Queensland University of Technology

EGB120

Practice Exam Solutions

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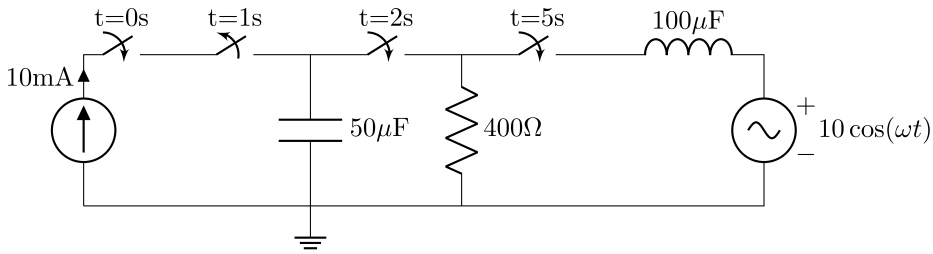
Contents

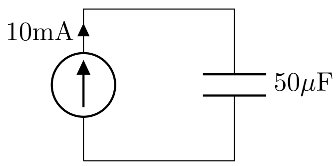
Chapter 1

Practice Exam 1

Cyril the Circuit Analyst is examining the interaction of both AC and DC sources with a circuit. The circuit is drawn below. Pay careful attention to the arrows on the switches which show which are initially open, and which are initially closed. Assume that the capacitor is initially discharged at t = 0. All times are in seconds.



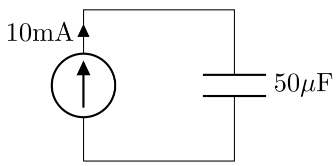
1. At t = 0, the first switch closes connecting the current source to the capacitor. Find the voltage on the capacitor at t = 1 second
   Solution:



$$\begin{aligned} v(t) &= \frac{1}{C}\int_0^t i(\tau)\ d\tau + v(0) \\ &= \frac{1}{50\times10^{-6}}\int_0^1 10\times10^{-3}\ d\tau + 0 \\ &= 200\text{V} \end{aligned}$$

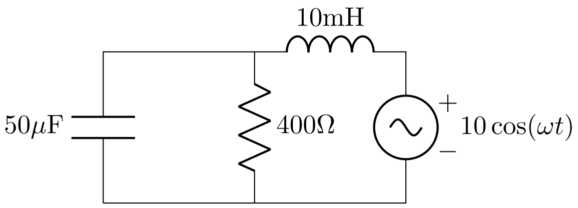
2. At t = 1, the second switch opens disconnecting the current source from the capacitor. Find the voltage on the capacitor at t = 2 seconds.
   Solution:
   The capacitor is not connected to any circuit elements. Therefore, the voltage across the capacitor will remain constant at 200V for $1 < t < 2$.

3. At t = 2, the third switch closes connecting the capacitor to the resistor. Find the voltage on the capacitor at t = 2.1 seconds
   Solution:



$$\begin{aligned} V(t) &= V(0)e^{-\frac{t}{RC}} \\ &= 200e^{-\frac{0.1}{400\times50\times10^{-6}}} \\ V(0.1) &= 200e^{-5\times10^{5}\times10^{-1}} = 1.348\text{V} \end{aligned}$$

4. At t = 5, the fourth switch closes connecting the AC source and inductor. The circuit reaches AC steady state at some $t >> 5$. Express the AC steady state voltage on the capacitor as a function of time using $\omega = 1000$ rad/s
   Solution:



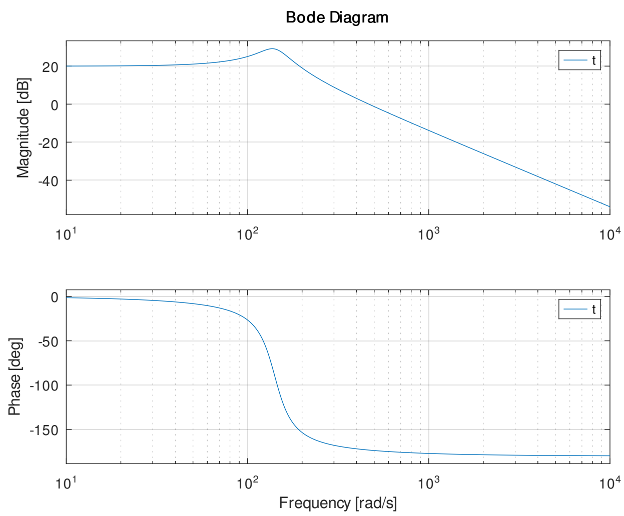
$$\begin{aligned} V_{\text{out}} &= \frac{Z_R||Z_C}{(Z_R||Z_C)+Z_L}V_{\text{in}} \qquad \text{(Voltage divider)} \\ &= \frac{\frac{1}{j\omega C}||R}{\frac{1}{j\omega C}||R+j\omega L}V_{\text{in}} \\ &= \frac{\left(j\omega C+\frac{1}{R}\right)^{-1}}{\left(j\omega C+\frac{1}{R}\right)^{-1}+j\omega L}V_{\text{in}} \\ &= \frac{\left(\frac{1}{400}+j\times 10^3\times 50^{-6}\right)^{-1}}{\left(\frac{1}{400}+j\times 10^3\times 50^{-6}\right)^{-1}+j\times 10^3\times 10^{-2}} \\ &= 19.95 - j0.998 \end{aligned}$$

