Local Filtering and Edge Detection

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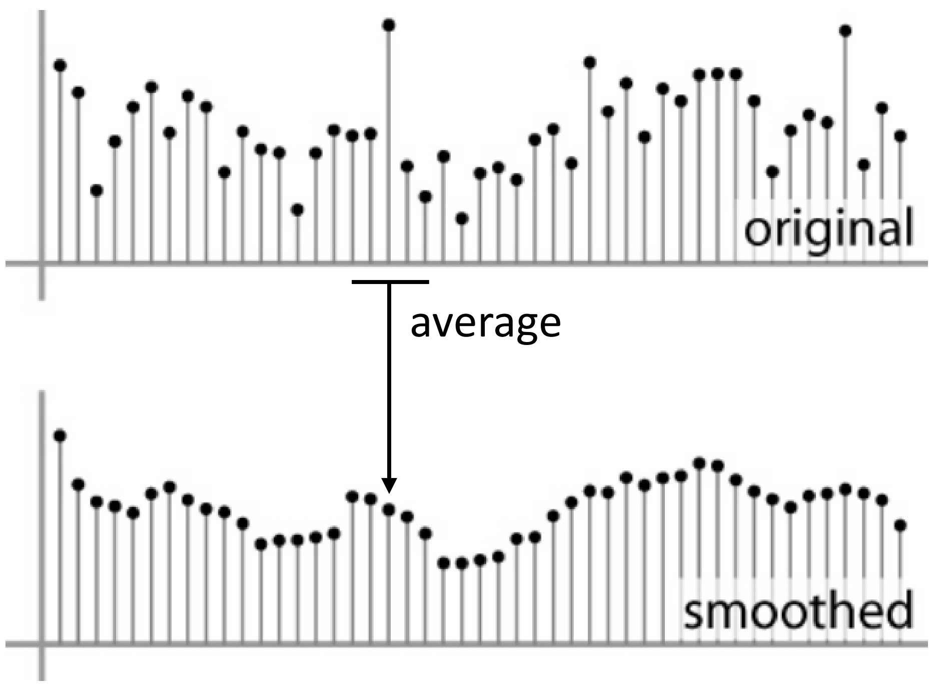
Local filtering

Simple filtering in 1D: moving average

Replace every value with the average of the pixels in its neighborhood



$$b_{\text{smooth}}[i] = \frac{1}{2r+1} \sum_{j=i-r}^{i+r} b[j]$$

Slide credit: S. Marschner

Convolution

A convolution is a **weighted moving average**

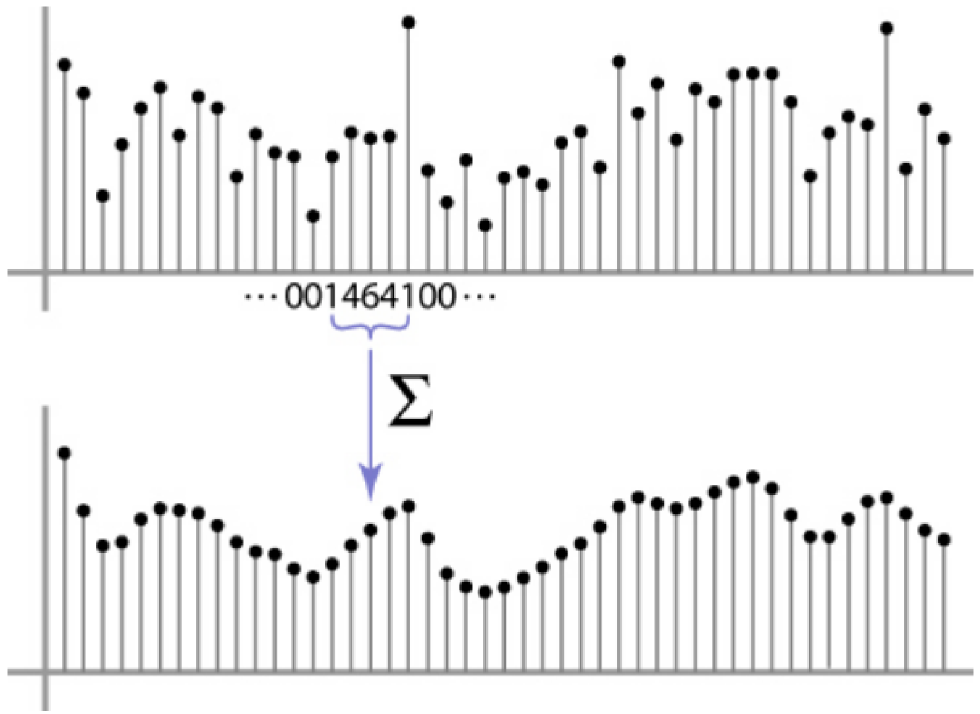
$$(a \star b)[i] = \sum_j a[j]b[i-j]$$

The sequence of weights a[j] is called filter or convolution kernel.

Slide credit: S. Marschner

1D convolution example

Using a bell-shaped kernel
a = [1, 4, 6, 4, 1] / 16

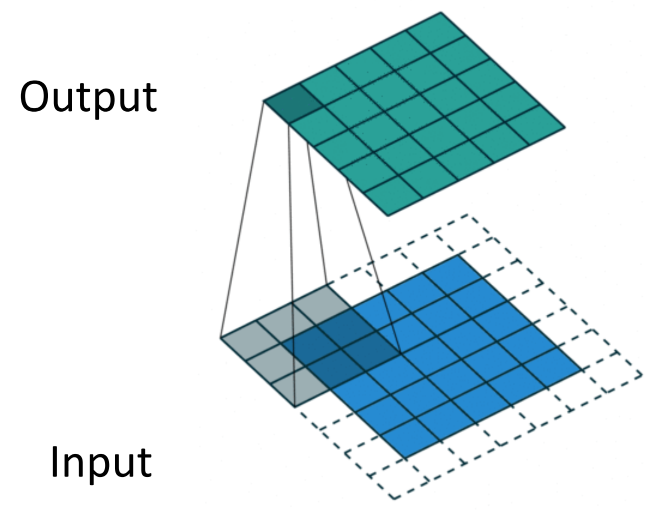


2D convolution

Same as 1D, with one more index

$$(a \star b)[i,j] = \sum_{i',j'} a[i',j']b[i-i',j-j']$$

Now the filter is a rectangle you slide around over a grid of numbers



Example

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

	0	10	20	30					

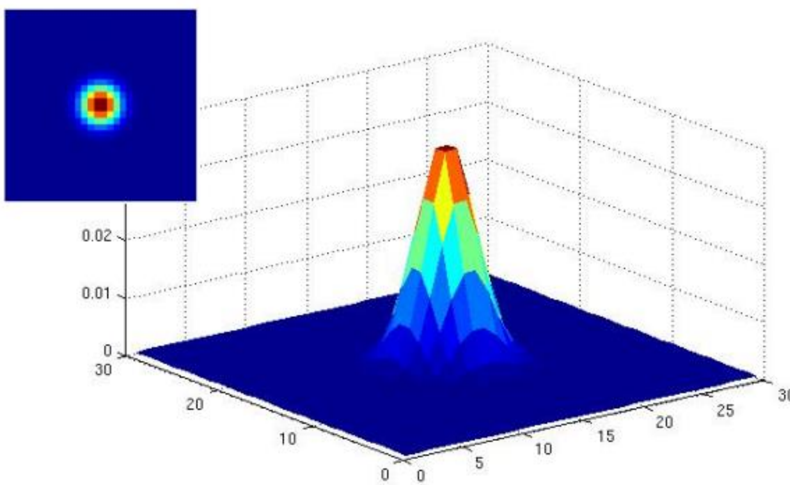
Example

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

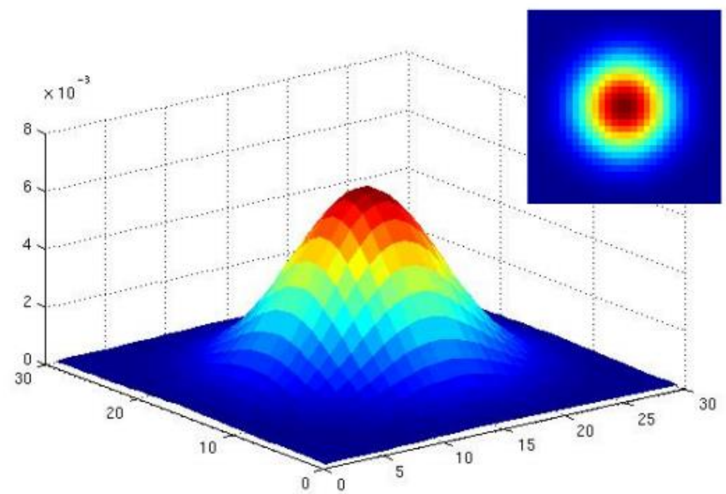
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Smoothing with gaussian filters

