Queensland University of Technology

EGB120
Foundations of Electrical Engineering

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Contents

Chapter 1

Circuits and Sources

1.1 Fundamental Electrical Quantities

1.2 Power and Energy

1.3 Circuits and Sources

1.4 Resistors and Ohm's Law

Chapter 2

Filters and Rectifiers

Chapter 3

Source Transformation

3.1 Thevenin Equivalent Circuits

3.2 Norton Equivalent Circuits

3.3 Superposition

3.4 Maximum Power Transfer

Chapter 4

Ohm's Law, Kirchoff's Laws, and Resistive Circuits

4.1 Kirchoff's Laws

4.2 Series and Parallel Elements

4.3 Voltage and Current Dividers

4.4 Measuring Voltages and Currents

Chapter 5

Diodes

5.1 Introduction to Diodes

5.2 Voltage and Current Characteristics

5.3 Operating Points and Load Lines

5.4 Practical Diodes

Chapter 6

Mesh Analysis

6.1 Nodes, Loops, and Meshes

6.2 Mesh Analysis

6.3 Current Source Mesh Analysis

6.4 Dependant Source Mesh Analysis

Chapter 7

Inductors and Capacitors

7.1 Capacitors

7.2 Inductors

7.3 Inductors and Capacitors in Steady State

Chapter 8

RL and RC circuits and Time Response

8.1 Switches

8.2 Natural Response

8.2.1 Capacitors and Inductors

Capacitors store energy as voltage

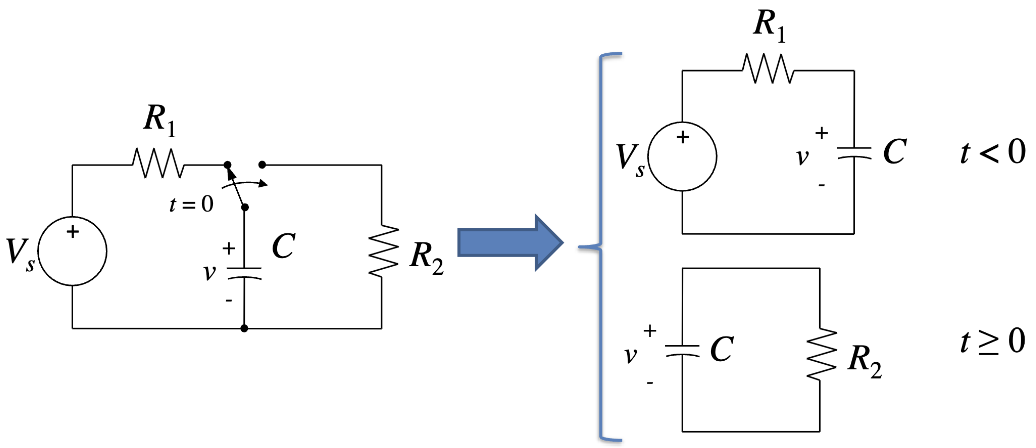
$$i = C\frac{\mathrm{d}v}{\mathrm{d}t}$$

Inductors store energy as current

$$v = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

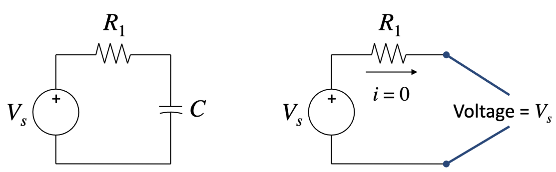
8.2.2 Switched RC Circuit

Assuming that the switch has been in the first position for a long time (till reached steady state). Find the voltage right before $t = 0$.



