The measurements in a circular array are stored in a defined interval without any gaps in between. Therefore, the value position and insertion can be calculated. The circular array has a $count$ attribute that is equal or smaller to $|W|$, which represents the number of measurements in the array. If the $count$ is equal to $|W|$ one measurement has to be deleted for every arriving measurement. Otherwise, there is no need to delete a value from the tree since no value dropped out of the sliding window W. The insertion of a new measurement is presented in Algorithm 2.

Algorithm 2: Shift($tree, array, \bar{t}, v$)

Input: Tree $tree$, the circular array $array$, the new time point $\bar{t}$ and the new value v

Output: The $array$ such that ($\bar{t}$,v) $\in array$

```
newPos ← 0
if array.count < |W| then
    if array.count ≠ 0 then
        newPos ← (array.lastPos + 1) % |W|
    end
    array.count++
else
    newPos ← (array.lastPos + 1) % |W|
    //delete measurement from tree
    Delete(tree, array.data[newPos].time, array.data[newPos].value)
end
array.data[newPos].time ← t̄
array.data[newPos].value ← v
array.lastPos ← newPos
AddMeasurement(tree, t̄, v)
```

Example 4.1.1 *We assume a new measurement arrives with time point 14:25 and value* 13.2. *The value* 41.5 *at time point 13:15 is replaced by the new measurement in the circular array. Figure 4.3 show the circular array before and after the update.*

		lastPos											
2.5	25.4	3.0	41.5	23.0	17.1	37.6	18.1	7.2	25.4	38.9	47.1	43.1	40.3
14:10	14:15	14:20	13:15	13:20	13:25	13:30	13:35	13:40	13:45	13:50	13:55	14:00	14:05

			lastPos										
2.5	25.4	3.0	13.2	23.0	17.1	37.6	18.1	7.2	25.4	38.9	47.1	43.1	40.3
14:10	14:15	14:20	14:25	13:20	13:25	13:30	13:35	13:40	13:45	13:50	13:55	14:00	14:05

Figure 4.3.: Update of a circular array.

4.1.3. Random Access

Due to the properties of a circular array the random access of a value at time t is very efficient. Since the position can be directly calculated without looping through the array, since the values arrive in a fixed interval (e.g. every 3 minutes). The $RandomAccess$ operation is used to fill a pattern $P(t)$ that is anchored at a time point t. The last update point can be used as reference time point for the calculation. The RandomAccess(t) is presented in Algorithm 3.

Algorithm 3: RandomAccess(t)

```
Input: The circular array array and the time point t
Output: Returns the value v at time point t
if array.count = 0 then
    //no values yet
    return NIL
end
step ← (t - array.data[array.lastPos].time)
if |step| < array.count then
    pos ← (array.lastPos + step)%|W|
    if array.data[pos].time = t then
        return array.data[pos].value
    end
end
return NIL
```

Example 4.1.2 *We want to find the measurement value at time point 14:10 in the circular array shown in Figure 4.4. The last update position was at time point 14:25, thus the step is calculated as* -3 *and the value at position* 0 *with time point 14:10 is* 2.5*. The value is returned by the* $RandomAccess$ *Algorithm.*

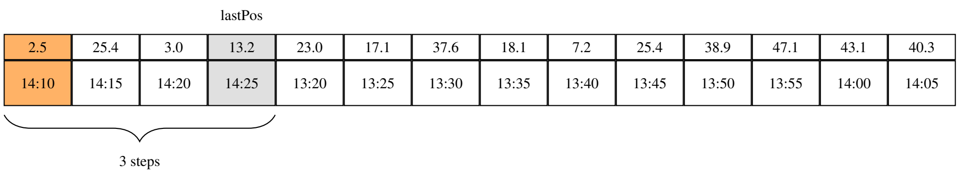


Figure 4.4.: 2.5 is the value at time point 14:10.

4.2. B^+tree

A B^+tree is organized in nodes which include search keys. The search key values in every node are kept in sorted order. A B^+tree is a balanced tree in which every path from the root of the tree to a leaf of the tree is of the same length. Each nonleaf node in the tree has between $\lceil n/2 \rceil$ and *n* children, where *n* is fixed for a particular tree. The used B^+tree and the implementation is based on the book of Silberschatz et al.[2]. A B^+tree is able to execute range queries very efficiently, since the leaves of a B^+tree are ordered from left to right and the leaf nodes are linked. Although insertion and deletion operations on B^+trees are complicated, they require relatively few I/O operations. Therefore, the use of B^+trees is popular in data base

systems since I/O operations are expensive. The speed of operations on B^+trees makes it a frequently used index structure in database implementations.
The B^+tree is a useful data structure to efficiently perform the sortedAccess($N(q_{i,j}), T$) operation described in Section 3.2. Because the B^+tree we use has leaves linked in both directions. The Section 4.2.1 presents the structure of the B^+tree for our implementation.

4.2.1. The Structure of the used B^+tree

We introduce the most important properties of a B^+tree, for further information please refer to the book [2].
The differences between the traditional B^+tree and the B^+tree we use are the following: On the one hand, the leaves in our B^+tree are linked to the succeeding as well as the preceding leaf to efficiently perform the sortedAccess($N(q_{i,j}), T$) operation. On the other hand, our B^+tree is able to handle duplicate values. How our tree handles duplicate values is described in Section 4.2.2. The other properties of our used B^+tree are presented in the following.
The parameter n determines the maximum number of pointers in a node and $n - 1$ represents the maximum number of search keys, hence the size of a tree node. The keys in a node are always sorted from left to right.

Example 2 *If n is set to 7, an internal node may have between $\lceil 7/2 \rceil = 4$ and 7 children and between 3 and 6 keys. Because $\lceil n/2 \rceil - 1 = 3$ for $n = 7$. The root may have between 2 and 7 children or if it is the only node in the tree it can have no children and just one key. A leaf node must have at least 3 keys and can have maximum 6 keys.*

A node contains m non-null pointers ($m \leq n$). For $i = 2, 3, ..., m - 1$, pointer P_i points to the subtree that contains search key values less than K_i and greater than or equal to K_{i-1}. Pointer P_m points to the part of the subtree that contains those key values greater than or equal to K_{m-1}.

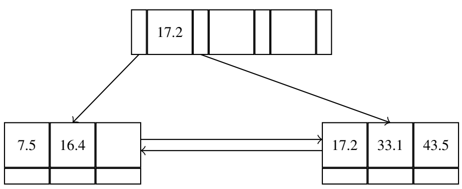


Figure 4.5.: Left children keys < 17.2 and right children keys $\geq$ 17.2.

Example 3 *Pointer P_0 in Figure 4.5 points to the left subtree, where all keys are smaller than 17.2 and pointer P_1 points to the subtree, where all keys greater than or equal to the root key* 17.2.

There are three types of nodes that may exist in a B^+tree: the root, interior nodes and leave nodes.

- (Leaf) A leaf node must have at least $\lceil (n-1)/2 \rceil$ keys and may hold at most $n-1$ keys.

- (Inner Node) The inner nodes can have at most $n-1$ search keys and n pointers, pointing to its child nodes. An inner node must have at least $\lceil n/2 \rceil$ pointers and can hold at most n pointers. Hence, it must have at least $\lceil n/2 \rceil - 1$ keys and at most $n-1$ keys.
- (Root) The root node is the only node that can contain less than $\lceil n/2 \rceil$ pointers. The root node must have at least one search key and two pointers to child nodes, unless the root is a leaf node and hence has no children.

A node contains the following attributes:

```
struct Node{
  struct Node *parent;
  void ** pointers;
  int numOfKeys;
  double * keys;
  bool isLeaf;
  struct Node *prev, *next;
};
```

The node structure can be used for every type of node, since the structure of the root, the inner nodes and the leaf nodes is similar. A $B+$tree is built out of multiple nodes that include: A pointer to the parent, which is *NIL* if there is no parent. Further, pointers to child nodes or in leaves pointers to measurement time points. Besides, the number of keys that a node holds at the moment is stored and of course, the actual keys. Additionally, a boolean is used to represent if a node is a leaf node. A node can have two pointers to the previous and the next node. These two pointers are only used if the node is a leaf node otherwise they are set to NIL.

Observation

The next higher value of a given value v in a B^+tree is at the same time the right neighbor of v and the next lower value is also the left neighbor of v, since the leaves and their keys are sorted from left to right.
Another property of the B^+tree we can exploit is that the path from the root to a leaf node always has the same length, thus, the number of nodes to traverse to find a specific value in a leaf is always the same.

