~~CP~~ CP312 Assignment #1

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1. $\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} > \frac{13}{24}$ ~~n≥0~~ $n > 1$ or $n \geq 2$

Base Case: $n = 2$

$$\frac{1}{(2+1)} + \frac{1}{(2+2)} = \frac{7}{12} = \frac{14}{24} > \frac{13}{24}$$

induction Hypothesis

$$S_K = \frac{1}{K+1} + \frac{1}{K+2} + \dots \frac{1}{2K} > \frac{13}{24}$$

Proof

$$S_{K+1} = \frac{1}{(K+1)+1} + \frac{1}{(K+1)+2} + \dots \frac{1}{2(K+1)}$$

$$= \frac{1}{K+2} + \frac{1}{K+3} + \dots \frac{1}{2K+2} + \frac{1}{2K+1} + \frac{1}{2K}$$

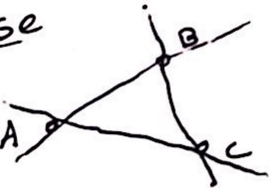
$$= \left(-\frac{1}{K+1} + \underbrace{\frac{1}{K+1}\right) + \frac{1}{K+2} + \frac{1}{K+3} + \dots + \frac{1}{2K}}_{S_K} \frac{1}{2K+2} + \frac{1}{2K+1}$$

$$= \frac{1}{2K+2} + \frac{1}{2K+1} - \frac{1}{K+1} + S_K = S_K + \frac{1}{2(2K+1)(K+1)}$$

$\therefore$ Since $\frac{1}{2(2K+1)(K+1)}$ is Always Positive by definition

$S_K + \frac{1}{2(2K+1)(K+1)} > S_K$ and $S_K > \frac{13}{24}$. ~~proof~~ proving the I.H.

2. <u>Base case</u>

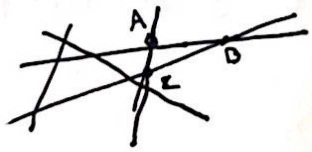


$n = 3$

creates a triangle, therefore ~~ka~~. true for $n = 3$

<u>induction Hypothesis</u>

We assume that for any region R_n we have 3 points/intersections Δabc that create a triangle

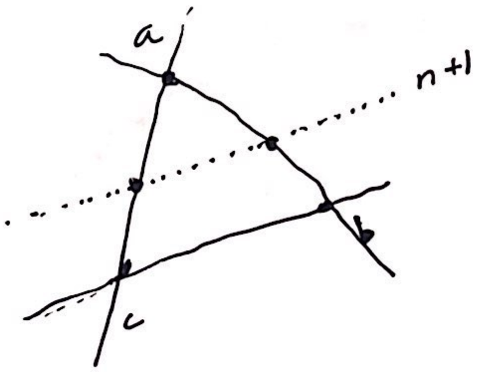


<u>Proof</u>

case 1: the new line R_{n+1} does not pass through Δabc. simple case we still have a triangle

case 2: if line $n+1$ intersects Δabc then it most intersect 2 other ~~b~~ lines of R_n.

By intersecting Δabc at two points the new ~~a~~ Δabc ~~will~~ will be a sub region of these two ~~where~~ of the old points and the singular point Δabc



3. a) $f(n) = \frac{n^2 - n}{2}$ $\quad g(n) = 6n$

$= \frac{n(n-1)}{2}$ $\quad g(n) = O(f(n))$ as $\lim_{n \to \infty}$

b) $f(n) = n + 2\sqrt{n}$ $\quad g(n) = n^2$

$f(n) = O(g(n))$ as $\lim_{n \to \infty}$ $\quad f(n) \leq c \cdot g(n) \ \forall \ n \geq n_0$

c) $f(n) = n + \log n$ $\quad g(n) = n\sqrt{n}$

$f(n) = O(g(n))$ $\quad f(n) \leq c \cdot g(n)$

d) $f(n) = n^2 + 3n + 4$ $\quad g(n) = n^3$

$f(n) = O(g(n))$ $\quad f(n) \leq g(n) \cdot c$ $\quad \lim_{n \to \infty}$

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4. $2n+1 = O(2^n)$

This is not a good upper bound as ~~it~~ it Quickly diverges from $2n+1$.

$0 \le c_1 n \le 2n+1$ $\quad c_1 = 1$ $\quad \boxed{n \le 2n+1 \le 3n}$

$2n+1 \le c_2 n$ $\quad c_2 = 3$ $\quad n \ge 1$

5. 1. a) $O(n)$ b) $n = 50000 \rightarrow 0.003847$ sec

$n = 500000 \rightarrow 0.03926$ sec

$n = 50000000 \rightarrow 3.8269$ sec

c) The results are consistant with $O(n)$ as ~~they~~ Time increases linearly with n

2. a) $O(n^2)$ b) $n = 50 \rightarrow 0.0002098$

$n = 500 \rightarrow 0.0200747$

$n = 5000 \rightarrow 2.079924$

c) as can be seen runtime increases 100x for every 10x increase to n. consistant with $O(n^2)$

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5. 3) $O(n^3)$

b) $n = 50 \to 0.0100792$ sec

$n = 100 \to 0.0869713$ sec

$n = 250 \to 1.249$ sec

$n = 500 \to 9.34875$ sec

c) as can be seen from $n = 50$ to $n = 500$ there is about ~ 1000x increase in run time which is consistant with $O(n^3)$

4) $n + \sum_{i=1}^{n} i = n + \frac{n(n+1)}{2} \leq n^3$ so $O(n^3)$

$n = 50 \to 0.00011993$ s

$n = 500 \to 0.00951$ s

$n = 5000 \to 0.9270575$ s

c) The upper bond of $O(n^3)$ is correct but the corritation is low. a big theta notation may be more useful in this case

$j = \frac{n(n+1)(2n+1)}{6}$

5. a) $n + \underbrace{\sum_{i=1}^{n} i^2}_{n^3} + \underbrace{\sum_{i=1}^{j} i}_{n^5}$

$O(n^5)$

b) $n=5 \rightarrow 0.00002309$ s

$n=20 \rightarrow 0.020711$ s

$n=25 \rightarrow 0.06735$ s

$n=50 \rightarrow 2.225565$ s

c) $O(n^5)$ holds as from $n=5$ to $n=50$ a 100,000 inclease in time occures

$j = \frac{n(n+1)(n+1)}{6}$

6. $n + \sum_{i=1}^{n} i^2 + \frac{\sum_{i=1}^{j} i}{2}$

$O(n^5)$

b) $n=10 \rightarrow 0.00010224$ sec

$n=20 \rightarrow 0.001565$ sec

$n=50 \rightarrow 0.0599186$ sec

$n=100 \rightarrow 1.000815$ sec

c) The $O(n^5)$ works in this but by dividing by Two is less accurate than 5.