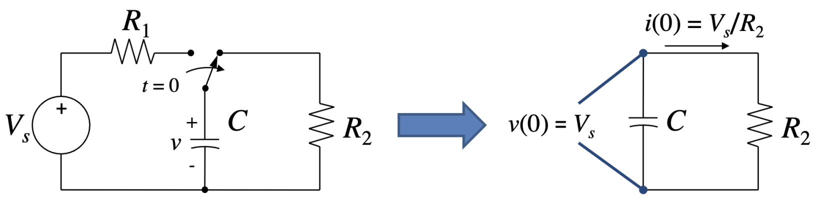
For the initial condition

- Perform steady state analysis by treating the capacitor as an open circuit



The voltage across the capacitor right before $t = 0$ is the same as source voltage V_s

Natural Response of an RC Circuit



Using KCL we get

$$-C\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{v}{R_2}$$

Rearranging and solving the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{v}{R_2C}$$
$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{R_2C}$$

Integrate both sides

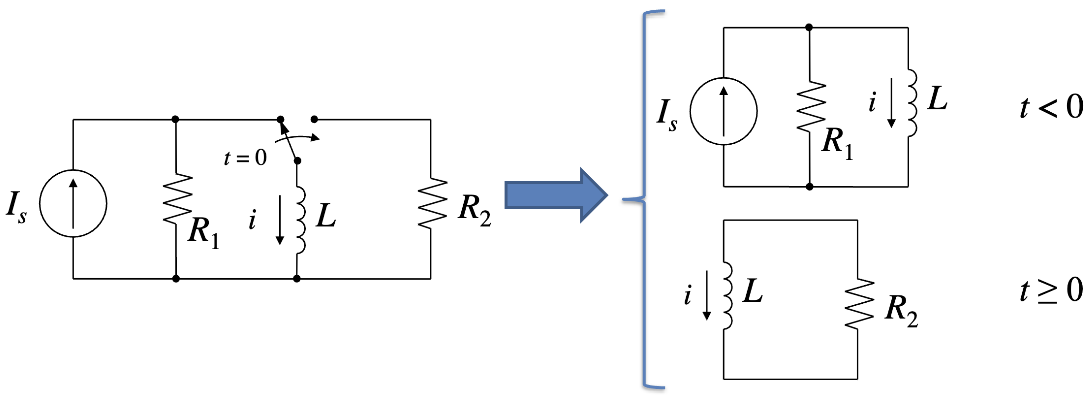
$$\int \frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}t}\,dt = \int -\frac{1}{R_2C}\,dt$$
$$\ln v = -\frac{t}{R_2C} + k$$
$$v = e^{-\frac{t}{R_2C}+k}$$
$$v = Ae^{-\frac{t}{R_2C}}$$

Noting that $A = v(0) = V_s$ we get

$$v = V_s e^{-\frac{t}{R_2C}}$$

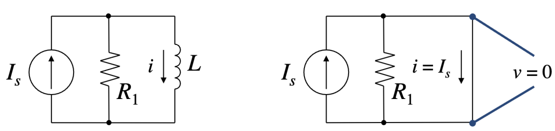
8.2.3 Switched RL Circuit

Assuming that the switch has been in the first position for a long time (till reached steady state). Find the current right before $t = 0$.



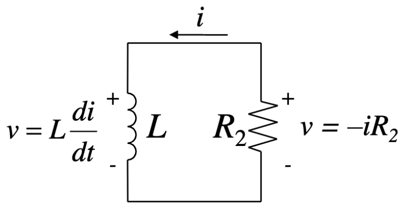
For the initial condition

- Perform steady state analysis by treating the inductor as a short circuit



The current through the inductor right before $t = 0$ is the same as source current I_s

Natural Response of an RL Circuit



Using KVL we get

$$-L\frac{\mathrm{d}i}{\mathrm{d}t} = R_2 i$$

Rearranging and solving the differential equation

$$\frac{\mathrm{d}i}{\mathrm{d}t} = -\frac{R_2}{L}i$$

Using the integrating factor method

$$\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{R_2}{L}i = 0$$

Multiplying both sides by $e^{\frac{R_2}{L}t}$

$$e^{\frac{R_2}{L}t}\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{R_2}{L}e^{\frac{R_2}{L}t}i = 0$$

Noting that $\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{\frac{R_2}{L}t}i\right) = e^{\frac{R_2}{L}t}\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{R_2}{L}e^{\frac{R_2}{L}t}i$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{\frac{R_2}{L}t}i\right) = 0$$

