Queensland University of Technology

EGB120

Foundations of Electrical Engineering

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Contents

Chapter 1

Electrical Circuits

1.1 Fundamental Units

Quantity	Definition	Symbol	SI Unit
Charge	Physical property of matter that causes it to experience a force when placed in an electromagnetic field	q	Coulomb (C)
Current	$$i(t) = \frac{\mathrm{d}q}{\mathrm{d}t}$$	i	Ampere (A)
Voltage	$$v(t) = \frac{\mathrm{d}w}{\mathrm{d}q}$$	v	Volt (V)
Power	$$p(t) = \frac{\mathrm{d}w}{\mathrm{d}t} = \frac{\mathrm{d}q}{\mathrm{d}t} \times \frac{\mathrm{d}w}{\mathrm{d}q} = vi$$	p	Watt (W)
Energy	$$w(\tau) = \int_{\tau}^{0} p(t)dt$$	e	Joule (J)

1.2 Basic Circuit Elements

1.2.1 Voltage Source

Produces or dissipates power at a specific voltage with whatever current is required

1.2.2 Current Source

Produces or dissipates power at a specific current with whatever voltage is required

1.2.3 Resistor

Dissipates power so that the voltage across the terminals is proportional to the current

$$v = Ri \tag{Ohm's Law}$$

Following this

$$p = vi = Ri^2 = \frac{v^2}{R}$$

Chapter 2

Simple Circuits

2.1 Physics Ignored

- Electrical effects are instantaneous
- Net charge on every component is zero
- No magnetic coupling between components

2.2 Series

Elements are connected end to end and **have the same current flowing through them**

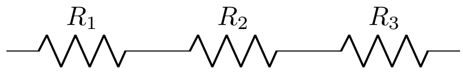


Figure 2.1: Series Circuit

2.3 Parallel

Both ends of one element are connected directly and **have the same voltage across them**

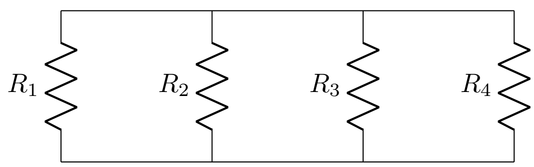


Figure 2.2: Parallel Circuit

2.4 Kirchoff's Laws

2.4.1 Kirchoff's Current Law

The sum of currents entering a node is equal to the sum of currents leaving a node

$$\sum_{k=1}^{n} i_k = 0$$

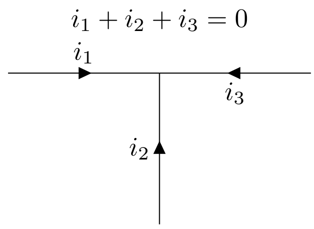
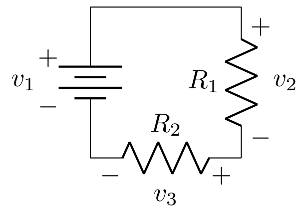


Figure 2.3: Kirchoff's Current Law

2.4.2 Kirchoff's Voltage Law

The sum of voltages around a closed loop is zero

$$\sum_{k=1}^{n} v_k = 0$$



$$v_1 - v_2 - v_3 = 0$$
$$v_1 = v_2 + v_3$$

Figure 2.4: Kirchoff's Voltage Law

2.5 Component Behaviours in Series and Parallel

Component	**Series**	**Parallel**
Voltage Source	$v_s = v_1 + v_2 + v_3$	$v_s = v_1 = v_2 = v_3$
Current Source	$i_s = i_1 = i_2 = i_3$	$i_s = i_1 + i_2 + i_3$
Resistor	$R_s = R_1 + R_2 + R_3$	$\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
Inductor	$L_s = L_1 + L_2 + L_3$	$\frac{1}{L_s} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$
Capacitor	$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$	$C_s = C_1 + C_2 + C_3$