Example 4 *Assume we want to know the most similar value to the key* 40 *in the* B^+*tree illustrated in Figure 4.6. We just have to compare* 40 *to two values, namely the right neighbor* 43 *and the left neighbor* 38.9*. Hence, we find out that* 38.9 *is the most similar value in the entire* B^+*tree.*

Application to our System

The B^+tree described above contains search keys. In our case these search keys are the measurement values in our circular array. A value in time series s can occur multiple times because

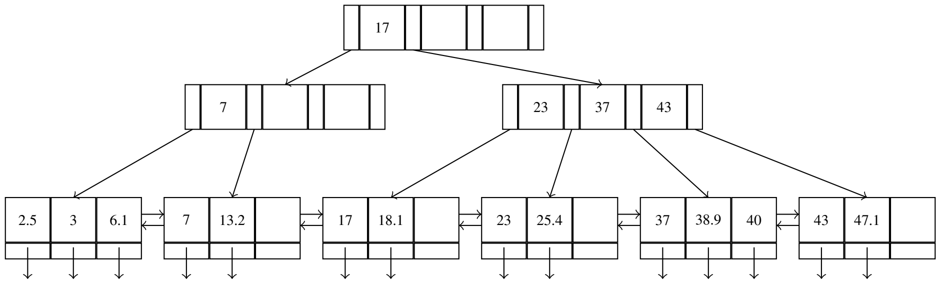


Figure 4.6.: Example of a complete B^+tree.

the values are not unique and since the values are used as search keys, the B^+tree must be able to handle possible duplicates. Section 4.2.2 proposes an approach that allows to use duplicate values in a B^+tree and it explains how the time points of measurements are stored in the tree.

4.2.2. Handling Duplicate Values: Associated Circular Doubly Linked List

This Section presents our approach to handle duplicate values in our B^+tree, in regard to our requirements. The idea is to associate a doubly, circular linked list to each key in leaf nodes. Cormen et al.[3] define three types of linked lists.

1. (Singly Linked List) A singly linked list can either be sorted in order of the keys or it can be unsorted. The order is determined by a pointer in every linked list element e. $e.next$ points to the successor element in the list. The first element, or $head$, of the list has no predecessor and the last element, or $tail$, has no predecessor.

2. (Doubly Linked List) A doubly linked list element has an additional pointer which points to the predecessor, namely $e.prev$. Each element of a doubly, linked list is an object with an attribute key and two pointer attributes: $next$ and $prev$.

3. (Circular Doubly Linked List) In a circular doubly linked list the $prev$ pointer of the head of the list points to the tail and the $next$ pointer ot the tail points to the $head$. If the element e is the only element in the list the pointers $next$ and $prev$ point to e itself.

We use a doubly, circular linked list for our system. For clarity, we name the circular doubly linked list in the following only linked list. The measurement time points represent the key of a list element. Every search key in a leaf node of our B^+tree has an associated linked list. If the same measurement value is added multiple times to the time series s, the time points of the measurements are added to the linked list associated to the search key. So instead of inserting the key again and using another position in the leaf, the new time point is inserted to the associated linked list.
The oldest value in a list, so the lowest time point, always is the element connected by a pointer from the leaf key to the list. Even though the linked list not literally has an end and

a beginning, we name the time point associated to the leaf the $head$ and we call the heads predecessor the $tail$.

Example 5 *Figure 4.7 illustrates the leaf level of a B^+tree and linked lists. It shows that the oldest time point, here 14:15, is connected to the tree and the newest time point, 14:50, the tail, is the predecessor. Also, the Figure illustrates that a single element in a linked list is linked to itself. The higher levels of the B^+tree and the additional linked lists are left away for clarity.*

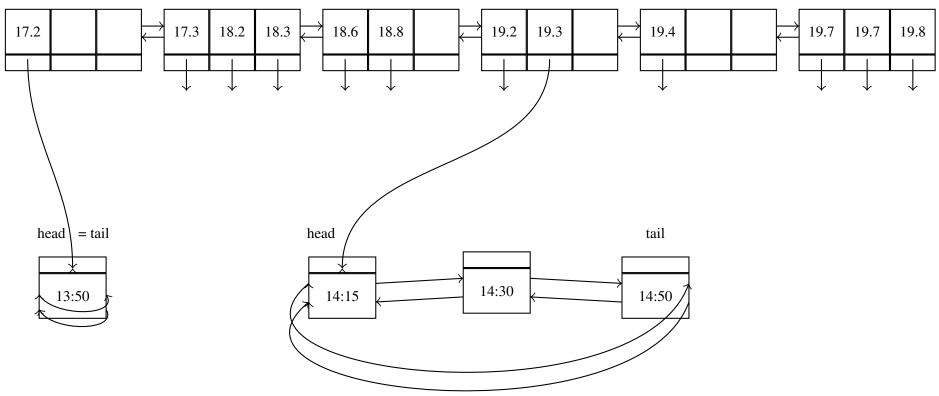


Figure 4.7.: Circular doubly linked lists associated to leaf nodes.

As described above, the leaf nodes in our B^+tree also have pointers, namely pointers to the associated linked list. The number of pointers in a leaf node is always equal to the number of search keys in the leaf. Hence, a pointer at position i points to the linked list associated with the leaf key at position i.
A linked list element consists of the time point and the pointers to the predecessor $prev$ and to the successor $next$. This can be represented as follows:

```
struct Element{
  timeStamp timestamp;
  struct Element *prev, *next;
};
```

Observation

The linked list associated to the measurement value stores the appropriate time point. The newest measurement with time point $\bar{t}$ that is added to an existing linked list always has a newer time point compared to all other time points in the same list and also compared to the other measurements in the tree. Because $\forall t : \underline{t} \leq t < \bar{t}$ holds for all time points in the tree. Consequently, a new time point is always inserted at the tail position. Therefore, the linked list is always sorted by the time from head to tail. A $shift$ operation on the circular array leads to a deletion of the oldest measurement in time series s, thus, the measurements time point, as explained, is always at the head position. Also, a new measurement can be inserted without looping through the list. It is always added to the tail position.

The insertion and the deletion of a list value from a linked list that contains multiple values is illustrated in Algorithm 4 and Algorithm 5, respectively.

Algorithm 4: AddNewTail$(node, i, t)$

Input: Leaf $node$, the index position i to the linked list L and the time point to insert t
Output: Linked List L such that $t \in L$
head ← node.pointers[i]
tail ← head.prev
newElement = CreateElement(t)
//insert new linked list element between head and tail
head.prev = newElement
tail.next = newElement
newElement.prev = tail
newElement.next = head

Algorithm 5: DeleteHead$(node, i)$

[a]**Input**: Leaf $node$ and index position i for the position of the associated Linked List L
Output: Linked List L such that $t \notin L$
head ← node.pointers[i]
nextElement ← head.next
prevElement ← head.prev
leaf.pointers[i] ← nextElement
prevElement.next ← nextElement
prevElement.prev ← prevElement

[a]Algorithm 5 is only used for linked lists that contain multiple elements.

4.3. B^+tree Operations

The data in the circular array is updated with every arriving measurement. Therefore, every $shift$ execution updates the array. The operation not only influences the array data but also the B^+tree. Therefore, the deletion and insertion of a measurement in the tree is executed within the update of the circular array.
The deletion and the insertion in a B^+tree are described in Section 4.3.2 and Section 4.3.3, respectively.