- Variance (sigma) determines the amount of smoothing



$\boxed{\sigma = 2}$ with
30 x 30 kernel



$\boxed{\sigma = 5}$ with
30 x 30 kernel

Example

Original

0	0	0
0	1	0
0	0	0

Kernel

Example

Original

0	0	0
0	1	0
0	0	0

Kernel

same as original

Example

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

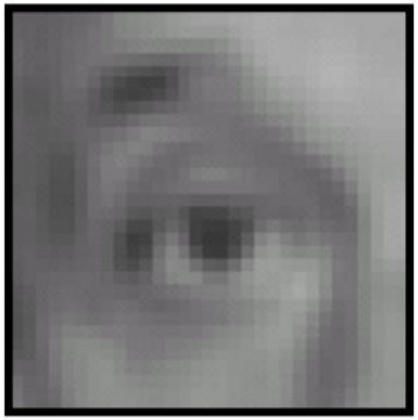
Kernel

Example

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

Kernel



smoothed

Example

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Kernel

Example

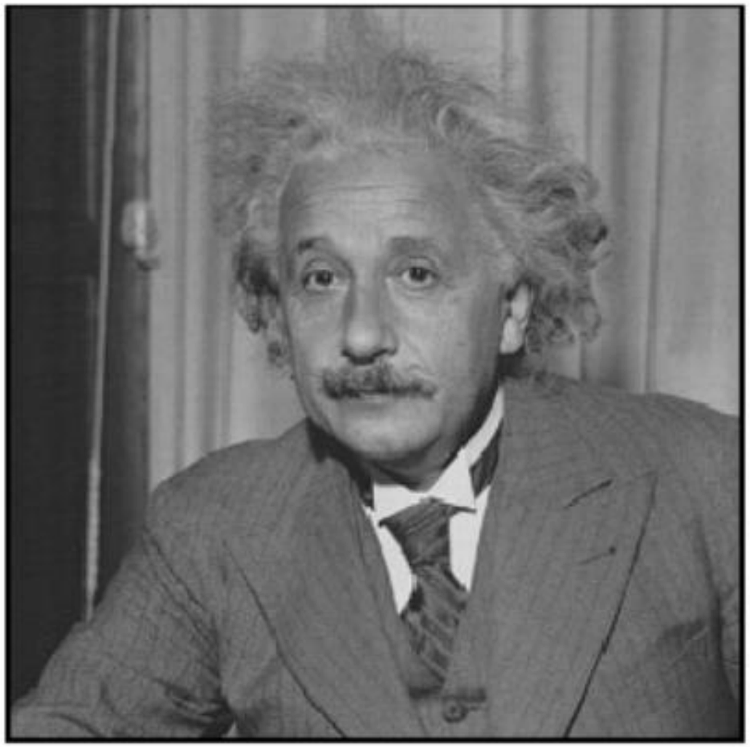
Original

$$\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

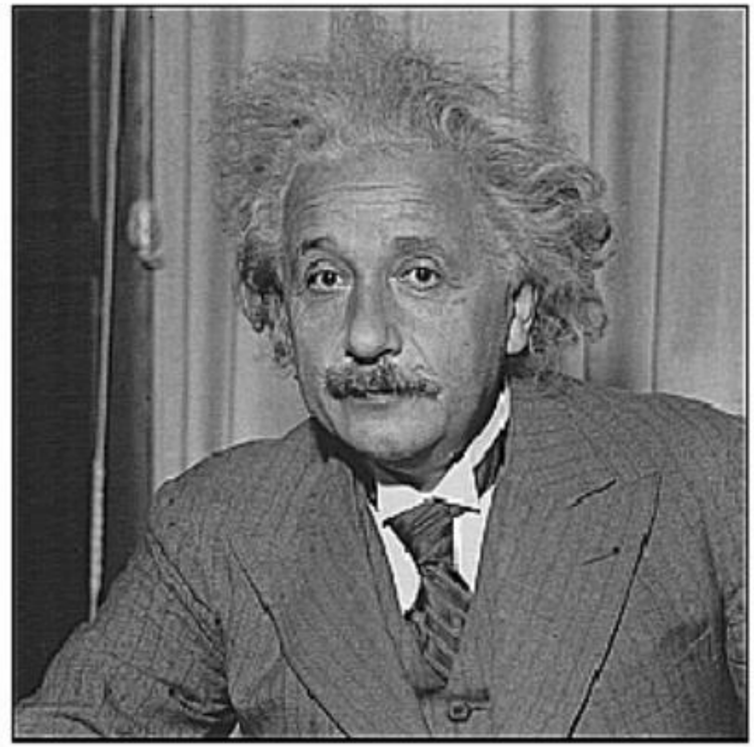
Kernel

sharpened

Sharpening

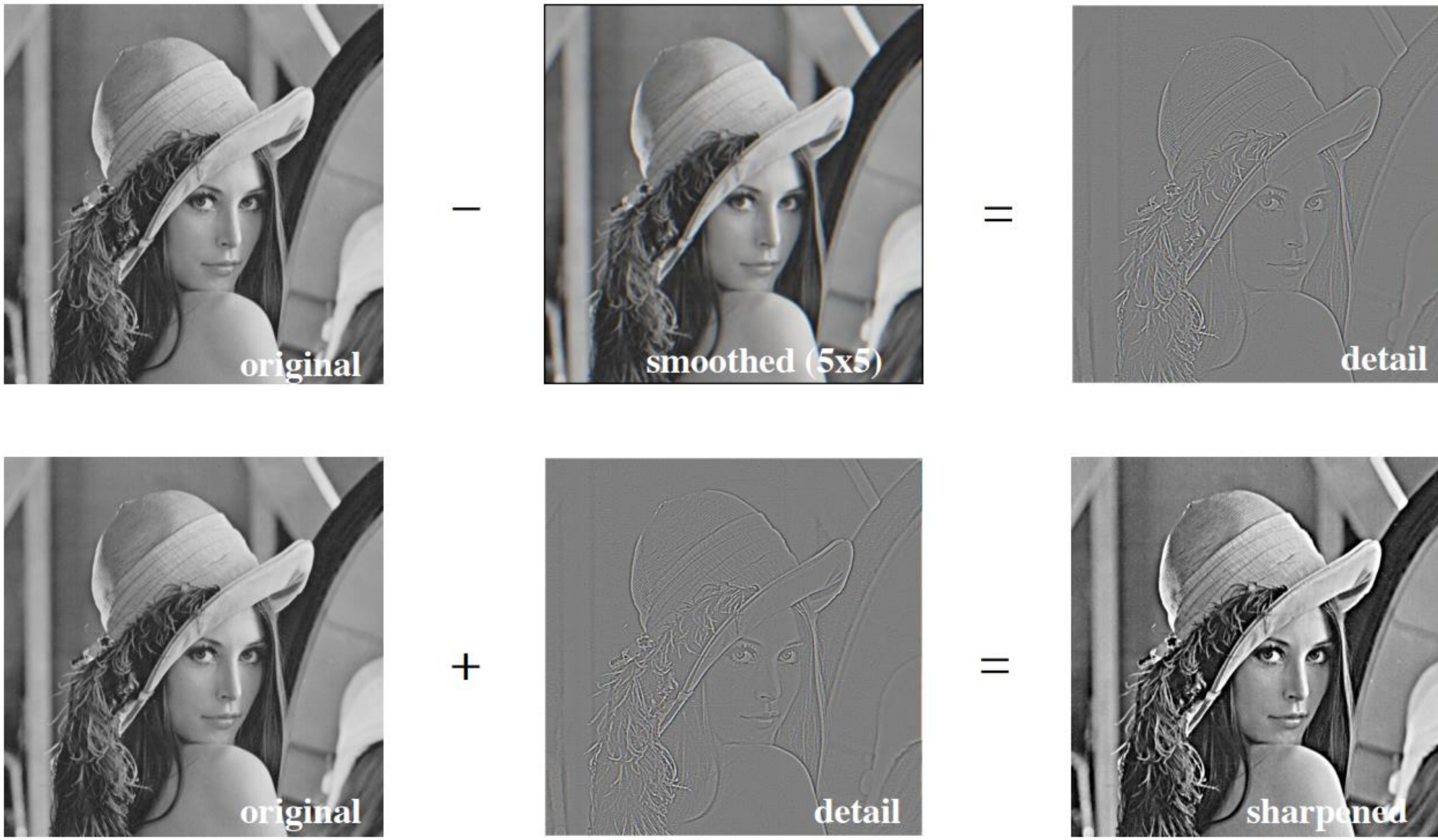


before



after

Why does sharpening work?



Edge detection

Edges are important features when we look at images!

