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SIGNAL PROCESSING

- Professor: Rubén Alvarez

CONTENT

- What is a signal?
- Fourier Transform?
- What is the convolution?
- Why we need signal processing in Computer Vision?

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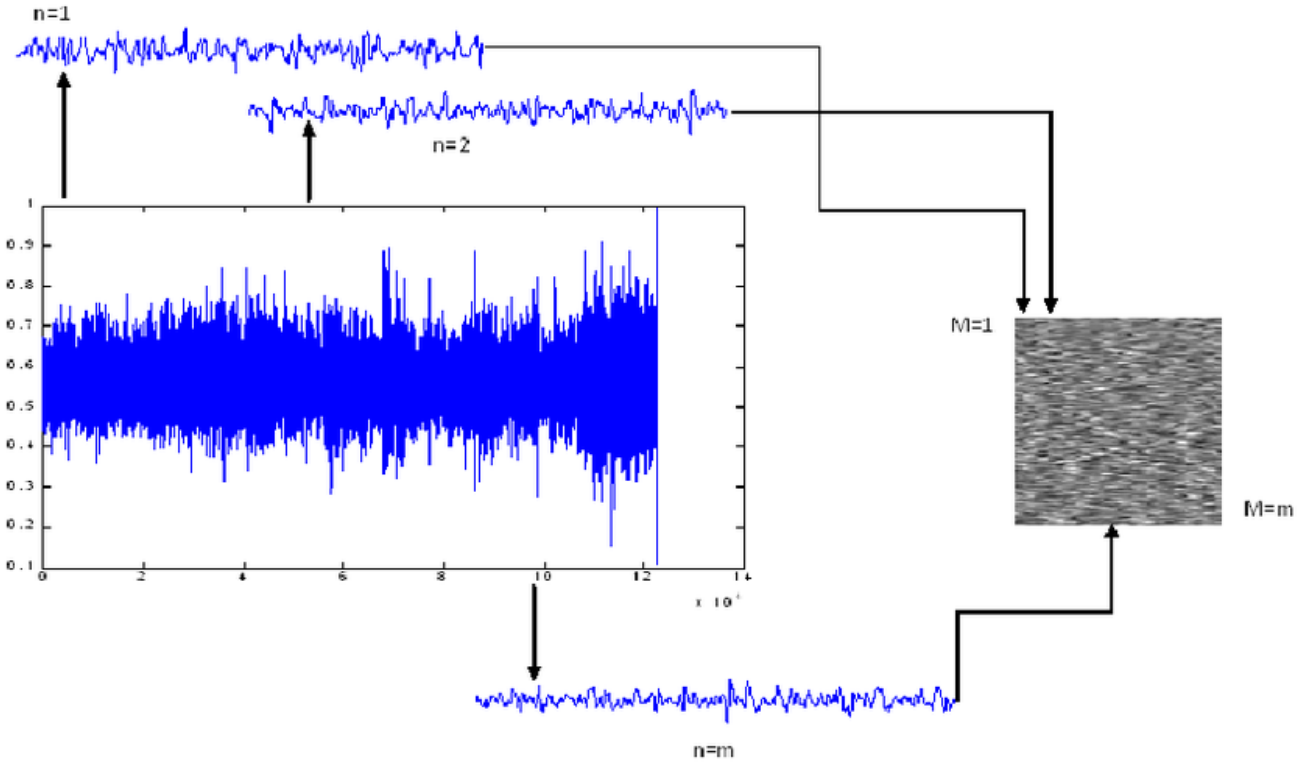
WHY DO I NEED TO KNOW SIGNAL PROCESSING?

WHY DO I NEED TO KNOW SIGNAL PROCESSING?

- A digital image can be seen as the result of applying a "sampling" to a continuous signal in two directions.

- I mean...

AN IMAGE IS A 2D SIGNAL



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WHAT IS A SIGNAL?

WHAT IS A SIGNAL?

- It's a flow of information
- A measure of amounts that vary in intensity with respect to time or position.
- An electrical signal received from a transducer (microphone, thermometer, accelerometer, antenna, etc.)
- An electrical signal that controls a process.
- And so on ..

TYPES OF SIGNALS

- Analog
- Digital

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¿ADC?