Integrating both sides

$$\int \frac{\mathrm{d}}{\mathrm{d}t}\left(e^{\frac{R_2}{L}t}i\right)\,dt = \int 0\,dt$$
$$e^{\frac{R_2}{L}t}i = k$$
$$i = ke^{-\frac{R_2}{L}t}$$

Noting that $k = i(0) = I_s$ we get

$$i = I_s e^{-\frac{R_2}{L}t}$$

8.2.4 Natural Response

$$v(t) = v(0)e^{-\frac{t}{R_2C}}$$ (Where $\tau = RC$ is the time constant)

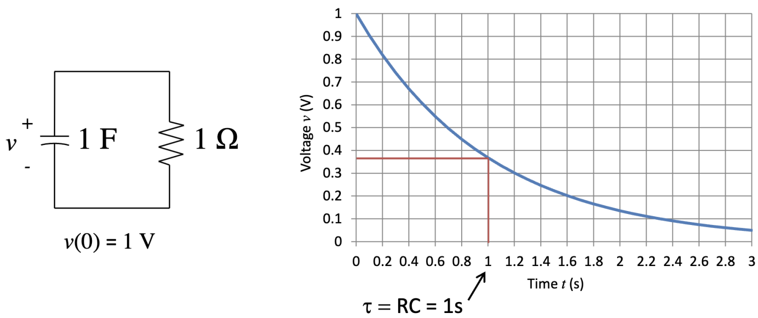


Figure 8.1: Natural Response of RC Circuit for $v(t) = v(0)e^{-\frac{t}{R_2C}}$ where $v(0) = 1V$, $R_2C = 1\Omega$, and $C = 1F$

$$i(t) = i(0)e^{-\frac{t}{R_2 C}} \qquad \text{(Where } \tau = \frac{L}{R_2} \text{ is the time constant)}$$

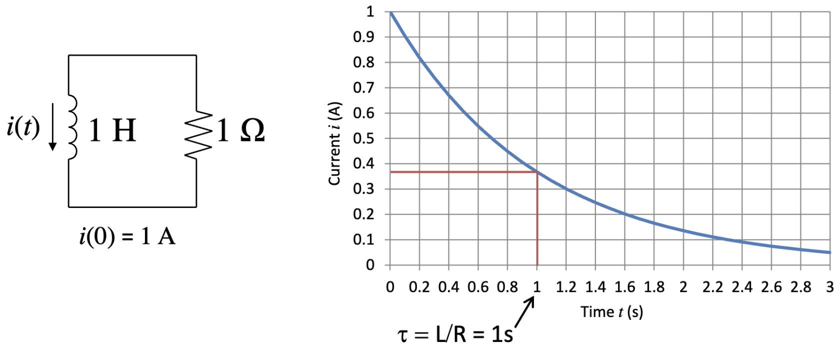


Figure 8.2: Natural Response of RL Circuit for $i(t) = i(0)e^{-\frac{t}{R_2 C}}$ where $i(0) = 1A$, $R_2C = 1\Omega$, and $L = 1H$

8.3 Step Response

Chapter 9

Operational Amplifiers

9.1 Introduction to Operational Amplifiers

9.2 Op Amp Analysis

9.3 Practical Op Amps

Chapter 10

Sinusoidal State Analysis

10.1 Sinusoidal Signals

10.2 RMS

10.3 Phasors

10.4 Circuit Analysis with Phasors

Chapter 11

Frequency Response of RL and RC Circuits

11.1 AC Circuit Analysis

11.2 Frequency Response and Transfer Functions

11.3 Bode Plots

Chapter 12

Filters and Rectifiers

12.1 Filter Introduction

12.2 Passive Filters

12.3 Active Filters

Chapter 13

Zener Diodes and Voltage Regulators

13.1 Rectifiers and Regulators

13.2 Zener Diode Regulators

13.3 Series Regulators

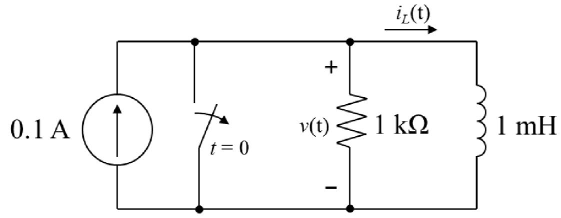
13.4 Op Amps as Regulators

Chapter 14

Tutorials

14.1 Tutorial 7 Sequential Switching

Consider the circuit below. The initial current in the inductor is $i_l(0-) = 0$. Find expressions for $iL(t)$ and $v(t)$ for $t \geq 0$ and sketch to scale versus time.