5. Perform frequency response analysis for the circuit for t >> 5 to calculate the gain in dB and phase in degrees for ω at 100 rad/s, 1000 rad/s and 10000 rad/s

$$\begin{aligned} H(\omega) = \frac{V_{\text{out}}}{V_{\text{in}}} &= \frac{Z_R||Z_C}{(Z_R||Z_C)+Z_L} \qquad \text{(Voltage divider)} \\ &= \frac{\frac{1}{j\omega C}||R}{\frac{1}{j\omega C}||R+j\omega L} \\ &= \frac{\left(j\omega C+\frac{1}{R}\right)^{-1}}{\left(j\omega C+\frac{1}{R}\right)^{-1}+j\omega L} \\ &= \frac{\left(\frac{1}{400}+j\omega\times 50\times 10^{-6}\right)^{-1}}{\left(\frac{1}{400}+j\omega\times 50\times 10^{-6}\right)^{-1}+j\omega\times 10^{-2}} \end{aligned}$$

ω	**Gain (dB)**	**Phase (degrees)**
100	0.435	-0.144
1000	5.998	-2.86
10000	-33.804	-179.71

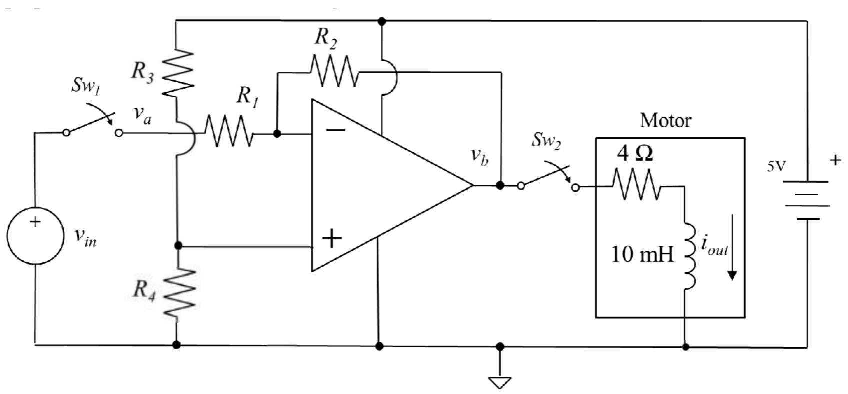
6. Plot the response calculated in part (3) in a Bode plot
Solution:



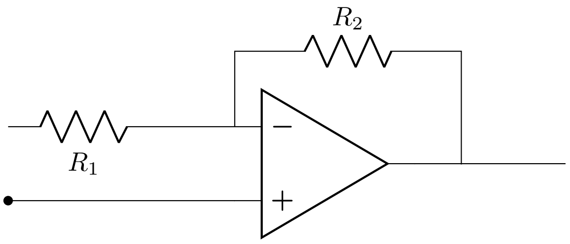
Chapter 2

Practice Exam 2

Michael is designing a motor driver for a wheeled mobile robot. The motor driver is designed as an inverting amplifier (using the circuit shown in Figure 1) to drive the motor in the opposite direction of the input voltage. Michael would like to select the components of the amplifier based on specific design requirements. Michael has installed a switch (Sw_1) to disconnect the motor driver input signal (v_{in}) to the system and a motor protection switch (Sw_2) to disconnect the motor from the amplifier when the motor is not used. A simplified model of a DC motor has been used, made up of a resistor and inductor in series. The DC motor generates a torque proportional to the current (i_{out}) through the inductor



1. Design an inverting amplifier by choosing suitable resistor values for R1 and R2 to produce a gain of 5 when both switches Sw1 and Sw2 are open.
Solution:

