Queensland University of Technology

EGB120
Foundations of Electrical Engineering

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Contents

Chapter 1

Electrical Circuits

1.1 Fundamental Units

Quantity	Definition	Symbol	SI Unit
Charge	Physical property of matter that causes it to experience a force when placed in an electromagnetic field	q	Coulomb (C)
Current	$$i(t) = \frac{\mathrm{d}q}{\mathrm{d}t}$$	i	Ampere (A)
Voltage	$$v(t) = \frac{\mathrm{d}w}{\mathrm{d}q}$$	v	Volt (V)
Power	$$p(t) = \frac{\mathrm{d}w}{\mathrm{d}t} = \frac{\mathrm{d}q}{\mathrm{d}t} \times \frac{\mathrm{d}w}{\mathrm{d}q} = vi$$	p	Watt (W)
Energy	$$w(\tau) = \int_{\tau}^{0} p(t)dt$$	e	Joule (J)

1.2 Basic Circuit Elements

1.2.1 Voltage Source

Produces or dissipates power at a specific voltage with whatever current is required

1.2.2 Current Source

Produces or dissipates power at a specific current with whatever voltage is required

1.2.3 Resistor

Dissipates power so that the voltage across the terminals is proportional to the current

$$v = Ri \qquad \text{(Ohm's Law)}$$

Following this

$$p = vi = Ri^2 = \frac{v^2}{R}$$

Chapter 2

Simple Circuits

2.1 Physics Ignored

- Electrical effects are instantaneous
- Net charge on every component is zero
- No magnetic coupling between components

2.2 Series

Elements are connected end to end and **have the same current flowing through them**

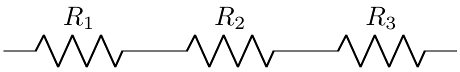


Figure 2.1: Series Circuit

2.3 Parallel

Both ends of one element are connected directly and **have the same voltage across them**

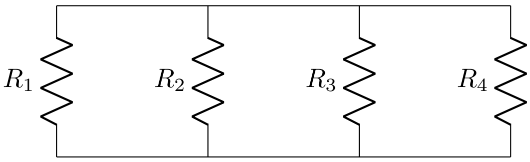


Figure 2.2: Parallel Circuit

2.4 Kirchoff's Laws

2.4.1 Kirchoff's Current Law

The sum of currents entering a node is equal to the sum of currents leaving a node

$$\sum_{k=1}^{n} i_k = 0$$

$$i_1 + i_2 + i_3 = 0$$

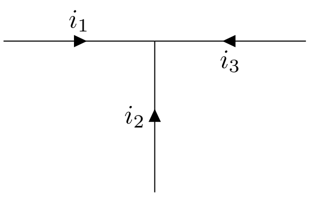
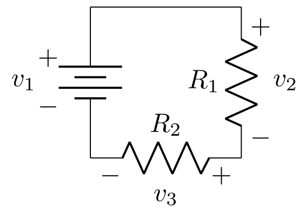


Figure 2.3: Kirchoff's Current Law

2.4.2 Kirchoff's Voltage Law

The sum of voltages around a closed loop is zero

$$\sum_{k=1}^{n} v_k = 0$$



$$v_1 - v_2 - v_3 = 0$$
$$v_1 = v_2 + v_3$$

Figure 2.4: Kirchoff's Voltage Law

2.5 Component Behaviours in Series and Parallel

Component	**Series**	**Parallel**
Voltage Source	$v_s = v_1 + v_2 + v_3$	$v_s = v_1 = v_2 = v_3$
Current Source	$i_s = i_1 = i_2 = i_3$	$i_s = i_1 + i_2 + i_3$
Resistor	$R_s = R_1 + R_2 + R_3$	$\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
Inductor	$L_s = L_1 + L_2 + L_3$	$\frac{1}{L_s} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$
Capacitor	$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$	$C_s = C_1 + C_2 + C_3$

Figure 2.5: Component Behaviours in Series and Parallel

2.6 Voltage Divider

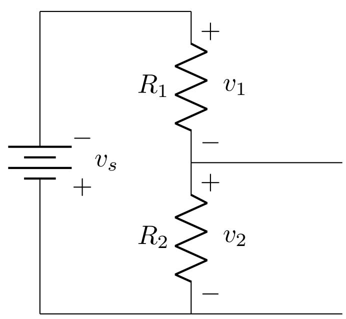


Figure 2.6: Voltage Divider

Current through resistors is

$$i = \frac{v_s}{R_1 + R_2}$$

Voltage across resistors is

$$v_1 = iR_1 = \frac{R_1}{R_1 + R_2} v_s$$

For more resistors

$$v_k = \frac{R_j}{R_{eq}} v_s$$

2.7 Current Divider

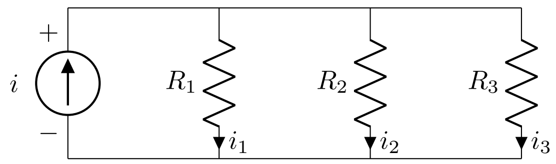


Figure 2.7: Current Divider

Voltage across resistors is

$$v = i(R_1||R_2||R_3)$$

Current through resistors is

$$i_j = \frac{v}{R_j} = \frac{R_1||R_2||R_3}{R_j}i$$

For more resistors

$$i_j = \frac{R_{eq}}{R_j}i$$

Chapter 3

Diodes

Current flows from the anode to the cathode and the voltage across the diode is positive.
A diode requires voltage to be applied across it to conduct current. This voltage is called the **forward voltage**.

