

In[1]:= **ClearAll**["Global`*"]

In[2]:= **f**[**x_**, **y_**, **z_**] = **3*x^2 + 8*x*z + 2*y^2 + 6*z^2**

Out[2]= $3 x^2 + 2 y^2 + 8 x z + 6 z^2$

(* матрицата а на квадратичната форма *)

In[3]:= **a** = $\begin{pmatrix} 3 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 6 \end{pmatrix}$;

(* намираме системата от собствени стойности и собствени вектори *)

In[4]:= **Eigensystem**[**a**]

Out[4]= $\left\{ \left\{ \frac{1}{2} (9 + \sqrt{73}), 2, \frac{1}{2} (9 - \sqrt{73}) \right\}, \left\{ \left\{ -\frac{3}{2} + \frac{1}{8} (9 + \sqrt{73}), 0, 1 \right\}, \{0, 1, 0\}, \left\{ -\frac{3}{2} + \frac{1}{8} (9 - \sqrt{73}), 0, 1 \right\} \right\} \right\}$

(* очевидно всичките собствени стойности са положителни *)

In[5]:= **Eigenvalues**[**a**]

Out[5]= $\left\{ \frac{1}{2} (9 + \sqrt{73}), 2, \frac{1}{2} (9 - \sqrt{73}) \right\}$

In[6]:= **N** [$\sqrt{73}$]

Out[6]= 8.544

In[7]:= **Eigenvectors**[**a**]

Out[7]= $\left\{ \left\{ -\frac{3}{2} + \frac{1}{8} (9 + \sqrt{73}), 0, 1 \right\}, \{0, 1, 0\}, \left\{ -\frac{3}{2} + \frac{1}{8} (9 - \sqrt{73}), 0, 1 \right\} \right\}$

(* ортогонални ли са векторите : взаимно ортогонални *)

In[8]:= **v1** = $\left\{ -\frac{3}{2} + \frac{1}{8} (9 + \sqrt{73}), 0, 1 \right\}$; **v2** = {0, 1, 0}; **v3** = $\left\{ -\frac{3}{2} + \frac{1}{8} (9 - \sqrt{73}), 0, 1 \right\}$;

In[9]:= **v1.v1 // Simplify**

Out[9]= $\frac{1}{32} (73 - 3 \sqrt{73})$

In[10]:= **v1.v2**

Out[10]= 0

In[11]:= **v1.v3 // Simplify**

Out[11]= 0

In[12]:= **v2.v2**

Out[12]= 1

In[13]:= **v2.v3**

Out[13]= 0

In[14]:= **v3.v3 // Simplify**

$$\text{Out[14]} = \frac{1}{32} (73 + 3 \sqrt{73})$$

(* нормираме векторите *)

In[15]:= **w1 = Normalize[v1] // Simplify**

$$\text{Out[15]} = \left\{ \frac{-3 + \sqrt{73}}{\sqrt{146 - 6\sqrt{73}}}, 0, 4 \sqrt{\frac{2}{73 - 3\sqrt{73}}} \right\}$$

In[16]:= **Norm[w1] // Simplify**

$$\text{Out[16]} = 1$$

In[17]:= **w2 = v2**

$$\text{Out[17]} = \{0, 1, 0\}$$

In[18]:= **w3 = Normalize[v3] // Simplify**

$$\text{Out[18]} = \left\{ -\frac{3 + \sqrt{73}}{\sqrt{146 + 6\sqrt{73}}}, 0, 4 \sqrt{\frac{2}{73 + 3\sqrt{73}}} \right\}$$

In[19]:= **(* Определяме матрица на прехода: *)**

U = Transpose[{w1, w2, w3}]

$$\text{Out[19]} = \left\{ \left\{ \frac{-3 + \sqrt{73}}{\sqrt{146 - 6\sqrt{73}}}, 0, -\frac{3 + \sqrt{73}}{\sqrt{146 + 6\sqrt{73}}} \right\}, \right. \\ \left. \{0, 1, 0\}, \left\{ 4 \sqrt{\frac{2}{73 - 3\sqrt{73}}}, 0, 4 \sqrt{\frac{2}{73 + 3\sqrt{73}}} \right\} \right\}$$

In[20]:= **MatrixForm[%]**

Out[20]//MatrixForm=

$$\begin{pmatrix} \frac{-3 + \sqrt{73}}{\sqrt{146 - 6\sqrt{73}}} & 0 & -\frac{3 + \sqrt{73}}{\sqrt{146 + 6\sqrt{73}}} \\ 0 & 1 & 0 \\ 4 \sqrt{\frac{2}{73 - 3\sqrt{73}}} & 0 & 4 \sqrt{\frac{2}{73 + 3\sqrt{73}}} \end{pmatrix}$$

In[21]:= **UT = Transpose[U]**

$$\text{Out[21]} = \left\{ \left\{ \frac{-3 + \sqrt{73}}{\sqrt{146 - 6\sqrt{73}}}, 0, 4 \sqrt{\frac{2}{73 - 3\sqrt{73}}} \right\}, \right. \\ \left. \{0, 1, 0\}, \left\{ -\frac{3 + \sqrt{73}}{\sqrt{146 + 6\sqrt{73}}}, 0, 4 \sqrt{\frac{2}{73 + 3\sqrt{73}}} \right\} \right\}$$

In[22]:= **U.UT // Simplify**

$$\text{Out[22]} = \{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$$

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In[23]:= MatrixForm[%]
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Out[23]//MatrixForm=
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$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(* оказва се че U е действително ортогонална *)