MZB125 Week 10

BallsBoy

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Question 8

$$
\begin{aligned}
r(z) &= \frac{(z+5)^3}{300}, z \in [0,1] \\
\therefore \frac{\mathrm{d}r}{\mathrm{d}z} &= \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{(z+5)^3}{300}\right) \\
&= \frac{(z+5)^2}{900}
\end{aligned}
$$

How much soil can the pot hold

V $= \int_{-\infty}^{\infty} A(z)dz \to \int_{-\infty}^{\infty} \pi r^2 dz \to \pi \int_{-\infty}^{\infty} r(z)^2 dz \therefore$

$$
\begin{aligned}
V &= \pi \int_0^1 r(z)^2 dz \\
&= \pi \int_0^1 \left(\frac{(z+5)^3}{300}\right)^2 dz \\
&= \pi \int_0^1 \frac{(z+5)^6}{300^2} dz \\
&= \frac{\pi}{90000}\left[\frac{(z+5)^7}{7}\right]_0^1 \\
&= \frac{\pi}{90000}\left[\frac{6^7 - 5^7}{7}\right] \\
&= \frac{\pi}{90000} \times \frac{201811}{7} \\
&= \frac{201811\pi}{630000}\mathrm{m}^3 \simeq 1.00636\mathrm{m}^3
\end{aligned}
$$

How much paint is needed to coat the exterior of the pot including the bottom? A litre of paint covers 1m^2

$$\begin{aligned}
S &= 2\pi\int_{-\infty}^{\infty} r(z)\sqrt{1+\frac{\mathrm{dr}}{\mathrm{dz}}^2}\,dz \\
&= 2\pi\int_{0}^{1} \frac{(z+5)^3}{300}\sqrt{1+\left(\frac{(z+5)^2}{900}\right)^2}\,dz \\
&= \frac{2\pi}{300}\int_{0}^{1} (z+5)^3\sqrt{1+\left(\frac{(z+5)^2}{900}\right)^2}\,dz
\end{aligned}$$

Apply u-substitution, let:

$$\begin{aligned}
u &= z+5 & u(1) &= 1+5 \\
\frac{\mathrm{d}u}{\mathrm{d}z} &= 1 & &= 6 \\
\mathrm{d}u &= \mathrm{d}z & u(0) &= 0+5 \\
& & &5
\end{aligned}$$

$$\begin{aligned}
&= \frac{2\pi}{300}\int_{5}^{6} u^3\sqrt{1+\frac{u^4}{900^2}}\,dz \\
&= \frac{2\pi}{300}\int_{5}^{6} u^3 \times \frac{1}{900}\sqrt{u^4+810000}\,dz \\
&= \frac{2\pi}{270000}\int_{5}^{6} u^3 \times \sqrt{u^4+810000}\,dz
\end{aligned}$$

Apply v-substitution, let:

$$\begin{aligned}
v &= u^4+810000 & v(6) &= 6^4+810000 \\
\frac{\mathrm{d}v}{\mathrm{d}z} &= 4u^3 & &= 811296 \\
\frac{1}{4}\mathrm{d}v &= u^3\mathrm{d}z & v(5) &= 5^4+810000 \\
& & &= 810625
\end{aligned}$$

$$\begin{aligned}
&= \frac{2\pi}{270000}\int_{810625}^{811296} \sqrt{v}\cancelto{\frac{1}{4}dv}{u^3dz} \\
&= \frac{2\pi}{1080000}\int_{810625}^{811296} v^{\frac{1}{2}}\,dv \\
&= \frac{2\pi}{1080000}\left[\frac{2}{3}v^{\frac{3}{2}}\right]_{810625}^{811296} \\
&= \frac{\pi}{810000}\left[811296^{\frac{3}{2}} - 810625^{\frac{3}{2}}\right] \\
&\simeq 3.51543\mathrm{m}^2 \text{ (needs 4 buckets of paint)}
\end{aligned}$$