3.1 Voltage-Current Characteristics

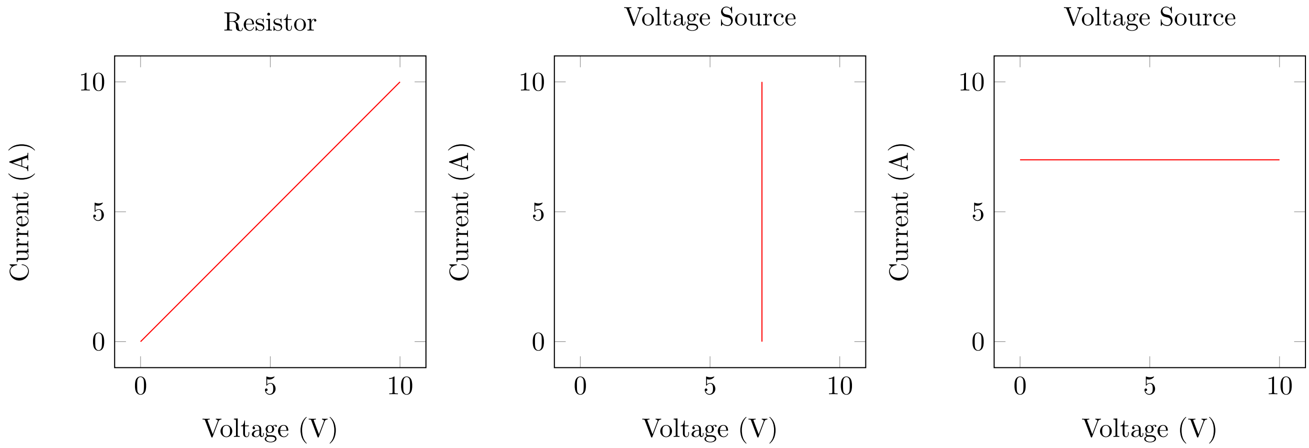


Figure 3.1: Diode VI Characteristics

3.1.1 Simplified Diode Characteristics

Diodes have non-linear characteristics. Therefore, they are simplified to make calculations easier.

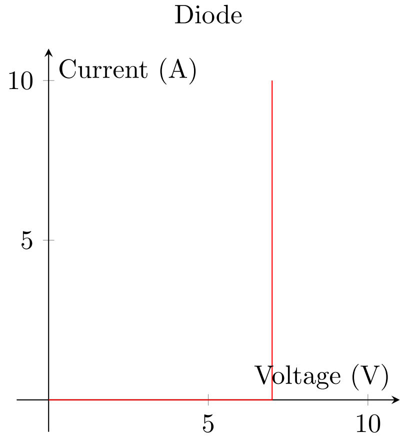


Figure 3.2: Simplified Diode Characteristics

3.1.2 Full Diode Characteristics

Diodes can be accurately modelled using Shcokley's equation

$$i_D = I_s \left(e^{\frac{v_D}{V_T}-1} \right) \qquad V_T = \frac{kT}{q}$$

where I_s is the saturation current, V_T is the thermal voltage, k is Boltzmann's constant, T is the temperature in Kelvin and q is the charge of an electron.

3.2 Operating Points

3.3 Load Lines

Chapter 4

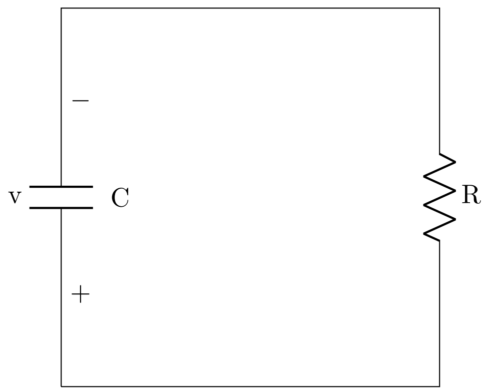
RL and RC Circuits

4.1 Natural Response

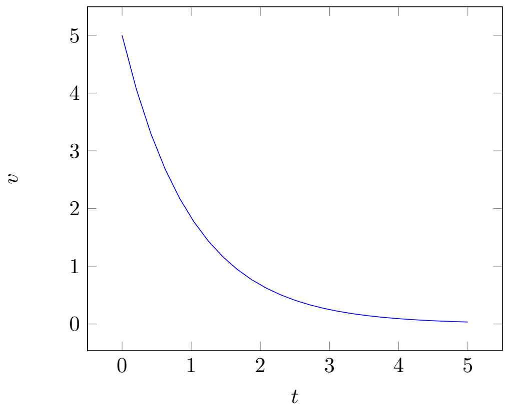
Is a decaying response for an RL and RC circuit

