

Problem 1:-

The proof

$$\because z = w_0 + w^T x, \quad p(c_1 | x) = \sigma'(w_0 + w^T x), \quad p(c_2 | x) = 1 - p(c_1 | x)$$

$$\therefore p(c_2 | x) = \frac{e^{(-w_0 - w^T x)}}{1 + e^{(-w_0 - w^T x)}}$$

$$L(w) = p(t^{(1)} | x^{(1)}, w) \dots p(t^{(n)} | x^{(n)}, w) = \prod_{\langle i \rangle} p(t^{(i)} | x^{(i)}, w)$$

$$\max_w L(w) = \max_{\langle i \rangle} \left( \prod p(t^{(i)} | x^{(i)}, w) \right)$$

$$\therefore p(t^{(i)} | x^{(i)}, w) = (p(c=1 | x^{(i)}, w))^{t^{(i)}} (p(c=0 | x^{(i)}, w))^{(1-t^{(i)})}$$

by taking log for both sides

$$\therefore \log(L(w)) = - \sum_{\langle i \rangle} \left( t^{(i)} \log(1 - p(c=0 | x^{(i)}, w)) + (1 - t^{(i)}) \log(p(c=0 | x^{(i)}, w)) \right)$$

$$\therefore \log(L(w)) = - \sum_{\langle i \rangle} \left( t^{(i)} \log \left( \frac{e^{-z}}{1 + e^{-z}} \right) + (1 - t^{(i)}) \log \left( \frac{1}{1 + e^{-z}} \right) \right)$$

$$\therefore \log(w) = \sum_{\langle i \rangle} t^{(i)} \log(1 + e^{-z^{(i)}}) + t^{(i)} z^{(i)} + (1 - t^{(i)}) \log(1 + e^{-z^{(i)}})$$

$$= \sum_{i=1}^N t^{(i)} z^{(i)} + \log(1 + e^{-z^{(i)}})$$

H&A



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$$\therefore \frac{\partial \log}{\partial w_j} = \sum_{i=1}^n t^{(i)} x_j^{(i)} - x_j^{(i)} * \frac{e^{(-z^{(i)})}}{1 + e^{-z^{(i)}}}$$

$$= \sum x_j^{(i)} (t^{(i)} - p(c=1 | x^{(i)}, w))$$

in MAP rule:-

we want to maximize  $P(w) \prod_{i=1}^n P(t^{(i)} | x^{(i)}, w)$

$$P(w) = \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{w_j^2}{2}\right)}$$

$$\therefore \frac{\partial \log(P(w))}{\partial w} = -w_j$$

for GA:

$$w_j = w_j + \alpha [-w_j + \sum x_j^{(i)} [t^{(i)} - p(c=1 | x_j^{(i)}, w)]]$$