Figure 2.5: Component Behaviours in Series and Parallel

2.6 Voltage Divider

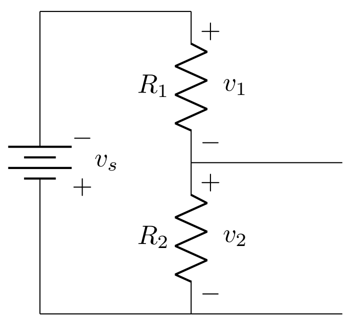


Figure 2.6: Voltage Divider

Current through resistors is

$$i = \frac{v_s}{R_1 + R_2}$$

Voltage across resistors is

$$v_1 = iR_1 = \frac{R_1}{R_1 + R_2} v_s$$

For more resistors

$$v_k = \frac{R_j}{R_{eq}} v_s$$

2.7 Current Divider

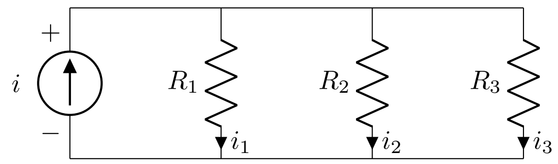


Figure 2.7: Current Divider

Voltage across resistors is

$$v = i(R_1||R_2||R_3)$$

Current through resistors is

$$i_j = \frac{v}{R_j} = \frac{R_1||R_2||R_3}{R_j} i$$

For more resistors

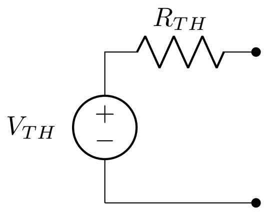
$$i_j = \frac{R_{eq}}{R_j} i$$

Chapter 3

Source Transformation

3.1 Thevenin Equivalent

Voltage source with a series resistance



Chapter 4

Diodes

Current flows from the anode to the cathode and the voltage across the diode is positive.
A diode requires voltage to be applied across it to conduct current. This voltage is called the **forward voltage**.

4.1 Voltage-Current Characteristics

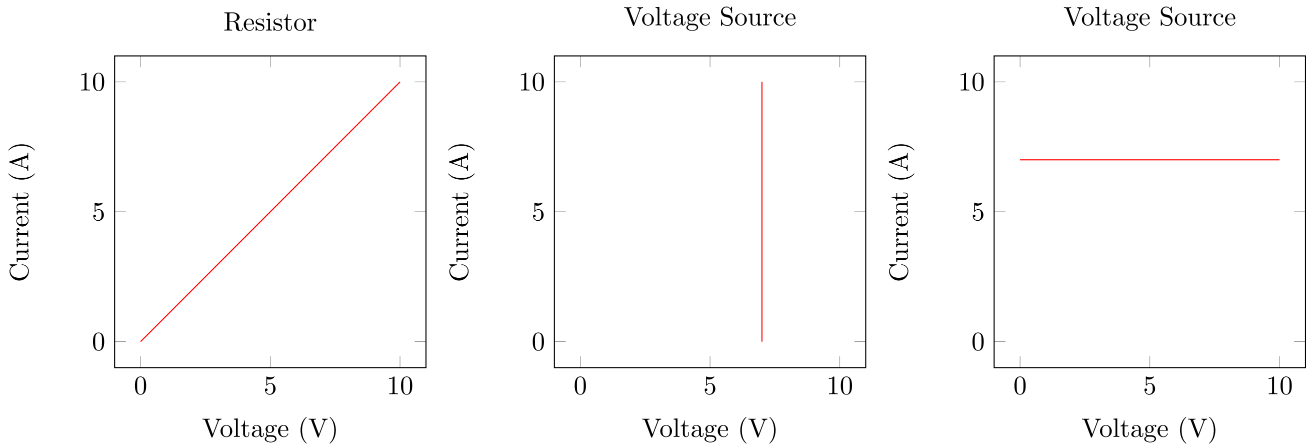


Figure 4.1: Diode VI Characteristics

4.1.1 Simplified Diode Characteristics

Diodes have non-linear characteristics. Therefore, they are simplified to make calculations easier.

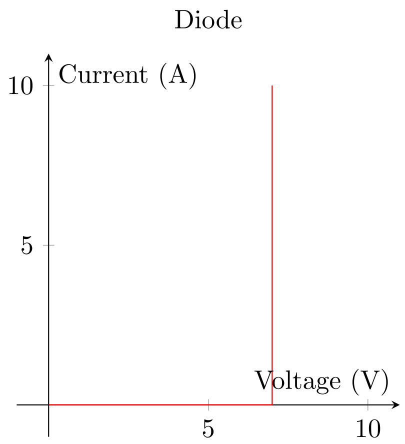


Figure 4.2: Simplified Diode Characteristics

4.1.2 Full Diode Characteristics

Diodes can be accurately modelled using Shcokley's equation

$$i_D = I_s \left(e^{\frac{v_D}{V_T}-1} \right) \qquad\qquad V_T = \frac{kT}{q}$$

where I_s is the saturation current, V_T is the thermal voltage, k is Boltzmann's constant, T is the temperature in Kelvin and q is the charge of an electron.