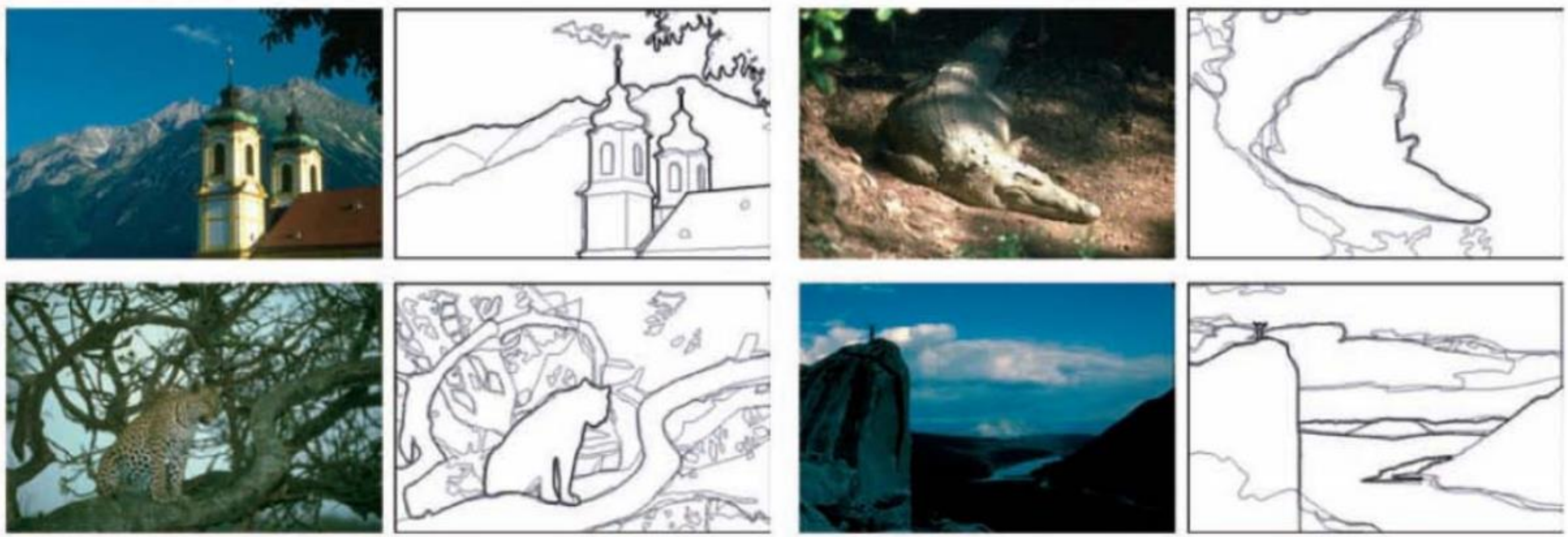


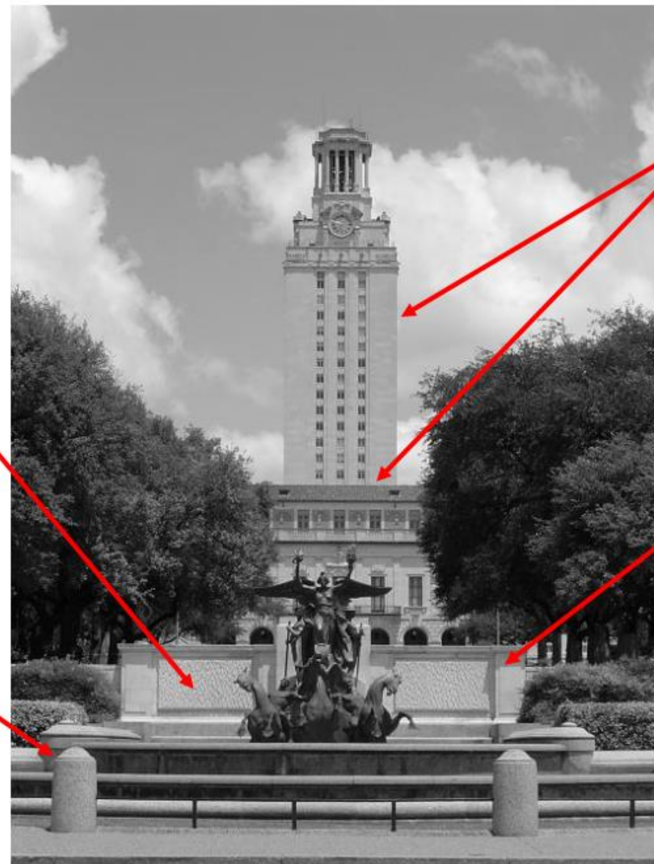
Figure 4.31 Human boundary detection (Martin, Fowlkes, and Malik 2004) © 2004 IEEE. The darkness of the edges corresponds to how many human subjects marked an object boundary at that location.

Szelinski: Computer Vision: theory and Applications

Causes of edges

Reflectance change: appearance information, texture

Change in surface orientation: shape

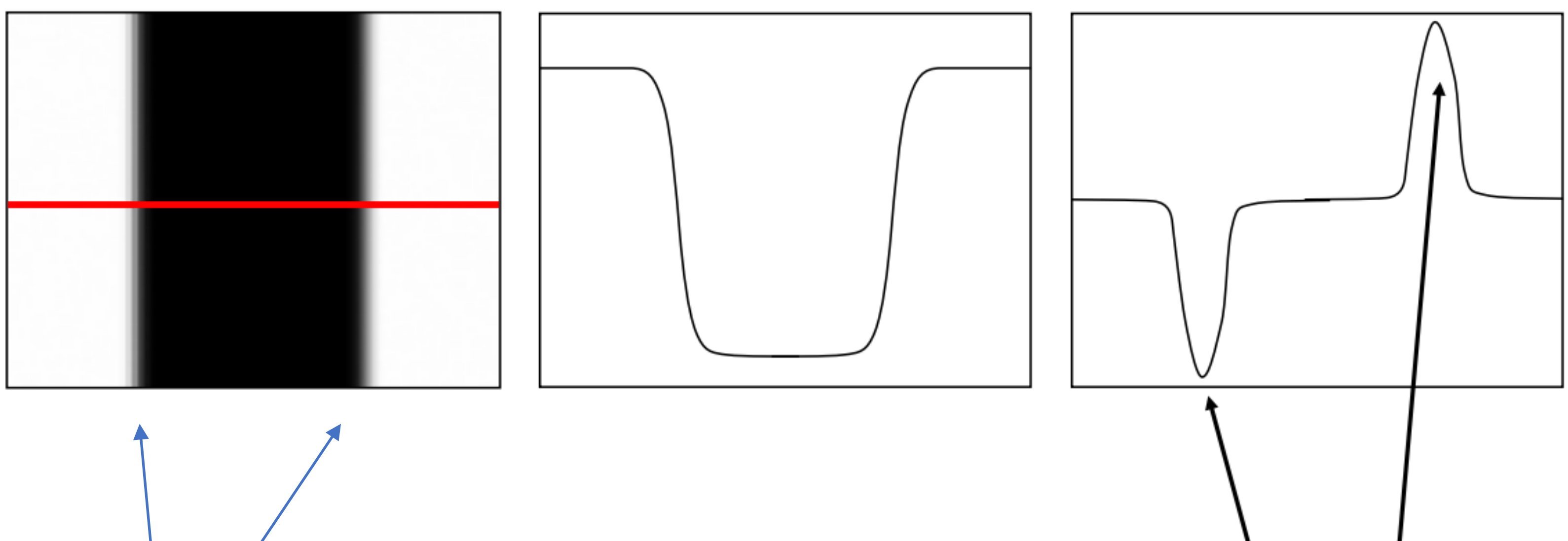


Depth discontinuity: object boundary

Cast shadows

Slide credit: K. Grauman

What is an edge?



Rapid changes in intensity
(edges)

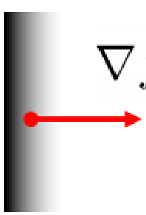
edges correspond to
extrema of derivative

Slide credit: K. Grauman

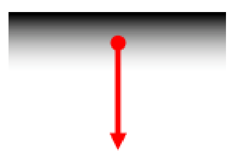
In 2D…

- In 2D, the derivative corresponds to the **gradient**
- The gradient is a vector defined for every point in the image
- It points to the direction of most rapid increase of the intensity

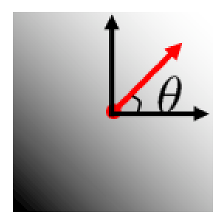
The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



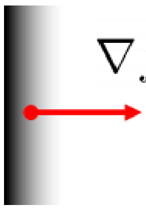
$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



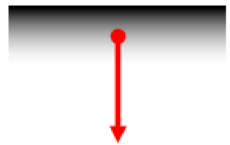
$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

Gradient direction and strength

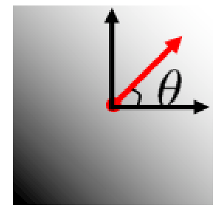
The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

- Gradient direction (orthogonal to the edge):

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

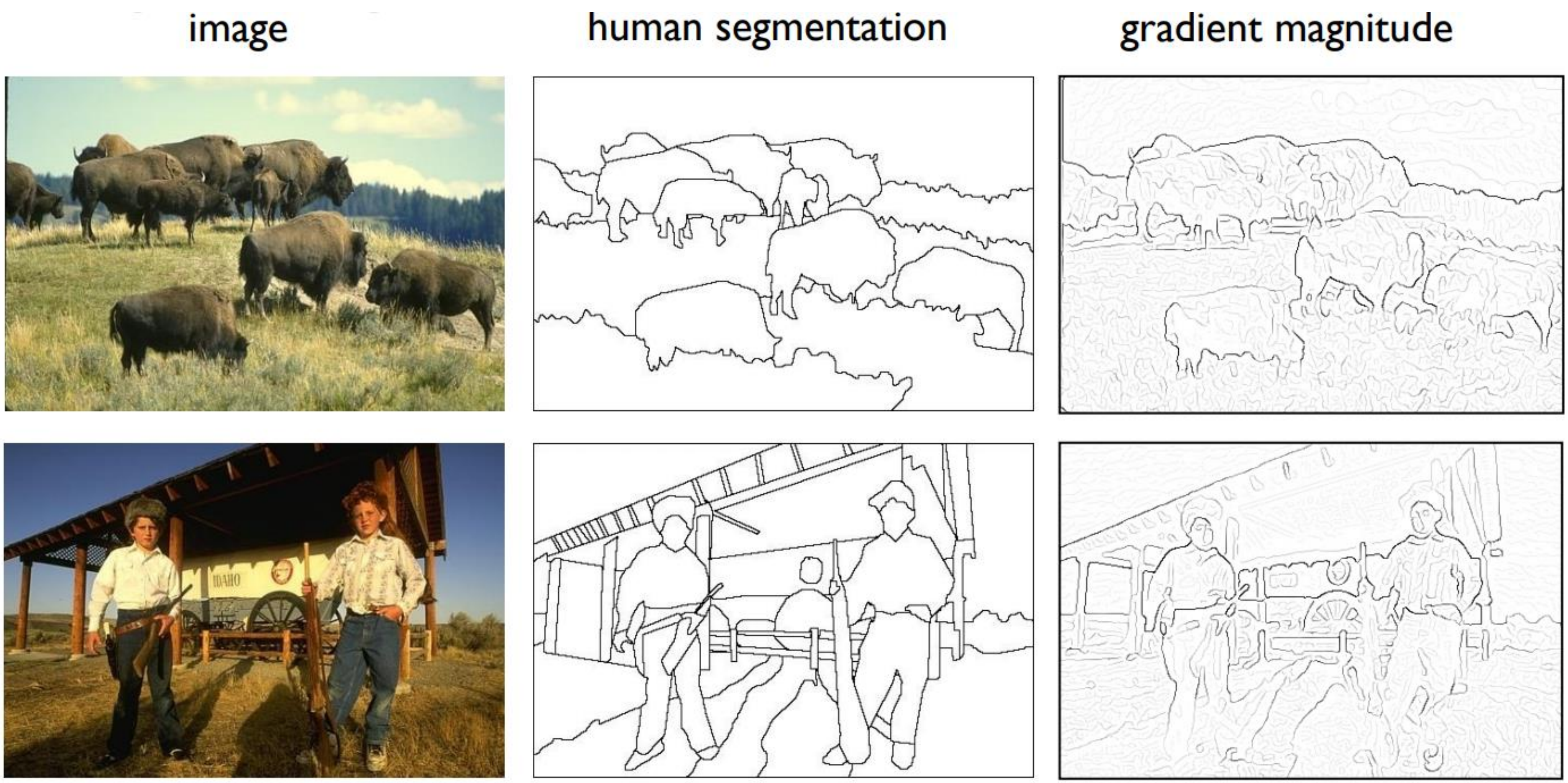
- Gradient magnitude (strength of the edge):

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Slide credit: S. Seitz

Keep in mind:

- Not all important edges have strong gradients
- Not all strong gradients are important edges



Computing the gradient on an image

- How do we approximate $\frac{\partial f}{\partial x}$?