$$v(t) = V_0 e^{-\frac{t}{RC}}$$

(a) Equation for RC Circuit Natural Response



(b) Natural Response RC Circuit

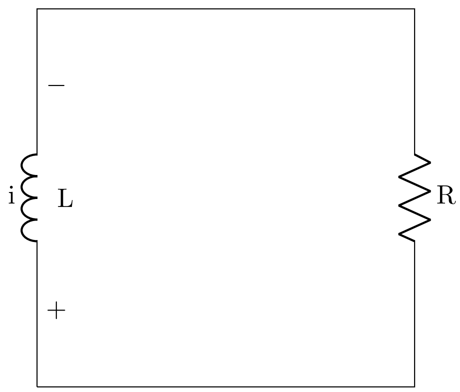


(c) VT Curve for RC Circuit Natural Response

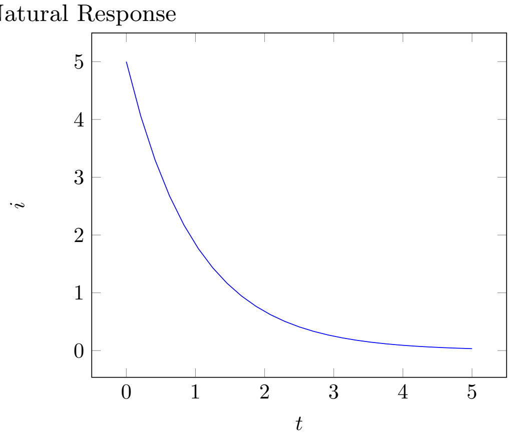
Figure 4.1: RC Natural Response

$$i(t) = I_0 e^{-\frac{Rt}{L}}$$

(a) Equation for RL Circuit Natural Response



(b) Natural Response RL Circuit



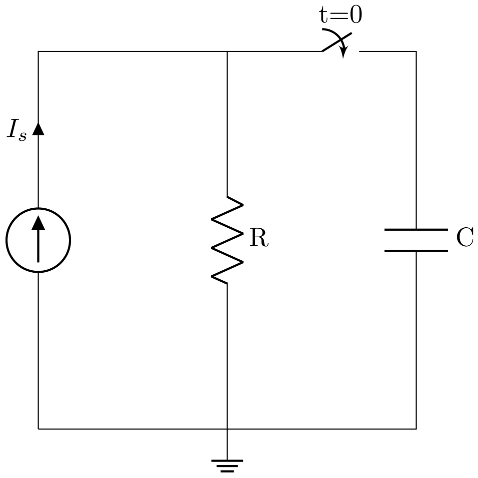
(c) VT Curve for RL Circuit Natural Response

Figure 4.2: RL Natural Response

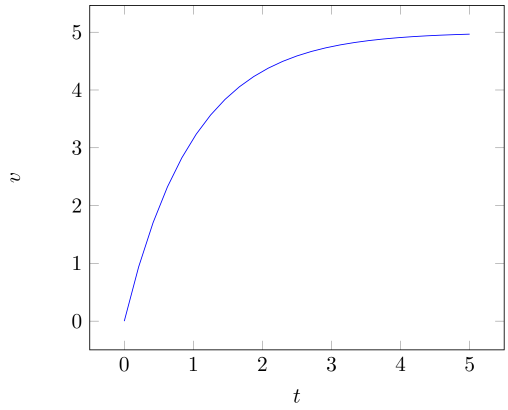
4.2 Step Response

$$v(t) = I_s R + (V_0 - I_s R)e^{-\frac{t}{RC}}$$

(a) Equation for RC Circuit Step Response



(b) Step Response RC Circuit

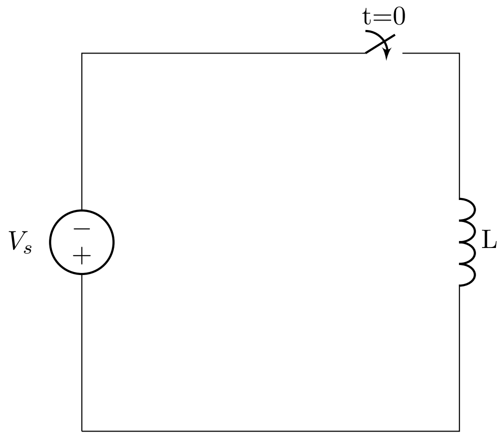


(c) VT Curve for RC Circuit Step Response

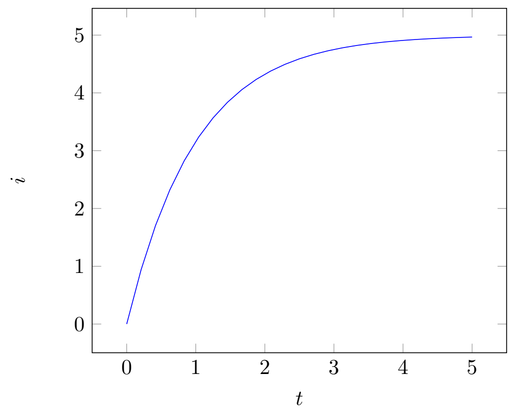
Figure 4.3: RC Step Response

$$i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right) e^{-\frac{Rt}{L}}$$

(a) Equation for RL Circuit Step Response



(b) Step Response RL Circuit



(c) VT Curve for RL Circuit Step Response

Figure 4.4: RL Step Response

Chapter 5

Filters and Rectifiers

Filters keep the desired frequency components of a signal and remove the unwanted frequency components. They do this in the frequency domain

5.1 Applications of Filters

- Audio Signals
  - Remove high frequency hiss (magnetic tape)
  - Remove low frequency rumble (vinyl)
- Medical Signals
  - EEG alpha waves are 8-12Hz
  - EEG beta waves are 12-30Hz
  - ECG waves are 1 - 40Hz
- 50Hz interference
  - Remove all signals near 50Hz
- Signal Processing