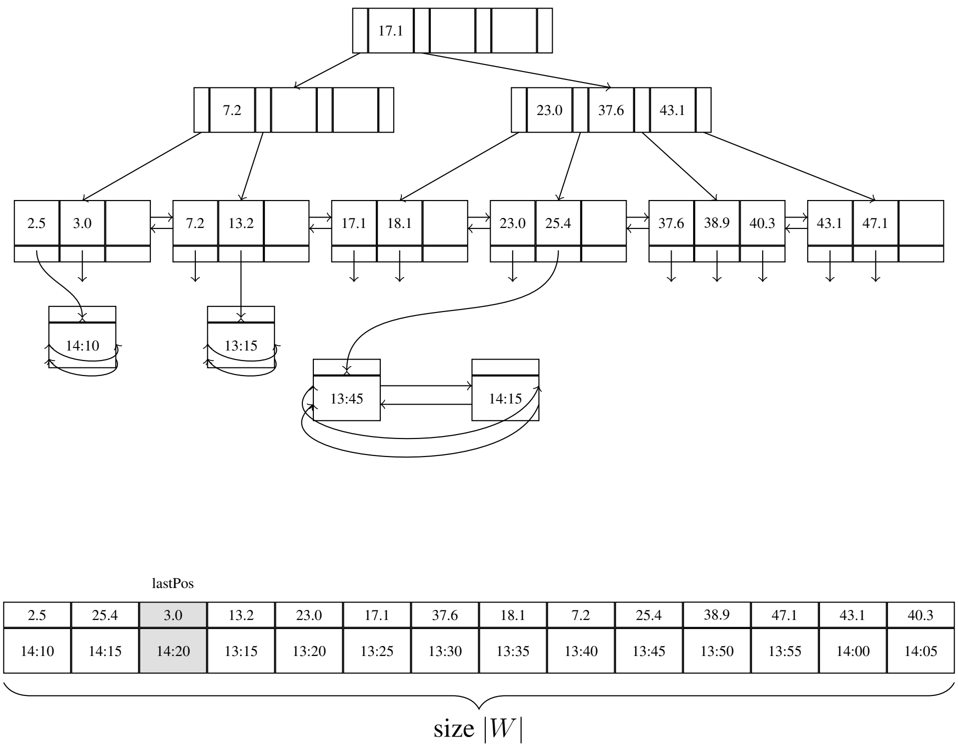


Figure 4.8.: Start situation

Example 6 *We assume we have the situation illustrated in Figure 4.8. Value* 25.4 *occurs two times. Hence, the associated list contains two elements. Further, all leaf keys have an associated time point, but some linked lists are not illustrated to improve clarity. The size of window* W *and the circular array is* $|W| = 14$*. The array is already full, hence for every new arriving measurement a value has to be added to the circular array and the* B^+*tree and another one has to be deleted from both data structures.*

4.3.1. Search in a B^+tree

Before we can delete or add a measurement, we have to find the right leaf. Algorithm 6 presents the pseudo-code to find the appropriate leaf. The method starts at the root of the tree and traverses the tree until it reaches the appropriate leaf node that would contain the value. The current node is examined by looking for the smallest i for which the search key value k is greater or equal to. The current node is updated to the child node at pointer P_i. This procedure is repeated until a leaf node is reached.

Algorithm 6: FindLeaf($tree, k$)

Input: Tree $tree$ and the search key k
Output: The appropriate leaf for the search key k

curNode ← tree.root
if *curNode = NIL* **then**
    return NIL
end
while *curNode is no leaf* **do**
    Let $i \leftarrow$ smallest number such that $k \leq$ curNode.K_i
    if *no such i exists* **then**
        $m \leftarrow$ last non-null pointer in the node
        curNode ← curNode.pointers[m]
    else
        curNode ← curNode.pointers[i]
    end
end
return curNode

Example 7 *We assume we want to find the key* 13.2 *in the* B^+*tree illustrated in Figure 4.8 because this was the value that dropped out of the sliding window* W.
The current node is examined by looking for the smallest i *for which the search key value* 13.2 *is greater or equal to. In this case, the first pointer comes from the root at index position* 0, *since* 13.2 *is smaller than* 17.1. *Then the new current node is set to the child node at pointer position* 0 *which includes the search key* 7.2. *Afterwards, the current node is updated again to the node at pointer position* $i = 1$, *hence* P_1. *Since the new current node is a leaf node, the algorithm returns the leaf node.*

4.3.2. Deletion in the B^+tree

In case there has been a measurement value that dropped out of the sliding window W, we first have to delete this measurement from the tree, before we can add a new one. We first present the deletion, because it is normally executed before the insertion.
First, the leaf containing the measurement to delete is located with Algorithm 6. Since our B^+tree accepts duplicates, it is afterwards checked as illustrated in Algorithm 7 if the associated linked list to the key has multiple linked list elements. If the linked list has multiple elements, the identified element is deleted and the deletion is already finished, as presented in Algorithm 5. If the entry time point is the single value in the linked list the leaf key and its belonging list is deleted.

Algorithm 7: Delete($tree, t, k$)

Input: Tree $tree$, the measurement time point t and key k
Output: Tree $tree$ such that t,k $\notin tree$

leaf ← findLeaf($tree$, k)
i ← position index in leaf such that keys remain ordered
if *list on pointer i has multiple elements* **then**
  DeleteHead(leaf, i)
else
  DeleteEntry(tree, leaf, leaf.keys[i])
end

The $DeleteEntry$ method illustrated in Algorithm 8 is called if a measurement is the only linked list element. A node has to be at least half-full to remain in the same state after a deletion. In detail, the minimum number of keys depends on the node properties. If the node is a leaf node it contains at least $\lceil (n-1)/2 \rceil$ keys. If the node is an internal node the minimum number of keys is $\lceil n/2 \rceil - 1$ and thus the minimum number of pointers is $\lceil n/2 \rceil$.
Hence, all nodes have to be half-full after is removed from a node. There are three possible cases to achieve a state where the B^+tree properties described in Section 4.2.1 are still satisfied. Hence, the tree must be still balanced and all nodes must be at least half-full after a deletion.

1. (The node is at least half-full) The node has still enough keys. Hence, the measurement is deleted and afterwards the algorithm stops. Figure 4.9 illustrates the case where the leaf has still enough keys.

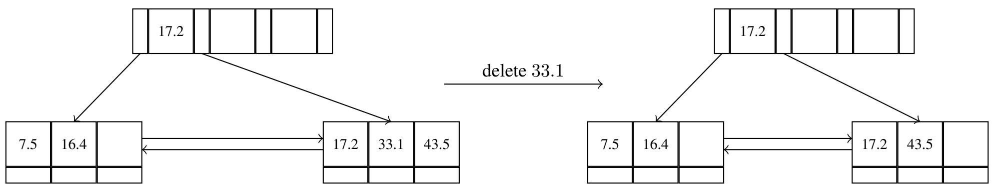


Figure 4.9.: The node has still enough keys.

2. (The values in a node are redistributed) Provided that, the node is not half-full any more and the node cannot be merged with a sibling, the nodes have to be redistributed. To ensure that each node is at least half-full and hence contains the minimum number of keys. Merging is not possible, if the sibling and node together have more than the allowed n pointers. Since inner nodes that have m keys have $m+1$ pointers, the nodes can only be merged if the sum of all keys in both nodes is smaller than the tree node size $n-1$. Leaf nodes can be merged if the sum of all keys in both leaves is smaller than n. Thus, if the keys and pointers do not fit into one node, the keys have to be redistributed. We redistribute the keys, such that each interior node has at least $\lceil n/2 \rceil - 1$ keys and

each leaf node has at least $\lceil n - 1/2 \rceil$. Therefore, we move the rightmost pointer from the left sibling to the under-full right sibling. Consequently, we also need to replace the key in the parent to ensure the parent pointer to the new organised node is still separable from the other pointers. Figure 4.10 illustrates the case where some keys have to be redistributed. The Algorithm 13 for the redistribution can be found in Appendix Section A.1.

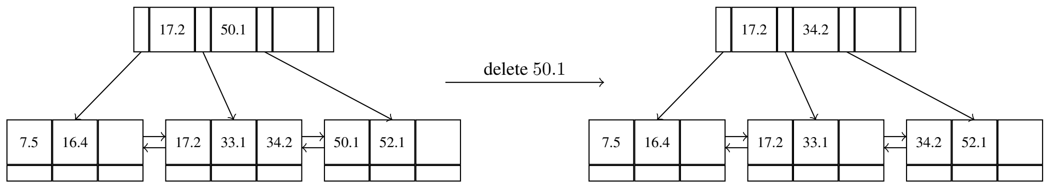


Figure 4.10.: Some entries must be redistributed.

3. (The node is merged with a sibling) Provided that, the keys and pointers of the node and its sibling fit into a single node and the node is not half-full after removing the entry, we merge them. We move the entries from the right sibling into the left sibling, and deleting the now empty right sibling. If there is no left sibling the right sibling is selected to receive the additional entries. Once a node is deleted, we must also delete the entry in the parent node that pointed to the deleted node. This is done by calling $DeleteEntry$ again. We traverse the tree recursively upwards until $DeleteEntry$ stops. Figure 4.11 illustrates the case where the node can be merged with a sibling. The Algorithm 14 for the redistribution can be found in Appendix Section A.1.

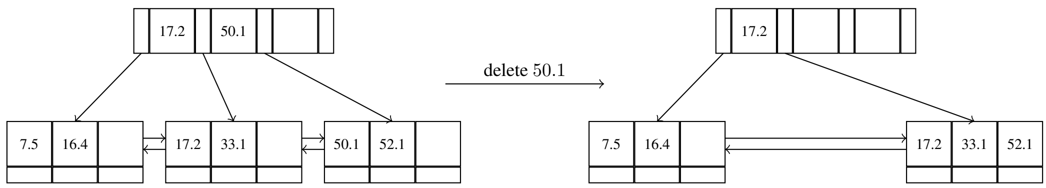


Figure 4.11.: The node can be merged with its sibling.