- How do we approximate $\frac{\partial f}{\partial y}$?

Computing the gradient on an image

- How do we approximate $\frac{\partial f}{\partial x}$? convolve with

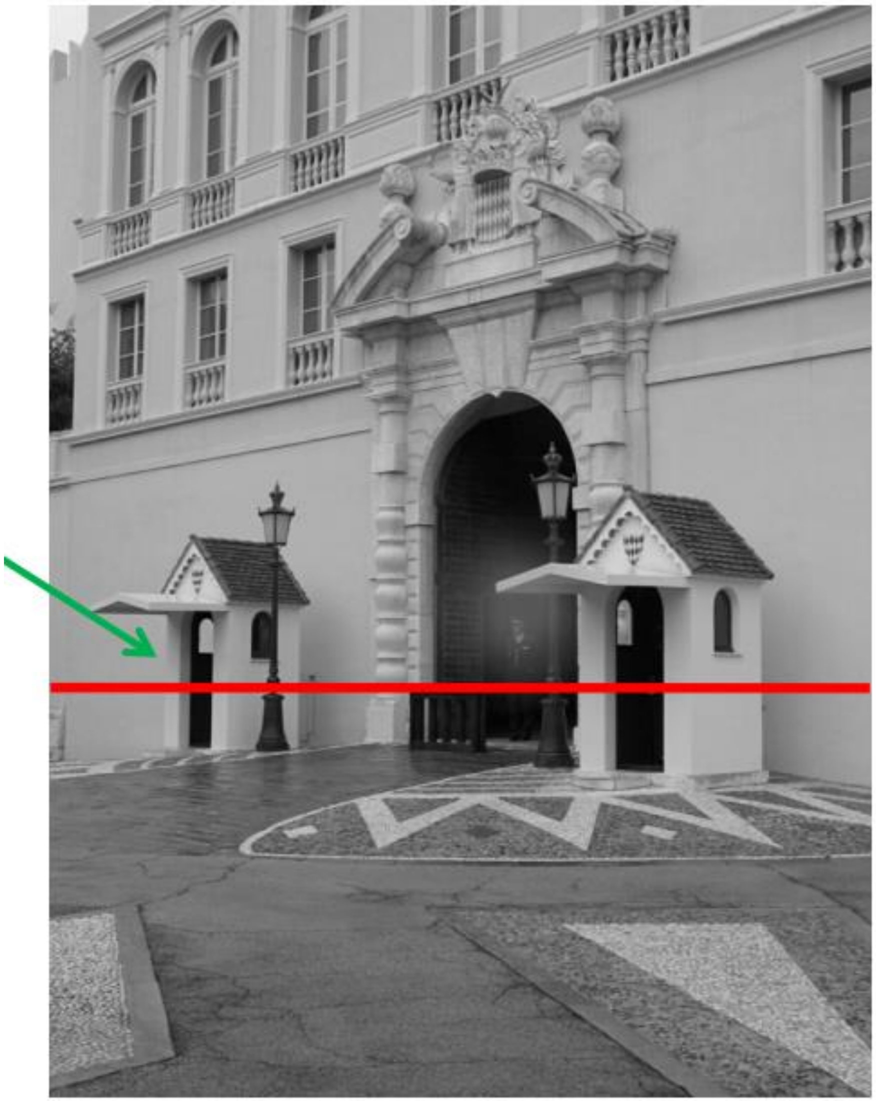
-1	1

- How do we approximate $\frac{\partial f}{\partial y}$? convolve with

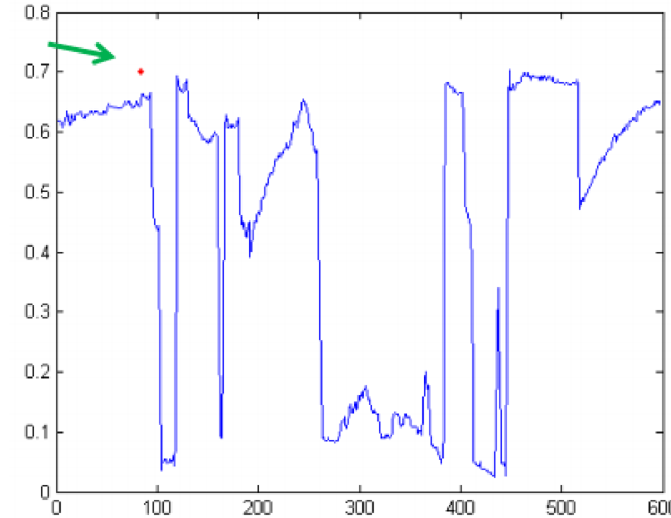
-1
1

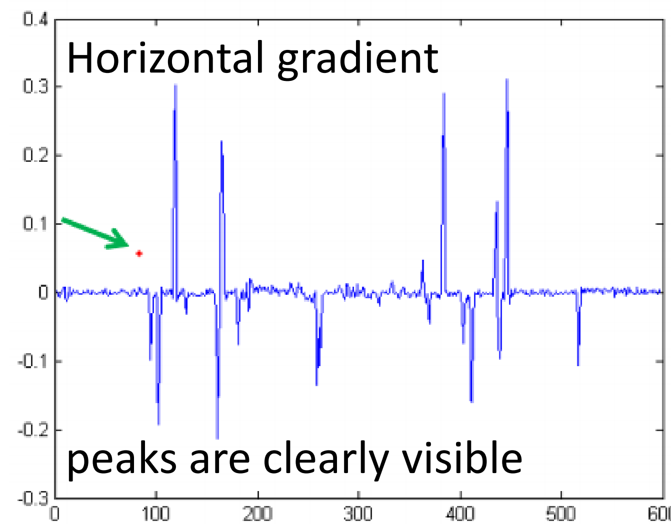
Finite differencing!

