Queensland University of Technology

# MZB221

## Electrical Engineering Mathematics

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# Chapter 1

# Infinite Series

## 1.1 Sequences, Infinite Series, Convergence

## 1.2 Taylor polynomials, Taylor series, Radius of convergence

## 1.3 Introduction to Fourier series

## 1.4 Constructing Fourier series

# Chapter 2

# Vector Calculus

## 2.1 Introduction to Vector Calculus, div, grad, curl

### 2.1.1 Scalar Fields

A scalar field is a function

$$f : \mathrm{R}^n \to \mathrm{R}$$

$\boxed{n = 2}$ $f = f(x, y)$

- 2 independent variables $x, y$
- $f$ is a function that has $x$ and $y$ as inputs and a single real number as the output

Phyiscal examples of scalar fields are

- The temperature $T(x, y, z)$, the pressure $p(x, y, z)$, the density $\rho(x, y, z)$ of a fluid
- Concentration of a pollutant in a lake $c(x, y, z)$
- Height of a surface or a mountain $h(x, y)$
- Charge density $\rho(x, y, z)$, electrical potential $V(x, y, z)$

#### Partial Derivatives

For $f : \mathrm{R}^2 \to \mathrm{R}$

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}, \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

- $f_x$ is the rate of change of $f$ in the $x$-direction (y is constant) (and vice-versa)
- Geometrically, $f_x$ and $f_y$ are the slopes of the surface $z = f(x, y)$ in the $x$ and $y$ directions.

For $f : \mathrm{R}^3 \to \mathrm{R}$

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}, \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$
$$\frac{\partial f}{\partial z} \lim_{h \to 0} \frac{f(z+h, y) - f(z, y)}{h}$$

No equivalent geometric representation as "surface" $(f(x, y, z))$ is a 4-dimensional hypersolid

#### Directional Derivative of a Scalar Field

## 2.2 Review of Multiple Integration, Change of Variables

## 2.3 Introduction to cylindrical and spheroidal coordinates, integration

## 2.4 Line Integrals, Surface Integrals

# Chapter 3

# Differential Equations

## 3.1 Introduction to Laplace transform, Strategy for Solving Linear ODEs

### 3.1.1 Definition of the Laplace Transform

The Laplace Transform of the function $f(t)$ is defined as

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$$

for values of $s$ that the integral converges
The notation $F(s)$ is often used to represent $\mathcal{L}\{f(t)\}$

## 3.2 Further Properties of Laplace Transforms, solving more complicated initial value problems

## 3.3 Non-Linear first-order ODEs, Phase lines, Stability, Bi-Furcation

## 3.4 Linear Systems of ODEs, Exact Solutions, Classification, Non-Homogeneous Systems

## 3.5 Non-Linear Systems of ODEs, Phase Plane, Nullclines, Stability

# Chapter 4

# Practice Exams

Show that $f(x) = \frac{1}{x-2}$ about $x = 3$ is

$$f(x) = \sum_{n=0}^{\infty} (-1)^n (x-3)^n, 3 - R < x < 3 + R$$

$$f'(x) = \frac{1}{(x-2)^2} \qquad f''(x) = \frac{-2}{(x-2)^3} \qquad f'''(x) = \frac{6}{(x-2)^4}$$

$$f'(3) = \frac{1}{(3-2)^2} \qquad f''(3) = \frac{-2}{(3-2)^3} \qquad f'''(3) = \frac{6}{(3-2)^4}$$

$$f'(3) = 1 \qquad f''(3) = -2 \qquad f'''(3) = 6$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

For