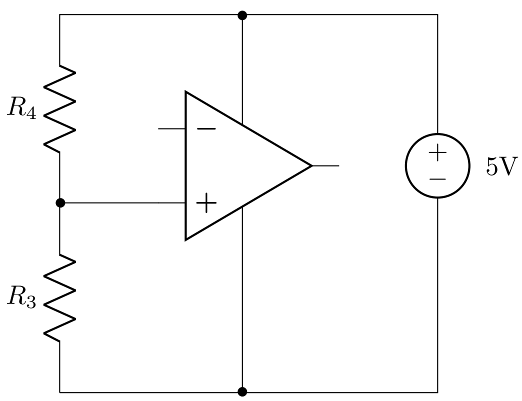


$$|\text{Gain}| = \left|-\frac{R_2}{R_1}\right| = 5$$

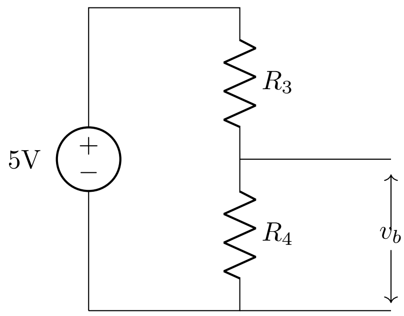
$$\text{Let } R_1 = 1k\Omega$$

$$\therefore R_2 = 5k\Omega$$

2. Design the bias input circuit by choosing suitable resistors R3 and R4 such that the voltage v_b will be 0.5V if the positive power supply of the op-amp is connected to a 5V battery. Again both switches are open.
Solution:

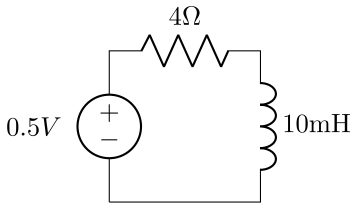


By removing the op amp as its outputs aren't connected we can identify that it is in a voltage divider configuration



$$\begin{aligned} v_b &= \frac{R_2}{R_1+R_2} v_{in} \\ 0.5 &= \frac{R_2}{R_1+R_2} 5 \\ 0.1 &= \frac{R_2}{R_1+R_2} \\ 10(R_1+R_2) &= R_2 \\ \text{Let } R_1 &= 1k\Omega \\ \therefore R_2 &= 9k\Omega \end{aligned}$$

3. Given Sw_1 is open, the amplifier is turned on for enough time, such that the voltages stabilise, Michael connects the motor by closing Sw2 and the motor starts to move. Find the step response of the motor current, i_{out}. Assume that the amplifier output resistance is negligible, it does not current limit and the motor inductor has been de-energised
Solution:
By closing Sw_2 the input of the op amp is connected to the positive node, changing it to a non-inverting amplifier. Therefore, v_b will be +0.5V.
Looking at the motor circuit we see the following



Noting that this is the step response of an RL circuit

$$\begin{aligned} i(t) &= \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) \\ &= \frac{0.5}{4}\left(1 - e^{-\frac{4}{0.01}t}\right) \\ &= 1.25\left(1 - e^{-40t}\right) \end{aligned}$$

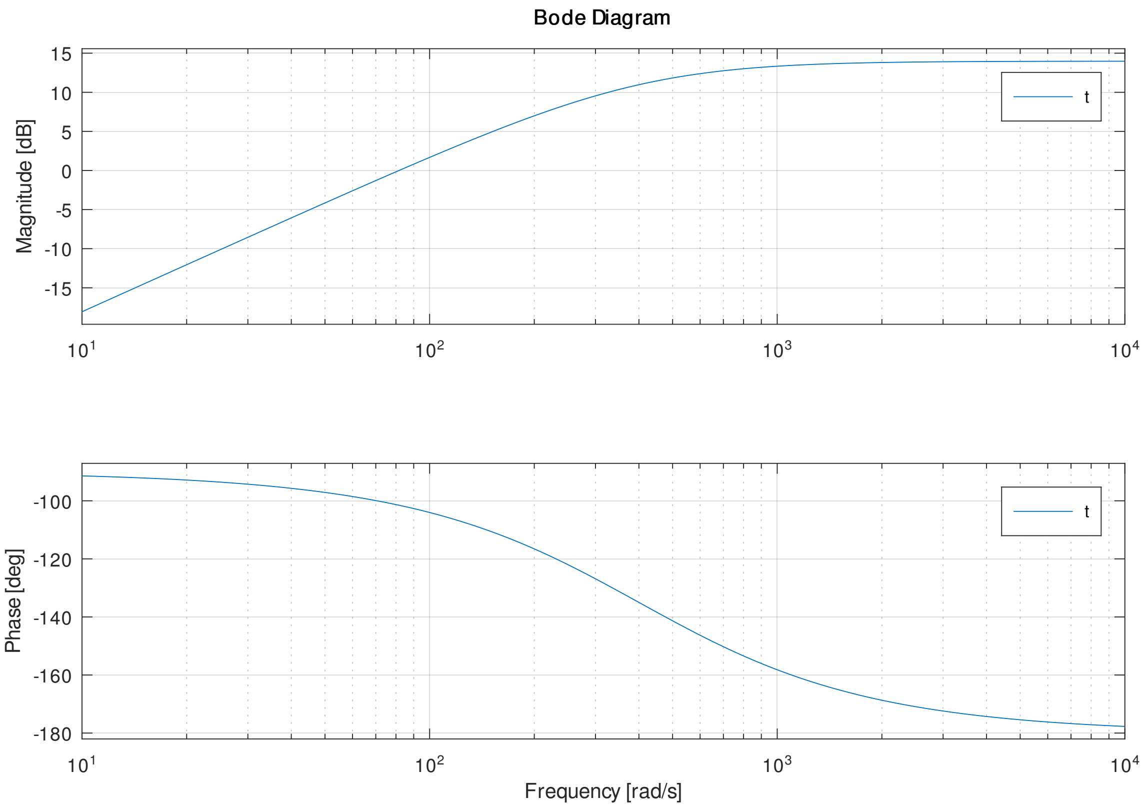
4. What is the steady state voltage across the motor?
Solution:
In steady state, the inductor is replaced with a short circuit.
As the resistor is the only element in the circuit, the steady state voltage across the motor will be 0.5V.

