Supergaussians

One dimension modified Gaussian

- Un-normalised

$$\mathrm{ModGauss}(x) = A\exp\left(-\frac{1}{2}\left(\frac{(x-\mu)^2}{\sigma^2}\right)^{1+\frac{\epsilon}{2}}\right)$$

or equivalently

$$\mathrm{ModGauss}(x) = A\exp\left(-\frac{1}{2}\left(\frac{|x-\mu|}{\sigma}\right)^{2+\epsilon}\right)$$

- To normalise

$$A = \frac{2^{-\frac{3+\epsilon}{2+\epsilon}}}{\sigma\Gamma\left[1+\frac{1}{2+\epsilon}\right]}$$

Γ is the gamma function

ModGauss is defined for $\epsilon > -1$

Tried ε=2 and ε=4, found ε=2 worked best, used this in the following

From Witold's presentation:
https://indico.cern.ch/materialDisplay.py?contribId=8&materialId=slides&confId=236932

Supergaussians (II)

- For $\varepsilon=0$: Standard Gaussian
  - To get this from formula on previous page, you need:
    - $\Gamma(1+z) = z * \Gamma(z)$
    - $\Gamma(1/2) = \mathrm{sqrt}(\pi)$
  - which yields: $A = 1/ (\mathrm{sqrt}(2\pi) * \sigma)$
- Tried fits with $\varepsilon=2$ and $\varepsilon=4$, found $\varepsilon=2$ gave best results, so for $\varepsilon=2$:
  - $A = 2^{3/4}/ (\sigma * \Gamma(1/4))$
  - where $\Gamma(1/4) \approx 3.6256099082219083...$ (no closed form exists)

More technical details

$$f = \mathrm{c}_2\left[c_3\exp\left(-\frac{1}{2}\left[\frac{x-c_4}{\dfrac{c_0c_1}{c_3c_1+1-c_3}}\right]^2\right)+\left(1-c_3\right)\exp\left(-\frac{1}{2}\left[\frac{x-c_4}{\dfrac{c_0}{c_3c_1+1-c_3}}\right]^2\right)\right]$$

$$CapSigma = c_0$$

$$\int f = \sqrt{2\pi}c_0c_2 = \sqrt{2\pi}\cdot peak\cdot CapSigma$$

$$f(c_4) = peak = c_2$$

Standard pylumi
double
Gaussian fit

More technical details (II)

$$f = \mathrm{c}_2 \left[c_3 \exp\left(-\frac{1}{2} \left[\frac{x - c_4}{\frac{c_0 c_1}{c_3 c_1 + 1 - c_3}} \right]^{2} \right) + (1 - c_3) \cdot \sqrt{2\pi} \cdot \frac{2^{\frac{3}{4}}}{\Gamma\left(\frac{1}{4}\right)} \cdot \exp\left(-\frac{1}{2} \left[\frac{x - c_4}{\frac{c_0}{c_3 c_1 + 1 - c_3}} \right]^{4} \right) \right]$$

$$CapSigma = \frac{c_0}{\left[c_3 + (1 - c_3) \cdot \sqrt{2\pi} \cdot \frac{2^{\frac{3}{4}}}{\Gamma\left(\frac{1}{4}\right)} \right]} = \frac{c_0 c_2}{peak}$$

$$\int f = \sqrt{2\pi} c_0 c_2 = \sqrt{2\pi} \cdot peak \cdot CapSigma$$

$$f(c_4) = peak = c_2 \left[c_3 + (1 - c_3) \cdot \sqrt{2\pi} \cdot \frac{2^{\frac{3}{4}}}{\Gamma\left(\frac{1}{4}\right)} \right]$$

Modified fit function: Sum of standard Gaussian plus ε=2 Supergaussian

Visible cross section

$$\sigma_{vis} = \frac{\int R(\Delta x, \Delta y_0) d\Delta x \cdot \int R(\Delta x_0, \Delta y) d\Delta y}{\nu N_1 N_2 R(\Delta x_0, \Delta y_0)}$$

$$\sigma_{vis} = \frac{\sqrt{2\pi} c_0^X c_2^X \cdot \sqrt{2\pi} c_0^Y c_2^Y}{11245 \cdot 0.5 \cdot \left(peak^X + peak^Y \right)}$$

$$\sigma_{vis} = \frac{2\pi \cdot CapSigma^X \cdot peak^X \cdot CapSigma^y \cdot peak^Y}{11245 \cdot 0.5 \cdot \left(peak^X + peak^Y \right)}$$

With the approximation peakX ~ peakY ~ 0.5*(peakX+peakY)

$$\sigma_{vis} = \frac{\pi \cdot CapSigma^X \cdot CapSigma^y \cdot \left(peak^X + peak^Y \right)}{11245}$$

NormChange

$$NormChange = \frac{1}{\sigma_{vis}} \cdot \frac{1}{oldNorm}$$