The given circuit is an RL circuit, therefore, step response is given by

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}$$

By applying a source transformation (Norton equivalent to Thevenin equivalent) we get

$$V_s = I_s R = 0.1 \times 1 \times 10^3 = 0.1 \times 10^3 V = 100V$$

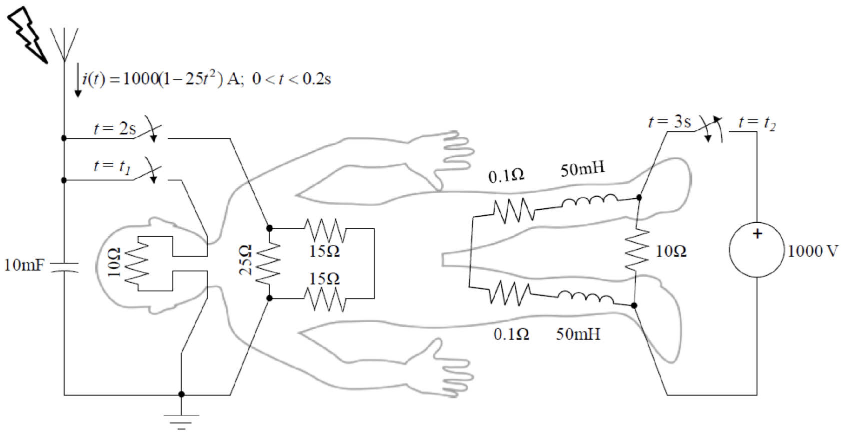
Using this we get that

$$i(t) = \frac{100}{1000} + \left(0 - \frac{100}{1000}\right) e^{-\frac{1000}{1\times -3}t} \qquad = 0.1 - 0.1e^{-1\times10^6 t}$$

$$v(t) = L\frac{\mathrm{d}i}{\mathrm{d}t} = 1 \times 10^{-3} \times \frac{\mathrm{d}}{\mathrm{d}t}\left(0.1 - 0.1e^{-1\times10^6 t}\right) = 0.1 \times 10^3 e^{-1\times10^6 t} \qquad = 100e^{-1\times10^6 t}$$

14.1.1 Tutorial Question

Igor the Mad Scientist is planning to reanimate a corpse (again). His plan is to capture energy from a lightning bolt into a capacitor, and then to discharge the capacitor into the torso and brain. The lower body is reanimated from a voltage supply. The electrical models of the body parts are shown in the figure below

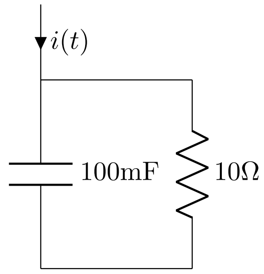


The steps for corpse re-animation are as follows:

- **Step 1**: Capture a lightning strike into a discharged capacitor. The moment of the lightning strike marks $t = 0$. The lightning strike has a duration of 200 ms, and creates a current $i(t) = 1000(1 - 25t^2)$ A for $0 < t < 0.2s$.
- **Step 2**: At t = 2s, start discharging the capacitor across the torso.
- **Step 3**: When the capacitor voltage falls to 1000 V, close the switch to discharge the capacitor into the brain (through the neck bolts).
- **Step 4**: At t = 3s, close the switch to connect the legs to the 1000 V source.
- **Step 5**: When the current reaches 500A, open the switch to disconnect the legs from the 1000 V source.
- **Step 6**: Once the leg current drops below 50 mA, and the capacitor voltage drops below 10 V the corpse will come to life. Disconnect the cables and feed your new monster some tea and cake.