Example of finite differencing



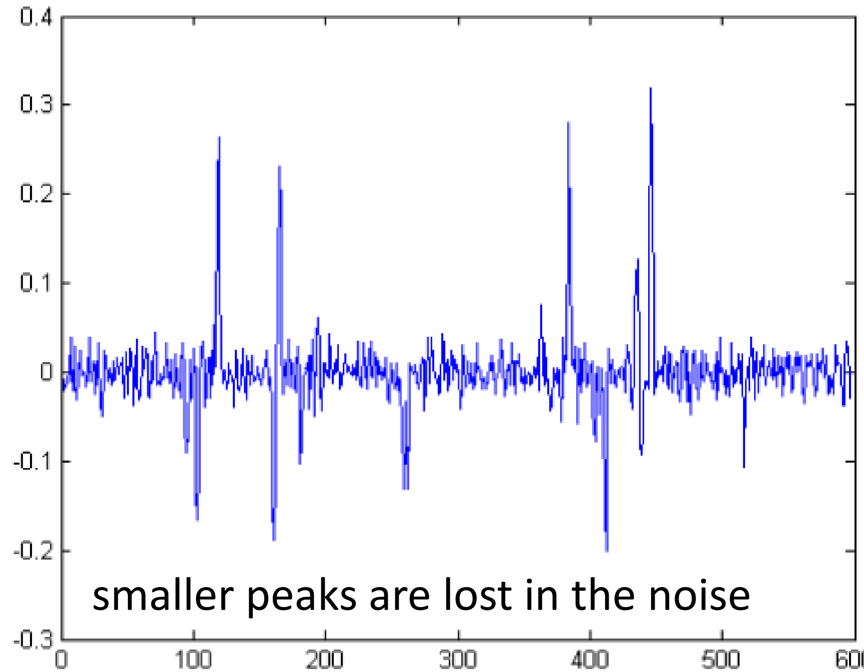
Intensity along profile





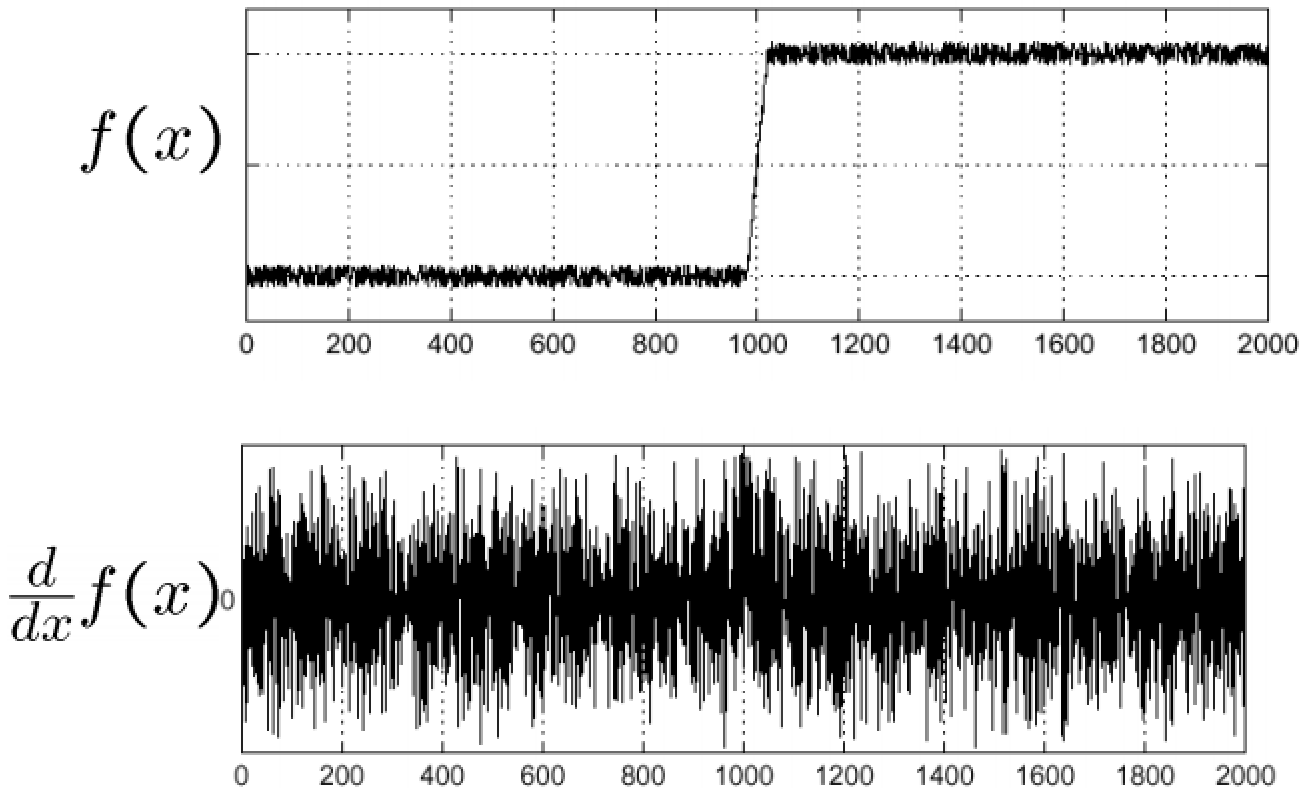
If we add a tiny amount of noise to the image...

Horizontal gradient



Slide credit: D. Hoiem

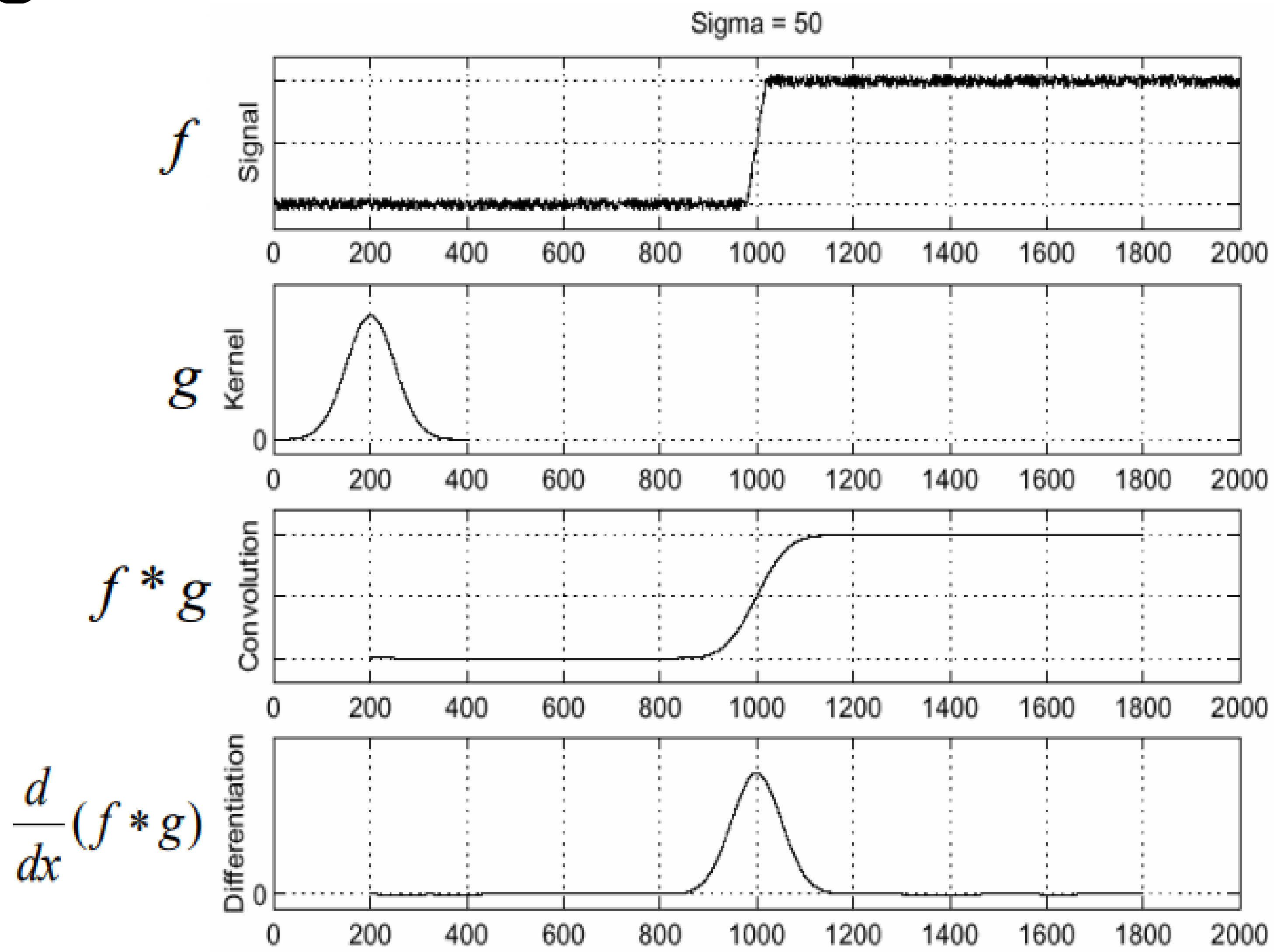
Another 1D example



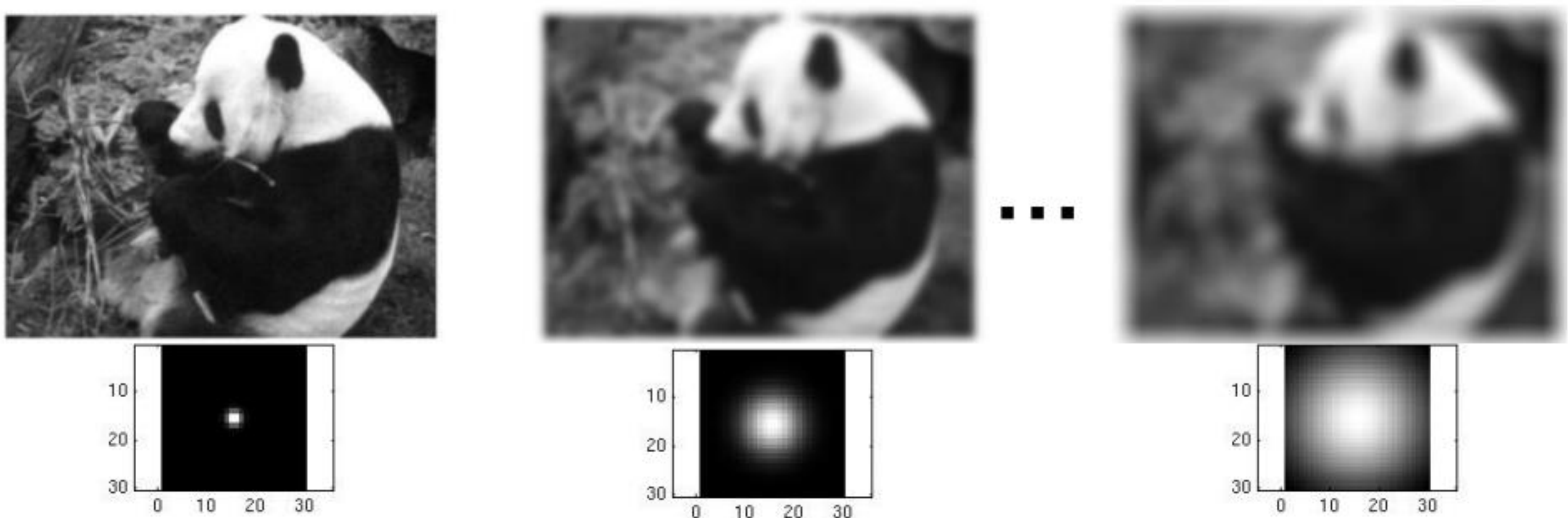
Where is the edge?!?

Slide credit: S. Seitz

Solution: smooth, then compute gradient



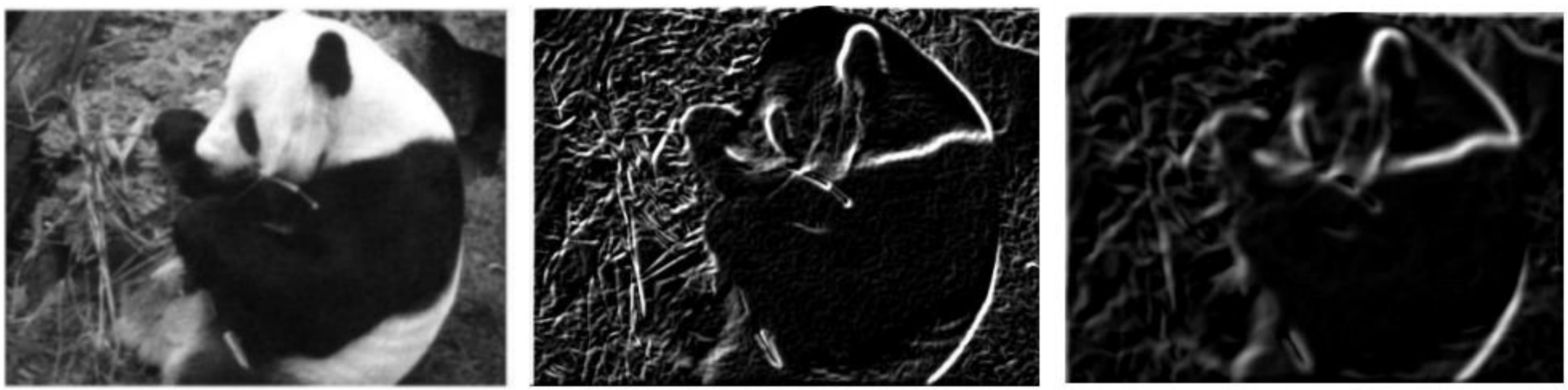
Different amounts of smoothing...



