

MZB125 Week 10

BallsBoy

March 2023

1 Question 8

$$r(z) = \frac{(z+5)^3}{300}, z \in [0, 1]$$

$$r = \frac{(z+5)^3}{300}$$

$$300r = (z+5)^3$$

$$\sqrt[3]{300r} = z+5$$

$$\sqrt[3]{300r} - 5 = z$$

$$z(r) = \sqrt[3]{300r} - 5r \in \left[r(0), r(1) \right]$$

$$z(r) = \sqrt[3]{300r} - 5r \in \left[\frac{5}{12}, \frac{18}{25} \right]$$

How much soil can the pot hold

$$V = \int_{-\infty}^{\infty} A(z)dz \rightarrow \int_{-\infty}^{\infty} \pi r^2 dz \rightarrow \pi \int_{-\infty}^{\infty} z(r)^2 dr \therefore$$

$$\begin{aligned} V &= \pi \int_{\frac{5}{12}}^{\frac{18}{12}} z(r)^2 dr \\ &= \pi \int_{\frac{5}{12}}^{\frac{18}{12}} \left(\sqrt[3]{300r} - 5 \right)^2 dr \\ &= \pi \int_{\frac{5}{12}}^{\frac{18}{12}} \left((300r)^{\frac{2}{3}} - 10(300r)^{\frac{1}{3}} + 25 \right) dr \\ &= \pi \left[300^{\frac{2}{3}} \times \frac{3}{5} r^{\frac{5}{3}} - 10 \times 300^{\frac{1}{3}} \times \frac{3r^{\frac{4}{3}}}{4} + 25r \right]_{\frac{5}{12}}^{\frac{18}{12}} \\ &= \pi \left[\left(300^{\frac{2}{3}} \times \frac{3}{5} \times \frac{18^{\frac{5}{3}}}{12} - 10 \times 300^{\frac{1}{3}} \times \frac{3 \times \frac{18^{\frac{4}{3}}}{12}}{4} + 25 \times \frac{18}{12} \right) - \left(300^{\frac{2}{3}} \times \frac{3}{5} \times \frac{5^{\frac{5}{3}}}{12} - 10 \times 300^{\frac{1}{3}} \times \frac{3 \times \frac{5^{\frac{4}{3}}}{12}}{4} + 25 \times \frac{5}{12} \right) \right] \\ &\simeq 9.74\text{m}^3 \end{aligned}$$

How much paint is needed to coat the exterior of the pot including the bottom? A litre of paint covers 1m^2

$$\begin{aligned} S &= 2\pi \int_{-\infty}^{\infty} z(r) \sqrt{1 + \frac{dz}{dr}^2} dz \\ &= 2\pi \int_{\frac{5}{12}}^{\frac{18}{12}} dr \end{aligned}$$