Igor is using computer controlled switching for precise timing. He needs your help to work out the right time for critical switching activities.

1. Find the voltage of the capacitor after the lightning strike for $0.2 < t < 2s$.
   By isolating the circuit for the step response we get

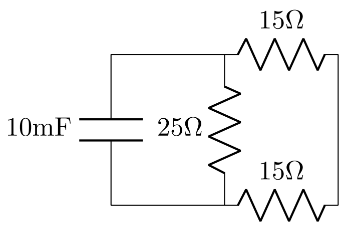


   As this is an RC Circuit

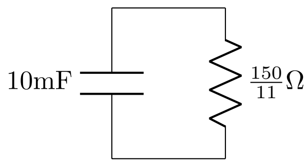
$$\begin{aligned} V(t) &= \frac{1}{C}\int_0^t i(\tau) + v(0)\, d\tau \\ &= \frac{1}{10^{10^{-3}}}\int_0^t 1000(1-25\tau^2)\, d\tau + 0 \\ &= 10^5\left(\tau - \frac{25}{3}\tau^3\right)\Big|_0^t \\ &= 10^5\left(t - \frac{25}{3}t^3\right) \end{aligned}$$

$$\begin{aligned} V(0.2) &= 10^5\left(0.2 - \frac{25}{3}(0.2)^3\right) \\ &= 10^5\left(0.2 - \frac{25}{3}(0.008)\right) \\ &= 10^5\,(0.2 - 0.066) \\ &= 10^5 \times 0.13\overline{3}\,\mathrm{V} \\ &= 13.33\overline{3}\,\mathrm{kV} \end{aligned}$$

2. Find an expression for the capacitor voltage while the capacitor is discharging into the torso only. When will the capacitor be sufficiently discharged to connect the capacitor to the brain?
   Capacitor will start discharging at $t = 2s$ and will be sufficiently discharged when $V(t) = 1000V$
   Find the circuit at $t = 2s$



   Note that the two 15Ω resistors are in series, therefore, we can replace them with a single 30Ω resistor.
   This 30Ω resistor is in parallel with the 25Ω resistor, therefore, we can replace them with a single $\left(\frac{1}{30}+\frac{1}{25}\right)^{-1} = \frac{150}{11}\Omega$ resistor.
   This gives the following circuit



   Using this RC circuit we can solve for when voltage is 1000, where $v(0) = 13.33\overline{3}\,\mathrm{kV}$

$$v(t) = v(0)e^{-\frac{t}{RC}}$$

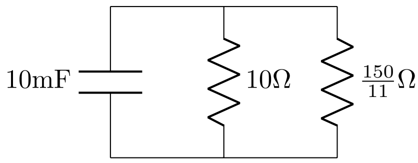
$$1000 = 13.33\overline{3}e^{-\frac{t}{\frac{150}{11}\times 10^{-3}}}$$

   Using calculator

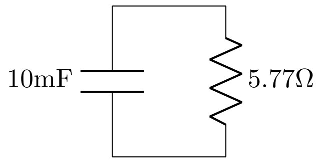
$$t = 0.3532s$$

   Noting that this time is relative to the 2 seconds since the lightning strike, therefore, the time since the lightning strike is $t = 2.3532s$

3. When will the capacitor voltage fall below 10V? First, include the head that is now connected



Note that the two resistors are in parallel therefore we can replace them with a single $\left(\frac{1}{10}+\frac{1}{\frac{150}{11}}\right)^{-1} = 5.77\Omega$ resistor. This gives the following circuit



Using this RC circuit we can solve for when voltage is 10, where $v(0) = 1000\,\mathrm{V}$

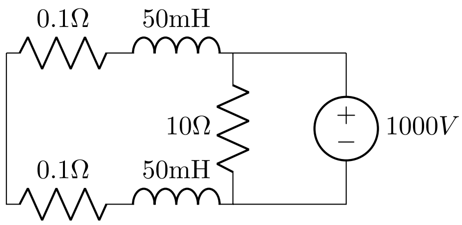
$$v(t) = v(0)e^{-\frac{t}{RC}}$$
$$10 = 1000e^{-\frac{t}{5.77\times10^{-3}}}$$