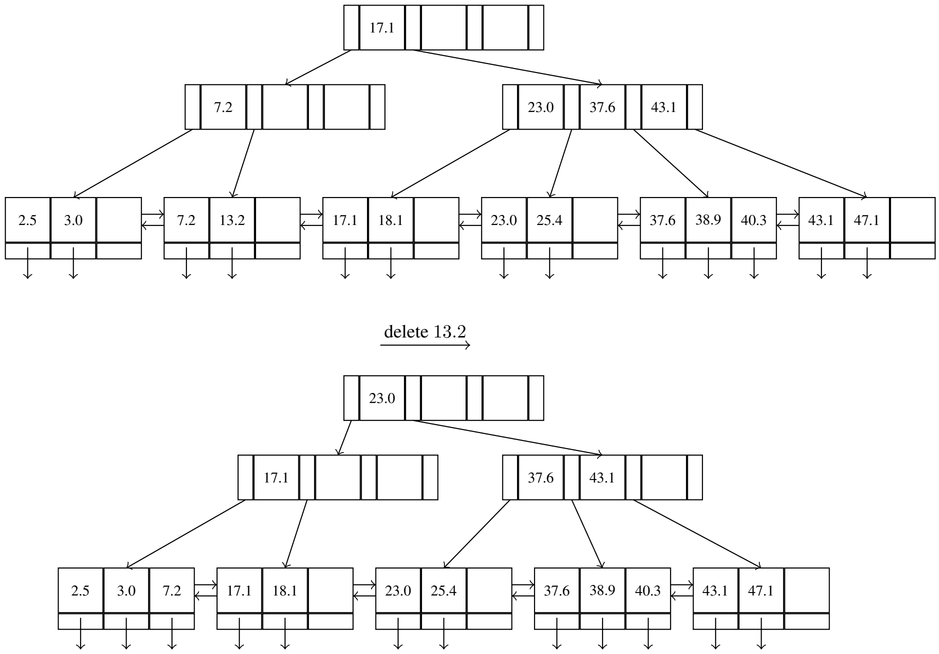


Figure 4.12.: Deleting 13.2 from the B^+tree.

Example 4.3.1 *We again refer to the example in Figure 4.8. The measurement with time point 13:15 dropped out of the sliding window* W*. Therefore, it needs to be deleted. First, the* $findLeaf$ *method finds the leaf that contains* 13.2*. Then it deletes the value and its associated linked list. The leaf after has just* 1 *key left and therefore is smaller than the minimum allowed keys of* $2 = \lceil (n-1)/2 \rceil$*. Since* $1 < \lceil (4-3)/2 \rceil$*. The left neighbor has still enough space for the only key left in the node, namely* 7.2*. The node is merged with its left sibling. As a consequence, the key in the parent is not correct any more, since* 7.2 *now is part of the left child. Key* 7.2 *in the parent is removed as well by calling* $DeleteEntry$ *recursively. Then the* $DeleteEntry$ *procedure checks weather this node can be merged with its sibling. Since the node has one pointer left to its now only child and the sibling has already three keys,* $23.0, 37.6, 43.1$ *and hence 4 pointers. So* $1+4$ *is more than the allowed* 4 *pointers in an inner node. As a consequence, the keys have to be redistributed. The node takes the root node key* 17.1 *as a new key and adds the leftmost child of the sibling. This is the leaf node with the keys* 17.1 *and* 18.1*. The root node takes the leftmost key of its right children* 23.0*. The right children now has the keys* 37.6 *and* 43.1 *left. The tree before and after deleting* 13.2 *is illustrated in Figure 4.12.*

Algorithm 8: DeleteEntry$(tree, node, k, pointer)$

Input: Tree $tree$, the node $node$ where the deletion key belongs to and k the key to delete
Output: The node $node$ such that $k \notin node$

node $\leftarrow$ remove k and associated linked list from $node$
if *node is the root* **then**
    AdjustRoot$(tree)$
    return
end
if *node is leaf* **then**
    minNrOfKeys $\leftarrow \lceil (n-1)/2 \rceil$
else
    minNrOfKeys $\leftarrow \lceil n/2 \rceil - 1$
end
if *minNrOfKeys* $\leq$ *number of keys left in node* **then**
    //node has still enough keys
    return
end
//node has not enough keys - merge or rearranges necessary
neighborIndex $\leftarrow$ get position of left sibling in parent or -1 if no left sibling
if *neighborIndex = -1* **then**
    kIndex $\leftarrow$ 0
    neighbor $\leftarrow$ node.parent.pointers[1]
else
    kIndex $\leftarrow$ nIndex;
    neighbor $\leftarrow$ node.parent.pointers[nIndex]
end
//innerKeyPrime is the value between pointers to $node$ and $neighbor$ in parent
innerKeyPrime $\leftarrow$ node.parent.pointers[kIndex]
capacity $\leftarrow$ n-1
if *node is a leaf* **then**
    capacity $\leftarrow$ n
end
//Merge if both nodes together have enough space
if *(neighbor.numOfKeys + node.numOfKeys) < capacity* **then**
    MergeNodes(tree, node, neighbor, nIndex, innerKeyPrime)
else
    RedestributeNodes(tree, node, neighbor, nIndex, kIndex, innerKeyPrime)
end

4.3.3. Insertion in the B^+tree

We now describe the insertion of a measurement to the B^+tree. The general technique for insertion into a B^+tree is to first determine the leaf node into which insertion must occur with the $FindLeaf$ method. If a split results, insert the new node into the parent of the leaf node. If this insertion causes a split, traverse recursively up the tree until either a new root is created or an insertion does not cause a split any more. More detailed, after the right leaf has been found, there are three possible consequences for our B^+tree: The key already exists in the leaf, the key does not exist yet and the leaf has still enough space or the key does not exist yet but the leaf is already full, since the $numOfKey = n - 1$.

Algorithm 9: AddMeasurement($tree, t, v$)

```
Input: Tree tree, the new time point t and the new value v
Output: The tree such that t, v ∈ tree
//the tree does not exist yet - create tree
if tree.root = NIL then
    newTree(tree, t, v)
    return
end
leaf ← findLeaf(tree, v)
//insert to leaf as linked list value
if key already exists in leaf then
    i ← getInserPoint(tree, leaf, v)
    AddNewTail(leaf, i, t)
else if leaf.numOfKeys < n-1 then
    //enough space for new key value pair
    InsertRecordIntoLeaf(tree, leaf, t, v)
else
    SplitAndInsertIntoLeaves(tree, leaf, t, v);
end
```

1. (Search key already exists in the leaf) If the search key already exists in the leaf node, the time point of the measurement is added to the already existing associated list and the search keys in the leaf remain unchanged. In this case, refer to Algorithm 4.

2. (Search key does not exist yet and the leaf has enough space) We insert the entry to the node such that the search keys are still in order and a new linked list is allocated where the measurement time point is inserted.

3. (Search key does not exist yet but the leaf has not enough space) In general, we take the n search key values (the values $n-1$ in the leaf node plus the value being inserted to the right position such that the keys remain ordered) and put the first $\lceil n/2 \rceil$ in the existing node and the remaining values in a newly created node. If a node is split, we consequently must insert the new leaf node into the B^+tree structure and re-link the leaves. We need to insert an entry with the leftmost key of the new leaf node, and a pointer to

the new node, into the parent of the leaf node that was split. If there is no room, the parent must be split, requiring an key to be added to its parent. In the worst case, all nodes in the path to the root must be split. If the root itself is split, the entire tree becomes one level deeper.Splitting of a nonleaf node is a different from splitting of a leaf node. If there is no space in the parent node to add a new key, and the parent node has to be split, the parent node is expanded temporarily, the entry is added, and the overfull node is then split. When an nonleaf node is split, the child pointers are divided among the original and the newly created node. The key values are handled a little differently if a inner node is split. The key values that lie between the pointers moved to the right node are moved along with the pointers, while those that lie between the pointers that stay on the left remain unchanged. However, the search key value that lies between the pointers that stay on the left, and the pointers that move to the right node is treated differently. This search key is not added to either of the two nodes. Instead, it is added to the parent node. The Algorithms for splitting a leaf node or an inner node can be found in Appendix Section A.2.