5. Perform frequency response analysis from vin, to the voltage across the motor inductor, when both switches are closed and Michael applies three different input signals of 10 Hz, 100 Hz and 1000 Hz through the system?
Solution:
Using the circuit from (3) we can see it is in a voltage divider configuration, where a gain of -5 is applied as the input is connected to the negative node of the op amp.

$$\begin{aligned} v_{out} &= -5\frac{Z_L}{Z_L+Z_R} v_{in} \\ &= -5\frac{j\omega L}{j\omega L + R} v_{in} \\ &= -5\frac{j\omega \times 10^{-2}}{j\omega \times 10^{-2} + 4} v_{in} \\ &= -5\frac{j\omega}{400 + j\omega} \end{aligned}$$

ω	$H(\omega)$	Gain	$\angle H(\omega)$
20π	$0.776\angle 81.07°$	-2.203	81.07
200π	$4.218\angle 32.48°$	21.203	32.48
2000π	$4.99\angle 3.64°$	41.203	3.64

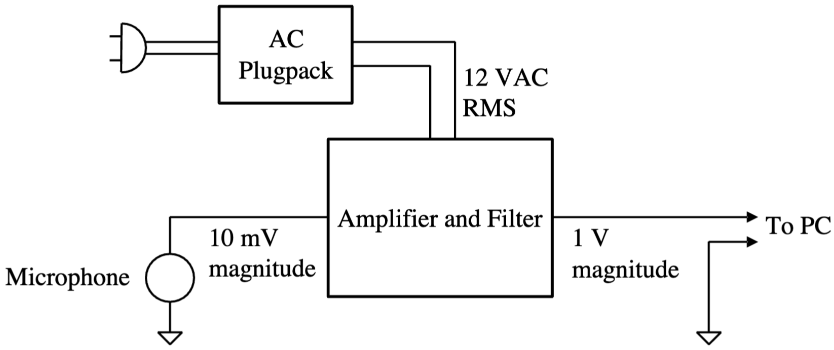
6. Plot the response calculated in part (e) in a Bode plot
Solution:


Bode Diagram
Magnitude [dB]
15
10
5
0
-5
-10
-15
t
Phase [deg]
-100
-120
-140
-160
-180
t
10^1
10^2
10^3
10^4
Frequency [rad/s]

Chapter 3

Practice Exam 3

Barry the Biologist has asked for your help to build an amplifier and filter to take small signals from his bird sensing microphones and amplify them so that he can capture the signals on his PC. The microphone produces AC signals at varying frequencies with 10 mV magnitude. His PC requires the signals to be 1 V magnitude. The signals that Barry is interested in are above 1 kHz Hz. He would like the filter to attenuate signals at frequencies below 1 kHz. The amplifier and filter circuit is to be powered from an AC plugpack. The AC plugpack provides 12 VAC RMS at 50 Hz.