5.2 Ideal Filters

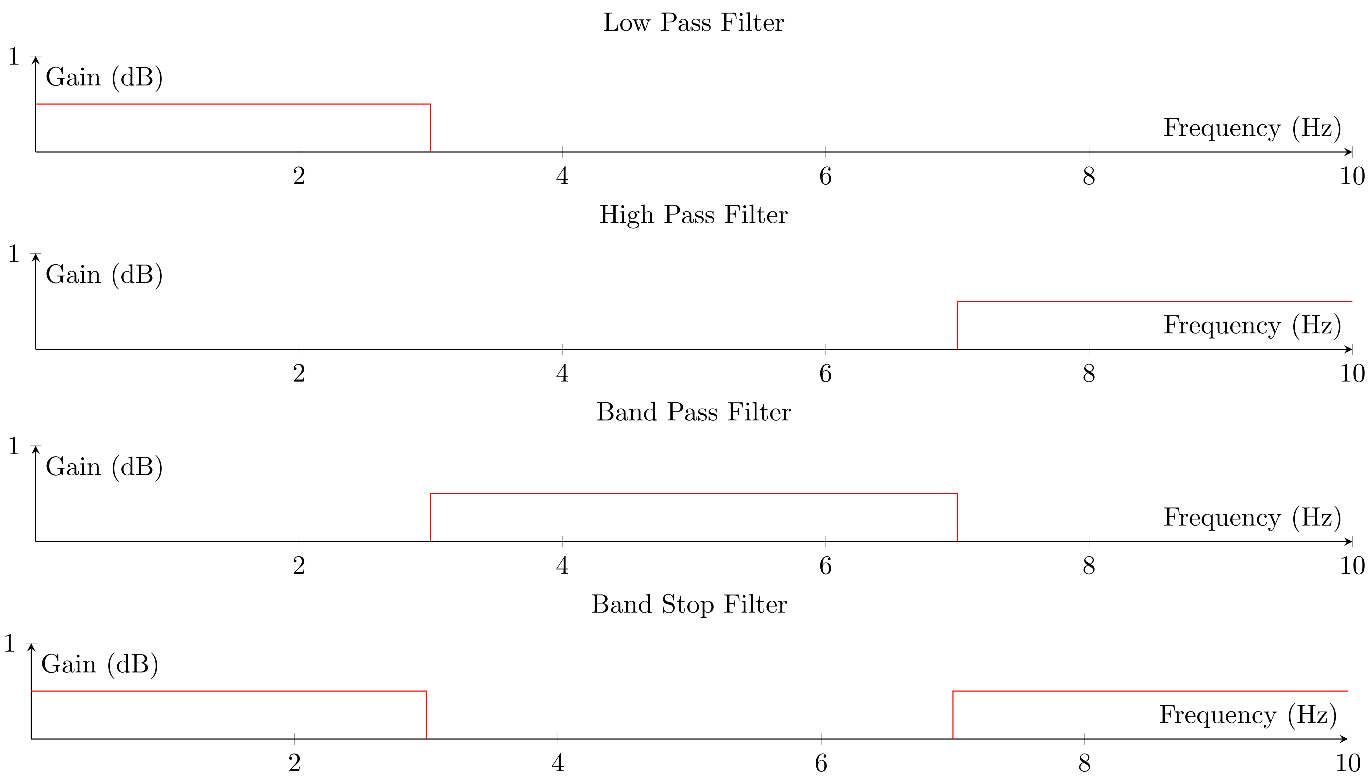


Figure 5.1: Ideal Filters

5.3 Passive Filters

5.3.1 Low Pass Filter

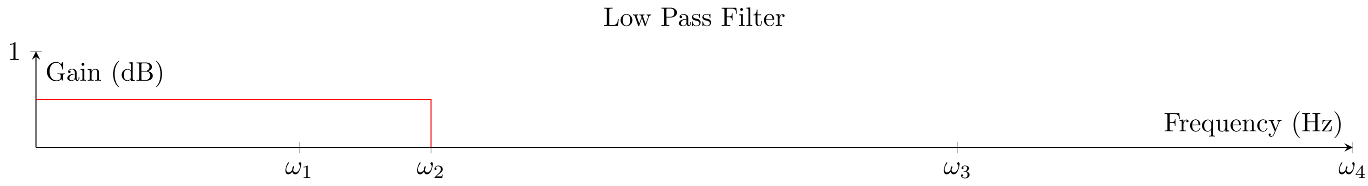
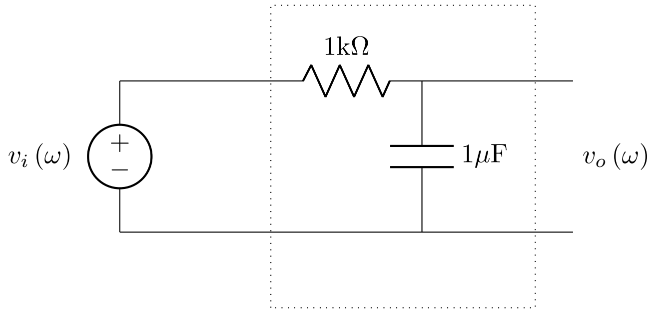


