CH-231-A

Algorithms and Data Structures

ADS

Lecture 29

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Design Concepts

- We have been looking into different algorithms and, in particular, emphasized one design concept, namely, the Divide & Conquer strategy, which was based on recursions and whose analysis was given by recurrences.
- Now, we are going to look into further design concepts.

Activity-Selection Problem (1)

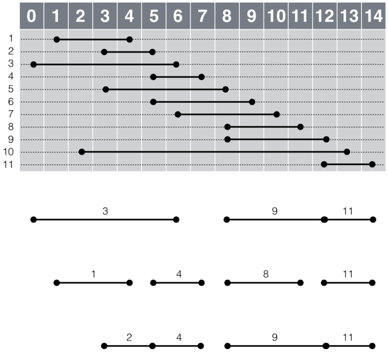
- Suppose we have a set $S = \{a_1, a_2, ..., a_n\}$ of n activities.
- The activities wish to use a resource, which can only be used by one activity at a time.
- Each activity a_i has a start time s_i and a finish time f_i, where $0 \leq s_i < f_i < \infty$.
- Two activities a_i and a_j are compatible, if $[s_i, f_i)$ and $[s_j, f_j)$ are disjoint.
- The activity-selection problem is to select a maximum-size subset of mutually compatible activities.

Activity-Selection Problem (2)

- Example:

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	8	9	10	11	12	13	14

- $\{a_3, a_9, a_{11}\}$ is a subset of mutually compatible activities.
- $\{a_1, a_4, a_8, a_{11}\}$ is a largest subset of mutually compatible activities.
- $\{a_2, a_4, a_9, a_{11}\}$ is another largest subset of mutually compatible activities.



Sorting

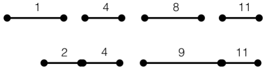
- We can apply a sorting algorithm to the finish times, which operates in $O(n \lg n)$ time.
- Then, we can assume that the activities are sorted, i.e., $f_1 \le f_2 \le ... \le f_n$.

Greedy Algorithm

- A greedy algorithm always makes the choice that looks best at the moment.
- I.e., it makes a locally optimal choice in the hope that it will lead to a globally optimal solution.

Greedy Approach (1)

- After sorting, a_1 has the earliest finish time f_1.
- A greedy approach starts with taking a_1 as a locally optimal choice.
- Lemma:
  The greedy choice of picking a_1 as first choice is optimal.
- Proof:
  - Suppose A is a globally optimal solution for set S.
  - Let $a_k \in A$ be the activity with earliest finish time f_k in A.
  - If $k = 1$, then $a_1 \in A$ and we are done.
  - If $k > 1$, then we can replace A by $(A \setminus \{a_k\}) \cup \{a_1\}$.
  - Since $f_1 \leq f_k$, this is still an optimal solution.
  - Hence, we can always start with a_1.



Greedy Approach (2)

- After the first step, we consider the subproblem $S' = \{a_i \in S : s_i \geq f_1\}$.
- We apply the same greedy strategy.
- Lemma:
  $A \setminus \{a_1\}$ is the optimal solution for S'.
- Proof:
  - Let B be a solution for S' that is larger than $A \setminus \{a_1\}$.
  - Then, $B \cup \{a_1\}$ would be solution for S that is larger than A.
  - Contradiction.

Greedy Approach (3)

Using the two lemmata we can prove by induction that the greedy approach delivers the globally optimal solution.

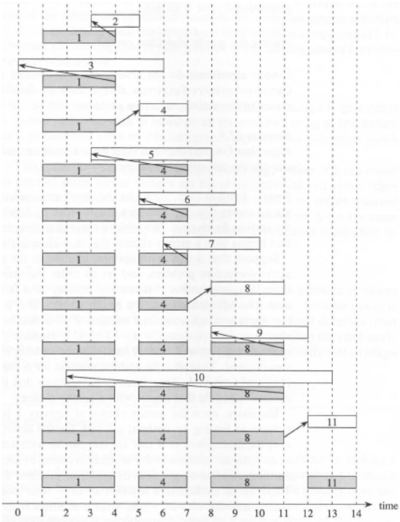
Greedy Algorithm

```
Greedy-Selector(S)
  // Assume S = {a[1], ..., a[n]}
  // with activities sorted by f[i].
  A := {a[1]}
  j := 1
  for i := 2 to n do
    if s[i] >= f[j]
      then A := A union {a[i]}
           j := i
  return A
```

Example

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	8	9	10	11	12	13	14

Alternative:
Recursive function



Greedy Algorithm (Recursive)

```
RECURSIVE-ACTIVITY-SELECTOR(s, f, k, n)
m = k + 1
while m ≤ n and s[m] < f[k]        // find the first activity in S_k to finish
    m = m + 1
if m ≤ n
    return {a_m} ∪ RECURSIVE-ACTIVITY-SELECTOR(s, f, m, n)
else return Ø
```

RECURSIVE-ACTIVITY-SELECTOR$(s, f, 0, n)$.

Time Complexity of Greedy Approach

- $O(n)$ – if already sorted
- $O(n \lg n)$ – if not sorted
- Comparison to brute-force approach:
  - Brute-force approach would try all combinations, reject all the combinations that have incompatibilities, and pick among the remaining ones one with maximum number of activities.
  - Time complexity: $O(2^n)$

Greedy Algorithm in General

- Greedy algorithms build upon solving subproblems.
- Greedy approaches make a locally optimal choice.
- There is no guarantee that this will lead to a globally optimal solution.
- Having found a global optimum requires a proof.