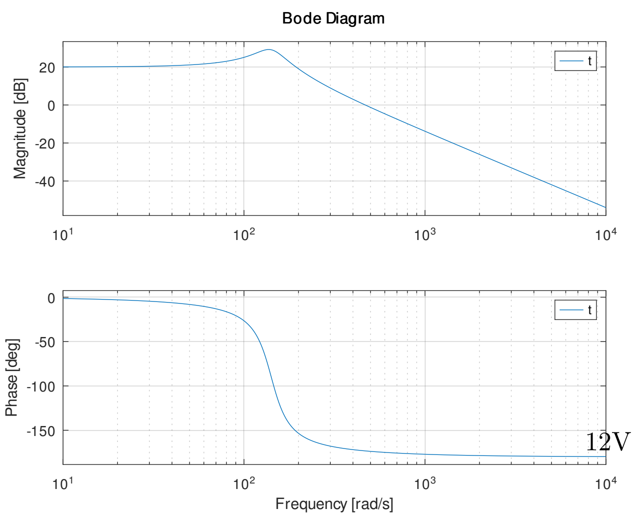
1. Design the power supply component of your amplifier and filter. You will need to produce a dual-sided power supply with +10V, 0V and -10V using the 12 VAC plug- pack as an AC source. Assume that your amplifier and filter will draw around 10 mA of current on both the positive and negative voltage supplies. Your design should include components to minimise voltage ripple. Show calculations for the selection of components where practical. Show a circuit diagram for the complete power supply circuit.
   Solution: In order to minimise voltage ripple, and to have seperate positive and negative voltage components we must use 2 half wave rectifiers and a voltage regulator. The voltage regulator will be used to regulate the voltage to 10V. The half wave rectifiers will be used to convert the AC voltage to DC.
   Calculate the capacitance required for the voltage regulator:

$$\Delta v = \frac{1}{fC} \Rightarrow C = \frac{i}{f\Delta v}$$

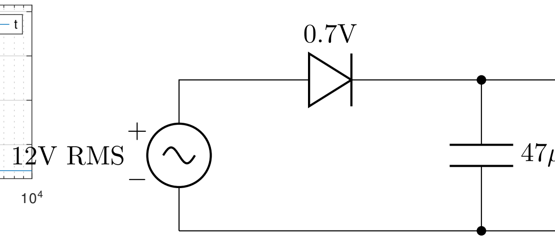
   Using diodes with a forw) at 50Hz, current of 10mA, and output voltage of 10V:

   C tput voltage for linear regulator)