Amount of smoothing depends on the value of sigma (width of the gaussian)

Slide credit: K. Grauman

.... show different structures in the gradients



σ = 1 pixel σ = 3 pixels

Larger values of sigma: larger scale edges detected

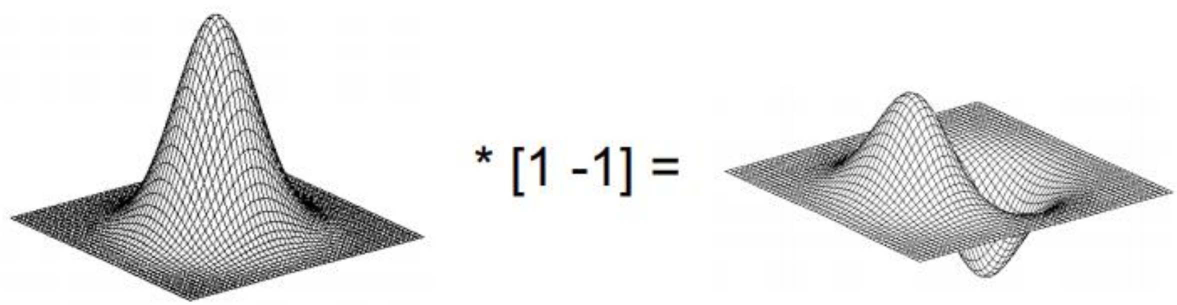
Smaller values of sigma: finer details detected

Slide credit: K. Grauman

In practice

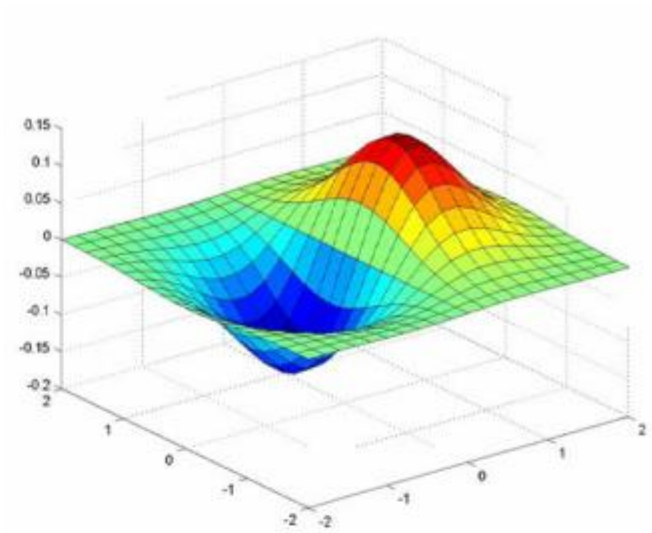
To compute gradients, convolve with a derivative-of-gaussian filter.

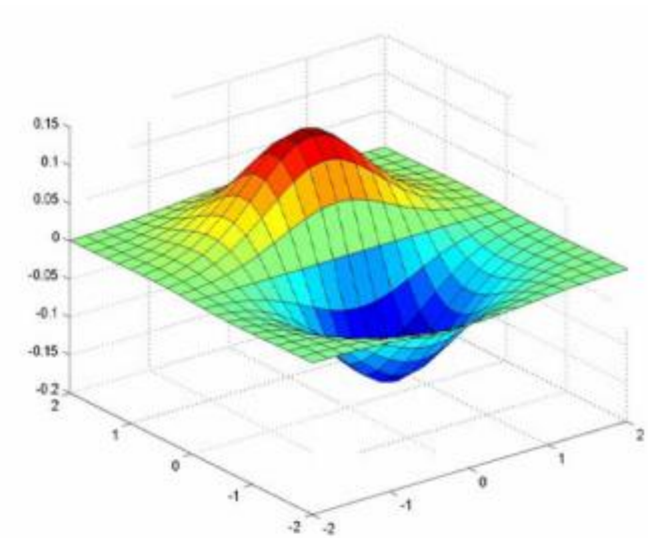
This is equivalent to smoothing with a gaussian and then taking the derivative.



Slide credit: S. Lazebnik

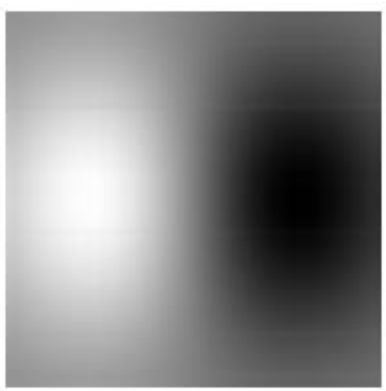
Derivative-of-gaussian filters

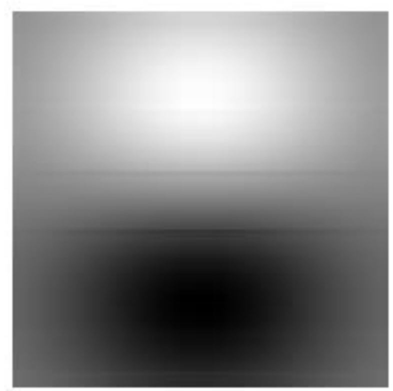




x-direction

y-direction





Slide credit: S. Lazebnik

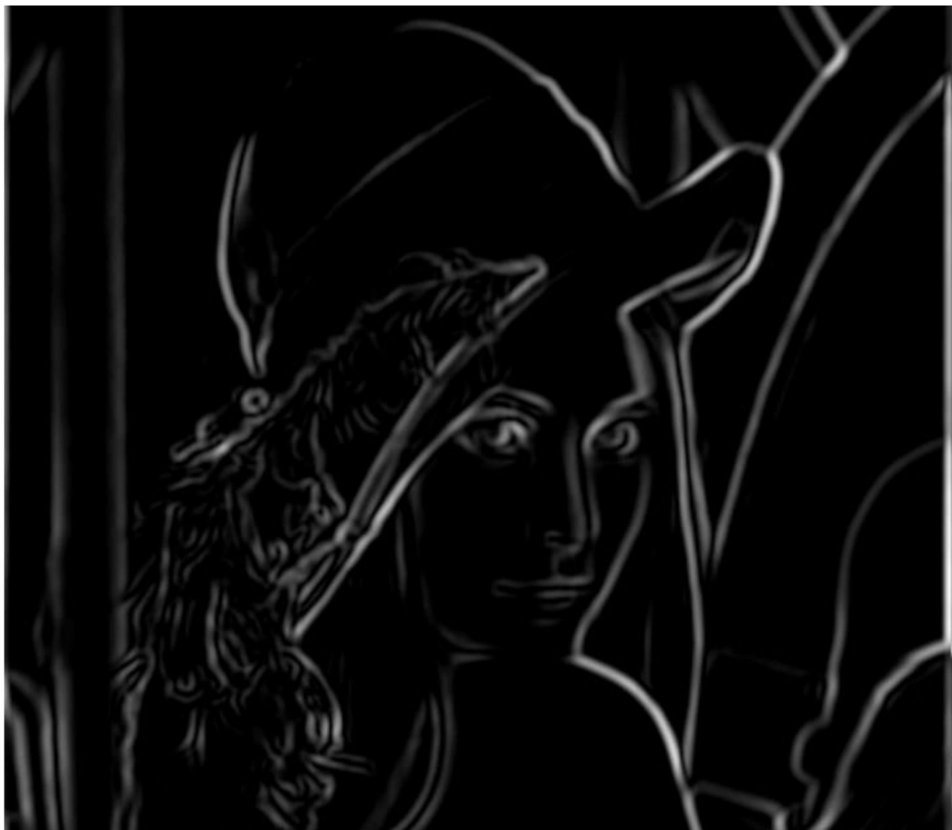
Notes

- Gaussian smoothing filter
  - removes “high-frequency” components
  - values sum to one
- Derivative filters
  - contain some negative values
  - values sum to 0
  - yield large responses at points with high contrast (e.g. edges)

Edge detection summary

$\frac{\partial f}{\partial x}$ $\frac{\partial f}{\partial y}$

- Approximate gradients along x and y by convolving with derivative-of-gaussian filters
- Compute gradient magnitude as $\|\nabla f\| = \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$



Canny edge detection algorithm

$\frac{\partial f}{\partial x}$ $\frac{\partial f}{\partial y}$

- Approximate gradients along x and y by convolving with derivative-of-gaussian filters
- Compute gradient magnitude as $\|\nabla f\| = \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$
- Make edges 1-pixel-wide (thinning):

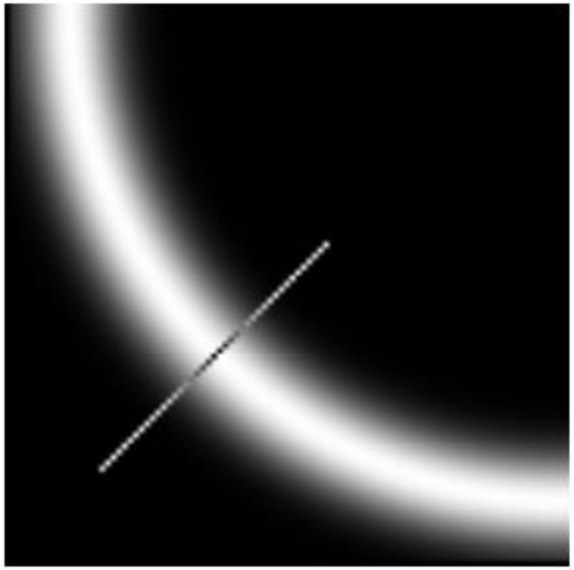
  non-maxima suppression along perpendicular direction to edge, i.e. direction of gradient
- Only keep strong edges