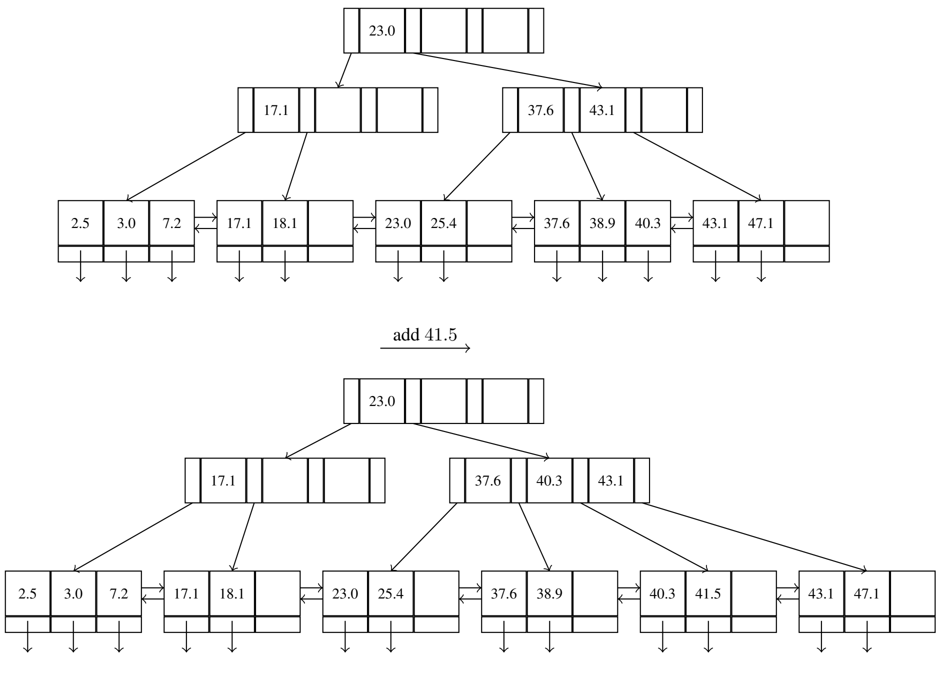


Figure 4.13.: B^{+}tree after the insertion of 41.5.

Example 4.3.2 *We again consider the example from the beginning after the deletion of* 13.2 *(Fig. 4.12). The value* 41.5 *has been inserted to the circular array and hence must be added to the* B^{+}*tree. The new tree after inserting the new measurement is illustrated in Figure 4.13. This measurement belongs to a leaf node which is already full. Hence, the leaf is split and the search key is passed to the parent and we find out that the parent is full as well. Therefore, the parent is split too. After, we see that the parent of the inner node is not full yet. The parent is* *at the same time the root node. The leftmost key in the new node is inserted to the root. We see that the insertion is recursive from the leaves to the root until a node with enough space is found. If the root node is full as well, the root node is split too and a new root node would be allocated. But in our case the root node just gets a new key. The child nodes are split and the new key is just inserted to the root node.*

4.4. Neighborhood

The sortedAccess($N(q_{i,j}), T$) method in Algorithm 11 uses the B^+tree to return the time point $t \in T$ such that $|v - s(t)|$ is minimal, given a value *v*. But before a neighborhood can grow it has to be initialized. The neighborhood initialization is described in Section 10.

4.4.1. Initialize Neighborhood

The Algorithm 10 initializes a new neighborhood. A neighborhood consists of the following attributes:

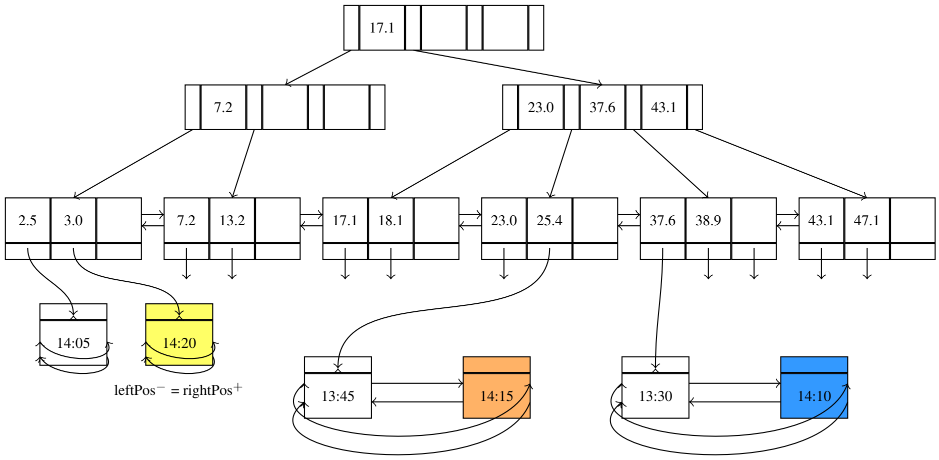
```
struct NeighborhoodPos{
  Element * timeStampPos;
  Node * LeafPos;
  int indexPos;
};

struct Neighborhood{
  int l;
  int j;
  double key;
  NeighborhoodPos leftPos;
  NeighborhoodPos rightPos;
};
```

A neighborhood $N(q_{ij})$ is defined around each value q_{ij} of a query pattern $Q(t)$, where leftPos$^-$ and rightPos$^+$ are neighborhood positions of type $NeighborhoodPos$ in leaves of the B^+tree. They represent the left and right border of the neighborhood. Initially the neighborhood positions are set to the position of q_{ij} in the B^+tree. In total $d \times l$ neighborhoods are initialized. Every time series r in query pattern $Q(t)$ initializes l neighborhoods. The index j represents the position of the measurement within the query pattern. If $j = 1$ the oldest time point in the query pattern is meant and if $j = l$, thus j is equal to the pattern length, the latest time point in the query pattern is meant.

Example 8 *If $d = 1$ just one time series is part of query pattern $Q(t)$. Figure 4.14 illustrates an example of a query pattern with $d = 1$. In total l neighborhoods are initialized in time series r_1. The Figure 4.15 shows the circular array and the B^+tree for the time series r_1 after initialization.*

	j=1	j=2	j=3	
i=1	37.6	25.4	3.0	r_1
	14:10	14:15	14:20	

Figure 4.14.: Query Pattern Q(t) of length $l = 3$ and $d = 1$ reference time series.



37.6	25.4	3.0	13.2	23.0	17.1	37.6	18.1	7.2	25.4	38.9	47.1	43.1	2.5
14:10	14:15	14:20	13:15	13:20	13:25	13:30	13:35	13:40	13:45	13:50	13:55	14:00	14:05

Figure 4.15.: The circular array and the B^+tree for time series r_1 after initialization.

Algorithm 10: NewNeighborhood$(q_{i,j}, t, j, l)$

Input: Tree $tree$, the query pattern value $q_{i,j}$ and its time point t and position j and the pattern length l

Output: The initialized neighborhood N

```
N.key ← q_{i,j}
N.j ← j
N.l ← l

leafNode ← FindLeaf(tree, q_{i,j})
pointerIndex ← GetInsertionIndex(leafNode, q_{i,j})
e ← leafNode.pointers[pointerIndex]

//Upper Bound: The value is at most pattern length away form first list value
maxSteps ← l
while e.timestamp ≠ t && maxSteps ≠ 0 do
    //go from newest value back towards oldest
    e ← e.prev
    maxSteps−−
end
N.leftPos ← set position to e
N.rightPos ← set position to e
return N;
```

4.4.2. Sorted Access

After the initialization of the $l \times d$ neighborhoods the k non-overlapping patterns are calculated. Thus, the sortedAccess$(N(q_{i,j}), T)$ operation is executed until k patterns are retrieved.
Sorted access searches the time point for the next most similar value to a query pattern cell $q_{i,j}$ of the query pattern $Q(t)$. What most similar means will be explained later. The neighborhood $N(q_{i,j})$ is expanded until such a value is retrieved. Some time points may have to be skipped because the time point anchored a pattern previously. The next unseen most similar value to $q_{i,j}$ is either at position t^- or t^+, which represent the time points to the direct left or right of leftPos$^-$ and rightPos$^+$. If the t^- is more or equally similar to $q_{i,j}$, this significances $|s_i(t^-) - q_{i,j}| \leq |s_i(t^+) - q_{i,j}|$. Than t^+, the leftPos$^-$ is decremented and $t = t^-$.