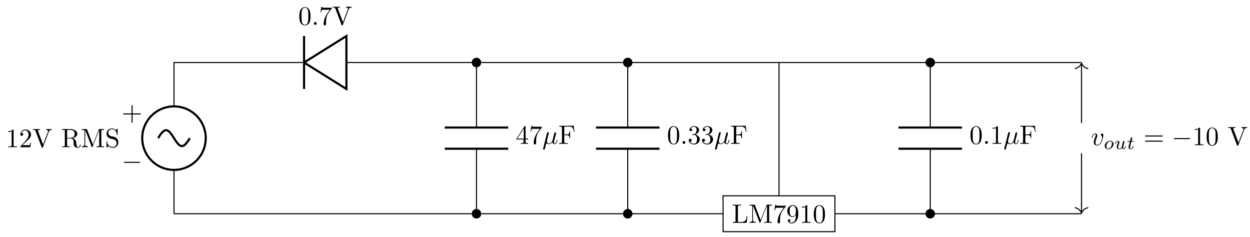
   C



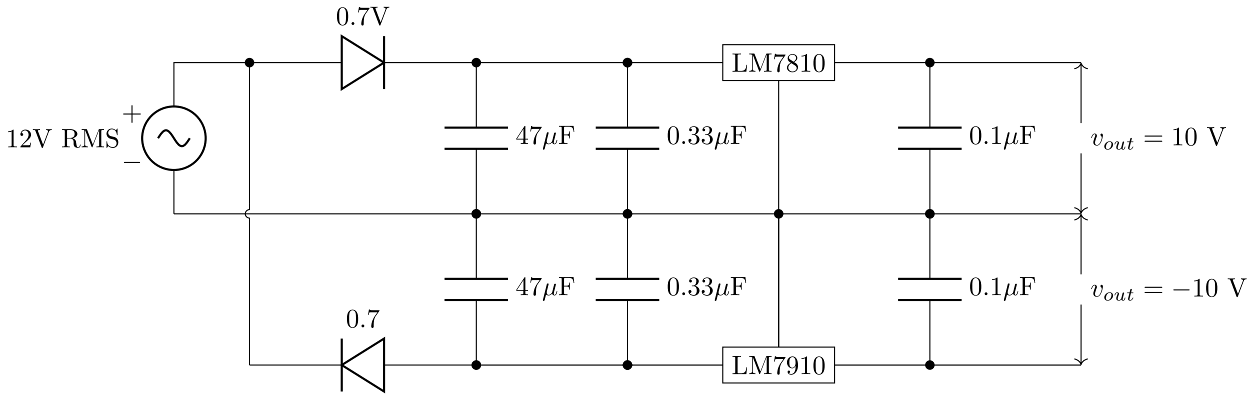
   Positive voltage regul



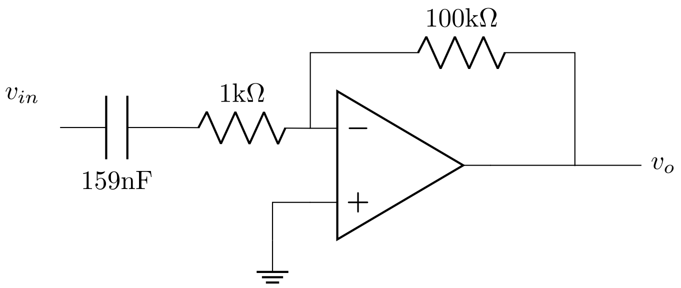
   Negative voltage regulator



Half wave rectifier with positive and negative output



2. Design the amplifier and filter circuit to meet Barry's requirements. The circuit must produce the required gain, and also attenuate the signals below 1 kHz. Show calculations for the selection of components where practical. Show a circuit diagram for the complete amplifier and filter circuit. You do not need to redraw the power supply from part (1)
Solution: In order to attenuate signals below 1kHz, and allow signals above 1kHz to pass through, we must use a high pass filter with a band pass of 1kHz.
As it needs to amplify signals from 10mV to 1V, the gain must be 100.
Active High pass filter:



$$|\text{Gain}| = \left| -\frac{R_2}{R_1} \right| = 100$$
$$\text{Let } R_1 = 1\text{k}\Omega$$
$$\therefore R_2 = 100\text{k}\Omega$$
$$C = \frac{1}{2\pi f R_1} = \frac{1}{2\pi \times 1\text{k} \times 1\text{k}} = 159\text{nF}$$

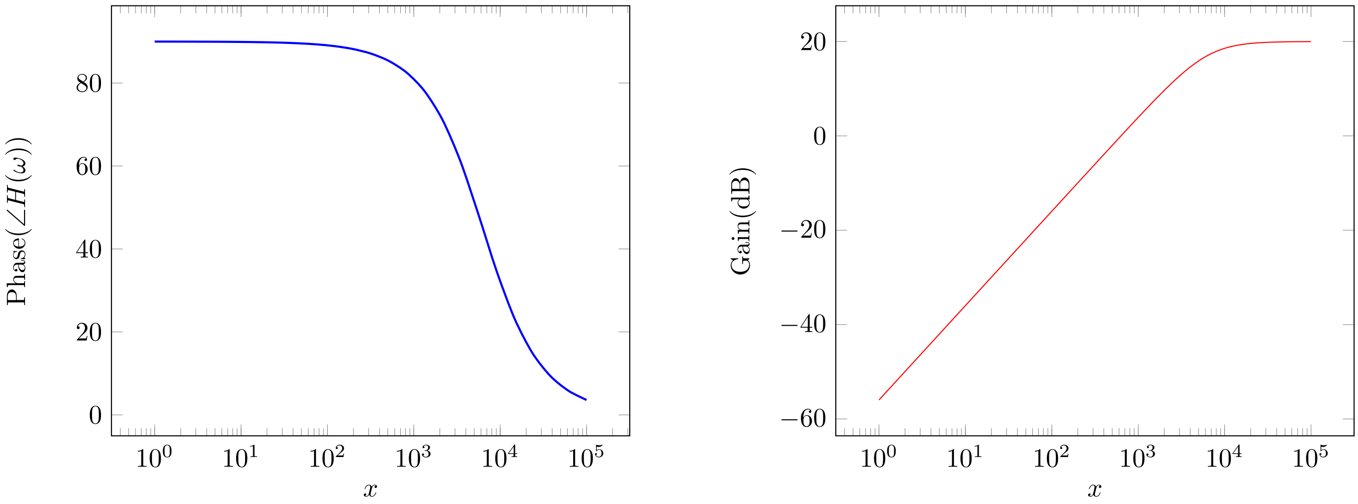
3. Perform frequency response analysis for the amplifier and filter you designed in part (2) to calculate the gain in dB and phase in degrees when the microphone sends a signal at 100 Hz, 1k Hz and 10 kHz
Solution: Using the general form for the transfer function of an active high pass filter:

$$H(\omega) = -\frac{R_2}{R_1}\frac{j\omega}{j\omega + \frac{1}{R_2C_1}} = -100\frac{j\omega}{j\omega + \frac{1}{1\times10^3\times159\times10^{-9}}} = -100\frac{j\omega}{j\omega + 6289.31}$$