Figure 5.2: Low Pass Filter



$$v_o(\Omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} v_i(\omega)$$

$$\frac{v_0(\omega)}{v_i(\omega)} = \frac{1}{j\omega RC + 1}$$

For $R = 1\text{k}\Omega$ and $C = 1\mu F$

$$H(\omega) = \frac{1000}{j\omega + 1000}$$

$$H(62.8) \approx 1 \qquad\qquad H(62800) = 0.016\angle - 89^\circ$$

Plotting the Response

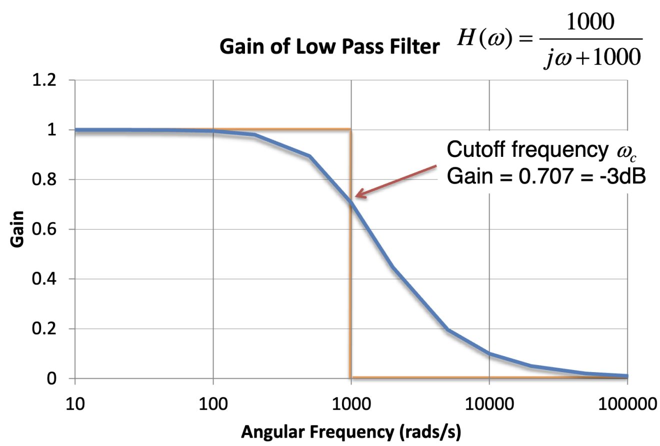
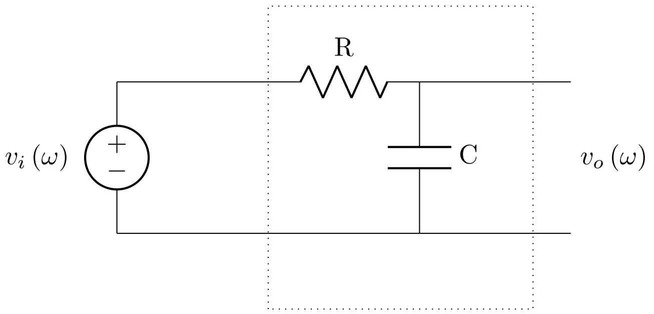


Figure 5.3: Low Pass Filter Bode Plot

Desigining a Low Pass Filter

Find the required **cutoff frequency** ω_c in radians per second (remember $\omega_c = 2\pi f$)
Choose a value for R and calculate C such that $\omega_c = \frac{1}{RC}$

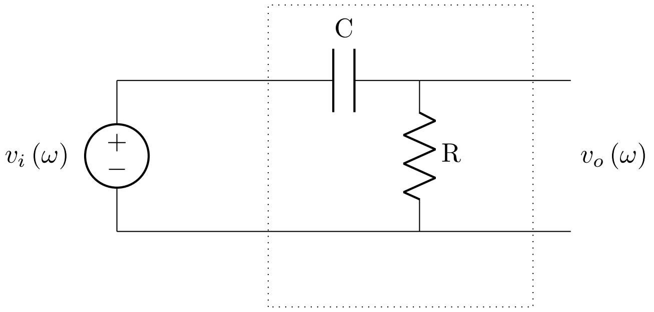


$$H(\omega) = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}}$$
$$\omega_c = \frac{1}{RC}$$

5.3.2 High Pass Filter

Similar to a low pass filter, but the capacitor and resistor are swapped

Desigining a High Pass Filter



$$v_o(\omega) = \frac{R}{R + \frac{1}{j\omega C}} v_i(\omega)$$
$$\frac{v_0(\omega)}{v_i(\omega)} = H(\omega) = \frac{j\omega RC}{j\omega RC + 1}$$
$$\omega_c = \frac{1}{RC}$$

Plotting the Response

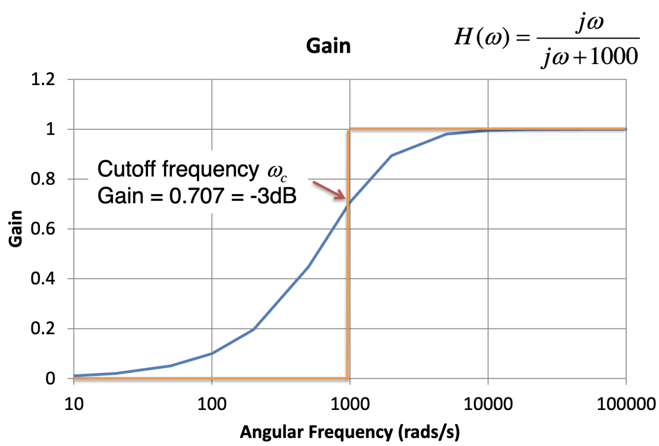
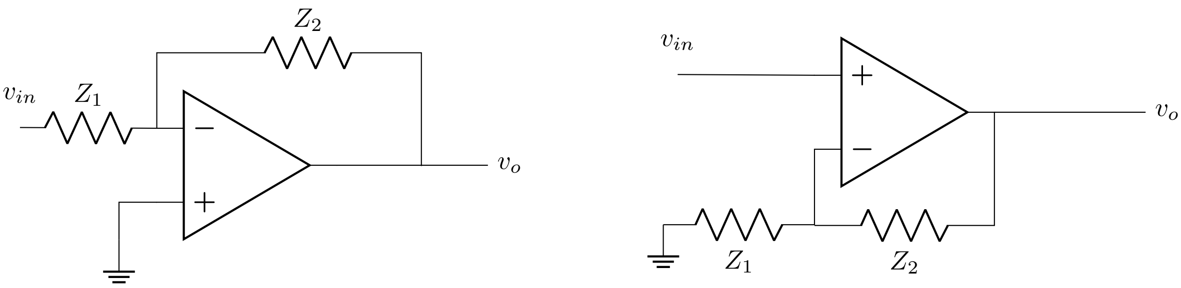