Example 9 *Figure 4.16 shows how the neighborhood $N(q_{1,2})$ is expanded. We assume that the other neighborhoods $N(q_{1,1})$ and $N(q_{1,3})$ has not been initialized yet. Hence, there are no additional time points anchored for a pattern previously and the timeset T is initially empty. If the sortedAccess$(N(q_{i,j}), T)$ is executed 3 times the resulting neighborhood will look as follows:*

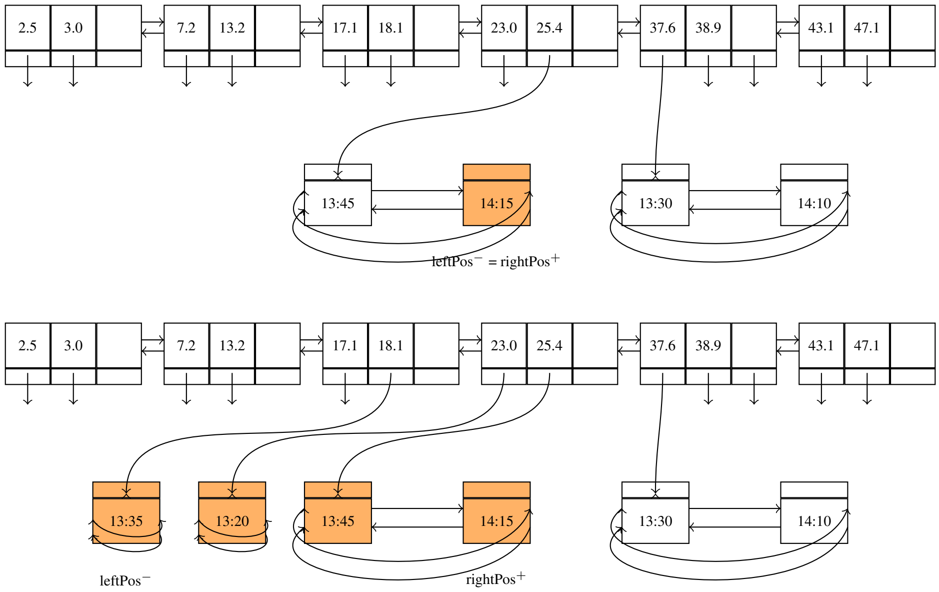


Figure 4.16.: Status of $N(q_{1,2})$ after growing 3 times.

Algorithm 11: SortedAccess$(N(q_{i,j}), T)$

Input: The neighborhood N around query pattern value $q_{i,j}$ and the set of visited time points T

Output: Time point of the next most similar value to $q_{i,j}$

leftPos$^-$ $\leftarrow$ N.leftPos
rightPos$^+$ $\leftarrow$ N.rightPos
while $t^- \neq$ *NIL and* $(t^- - (j+l)) \in T$ **do**
  leftPos$^-$ $\leftarrow$ prev(leftPos$^-$)
end
while $t^- \neq$ *NIL and TimeSetContains*$(t^- - (j+l))$ **do**
  rightPos$^+$ $\leftarrow$ next(rightPos$^+$)
end
if $t^- \neq$ *NIL and* $t^+ \neq$ *NIL* **then**
  if $|r_i(t^-)$ - $q_{i,j}| \leq |r_i(t^+)$ - $q_{i,j}|$ **then**
    leftPos$^-$ $\leftarrow$ prev(leftPos$^-$); $t \leftarrow t^-$
  else
    rightPos$^+$ $\leftarrow$ next(rightPos$^+$); $t \leftarrow t^+$
  end
else if $t^- \neq$ *NIL* **then**
  leftPos$^-$ $\leftarrow$ prev(leftPos$^-$); $t \leftarrow t^-$
else if $t^+ \neq$ *NIL* **then**
  rightPos$^+$ $\leftarrow$ next(rightPos$^+$); $t \leftarrow t^+$
else
  return NIL
end
return t

---

5. Complexity Analysis

This analysis refers to a single time series s for which a B^+tree and a circular array are maintained and the complexity calculations are valid for our context, e.g. the linked list elements are traversed at most l times.

5.1. Runtime Complexity

Circular, Doubly Linked List

Lemma 1 *The insertion of a new time point to a linked list needs $O(1)$ time.*

Proof: For each new value produced by a time series s a linked list in the B^+tree is expanded. Since the new value is always inserted at the tail position it needs $O(1)$ time. $\square$

Lemma 2 *The deletion of a time point from a linked list needs $O(1)$ time.*

Proof: For each deletion in a B^+tree a time point needs to be removed from a linked list. Since the oldest value is always at the head position, the deletion needs $O(1)$ time. $\square$

Lemma 3 *The search of a specific time point in a linked list for newNeighborhood$(q_{i,j}, t, j, l)$ is executed takes at most $O(l)$ time.*

Proof: A time series s in a pattern $P(t)$ initializes l neighborhoods. Hence in the worst case, if the l newest measurements in the circular array of s have the same value and thus are stored in the same linked list in the B^+tree, l list elements have to be traversed to find the specific measurement time point, starting from the tail and scanning backwards. $\square$

Circular Array Operations

Lemma 4 *The update of the circular array takes $O(1)$ time.*

Proof: The next update position in a circular array is computed in $O(1)$ time, hence the update takes $O(1)$ time. $\square$

Lemma 5 *The random access of a time point t takes $O(1)$ time.*

Proof: The random access algorithm needs to calculate the position for the time point t. Since the position can be directly calculated the calculation is done in $O(1)$ time. $\square$

B^+tree Operations

Normally, the complexity of operations on B^+trees is dependent on the required I/O operations since I/O operations are expensive. The speed of operations on B^+trees makes it a frequently used index structure in database implementations. But since we do not have I/O operations the complexity of our implementation is also dependent on the tree node size which usually is neglected because it is not as expensive as I/O operations.
Since we have no I/O operation the parameter n has a influence to our complexity. For every node visited we pay $O(n)$ time to find a specific position. The parameter $|W|$ has an influence as well, since there are at most $|W|$ keys in the leaves of the B^+tree (provided that there are no duplicate values).

Lemma 6 *Finding a leaf in the B^+tree takes at most $O(n \times \log_n(|W|))$ time.*

Proof: The nodes are traversed, recursively downwards from the root of the tree. In the worst case, the number of nodes that must be traversed is $log_n(|W|)$. This leads to $log_n(|W|)$ complexity, where $|W|$ is the maximum total number of keys in the leaves and n is representing the maximum number of pointers in an inner node.
Additionally you pay for every node that is visited $O(n)$ time to find the index position to the child node. Hence, the total complexity to find a leaf takes $O(n \times \log_n(|W|))$ time. □

Lemma 7 *Finding a measurement in the B^+tree takes at most $O(l + (n \times \log_n(|W|)))$ time.*

Proof:

- Lemma 6 states that finding a leaf takes $O(n \times \log_n(|W|))$ time.
- Lemma 3 states that the pattern length l provides an upper bound to the required value lookups in a linked list when the sortedAccess($N(q_{i,j}), T$) operation is executed.
- To search the insertion and deletion position in a B^+tree takes $O(n \times \log_n(|W|))$ according to Lemma 1 and 2, respectively.

Due to this information we can state that the overall runtime complexity to find a specific measurement in the B^+ tree is $O(l + (n \times \log_n(|W|)))$. □

Lemma 8 *The insertion of a measurement to the B^+tree takes $O(n \times \log_n(|W|))$ time.*

Proof: Before a measurement can be inserted to the tree the appropriate leaf has to be found. Lemma 6 states that this takes $O(n \times \log_n(|W|))$ time. The addition of a measurement can cause three different cases as explained in Section 4.3.3.

- (The key already exists)Lemma 1 states that a measurement can be inserted in $O(1)$ time to the linked list.

- (The node has still room and the measurement is inserted to the node) This causes that the insertion place has to be found which takes at most $O(n)$ time, since the keys have to remain ordered.

- (The node is already full and the node has to split) To split the node it takes $O(n)$ time. Because the keys have to remain ordered, the insertion point is found by traversing the keys. After the keys are distributed to both nodes in $O(n)$ time. If the tree is recursively traversed upwards and more nodes have to split, it takes again $O(n)$ time for every node to split.

Since $O(n \times \log_n(|W|)) > O(n)$ the overall runtime complexity of an insertion is $O(n \times \log_n(|W|))$. $\square$

Lemma 9 *The deletion of a measurement from the B^+tree takes $O(n \times \log_n(|W|))$ time.*

Proof: Before a measurement is deleted to from tree the appropriate leaf has to be found. Lemma 6 states that this takes $O(n \times \log_n(|W|))$ time. The deletion of a measurement can cause four different cases as explained in Section 4.3.2.

- (The measurement time point is a linked list element which contains multiple elements) Lemma 2 states that a measurement can be deleted in $O(1)$ time from the linked list. Hence, duplicates have no influence to the complexity of a deletion.

- (The node is at least half-full) If the node has still enough keys the entry is searched in $O(n)$ time by traversing the node keys and then is deleted.

- (The values in a node are redistributed) The redistribution is done by moving keys and pointers. At most n nodes have to be shifted which cost $O(n)$ time.

- (The node is merged with a sibling) To merge two nodes the keys and pointers have to be moved, which costs at most $O(n)$ time.