Chapter 5

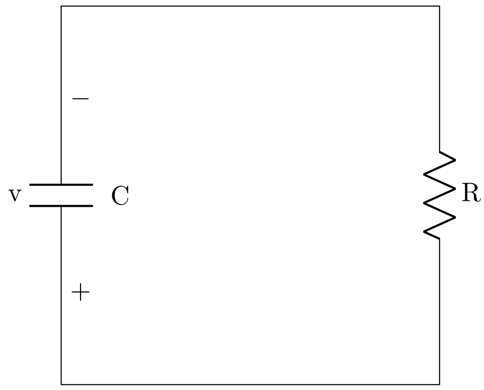
RL and RC Circuits

5.1 Natural Response

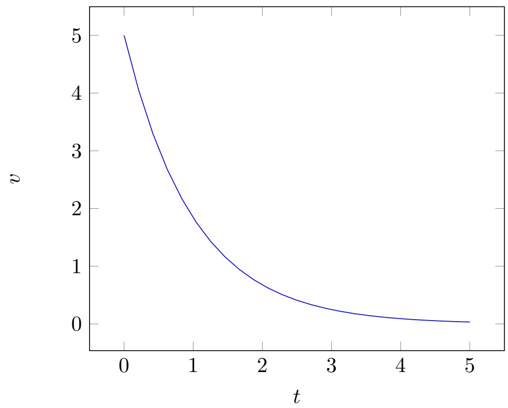
Is a decaying response for an RL and RC circuit

$$v(t) = V_0 e^{-\frac{t}{RC}}$$

(a) Equation for RC Circuit Natural Response



(b) Natural Response RC Circuit

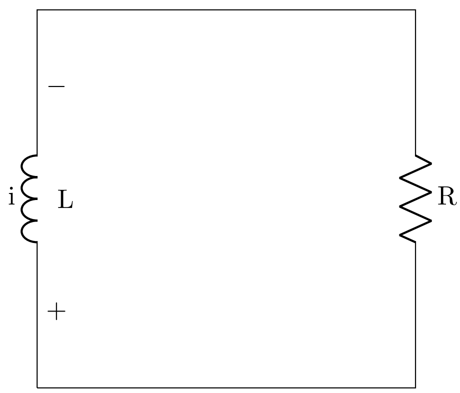


(c) VT Curve for RC Circuit Natural Response

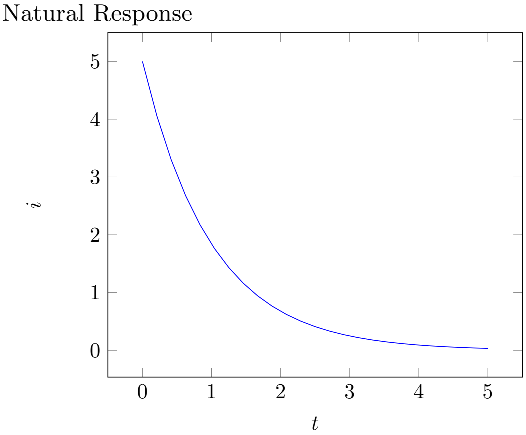
Figure 5.1: RC Natural Response

$$i(t) = I_0 e^{-\frac{Rt}{L}}$$

(a) Equation for RL Circuit Natural Response



(b) Natural Response RL Circuit



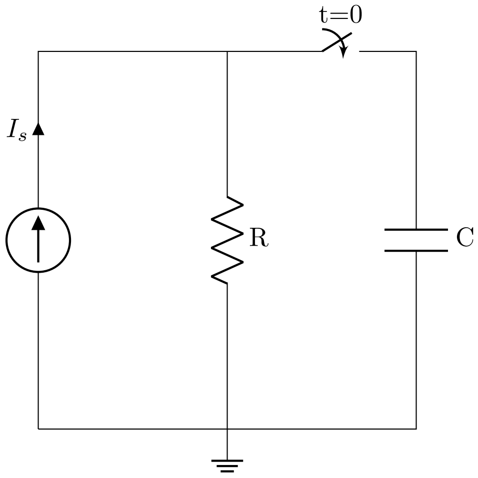
(c) VT Curve for RL Circuit Natural Response

