Queensland University of Technology

MZB221

Electrical Engineering Mathematics

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Contents

Chapter 1

Infinite Series

1.1 Sequences, Infinite Series, Convergence

1.2 Taylor polynomials, Taylor series, Radius of convergence

1.3 Introduction to Fourier series

1.4 Constructing Fourier series

Chapter 2

Vector Calculus

2.1 Introduction to Vector Calculus, div, grad, curl

2.1.1 Scalar Fields

A scalar field is a function

$$f : \mathrm{R}^n \to \mathrm{R}$$

$\boxed{n = 2}$ $f = f(x, y)$

- 2 independent variables x, y
- f is a function that has x and y as inputs and a single real number as the output

Phyiscal examples of scalar fields are

- The temperature $T(x, y, z)$, the pressure $p(x, y, z)$, the density $\rho(x, y, z)$ of a fluid
- Concentration of a pollutant in a lake $c(x, y, z)$
- Height of a surface or a mountain $h(x, y)$
- Charge density $\rho(x, y, z)$, electrical potential $V(x, y, z)$

Partial Derivatives

For $f : \mathrm{R}^2 \to \mathrm{R}$

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}, \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

- f_x is the rate of change of f in the x-direction (y is constant) (and vice-versa)
- Geometrically, f_x and f_y are the slopes of the surface $z = f(x, y)$ in the x and y directions.

For $f : \mathrm{R}^3 \to \mathrm{R}$

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}, \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$
$$\frac{\partial f}{\partial z} \lim_{h \to 0} \frac{f(z+h, y) - f(z, y)}{h}$$

No equivalent geometric representation as "surface" $(f(x, y, z))$ is a 4-dimensional hypersolid

Directional Derivative of a Scalar Field

2.2 Review of Multiple Integration, Change of Variables

2.3 Introduction to cylindrical and spheroidal coordinates, integration

2.4 Line Integrals, Surface Integrals

Chapter 3

Differential Equations

3.1 Introduction to Laplace transform, Strategy for Solving Linear ODEs

3.1.1 Definition of the Laplace Transform

The Laplace Transform of the function $f(t)$ is defined as

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$$

for values of s that the integral converges
The notation $F(s)$ is often used to represent $\mathcal{L}\{f(t)\}$

3.2 Further Properties of Laplace Transforms, solving more complicated initial value problems

3.3 Non-Linear first-order ODEs, Phase lines, Stability, Bi-Furcation

3.4 Linear Systems of ODEs, Exact Solutions, Classification, Non-Homogeneous Systems

3.5 Non-Linear Systems of ODEs, Phase Plane, Nullclines, Stability