Since $O(n \times \log_n(|W|)) > O(n)$ the overall runtime complexity of a deletion is $O(n \times \log_n(|W|))$. $\square$

Neighborhood Operations

The newNeighborhood$(q_{i,j}, t, j, l)$ operation searches a specific measurement. Therefore, it cannot just take the oldest or newest time point position like the insertion or deletion method. In the worst case the entire linked list must be searched for the specific time point. We initialize $l \times d$ neighborhoods, one for each value in a query pattern $Q(t)$. Thus, l neighborhoods in every time series r which is part of the pattern. Hence, the newNeighborhood$(q_{i,j}, t, j, l)$ operation always is executed at the l newest measurements in an involved time series. Thus, we can give an upper bound, namely the pattern length l. As a consequence, at most l element examinations in a linked list are necessary, where l represents the pattern length.

Lemma 10 *The initialization of a neighborhood in time series s takes $O(l+(n \times \log_n(|W|))))$ time.*

Proof: The initialization of the neighborhood needs to search a specific measurement in the B^+tree. Hence the complexity to find the measurement leaf is equal to the find leaf operation defined in Lemma 6. At most l linked list elements are traversed as stated in Lemma 3. □

Lemma 11 *The sortedAccess$(N(q_{i,j}), T)$ takes at most $O(|T|)$ time.*

Proof: If the time set contains $|T|$ time points and since at most $|T|$ time points have to be skipped the sortedAccess$(N(q_{i,j}), T)$ takes at most at most $O(|T|)$ time. □

5.1.1. Space Complexity

Lemma 12 *The space complexity of a circular array is $O(|W|)$.*

Proof: Every circular array for a time series s has a size $|W|$. Hence the space complexity of one circular array is $O(|W|)$. □

The B^+tree structure adds space overhead. Furthermore, since nodes may be as much as half empty (if they have the minimum number of children), there is some wasted space. This space overhead, is acceptable given the performance benefits of the B^+tree structure. And the cost of reorganization of the measurements is avoided.

Lemma 13 *The space complexity of a B^+tree is $O(|W|)$.*

Proof: Each measurement is stored once in the tree. In the worst case, all values are unique and $|W|$ values are stored in leaves which cause a space complexity of $O(|W|)$. The number of keys in inner nodes and the root is always smaller than the sum of keys in the leaf level. Hence, the space complexity of the leaf level is the highest. □

6. Experimental Evaluation

This chapter describes our experimental setup and results. We evaluated the running time of the $shift$ algorithm with different values for the tree node size $n-1$ and parameter $|W|$.

6.1. Setup

In the experiments we construct a data set with measurement values with a time span of 100 years between the newest and the oldest value. We use one time series r, thus $d = 1$. The interval between two values is set to 3 minutes. Since 20 measurements arrive per hour the data set contains in total 17'520'000 measurements in 100 years ($100 \times 365 \times 24 \times 20$). Assuming every of these 100 years has 365 days. We assume the data set contains values between 0 and 10000 and duplicates are possible. The values are random but always between these bounds.

6.2. Runtime

Tree Node Size. The size of a node determines the number of values it can hold. We set the window size $|W|$ to 3 years for this experiment and show the effect of an increasing tree node size $n-1$ in Figure 6.1. It shows that the parameter n has a influence on our running time. We observe that the algorithm had the best running time with a tree node size around 10. A very small tree node size is not recommended.

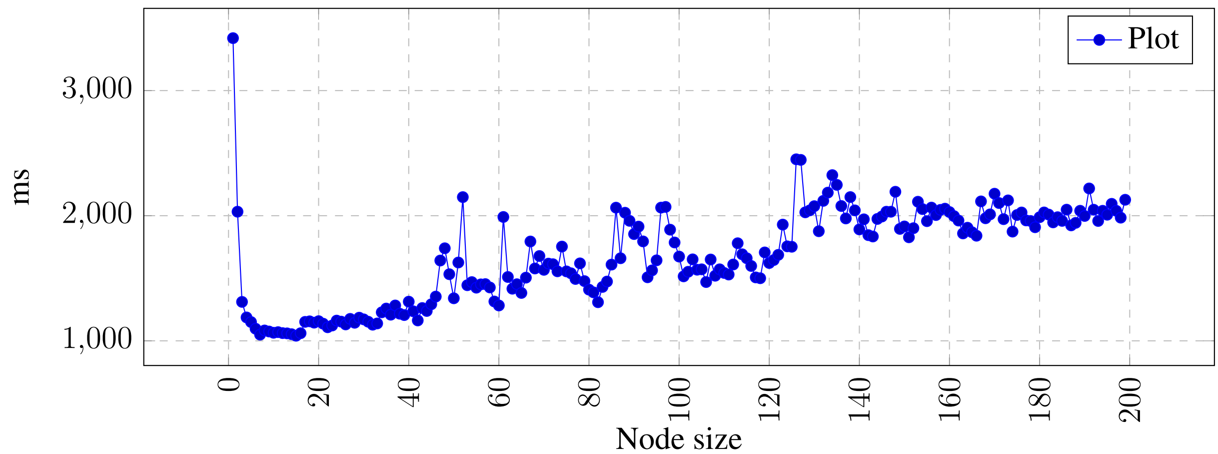


Table 6.1.: Shift operation with increasing values for $n-1$.

Window Size. The window size $|W|$ determines the number of measurements stored in the circular array and the B^+tree. Figure 6.2 illustrates the effect of an increasing window size on the running time of the $shift$ operation. The window size is between 60 days, hence 28800 measurements (60×24×20) and 3 years, 525600 measurements (3×365×24×20). The tree node size is set to 40. The shift operation seems to be not highly influenced by the window size $|W|$. The differences are probably more influenced by the value distribution due to a different window size. Hence, the performance of the shift operation can still be good for a large window size W.

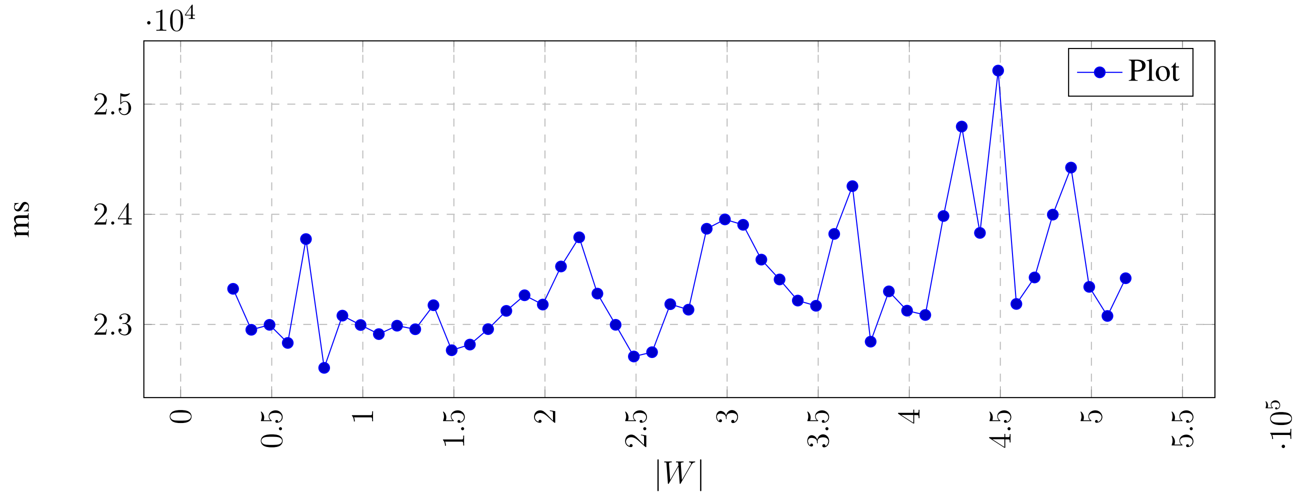


Table 6.2.: Shift operation with increasing values for $|W|$.

7. Summary and Conclusion

We studied the requirements to keep a portion of a streaming time series in main memory and simultaneously provide efficient access possibilities to the time series data. The system we presented uses two different data structures to achieve efficient access operations: a B^+tree and a circular array. Random access is efficiently performed on the circular array and sorted access is efficiently performed on a B^+tree with leaves linked in both directions, thus, to the respective successor and predecessor.
Furthermore, the system can handle duplicate values with a simple but powerful linked list. The duplicate handling is not only simple to implement but also effective in terms of update velocity. Moreover, retrieving a specific value in the linked list has an upper bound and therefore can be efficiently performed. We presented the algorithms to implement this system and evaluated its performance.