Figure 5.2: RL Natural Response

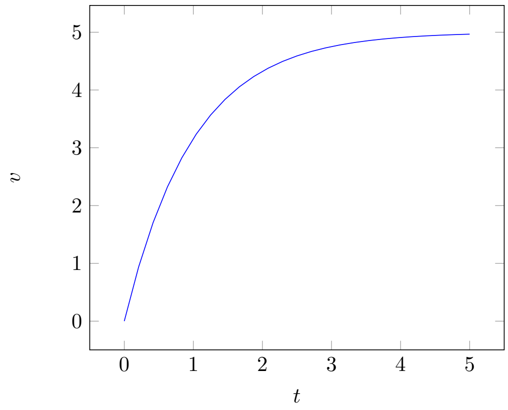
5.2 Step Response

$$v(t) = I_s R + (V_0 - I_s R)e^{-\frac{t}{RC}}$$

(a) Equation for RC Circuit Step Response



(b) Step Response RC Circuit

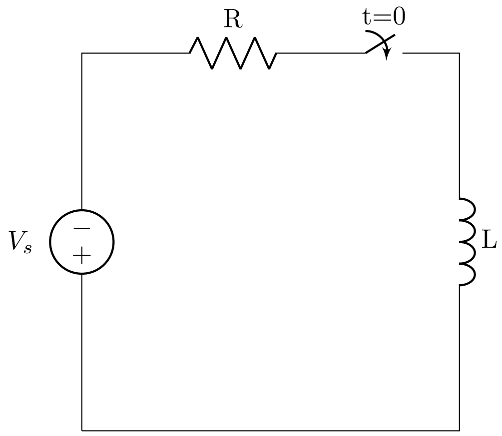


(c) VT Curve for RC Circuit Step Response

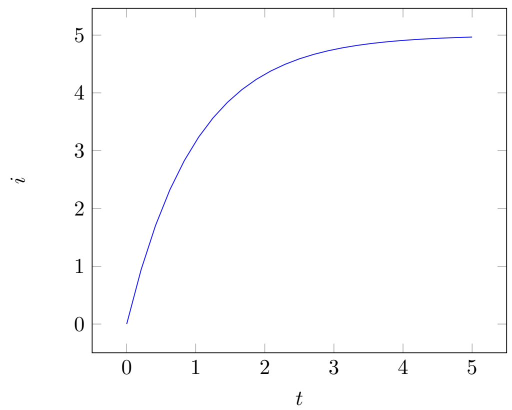
Figure 5.3: RC Step Response

$$i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-\frac{Rt}{L}}$$

(a) Equation for RL Circuit Step Response



(b) Step Response RL Circuit



(c) VT Curve for RL Circuit Step Response

Figure 5.4: RL Step Response

Chapter 6

AC

6.1 AC Circuit Analysis

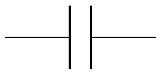
Element	Impedance	Diagram
Resistor	R	
Capacitor	$\frac{1}{j\omega C}$ or $\omega L\angle 90°$	C 
Inductor	$j\omega L$ or $\frac{1}{\omega L}\angle -90°$	L 

Figure 6.1: Impedance of Circuit Elements

6.2 Bode Plots

Plot frequency on x axis using the log of frequency

- 1, 10, 100, 1000, 10000 are equally spaced
- 3, 30, 300, 3000, 30000 are midpoints

Plot the log of gain (decibels) on one y axis and the phase (degrees) on another y axis

$$\omega = 2\pi f$$

(a) Hz to Radians per Second

$$\text{Gain}_{\text{dB}} = 20\log_{10}(\text{Gain})$$

(b) Gain to Decibels

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

(c) Transfer Function

$$\begin{aligned} \text{Phase} &= \angle H(\omega) \\ &= \arctan\frac{\text{imag}(H(\omega))}{\text{real}(H(\omega))} \end{aligned}$$

(d) Transfer Function to Phase

$$\begin{aligned} \text{Gain} &= |(H(\omega))| \\ &= \sqrt{(\text{real}(H(\omega)))^2 + (\text{imag}(H(\omega)))^2} \end{aligned}$$