Figure 5.4: High Pass Filter Bode Plot

5.4 Active Filters

5.4.1 Op-Amps

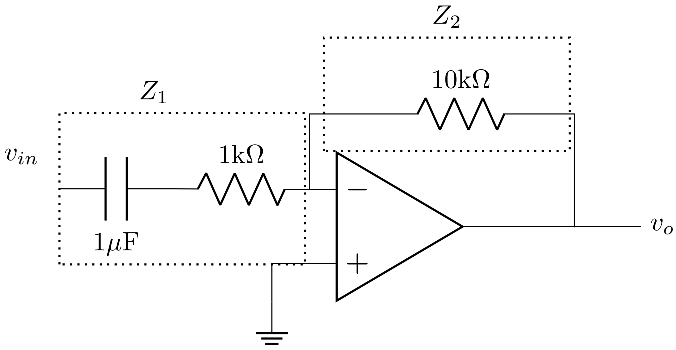


(a) Inverting Amplifier (b) Non-Inverting Amplifier

Figure 5.5: Active Filters

5.4.2 Using an Op Amp

Active High Pass Filter



(a) Active High Pass Filter

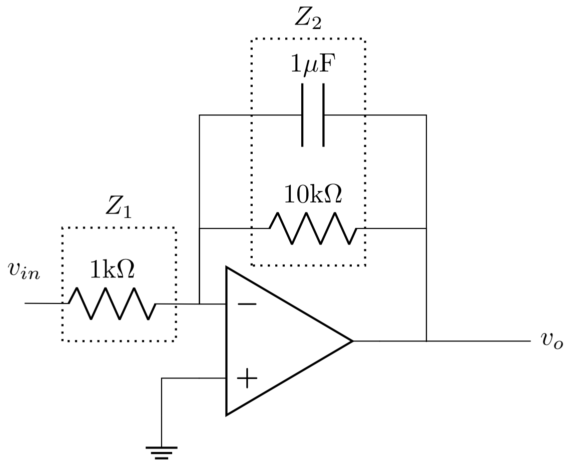
$$v_o = -\frac{Z_2}{Z_1}v_{in}$$

$$\frac{v_o}{v_{in}} = -\frac{R_2}{R_1 + \frac{1}{j\omega C_1}} = -\frac{R_2}{R_1}\frac{j\omega}{j\omega + \frac{1}{R_1 C_1}}$$

High pass filter with
$\omega_c = \frac{1}{R_1 C_1} = \frac{1}{10^3 \times 10^{-6}} = 1000\text{rad/s}$

Gain of $-\frac{R_2}{R_1} = -10$ in passband

Active Low Pass Filter



(a) Active Low Pass Filter

$$\begin{aligned}
v_o &= -\frac{Z_2}{Z_1}v_{in} \\
&= \frac{\frac{1}{\frac{1}{R_2}+j\omega C_2}}{R_1}v_{in} \\
\frac{v_o}{v_{in}} = H(\omega) &= -\frac{R_2}{R_1}\frac{1}{1+j\omega R_2 C_2} \\
&= -\frac{R_2}{R_1}\frac{\frac{1}{R_2 C_2}}{j\omega + \frac{1}{R_2 C_2}}
\end{aligned}$$

Low pass filter with
$\omega_c = \frac{1}{R_2 C} = \frac{1}{10^4 \times 10^{-6}} = 100\text{rad/s}$

Gain of $-\frac{R_2}{R_1} = -10$ in passband

5.5 Band Pass and Band Stop Filters

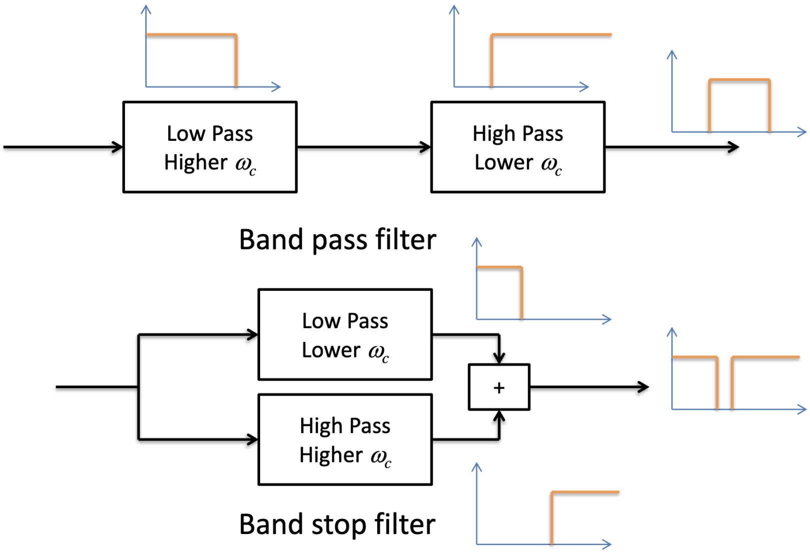


Figure 5.8: Band Pass and Band Stop Filters

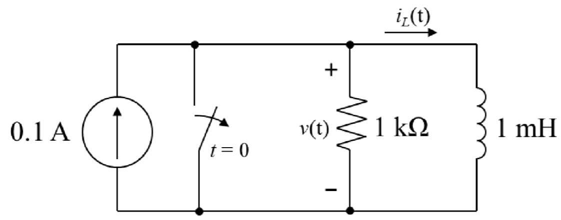
List of Figures

Chapter 6

Tutorials

6.1 Tutorial 7 Sequential Switching

Consider the circuit below. The initial current in the inductor is $i_l(0-) = 0$. Find expressions for $iL(t)$ and $v(t)$ for $t \geq 0$ and sketch to scale versus time.