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Appendices

A. Algorithms

A.1. Deletion

Algorithm 12: AdjustRoot($tree$)

```
Input: Tree tree
Output: The tree with an adjusted root node
//enough keys in the root
if 0 < tree.root.numOfKeys then
    return
end
//if the root has a child, promote the first (only) child as the new root
if root is not a leaf then
    newRoot ← tree.root.pointers[0]
    newRoot.parent ← NIL
else
    newRoot ← NIL
end
tree.root ← newRoot
```

Algorithm 13: Redistribute($tree, node, neighbor, nIndex, kIndex, kPrime$)

Input: Tree $tree$, the node $node$ and its neighbor $neighbor$, the neighborIndex $nIndex$, the $kIndex$ and the the key $kPrime$

Output: The keys in the $node$ and its $neighbor$, as well as the $parents$ keys are redestributed

```
//node has neighbor to the left side
if nIndex != -1 then
    //Pull neighbor's last key-pointer pair
    over from the neighbor's right end to n
    if node is not a leaf then
        m ← neighbor.pointers[neighbor.numOfKeys]
        insert neighbor.pointers[m] and kPrime to first position in node and shift other
        pointers and values right
        remove neighbor.key[m-1], neighbor.pointers[m] from neighbor
        replace kPrime in node.parent by neighbor.keys[m-1]
    else
        //last value pointer pair in the node
        m ← neighbor.pointers[neighbor.numOfKeys -1]
        insert neighbor.pointers[m] and neighbor.keys[m] to first position in node and
        shift other pointers and values right
        remove neighbor.key[m], neighbor.pointers[m] from neighbor
        replace kPrime in node.parent by node.keys[0]
    end
else
    //node is leftmost child. Take a key-pointer pair from the neighbor to the right
    //Move the neighbor's leftmost key-pointer pair to n's rightmost position
    if node is not a leaf then
        node.keys[node.numOfKeys] ← kPrime
        node.pointers[node.numOfKeys +1] ← neighbor.pointers[0]
        replace kPrime in node.parent by neighbor.keys[0]
        remove neighbor.keys[0], neighbor.pointers[0] from neighbor
    else
        node.keys[node.numOfKeys] ← neighbor.keys[0]
        node.pointers[node.numOfKeys +1] ← neighbor.pointers[0]
        node.parent.keys[kIndex] = neighbor.keys[1]
        remove neighbor.keys[0], neighbor.pointers[0] from neighbor
    end
end
```

Algorithm 14: MergeNodes($tree, node, neighbor, nIndex, kPrime$)

Input: Tree $tree$, the $node$ and its neighbor $neighbor$, the neighborIndex $nIndex$ and the key $kPrime$

Output: $node$ and its $neighbor$ are merged to one node

```
//Swap neighbor with node if node is on the extreme left and neighbor is to its right
if nIndex = -1 then
    swap neighbor with node
end
neighborInsertionIndex ← neighbor.numOfKeys
if node is no leaf then
    neighbor.keys[neighborInsertionIndex] ← kPrime
    neighbor.numOfKeys++
    decreasingIndex ← 0
    numOfKeysBefore ← node.numOfKeys
    for i ← neighborInsertionIndex + 1, j ← 0; j < node.numOfKeys do
        neighbor.keys[i] ← node.keys[j]
        neighbor.pointers[i] ← node.pointers[j]
        neighbor.numOfKeys++
        decreasingIndex++, i++, j++
    end
    node.numOfKeys ← numOfKeysBefore - decreasingIndex
    neighbor.pointers[i] ← node.pointers[j]
    //All children must now point up to the same parent
    for i ← 0; i < neighbor.numOfKeys + 1; i++ do
        tmp ← neighbor.pointers[i]
        tmp.parent ← neighbor
    end
else
    // a leaf, append the keys and pointers of the node to the neighbor
    //Set the neighbor's last pointer to point to what had been the node's right neighbor
    for i ← neighborInsertionIndex, j ← 0; j < node.numOfKeys do
        neighbor.keys[i] ← node.keys[j]
        neighbor.pointers[i] = node.pointers[j]
        neighbor.numOfKeys++, i++, j++
    end
    relink leaves
end
deleteEntry(tree, node.parent, kPrime, node)
```

A.2. Insertion

Algorithm 15: SplitLeaves$(tree, leaf, t, v)$

Input: Tree $tree$, the insertion node $leaf$, the time point t and the value v

Output: The leaf is split into two leaves

```
insertPoint ← 0
nrOfTempKeys ← 0
insertPoint ← getInsertPoint(tree, leaf, v)
//fills the keys and pointers
for i ← 0, j ← 0; i < oldNode.numOfKeys; do
    if j = insertPoint then
        j++
    end
    tempKeys[j] ← oldNode.keys[i]
    tempPointers[j] ← oldNode.pointers[i]
    nrOfTempKeys++, i++, j++
end
//enter the record to the right position
tempKeys[insertPoint] ← v
newList ← create list and insert t
tempPointers[insertPoint] ← newList
nrOfTempKeys++
newNode.numOfKeys ← 0
oldNode.numOfKeys ← 0
//calculate splitpoint by ⌈n/2⌉
split = GetSplitPoint(n-1)
//fill first leaf
for i ← 0; i < split do
    oldNode.keys[i] ← tempKeys[i]
    oldNode.pointers[i] ← tempPointers[i]
    oldNode->numOfKeys++, i++
end
//fill second leaf
for j ← 0, i ← split; i < nrOfTempKeys; do
    newNode.keys[j] ← tempKeys[i]
    newNode.pointers[j] ← tempPointers[i]
    newNode->numOfKeys++, i++, j++
end
link leaves
newNode.parent ← oldNode.parent
keyForParent ← newNode.keys[0]
insertIntoParent(tree, oldNode, keyForParent, newNode)
```

Algorithm 16: SplitInnerNodes($tree, oldInnerNode, index, key, childNode$)

Input: Tree $tree$, the node $oldInnderNode$ and the child node $childNode$, the index $index$ and in addition the key

Output: The inner node is split into two nodes

```
nrOfTempKeys ← 0, x ← 0
for i ← 0, j ← 0; i < oldNode.numOfKeys; do
    if j = index then
        j++
    end
    tempKeys[j] ← oldInnerNode.keys[i], nrOfTempKeys++, i++, j++
end
for i ← 0, j ← 0; i < oldInnerNode.numOfKeys + 1; do
    if j = index + 1 then
        j++
    end
    tempPointers[j] ← oldInnerNode.pointers[i], i++, j++
end
newInnerKey ← key
tempKeys[index] ← newInnerKey, tempPointers[index + 1] ← childNode
nrOfTempKeys++
newInnerNode.numOfKeys ← 0, oldInnerNode.numOfKeys ← 0
split ← getSplitPoint(n)
for x < split; do
    oldInnerNode.keys[x] ← tempKeys[x]
    oldInnerNode.pointers[x] ← tempPointers[x]
    oldInnerNode.numOfKeys++, x++
end
oldInnerNode.pointers[x] ← tempPointers[x]
leftMostKey ← tempKeys[x]
newInnerNode.parent ← oldInnerNode.parent
newInnerNode.numOfKeys ← (nrOfTempKeys - oldInnerNode.numOfKeys-1)
for ++x, j ← 0; j < newInnerNode.numOfKeys; do
    //first key is not inserted to this node - it is inserted to upper node
    newInnerNode.pointers[j] ← tempPointers[x]
    newInnerNode.keys[j] ← tempKeys[x], j++, x++
end
newInnerNode.pointers[j] ← tempPointers[x]
for i ← 0; i < newInnerNode.numOfKeys + 1; do
    childOfNewNode ← newInnerNode.pointers[i]
    childOfNewNode.parent ← newInnerNode, i++
end
insertIntoParent(tree, oldInnerNode, leftMostKey, newInnerNode)
```

Algorithm 17: InsertIntoParent($tree, oldChild, k, newChild$)

Input: Tree $tree$, the newly created $newChild$ and the $oldChild$ and the key k
Output: The key k is inserted to the parent or the parent is split

```
parent ← oldChild.parent
if parent = NIL then
    insertIntoANewRoot(tree, oldChild, k, newChild)
    return
end
pointerPos ← pointer position index from parent to oldChild
//the new key fits into the node
if parent.numOfKeys < n-1 then
    insertIntoTheNode(parent, pointerPos, k, newChild)
else
    splitAndInsertIntoInnerNode(tree, parent, pointerPos, k, newChild)
end
```

The entire source code can be found here:
`https://github.com/memast2/BA-TimeSeriesData`

B. Experimental Results

The entire source code can be found here:
https://github.com/memast2/BA-TimeSeriesData/tree/master/Experimental_Results