(e) Transfer Function to Gain

Figure 6.2: Bode Plot Equations

6.3 Transfer Function $H(\omega)$ - The 3dB Point

The 3dB point is the frequency at which the gain is $\frac{1}{\sqrt{2}}$ of the maximum gain. Also called the **half power point** or **half power frequency** or **break frequency** or **corner frequency**

Chapter 7

Filters and Rectifiers

Filters keep the desired frequency components of a signal and remove the unwanted frequency components. They do this in the frequency domain

7.1 Applications of Filters

- Audio Signals
  - Remove high frequency hiss (magnetic tape)
  - Remove low frequency rumble (vinyl)
- Medical Signals
  - EEG alpha waves are 8-12Hz
  - EEG beta waves are 12-30Hz
  - ECG waves are 1 - 40Hz
- 50Hz interference
  - Remove all signals near 50Hz
- Signal Processing

7.2 Ideal Filters

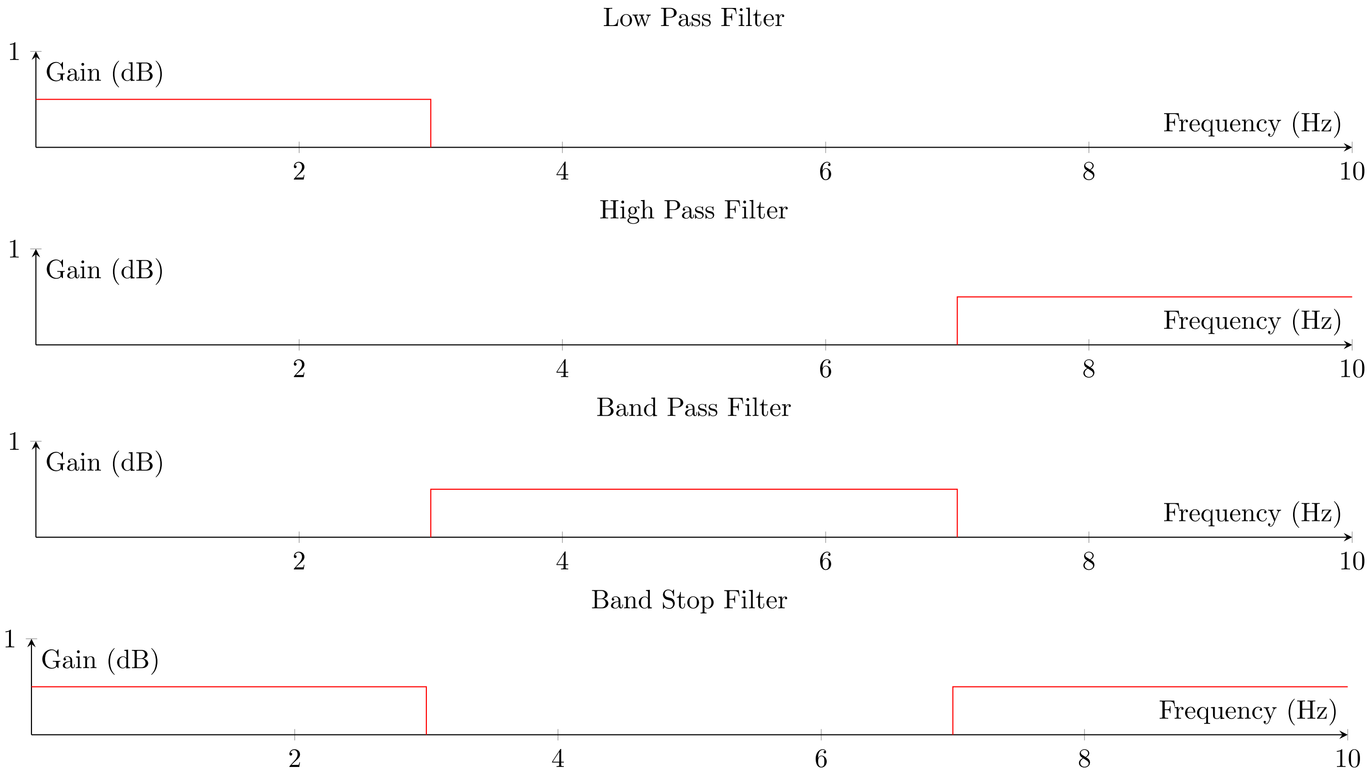


Figure 7.1: Ideal Filters

7.3 Passive Filters

7.3.1 Low Pass Filter

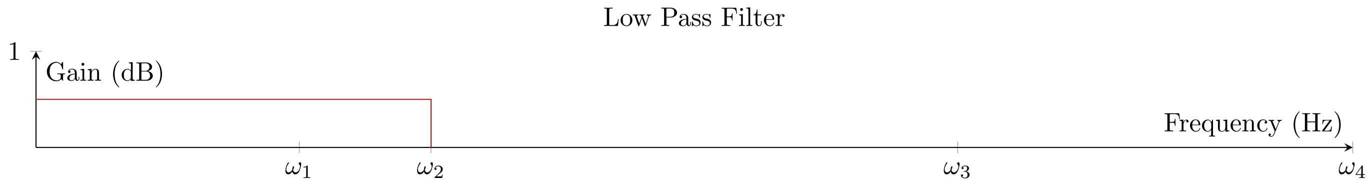
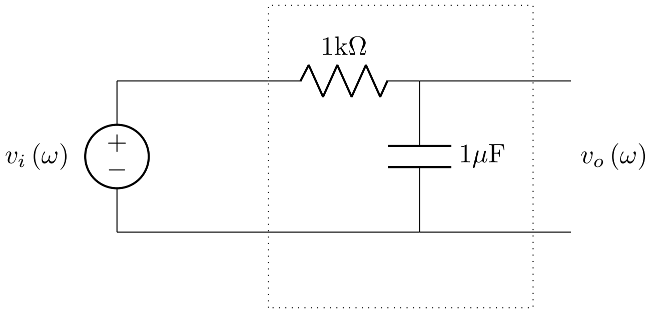


Figure 7.2: Low Pass Filter



$$v_o(\Omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} v_i(\omega)$$

$$\frac{v_0(\omega)}{v_i(\omega)} = \frac{1}{j\omega RC + 1}$$

For $R = 1\text{k}\Omega$ and $C = 1\mu F$

$$H(\omega) = \frac{1000}{j\omega + 1000}$$

$$H(62.8) \approx 1 \qquad\qquad H(62800) = 0.016\angle - 89°$$

Plotting the Response

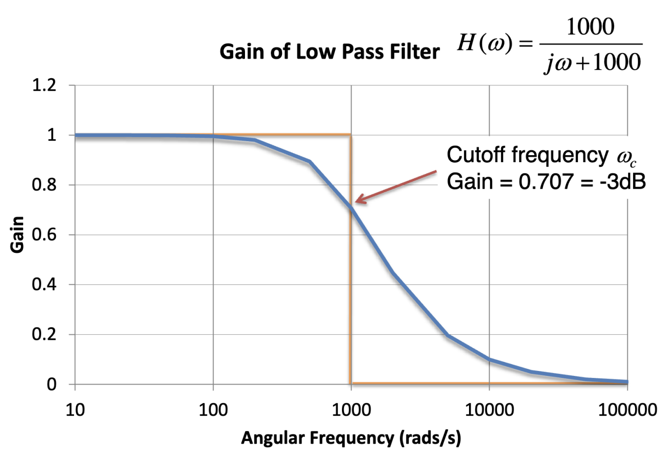
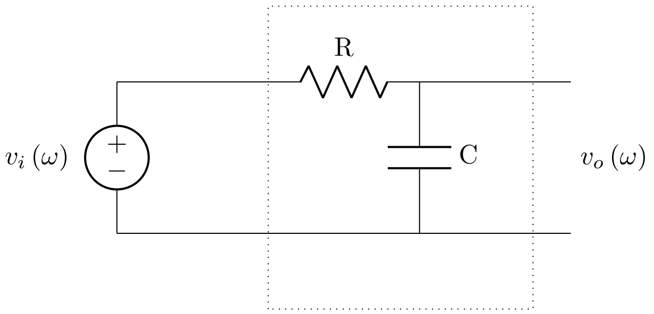


Figure 7.3: Low Pass Filter Bode Plot

Desigining a Low Pass Filter

Find the required **cutoff frequency** ω_c in radians per second (remember $\omega_c = 2\pi f$)
Choose a value for R and calculate C such that $\omega_c = \frac{1}{RC}$