The given circuit is an RL circuit, therefore, step response is given by

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}$$

By applying a source transformation (Norton equivalent to Thevenin equivalent) we get

$$V_s = I_s R = 0.1 \times 1 \times 10^3 = 0.1 \times 10^3 V = 100V$$

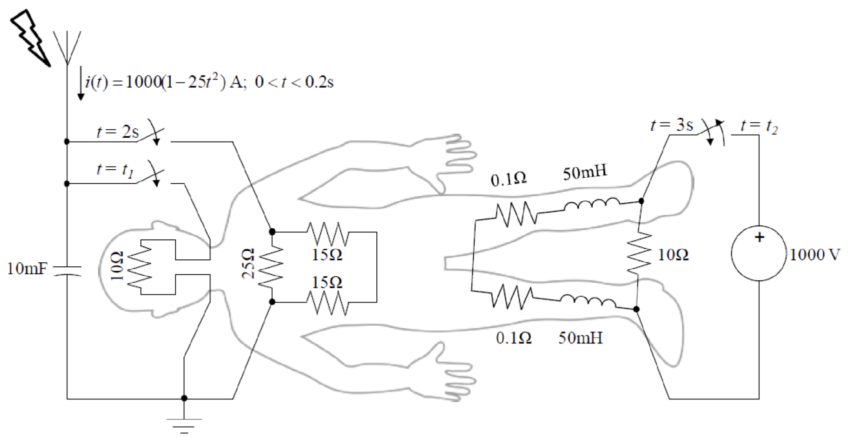
Using this we get that

$$i(t) = \frac{100}{1000} + \left(0 - \frac{100}{1000}\right) e^{-\frac{1000}{1\times -3}t} \qquad = 0.1 - 0.1e^{-1\times10^6 t}$$

$$v(t) = L\frac{\mathrm{d}i}{\mathrm{d}t} = 1 \times 10^{-3} \times \frac{\mathrm{d}}{\mathrm{d}t}\left(0.1 - 0.1e^{-1\times10^6 t}\right) = 0.1 \times 10^3 e^{-1\times10^6 t} \qquad = 100e^{-1\times10^6 t}$$

6.1.1 Tutorial Question

Igor the Mad Scientist is planning to reanimate a corpse (again). His plan is to capture energy from a lightning bolt into a capacitor, and then to discharge the capacitor into the torso and brain. The lower body is reanimated from a voltage supply. The electrical models of the body parts are shown in the figure below

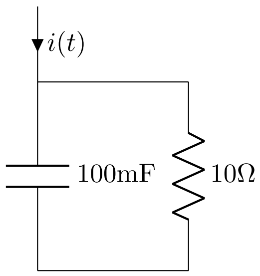


The steps for corpse re-animation are as follows:

- **Step 1**: Capture a lightning strike into a discharged capacitor. The moment of the lightning strike marks $t = 0$. The lightning strike has a duration of 200 ms, and creates a current $i(t) = 1000(1 - 25t^2)$ A for $0 < t < 0.2s$.
- **Step 2**: At t = 2s, start discharging the capacitor across the torso.
- **Step 3**: When the capacitor voltage falls to 1000 V, close the switch to discharge the capacitor into the brain (through the neck bolts).
- **Step 4**: At t = 3s, close the switch to connect the legs to the 1000 V source.
- **Step 5**: When the current reaches 500A, open the switch to disconnect the legs from the 1000 V source.
- **Step 6**: Once the leg current drops below 50 mA, and the capacitor voltage drops below 10 V the corpse will come to life. Disconnect the cables and feed your new monster some tea and cake.

Igor is using computer controlled switching for precise timing. He needs your help to work out the right time for critical switching activities.

1. Find the voltage of the capacitor after the lightning strike for $0.2 < t < 2s$.
   By isolating the circuit for the step response we get

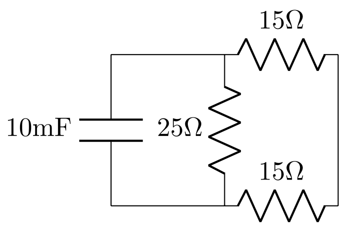


As this is an RC Circuit

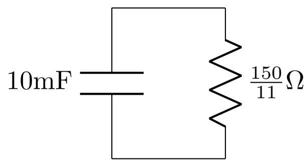
$$
\begin{aligned}
V(t) &= \frac{1}{C}\int_0^t i(\tau) + v(0)\, d\tau \\
&= \frac{1}{10^{10^{-3}}}\int_0^t 1000(1 - 25\tau^2)\, d\tau + 0 \\
&= 10^5 \left(\tau - \frac{25}{3}\tau^3\right)\Big|_0^t \\
&= 10^5 \left(t - \frac{25}{3}t^3\right)
\end{aligned}
$$

$$
\begin{aligned}
V(0.2) &= 10^5 \left(0.2 - \frac{25}{3}(0.2)^3\right) \\
&= 10^5 \left(0.2 - \frac{25}{3}(0.008)\right) \\
&= 10^5\,(0.2 - 0.066) \\
&= 10^5 \times 0.13\overline{3}\,\mathrm{V} \\
&= 13.33\overline{3}\,\mathrm{kV}
\end{aligned}
$$

2. Find an expression for the capacitor voltage while the capacitor is discharging into the torso only. When will the capacitor be sufficiently discharged to connect the capacitor to the brain?
   Capacitor will start discharging at $t = 2s$ and will be sufficiently discharged when $V(t) = 1000V$
   Find the circuit at $t = 2s$



Note that the two 15Ω resistors are in series, therefore, we can replace them with a single 30Ω resistor.
This 30Ω resistor is in parallel with the 25Ω resistor, therefore, we can replace them with a single $\left(\frac{1}{30} + \frac{1}{25}\right)^{-1} = \frac{150}{11}\Omega$ resistor.
This gives the following circuit



Using this RC circuit we can solve for when voltage is 1000, where $v(0) = 13.33\overline{3}\,\text{kV}$

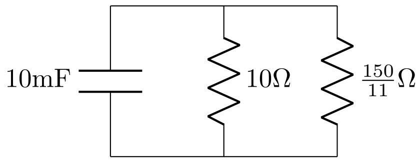
$$v(t) = v(0)e^{-\frac{t}{RC}}$$
$$1000 = 13.33\overline{3}e^{-\frac{t}{\frac{150}{11}\times 10^{-3}}}$$