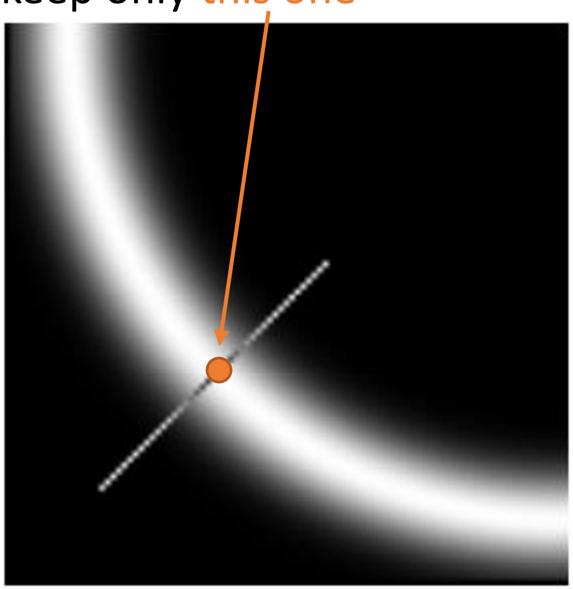
  hysteresis thresholding

Non-maxima suppression

Intuitively:

of all the possible edge pixels along the line... keep only this one

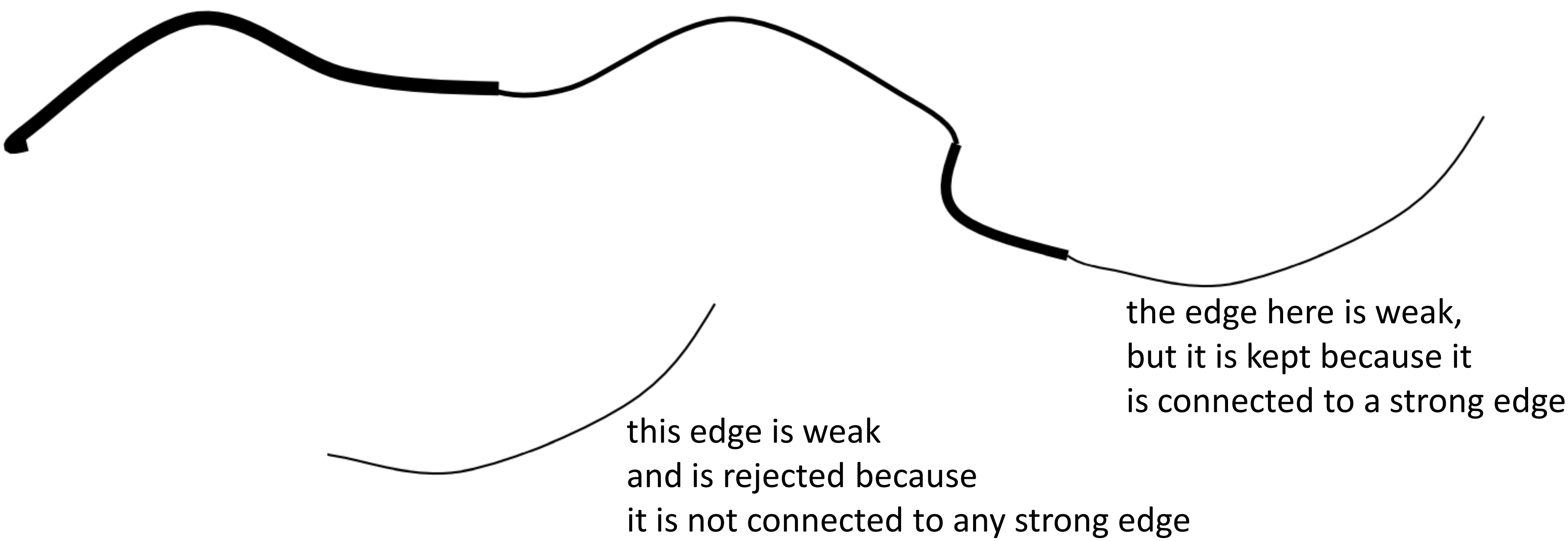




... then repeat along the whole edge. Results in thinning the edge to a 1-pixel width

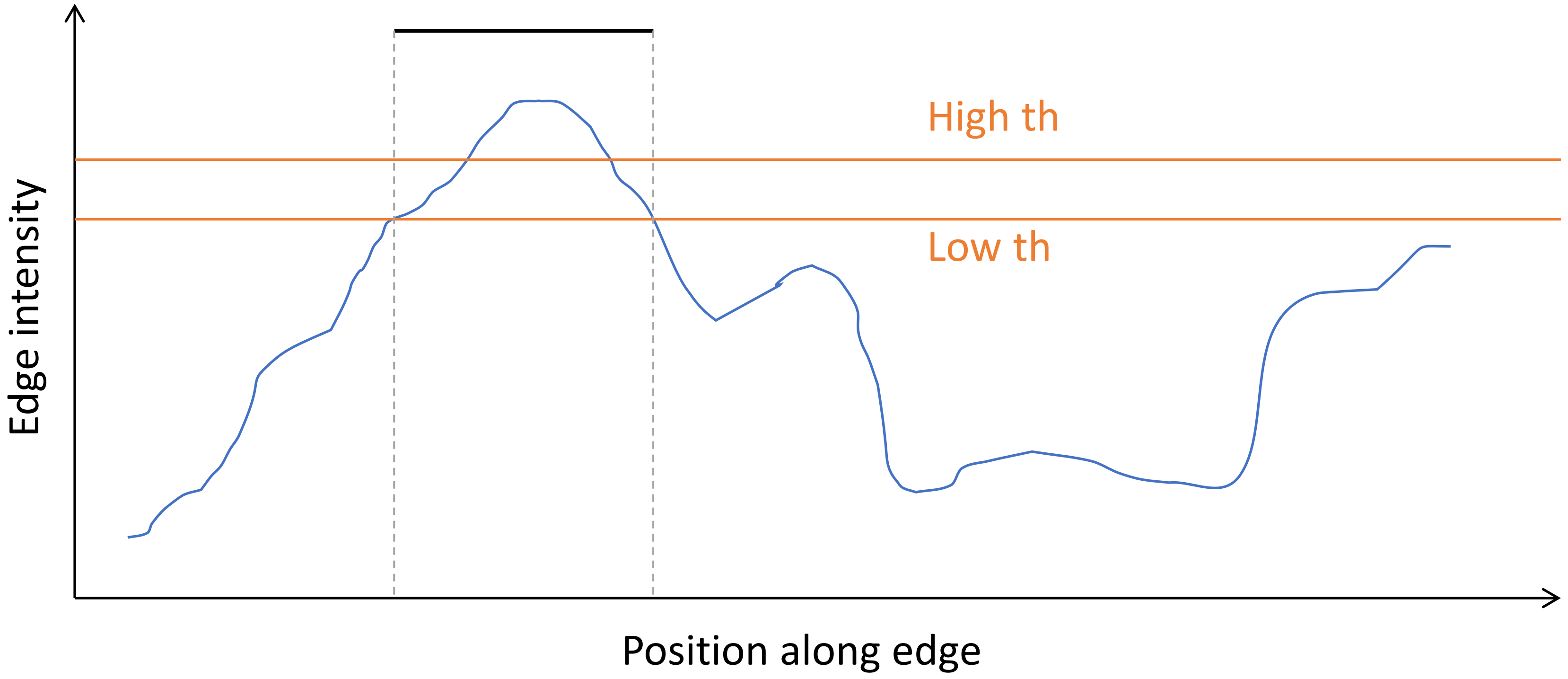
Hysteresis thresholding

Intuitively:
Use a high threshold to start curves and a low threshold to continue them



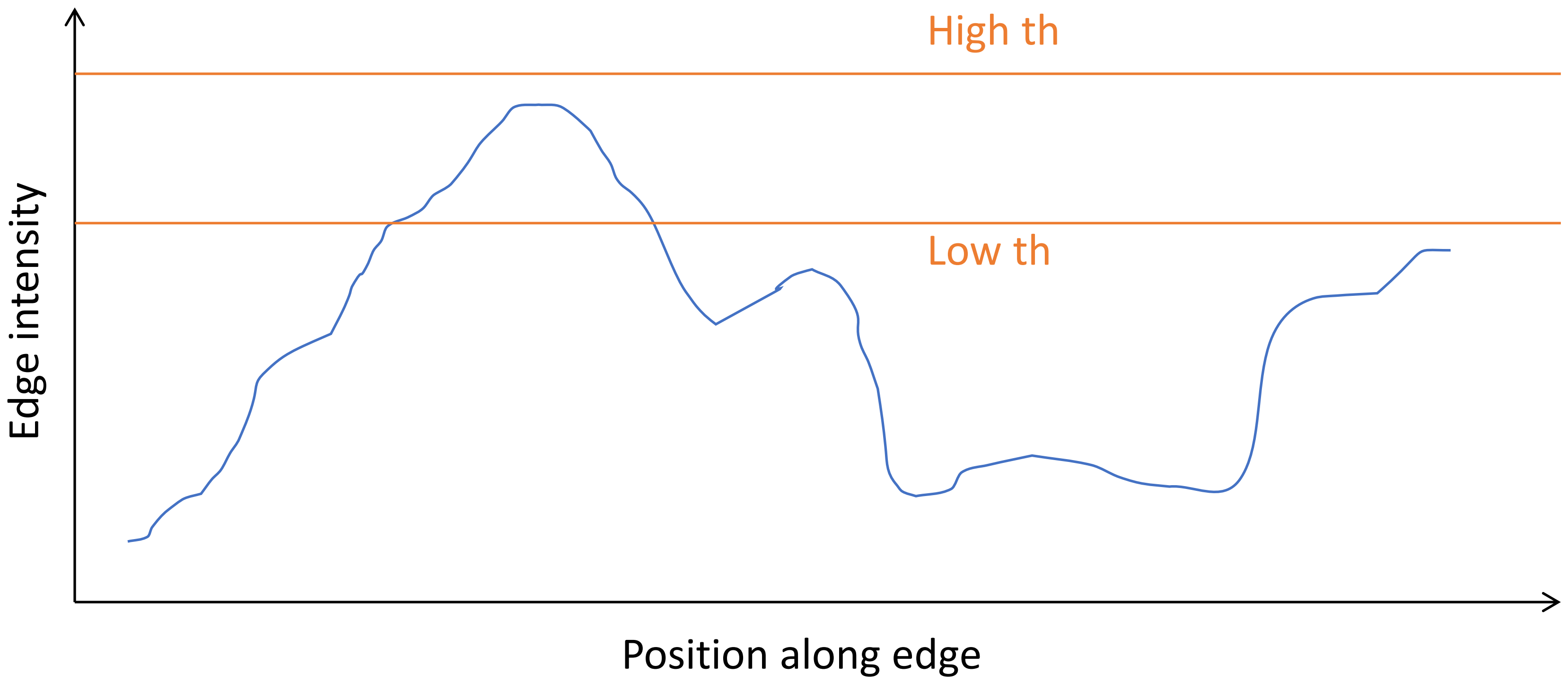
Hysteresis thresholding

Quiz: which parts of the edge would survive?