Note that angles are shifted to the positive quadrant to simplify plotting (still applies the same phase shift)

Frequency	$\boldsymbol{H(\omega)}$	**Gain**	$\angle \boldsymbol{H(\omega)}$
100Hz	$9.947\angle 84.2949°$	19.948	84.2949
1kHz	$70.68\angle 45.03°$	36.985	45.03
10kHz	$99.5\angle 5.72°$	39.957	5.72

4. Plot the response calculated in part (3) in a Bode plot.
Solution:



(a) Phase plot

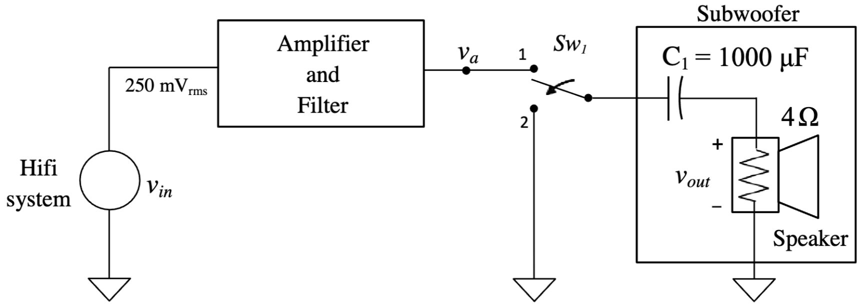
(b) Magnitude plot

Figure 3.1: Phase and Magnitude plots

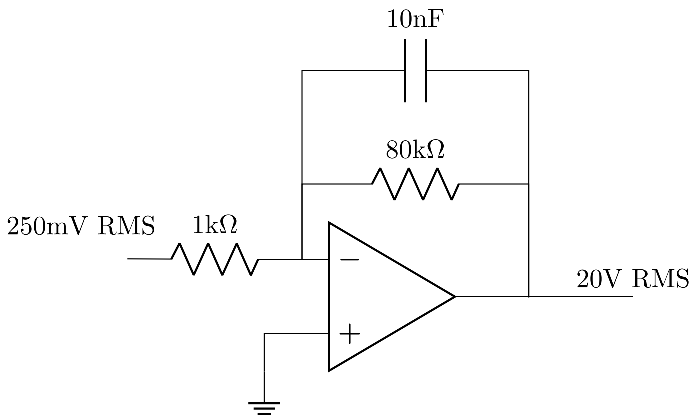
Chapter 4

Practice Exam 4

Anna the Audiophile has asked for your help to build an amplifier and filter to take small signals from her hifi system and amplify them so that she can drive her new subwoofer. The hifi system produces AC signals at varying frequencies with 250 mVrms maximum magnitude. Her subwoofer requires the signals to be 20 Vrms maximum magnitude. The signals that Anna is interested in are below 200 Hz. She would like the filter to attenuate signals at frequencies above 200 Hz.



1. Design the amplifier and filter circuit to meet Anna's requirements. The circuit must produce the required gain, and also attenuate the signals above 200 Hz. Show calculations for the selection of components where practical. Show a circuit diagram for the complete amplifier and filter circuit
   Solution:
   The filter must attenuate all frequencies above 200 Hz. And must increase the gain of the input signal. Therefore, an active low-pass filter was chosen.
   As it needs to amplify signals from 250 mVrms to 20 Vrms, the gain of the amplifier must be 80.



$$
\begin{aligned}
|\text{Gain}| &= \left|-\frac{R_2}{R_1}\right| = 80 \\
\text{Let } R_1 &= 1k\Omega \\
\therefore R_2 &= 80k\Omega \\
C &= \frac{1}{2\pi f R_2} \\
&= \frac{1}{2\pi \times 200 \times 80 \times 10^3} \\
&= 10\text{nF}
\end{aligned}
$$

2. Anna reads the subwoofer manual and notes the value of DC blocking capacitor C1 = 1000 μF. She wonders how the capacitor will affect the frequency response of the subwoofer. What kind of filter is formed by C1 and the speaker resistance? Calculate the cut-off frequency of this filter. Describe the behaviour of this filter in words.
   Solution:
   The capacitor and the speaker resistance form a high-pass filter with a cut-off frequency defined by

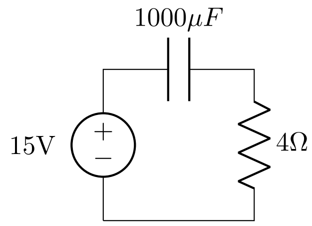
$$
\begin{aligned}
f_c &= \frac{1}{2\pi RC} \\
&= \frac{1}{4 \times 1000 \times 10^{-6} \times 2\pi} \\
&= 40\text{Hz}
\end{aligned}
$$

   This filter will attenuate all frequencies below 40 Hz.

