Queensland University of Technology

---

# EGB241

Electromagnetics and Machines

---

*Jacob Coetzee*

Dinal Atapattu

March 8, 2024

# Contents

# Chapter 1

# Alternating Current

## 1.1 AC Signals

- Direct Current: DC has polarity (direction) which stays the same. Amplitude may vary, but charge always flows in the same direction
- Alternating Current: Voltage polarity and current direction reverses periodically
- Period: Length of time (seconds) for one repetition of a cycle
- Frequency: $f = \frac{1}{T}$ [Hz]

## 1.2 Sine Wave

In its purest form, AC signals are sine waves, for example:
$v(t) = V_o \sin(2\pi f t) = V_o \sin(\omega t)$ Any AC wave that consists of only one frequency is sinusoidal
Utility AC is a sine wave with frequency 50Hz

## 1.3 Amplitude of an AC wave

- Peak Amplitude ($V_p$): Maximum positive deviation
- Peak-to-Peak Amplitude ($V_{p-p}$): Net difference between positive and negative peak amplitude

$$V_{p-p} = 2 \times V_p \tag{1.1}$$

Figure 1.1: Peak to Peak Amplitude

- Root-Mean-Square Amplitude ($V_{rms}$): Effective amplitude of a sinusoidal AC wave. Average power dissipated in a resistor with an AC voltage = power dissipated with DC voltage of $V_{rms}$
  $V_{rms} \leq V_p \forall$ AC Waves

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{1}{T}\int_\alpha^{T+\alpha} v^2(t)dt} \tag{1.2}$$

Figure 1.2: RMS Amplitude

| | |
|---|---|
| Sinusoidal AC Wave | $V_{rms} = \frac{V_o}{\sqrt{2}} \approx 0.707V_o$ |
| Square Ac Wave | $V_{rms} = V_p$ |
| Triangular Wave | $V_{rms} = \frac{V_o}{\sqrt{3}} \approx 0.577V_o$ |

Figure 1.3: AC RMS Values

## 1.4 Phase Shifting

$f(x+\alpha)$ is a horizontal translation of $f(x)$
Where:

If $a > 0$, $f(x)$ is shifted to the left
If $a < 0$, $f(x)$ is shifted to the right

If $v_1(t) = V_o \sin(\omega t)$, then $v_2 = V_o \sin(\omega t + \phi) = V_o \sin\left[\omega\left(t + \frac{\phi}{\omega}\right)\right]$ is a shifted version of $v_1(t)$ by $\frac{\phi}{\omega}$ seconds to the left (where $\phi$ is in radians and positive)
v$_2(t)$ leads $v_1(t)$ by $\phi$ radians ($180 \times \frac{\phi}{\pi}$ degrees)
$v_1(t)$ lags $v_2(t)$ by $\phi$ radians ($180 \times \frac{\phi}{\pi}$ degrees)

## 1.5 Complex Numbers

### 1.5.1 Polar Form

$$\text{Rectangular form} := z = x + jy$$
$$\text{Polar form} := z = re^{j\theta} = r\angle\theta$$

- r is called the **magnitude** of $z : r = |z| = \sqrt{x^2 + y^2}$
- $\theta$ is called the **argument** of $z : \theta = (\frac{y}{x})$ or more accurately $\theta = \arg(x + jy)$, $-\pi < \theta \leq \pi$ radians (Convention)

### 1.5.2 Properties of Complex Numbers

$$(x+jy) \pm (p+jq) = (x \pm p) + j(y \pm q)$$

(add real and imaginary parts)

(a) Addition and Subtraction

$$(x+jy) \times (p+jq) = (xp - yq) + j(yp + xq)$$

Or in polar form

$$Ae^{j\theta} \times Be^{j\phi} = ABe^{j(\theta+\phi)}$$

(multiply magnitudes, add angles)

(b) Multiplication

By rationalisation

$$\frac{(x+jy)}{(p+jq)} = \frac{(x+jy) \times (p-jq)}{(p+jq) \times (p-jq)} = \frac{(x+jy) \times (p-jq)}{(p^2+q^2)}$$

In polar form:

$$\frac{Ae^{j\theta}}{Be^{j\phi}} = \frac{A}{B} e^{j(\theta-\phi)}$$ (divide magnitudes, subtract angles)

(c) Division

$$\begin{aligned} e^{j(\theta+2\pi n)} &= e^{j\theta} e^{j2\pi n} \\ &= e^{j\theta} \times 1 \\ &= e^{j\theta} \end{aligned}$$

(d) Cyclicity

Figure 1.4: Complex Number Operations

## 1.6 Magnetic Circuits

### 1.6.1 Phasors

$$v(t) = V_0 \cos(\omega t + \theta) = Re[Ve^{j\omega t}] = V_0 \angle \theta = V_0 e^{j\theta}$$

Only for sinusoids, component signals canot be represented as phasors (square, triangle waves, etc)

#### 1.6.1.1 Solving RL RC Circuits

$$\begin{aligned} v(t) &= L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri(t) \\ Re\left[Ve^{j\omega t}\right] &= L\frac{\mathrm{d}}{\mathrm{d}t}Re\left[Ie^{j\omega t}\right] \\ &= LRe\left[I\frac{\mathrm{d}}{\mathrm{d}t}e^{j\omega t}\right] \\ Ve^{j\omega t} &= j\omega Ie^{j\omega t} \\ V &= j\omega LI \end{aligned}$$

Given an RL circuit with the following values

$$v_s(t) = 50\cos(100t) \tag{1.3}$$
$$C = 0.2H \tag{1.4}$$
$$R = 30\Omega \tag{1.5}$$

Using KCL

$$\begin{aligned} V_s &= V_R + V_L \\ &= IR + Ij\omega L \\ &= I\left(30 + \times j100 \times 0.2\right) \\ 50 &= I\left(30 + j20\right) \\ \frac{50}{30 + j20} &= I \\ 1.138 - j0.76923 &= I \\ 1.386\angle - 33.7\deg &= I \\ \therefore i(t) &= 1.386\cos(100t - 0.588)[A] \end{aligned}$$

### 1.6.2 Oersted's Rules

- The magnetic field lines run encircle the current-carrying wire (Right hand screw rule)
- The magnetic field lines lie in a plan perpendicular to the wire
- If the direction of the current is reversed, the direction of the magnetic force reverses
- The strength of a field is directly proportional to the magnitude of the current
- The strength of the field at any point is inversely proportional to the distance of the point from the wire

This leads to Ampere's Law: The line integral of the magnetic field $B(x)$ around any closed curve $C$ is proportional to the total current $I$ passing through any surface bounded by the curve

$$\oint \mathbf{B} \cdot dl = \mu_0 I$$

Figure 1.5: Ampere's Law

### 1.6.3 Electric Ohm's Law

# List of Figures