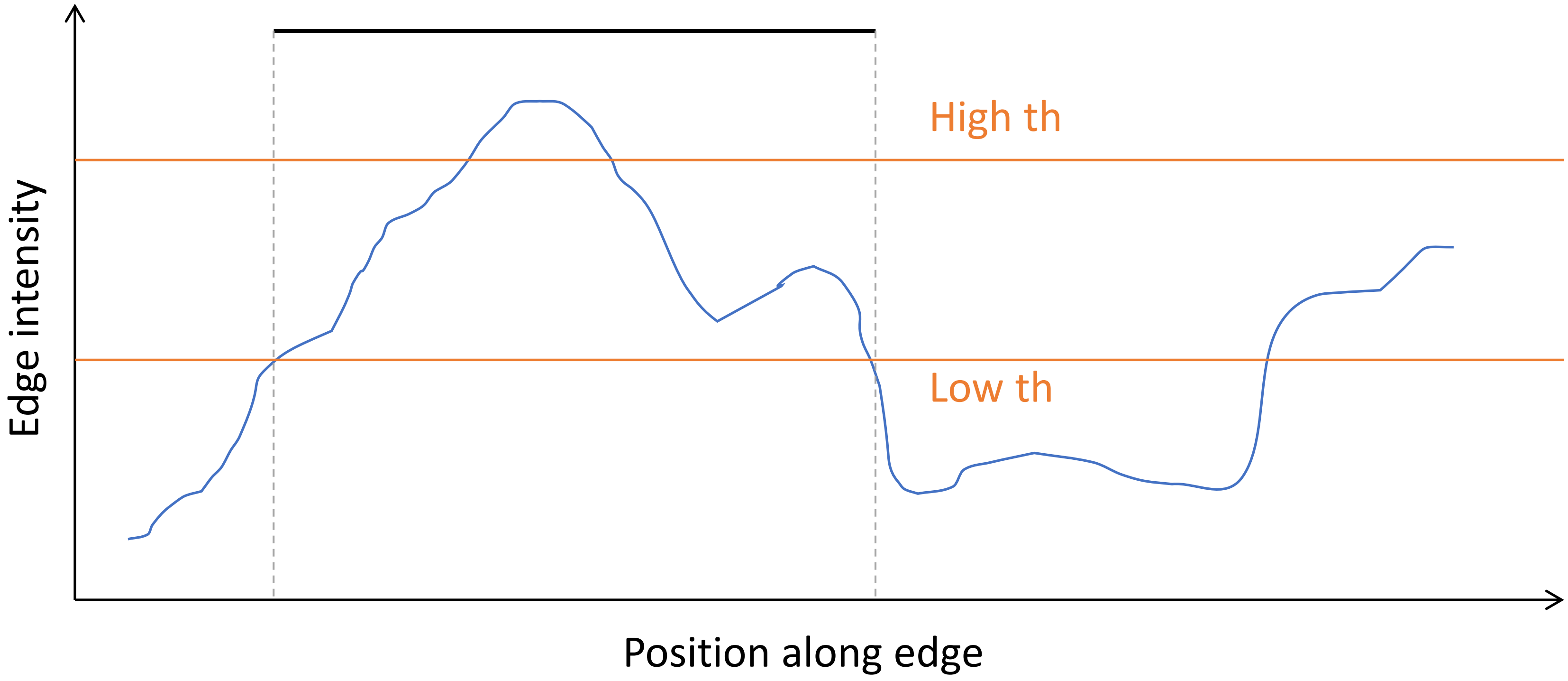
Hysteresis thresholding

Quiz: which parts of the edge would survive? None!



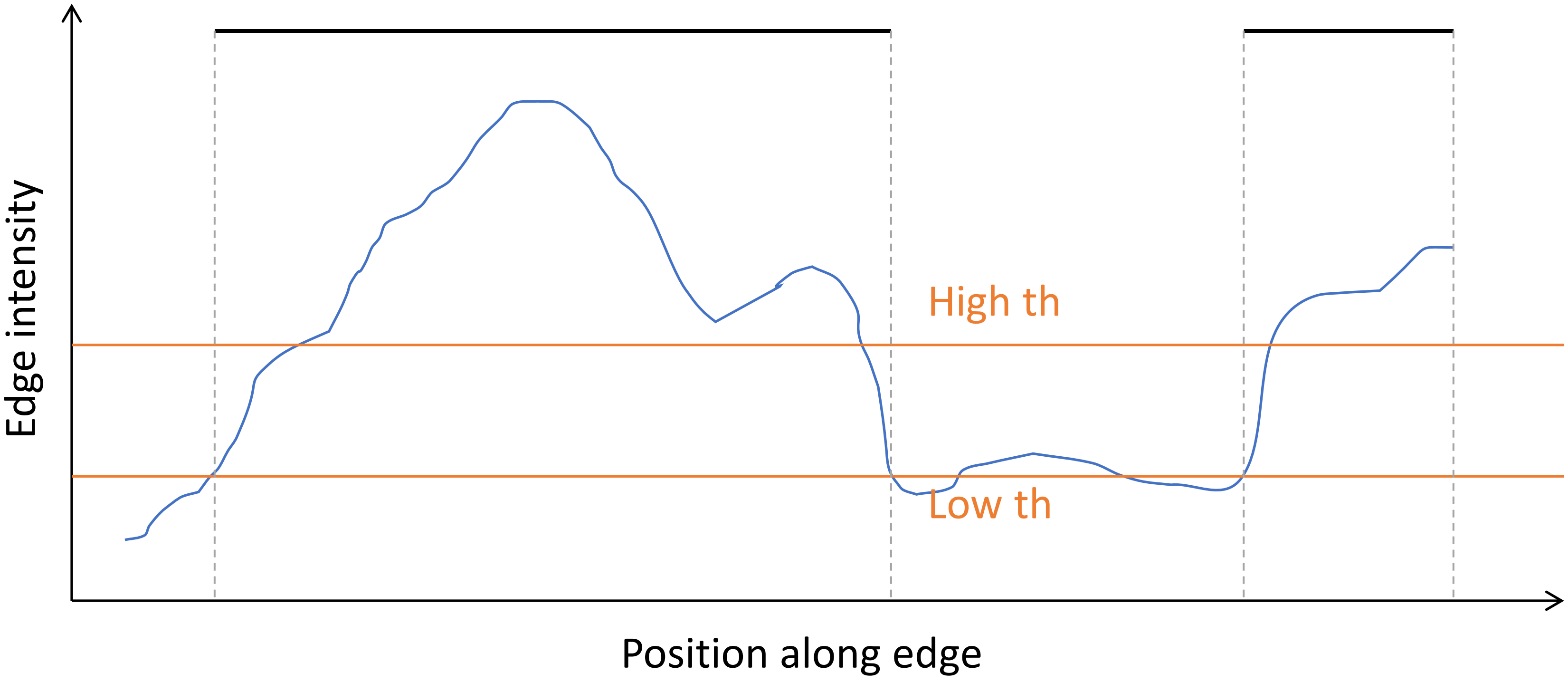
Hysteresis thresholding

Quiz: which parts of the edge would survive?



Hysteresis thresholding

Quiz: which parts of the edge would survive?



original image

high threshold
(strong edges)

low threshold
(weak edges)

hysteresis threshold

Slide credit: L. Fei-Fei

high threshold
(strong edges)

low threshold
(weak edges)

hysteresis threshold

Slide credit: L. Fei-Fei

original image

high threshold
(strong edges)

low threshold
(weak edges)

hysteresis threshold

Slide credit: L. Fei-Fei

Effect of gaussian sigma

original | Canny with $\sigma = 1$ | Canny with $\sigma = 2$

canny

`skimage.feature.canny`(*image*, *sigma=1.0*, *low_threshold=None*, *high_threshold=None*, *mask=None*, *use_quantiles=False*) [source]

Edge filter an image using the Canny algorithm.

Parameters:

image : 2D array

Grayscale input image to detect edges on; can be of any dtype.

sigma : float

Standard deviation of the Gaussian filter.

low_threshold : float

Lower bound for hysteresis thresholding (linking edges). If None, low_threshold is set to 10% of dtype's max.

high_threshold : float

Upper bound for hysteresis thresholding (linking edges). If None, high_threshold is set to 20% of dtype's max.

mask : array, dtype=bool, optional

Mask to limit the application of Canny to a certain area.

use_quantiles : bool, optional

If True then treat low_threshold and high_threshold as quantiles of the edge magnitude image, rather than absolute edge magnitude values. If True then the thresholds must be in the range [0, 1].

Returns:

output : 2D array (image)

The binary edge map.