Using calculator

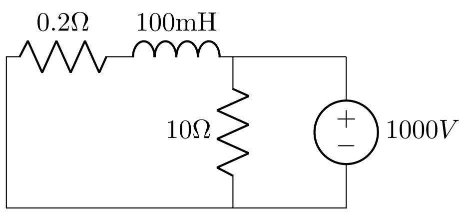
$$t = 0.2657s$$

As this time is relative to the time from part 3, the time since the lightning strike is $t = 2.6189s$

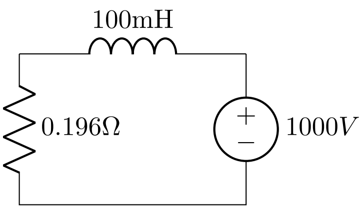
4. Find an expression for the leg current when the voltage source is connected to the legs. When should the switch connecting the voltage source to the legs be opened?
Looking at the torso circuit we see the following



Noting that the two 0.1Ω resistors are in series, we can combine them into a single 0.2Ω resistor.
Noting that the two 50mH inductors are in series, we can combine them into a single 100mH inductor.
This gives the following circuit



Noting that the 0.2Ω resistor and 10Ω resistor are in parallel, we can combine them into a single $\left(\frac{1}{0.2}+\frac{1}{10}\right)^{-1} = 0.196\Omega$ resistor.
This gives the following circuit



$$
\begin{aligned}
i(t) &= \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t} \\
&= \frac{1000}{0.196} + \left(0 - \frac{1000}{0.196}\right) e^{-\frac{0.196}{100\times 10^{-3}}t} \\
&= 5102.04 - 5102.04e^{-1.96t} \\
v(t) &= L\frac{\mathrm{d}i}{\mathrm{d}t} \\
&= 100 \times 10^{-3} \times \frac{\mathrm{d}}{\mathrm{d}t}\left(5102.04 - 5102.04e^{-1.96t}\right) \\
&= 100 \times 10^{-3} \times 5102.04 \times 1.96e^{-1.96t} \\
&= 100007e^{-1.96t}
\end{aligned}
$$

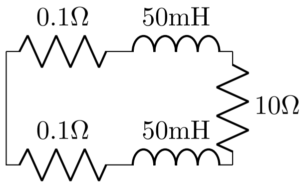
When current is 500A, open the switch to disconnect legs

$$500 = 5102.04 - 5102.04e^{-1.96t}$$

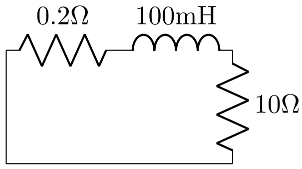
Using calculator

$$
\begin{aligned}
t &= 0.05262s && \text{(Relative to 3 seconds)} \\
t_{\text{since lightning strike}} &= 3.05262s
\end{aligned}
$$

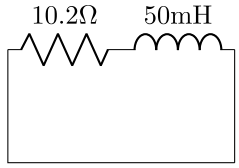
5. When will the leg current fall below 50mA? Now that the legs are disconnected, we get the following circuit



Noting that the two 0.1Ω resistors are in series, we can combine them into a single 0.2Ω resistor.
Noting that the two 50mH inductors are in series, we can combine them into a single 100mH inductor.
This gives the following circuit



Noting that the 0.2Ω resistor and 10Ω resistor are in series, we can combine them into a single 10.2Ω resistor.
This gives the following circuit



Natural response of an RL circuit

$$
\begin{aligned}
i(t) &= i(0)e^{-\frac{t}{R_2 C}} \\
&= 500e^{-\frac{t}{10.2\times 10^{-3}}}
\end{aligned}
$$

When current is 50mA, open the switch to disconnect legs

$$0.05 = 500e^{-\frac{t}{10.2\times 10^{-3}}}$$

Using calculator

$$
\begin{aligned}
t &= 0.0903s && \text{(Relative to } t_{\text{since lightning strike}} \text{ from 4)} \\
t_{\text{since lightning strike}} &= 3.14292s
\end{aligned}
$$