Using calculator

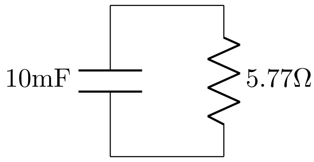
$$t = 0.3532s$$

Noting that this time is relative to the 2 seconds since the lightning strike, therefore, the time since the lightning strike is $t = 2.3532s$

3. When will the capacitor voltage fall below 10V? First, include the head that is now connected



Note that the two resistors are in parallel therefore we can replace them with a single $\left(\frac{1}{10} + \frac{1}{\frac{150}{11}}\right)^{-1} = 5.77\Omega$ resistor. This gives the following circuit



Using this RC circuit we can solve for when voltage is 10, where $v(0) = 1000\,\text{V}$

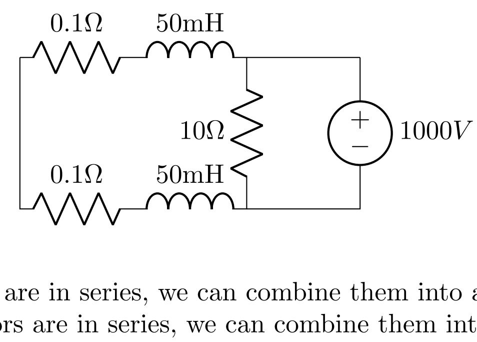
$$v(t) = v(0)e^{-\frac{t}{RC}}$$
$$10 = 1000e^{-\frac{t}{5.77\times 10^{-3}}}$$

Using calculator

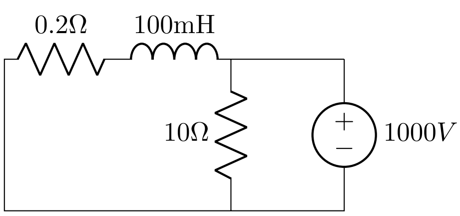
$$t = 0.2657s$$

As this time is relative to the time from part 3, the time since the lightning strike is $t = 2.6189s$

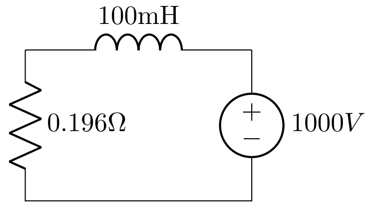
4. Find an expression for the leg current when the voltage source is connected to the legs. When should the switch connecting the voltage source to the legs be opened?
Looking at the torso circuit we see the following



Noting that the two 0.1Ω resistors are in series, we can combine them into a single 0.2Ω resistor.
Noting that the two 50mH inductors are in series, we can combine them into a single 100mH inductor.
This gives the following circuit



Noting that the 0.2Ω resistor and 10Ω resistor are in parallel, we can combine them into a single $\left(\frac{1}{0.2}+\frac{1}{10}\right)^{-1}=0.196\Omega$ resistor.
This gives the following circuit



$$
\begin{aligned}
i(t) &= \frac{V_s}{R}+\left(I_0-\frac{V_s}{R}\right)e^{-\frac{R}{L}t}\\
&= \frac{1000}{0.196}+\left(0-\frac{1000}{0.196}\right)e^{-\frac{0.196}{100\times10^{-3}}t}\\
&= 5102.04-5102.04e^{-1.96t}\\
v(t) &= L\frac{\mathrm{d}i}{\mathrm{d}t}\\
&= 100\times10^{-3}\times\frac{\mathrm{d}}{\mathrm{d}t}\left(5102.04-5102.04e^{-1.96t}\right)\\
&= 100\times10^{-3}\times5102.04\times1.96e^{-1.96t}\\
&= 100007e^{-1.96t}
\end{aligned}
$$

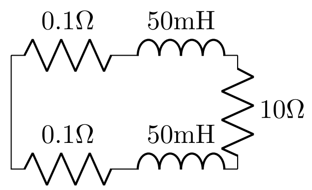
When current is 500A, open the switch to disconnect legs

$$500 = 5102.04-5102.04e^{-1.96t}$$

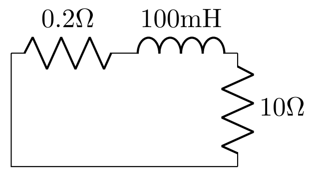
Using calculator

$$
\begin{aligned}
t &= 0.05262s && \text{(Relative to 3 seconds)}\\
t_{\text{since lightning strike}} &= 3.05262s
\end{aligned}
$$

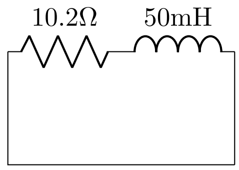
5. When will the leg current fall below 50mA? Now that the legs are disconnected, we get the following circuit



Noting that the two 0.1Ω resistors are in series, we can combine them into a single 0.2Ω resistor.
Noting that the two 50mH inductors are in series, we can combine them into a single 100mH inductor.
This gives the following circuit



Noting that the 0.2Ω resistor and 10Ω resistor are in series, we can combine them into a single 10.2Ω resistor.
This gives the following circuit



Natural response of an RL circuit

$$
\begin{aligned}
i(t) &= i(0)e^{-\frac{t}{R_2 C}} \\
&= 500e^{-\frac{t}{10.2\times10^{-3}}}
\end{aligned}
$$

When current is 50mA, open the switch to disconnect legs

$$0.05 = 500e^{-\frac{t}{10.2\times10^{-3}}}$$

Using calculator

$$
\begin{aligned}
t &= 0.0903s && \text{(Relative to } t_{\text{since lightning strike}} \text{ from 4)} \\
t_{\text{since lightning strike}} &= 3.14292s
\end{aligned}
$$