

Queensland University of Technology

MZB221
Electrical Engineering Mathematics

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October 1, 2023

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Chapter 1

Infinite Series

- 1.1 Sequences, Infinite Series, Convergence
- 1.2 Taylor polynomials, Taylor series, Radius of convergence
- 1.3 Introduction to Fourier series
- 1.4 Constructing Fourier series

Chapter 2

Vector Calculus

2.1 Introduction to Vector Calculus, div, grad, curl

2.1.1 Scalar Fields

A scalar field is a function

$$f : \mathbb{R}^n \rightarrow \mathbb{R}$$

$$n = 2 \quad f = f(x, y)$$

- 2 independent variables x, y
- f is a function that has x and y as inputs and a single real number as the output

Physical examples of scalar fields are

- The temperature $T(x, y, z)$, the pressure $p(x, y, z)$, the density $\rho(x, y, z)$ of a fluid
- Concentration of a pollutant in a lake $c(x, y, z)$
- Height of a surface or a mountain $h(x, y)$
- Charge density $\rho(x, y, z)$, electrical potential $V(x, y, z)$

Partial Derivatives

For $f : \mathbb{R}^2 \rightarrow \mathbb{R}$

$$\frac{\partial f}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x+h, y) - f(x, y)}{h}, \quad \frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{f(x, y+h) - f(x, y)}{h}$$

- f_x is the rate of change of f in the x -direction (y is constant) (and vice-versa)
- Geometrically, f_x and f_y are the slopes of the surface $z = f(x, y)$ in the x and y directions.

For $f : \mathbb{R}^3 \rightarrow \mathbb{R}$

$$\frac{\partial f}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x+h, y) - f(x, y)}{h}, \quad \frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{f(x, y+h) - f(x, y)}{h}$$
$$\frac{\partial f}{\partial z} = \lim_{h \rightarrow 0} \frac{f(x, y, z+h) - f(x, y, z)}{h}$$

No equivalent geometric representation as "surface" ($f(x, y, z)$) is a 4-dimensional hypersolid

Directional Derivative of a Scalar Field

2.2 Review of Multiple Integration, Change of Variables

2.3 Introduction to cylindrical and spheroidal coordinates, integration

2.4 Line Integrals, Surface Integrals

Chapter 3

Differential Equations

- 3.1 Introduction to Laplace transform, Strategy for Solving Linear ODEs
- 3.2 Further Properties of Laplace Transforms, solving more complicated initial value problems
- 3.3 Non-Linear first-order ODEs, Phase lines, Stability, Bi-Furcation
- 3.4 Linear Systems of ODEs, Exact Solutions, Classification, Non-Homogeneous Systems
- 3.5 Non-Linear Systems of ODEs, Phase Plane, Nullclines, Stability