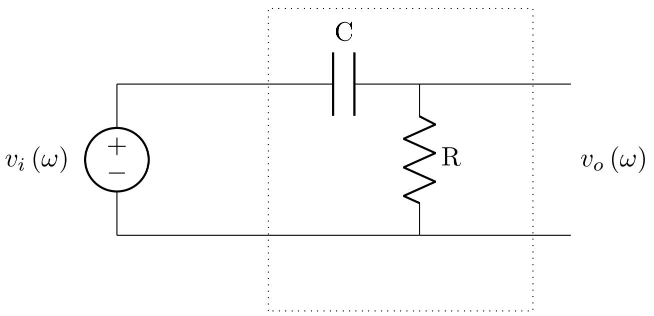


$$H(\omega) = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}}$$
$$\omega_c = \frac{1}{RC}$$

7.3.2 High Pass Filter

Similar to a low pass filter, but the capacitor and resistor are swapped

Desigining a High Pass Filter



$$v_o(\omega) = \frac{R}{R + \frac{1}{j\omega C}} v_i(\omega)$$
$$\frac{v_0(\omega)}{v_i(\omega)} = H(\omega) = \frac{j\omega RC}{j\omega RC + 1}$$
$$\omega_c = \frac{1}{RC}$$

Plotting the Response

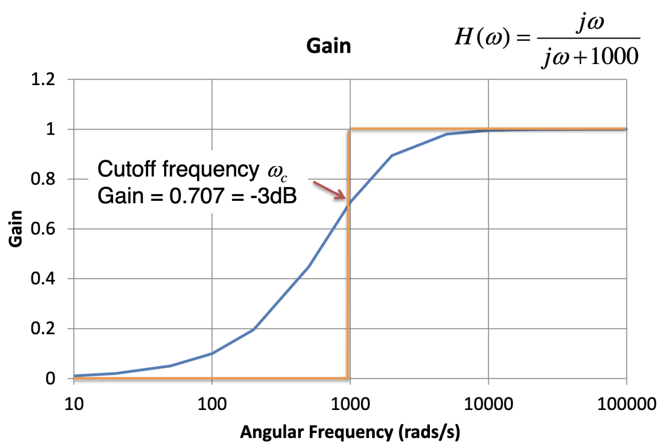
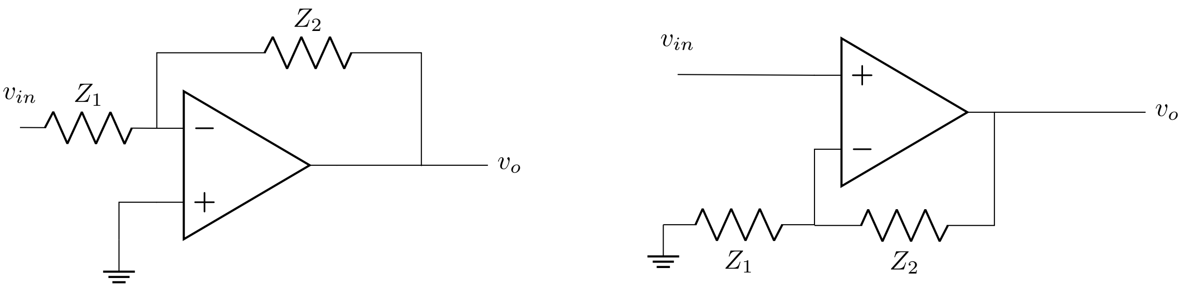


