Queensland University of Technology

CA203
Discrete Structures

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Chapter 1

Sets

1.1 Set Theory

- A set S is a collection of items (elements)
- Unordered and unique
- One basic property, *membership*:
  - $x \in S$ x is in S
  - $x \notin S$ x is not in S
- For $x \in S$ we also say x is an element/member of S

1.1.1 Defining Sets

Sets can be defined by the following methods

- Listing elements

$$\text{SMALLPRIMES} = 1, 3, 5, 7$$

- Setbuilder Notation

$$\text{SQUARES} = \left\{x \in \mathrm{Z} : x = y^2 : \text{ for some } y \in \mathrm{Z}\right\}$$

- Implied patterns

$$\text{EVENS} = 2, 4, 6, 8$$

  Implied conditions are bad as they are ambigious ($6 \in 2, 4, ...$ or $6 \notin 2, 4, ...$ depending if condition is even numbers or powers of 2)

1.1.2 Membership

$x \in S$ if

- x is in the list if S is given explicitly

  $1 \in 1, 2, 3$ because it exists in the list

- x satisfies the conditions for S if given in setbuilder notation

  $12 \in \{x \in \mathrm{Z} : x|60\}$ because 12 is an intger that divides 60

- x satisfies the implied condition for the pattern

  $15 \in \{1, 3, 5, ...\}$ because it follows the implied condition (odd numbers)

1.1.3 Equality of Sets

Two sets are equal if they contain the same elements

$$S = T \Rightarrow x \in S \cap T$$

1.1.4 Subsets

A is a subset of B if and only if every x in A exists in B (all the elements in A are in B)

$$A \subseteq B \Leftrightarrow \forall x : x \in A \Rightarrow x \in B$$

A proper subset of B is a subset of B that is not equal to B

$$A \subset B \equiv A \subseteq B \wedge A \neq B$$

Figure 1.1: Improper and Proper Subset definition

The collection of all subsets of set A is defined as the power set of set A

$$2^A = \mathcal{P}\left(A\right) = \{x | x \subseteq A\}$$

For example:

$$\mathcal{P}\left(\varnothing \right. = \{\varnothing\}$$
$$\mathcal{P}\left(\{a\}\right) = \{\varnothing, \{a\}\}$$

Figure 1.2: Power Set Definition