MZB125 Week 11

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Question 9

$$z_1 = 3 + 3i \qquad z_2 = 4 - 4i$$

$$\begin{aligned}\frac{1}{z_1} &= \frac{\tilde{z_1}}{|z_1|^2} \\ &= \frac{3-3i}{(\sqrt{3^2+3^2})^2} \\ &= \frac{3-3i}{3^2+3^2} \\ &= \frac{3}{18} - \frac{3}{18}i \\ &= \frac{1}{6} - \frac{1}{6}i\end{aligned}$$

$$\begin{aligned}\frac{1}{z_2} &= \frac{\tilde{z_2}}{|z_2|^2} \\ &= \frac{4+4i}{(\sqrt{4^2+(-4)^2})^2} \\ &= \frac{4+4i}{16+16} \\ &= \frac{4}{32} + \frac{4}{32}i \\ &= \frac{1}{8} + \frac{1}{8}i\end{aligned}$$

$$\begin{aligned}\frac{1}{z} &= \frac{1}{z_1} + \frac{1}{z_2} \\ &= \left(\frac{1}{6} - \frac{1}{6}i\right) + \left(\frac{1}{8} + \frac{1}{8}i\right) \\ &= \left(\frac{1}{6} + \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{6}\right)i \\ &= \left(\frac{8}{48} + \frac{6}{48}\right) + \left(\frac{6}{48} - \frac{8}{48}\right)i \\ &= \frac{14}{48} - \frac{2}{48}i \\ &= \frac{7}{24} - \frac{1}{24}i\end{aligned}$$

$$\frac{1}{\frac{1}{z}} = \frac{\tilde{\frac{1}{z}}}{|\frac{1}{z}|^2} = z$$

$$\begin{aligned}\therefore z &= \frac{\frac{7}{24} + \frac{1}{24}i}{(\sqrt{\frac{7}{24}^2 + \frac{1}{24}^2})^2} \\ &= \frac{\frac{7}{24} + \frac{1}{24}i}{\frac{49}{576} + \frac{1}{576}} \\ &= \frac{\frac{7}{24} + \frac{1}{24}i}{\frac{50}{576}} \\ &= \frac{288}{25}\left(\frac{7}{24} + \frac{1}{24}i\right) \\ &= \frac{12}{25}\left(7 + i\right) \\ &= \frac{84}{25} + \frac{12}{25}i\end{aligned}$$

$$
\begin{aligned}
\therefore |z| &= \sqrt{\frac{84}{25}^2 + \frac{12}{25}^2} = \frac{12\sqrt{2}}{5} \\
\tan(\theta) &= \frac{\frac{12}{25}}{\frac{84}{25}} \\
&= \frac{1}{7} \\
\theta &= \arctan\left(\frac{1}{7}\right) \\
&\approx 0.142
\end{aligned}
$$