Figure 7.4: High Pass Filter Bode Plot

7.4 Active Filters

7.4.1 Op-Amps



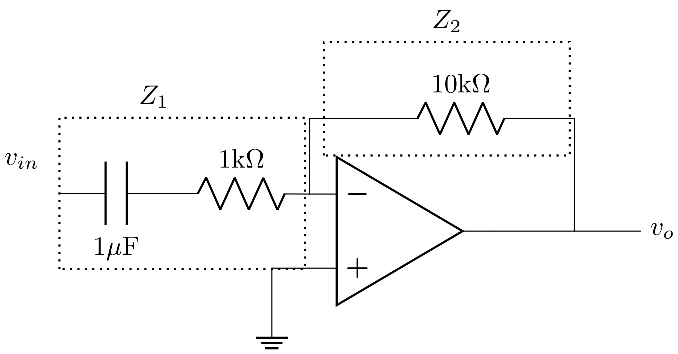
(a) Inverting Amplifier

(b) Non-Inverting Amplifier

Figure 7.5: Active Filters

7.4.2 Using an Op Amp

Active High Pass Filter



(a) Active High Pass Filter

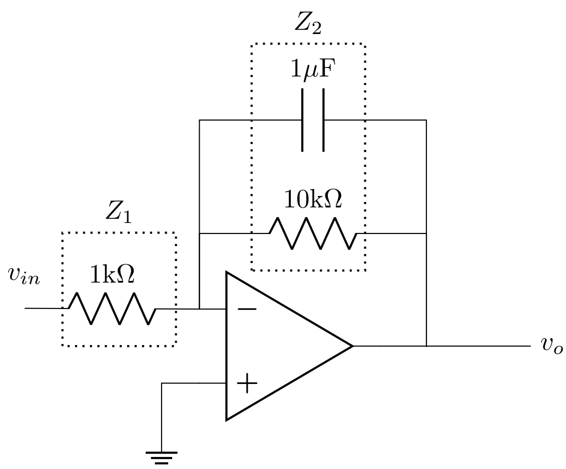
$$v_o = -\frac{Z_2}{Z_1} v_{in}$$

$$\frac{v_o}{v_{in}} = -\frac{R_2}{R_1 + \frac{1}{j\omega C_1}} = -\frac{R_2}{R_1} \frac{j\omega}{j\omega + \frac{1}{R_1 C_1}}$$

High pass filter with
$\omega_c = \frac{1}{R_1 C} = \frac{1}{10^3 \times 10^{-6}} = 1000\text{rad/s}$

Gain of $-\frac{R_2}{R_1} = -10$ in passband

Active Low Pass Filter



(a) Active Low Pass Filter

$$\begin{aligned} v_o &= -\frac{Z_2}{Z_1} v_{in} \\ &= \frac{\frac{1}{\frac{1}{R_2} + j\omega C_2}}{R_1} v_{in} \\ \frac{v_o}{v_{in}} = H(\omega) &= -\frac{R_2}{R_1} \frac{1}{1 + j\omega R_2 C_2} \\ &= -\frac{R_2}{R_1} \frac{\frac{1}{R_2 C_2}}{j\omega + \frac{1}{R_2 C_2}} \end{aligned}$$

Low pass filter with
$\omega_c = \frac{1}{R_2 C} = \frac{1}{10^4 \times 10^{-6}} = 100\text{rad/s}$

Gain of $-\frac{R_2}{R_1} = -10$ in passband

7.5 Band Pass and Band Stop Filters

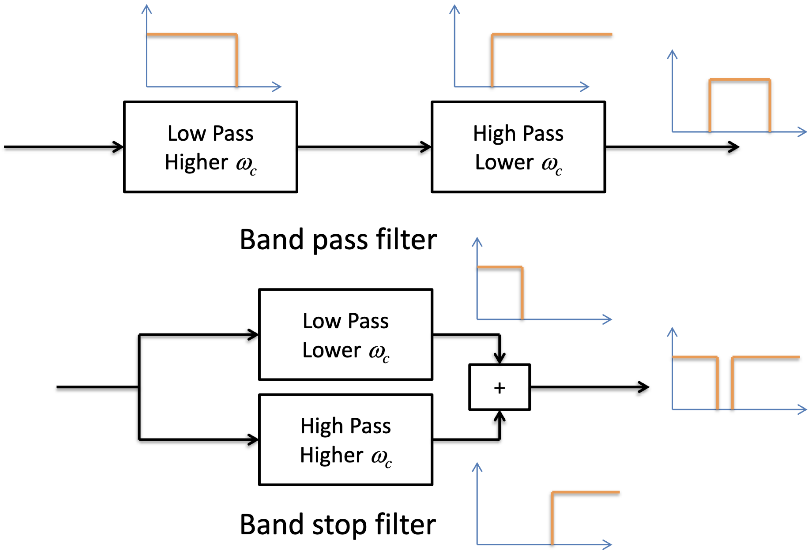


Figure 7.8: Band Pass and Band Stop Filters

List of Figures