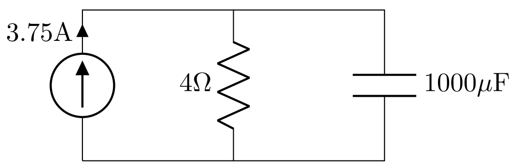
3. When first turned on, Anna's amplifier fails and applies +15V DC directly to the output (va). After 30 milliseconds, the speaker protection circuit operates Sw1 to disconnect the speaker and connect it to ground (Sw1 changes from position '1' to position '2'). Taking the time the amplifier was turned on as t=0 seconds, calculate and plot the voltage across the speaker for the first 60 milliseconds. Ensure your drawing is to scale and has labelled key values

Solution:

First isolate the circuit at $t = 0$.



Apply source transformation from Thevenin to Norton equivalent ($I = \frac{V}{R} = \frac{15}{4} = 3.75A$)



For step response ($0 < t < 0.03$) (capacitor charging)

$$\begin{aligned} v(t) &= I_sR + (V_o - I_sR)e^{-\frac{t}{RC}} \\ &= 3.75 \times 4 + (0 - 3.75 \times 4)e^{-\frac{t}{100\times10^{-6}\times4}} \\ &= 15 - 15e^{-250t} \\ v(0.03) &= 15 - 15e^{-250\times0.03} = 14.9917 \end{aligned}$$

For natural response ($t > 0.03$) (capacitor discharging), $V_o = 14.9917$

$$\begin{aligned} v(t) &= V_oe^{-\frac{t-0.03}{RC}} \\ &= 14.9917e^{-250(t-0.03)} \end{aligned}$$

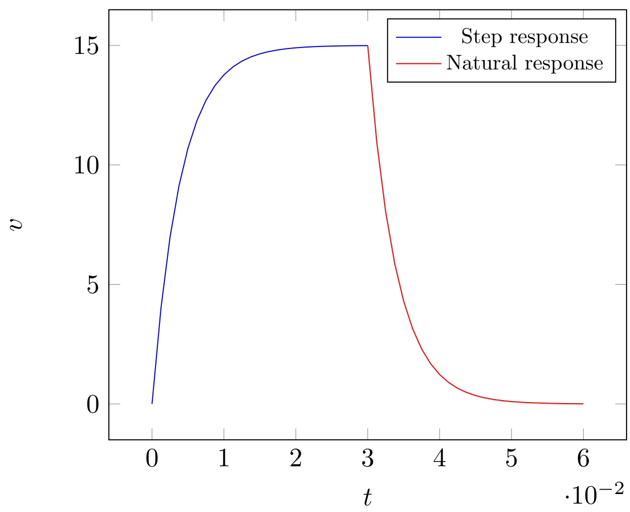


Figure 4.1: Voltage across the speaker

4. The amplifier fault has now been fixed and the system works normally (Sw1 is in position '1'). Perform frequency response analysis for the amplifier and filter you designed in part (a) to calculate the gain in dB and phase in degrees when the hifi system sends a signal at 10 Hz, 100 Hz and 1000 Hz.

Solution:

$$\begin{aligned} H(\omega) &= -\frac{R_2}{R_1}\frac{\frac{1}{R_2C}}{j\omega + \frac{1}{R_2C}} \\ &= -80\frac{1250}{j\omega + 1250} \end{aligned}$$

$$\left(\frac{1}{R_2C} = \frac{1}{80\times10^3\times10^{-9}} = 1250\right)$$

ω	$\mathbf{H}(\omega)$	Gain (dB)	Phase (degrees)
20π	$79.899\angle177.122°$	38.051	177.122
200π	$71.4781\angle153.313°$	37.084	153.313
2000π	$15.6096\angle101.252°$	23.868	101.252

Figure 4.2: Frequency response analysis

5. Calculate the output voltage of the amplifier and filter circuit (va) when a 250 mVrms, 100 Hz signal is applied at its input (v_{in})

Solution:

$$
\begin{aligned}
V_{out} &= V_{in}H(\omega) \\
&= 0.25 \times \sqrt{2} \times H(200\pi) \\
&= 0.25 \times \sqrt{2} \times 71.4781\angle 153.313^\circ && \text{(From part (d))} \\
&= 25.2713\angle 153.313^\circ
\end{aligned}
$$