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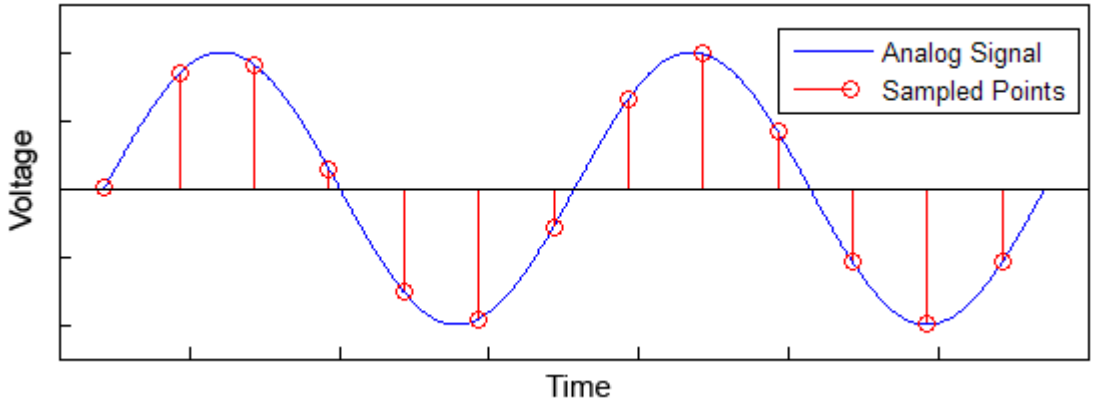
HOW DO WE GO FROM AN ANALOG SIGNAL TO A DIGITAL SIGNAL?

¿SAMPLING?



SAMPLING

- Sampling is commonly a periodic.
- $x\,[n]\; =\; x_c(nT)$ con $-\infty <\; n\; < \infty$
- T It's the "sampling" period in seconds
- $fs\; = \frac{1}{T}$ is the frequency of "sampling" in Hz
- The frequency of "sampling" in radians will be $\Omega_s = 2\pi f_s\, rad/sec$
- It is commonly used [.] for signals in discrete time and (.) for continuous-time signals.

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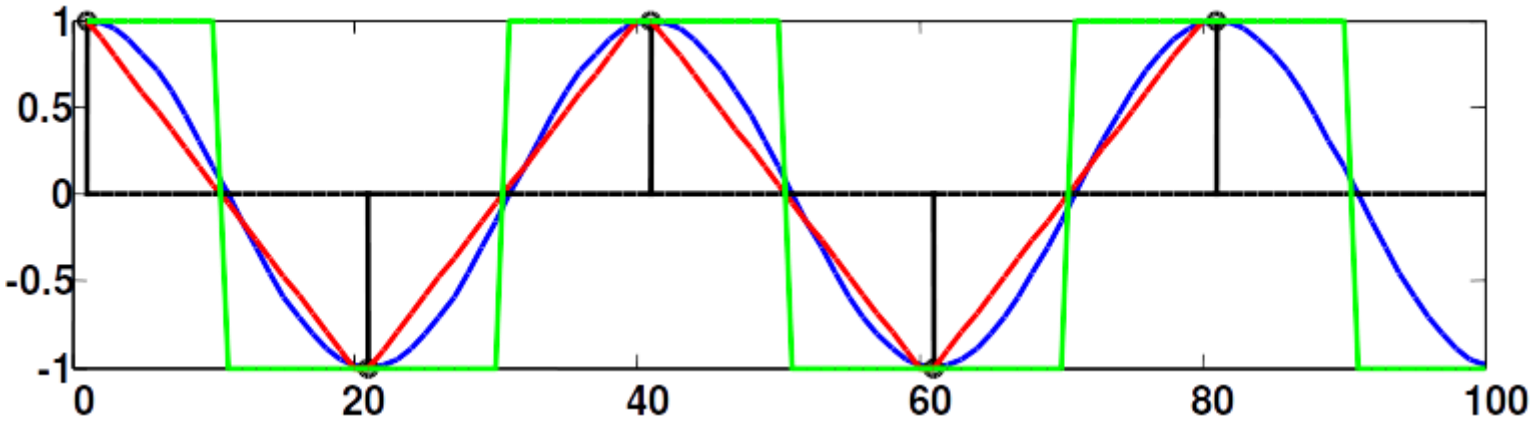
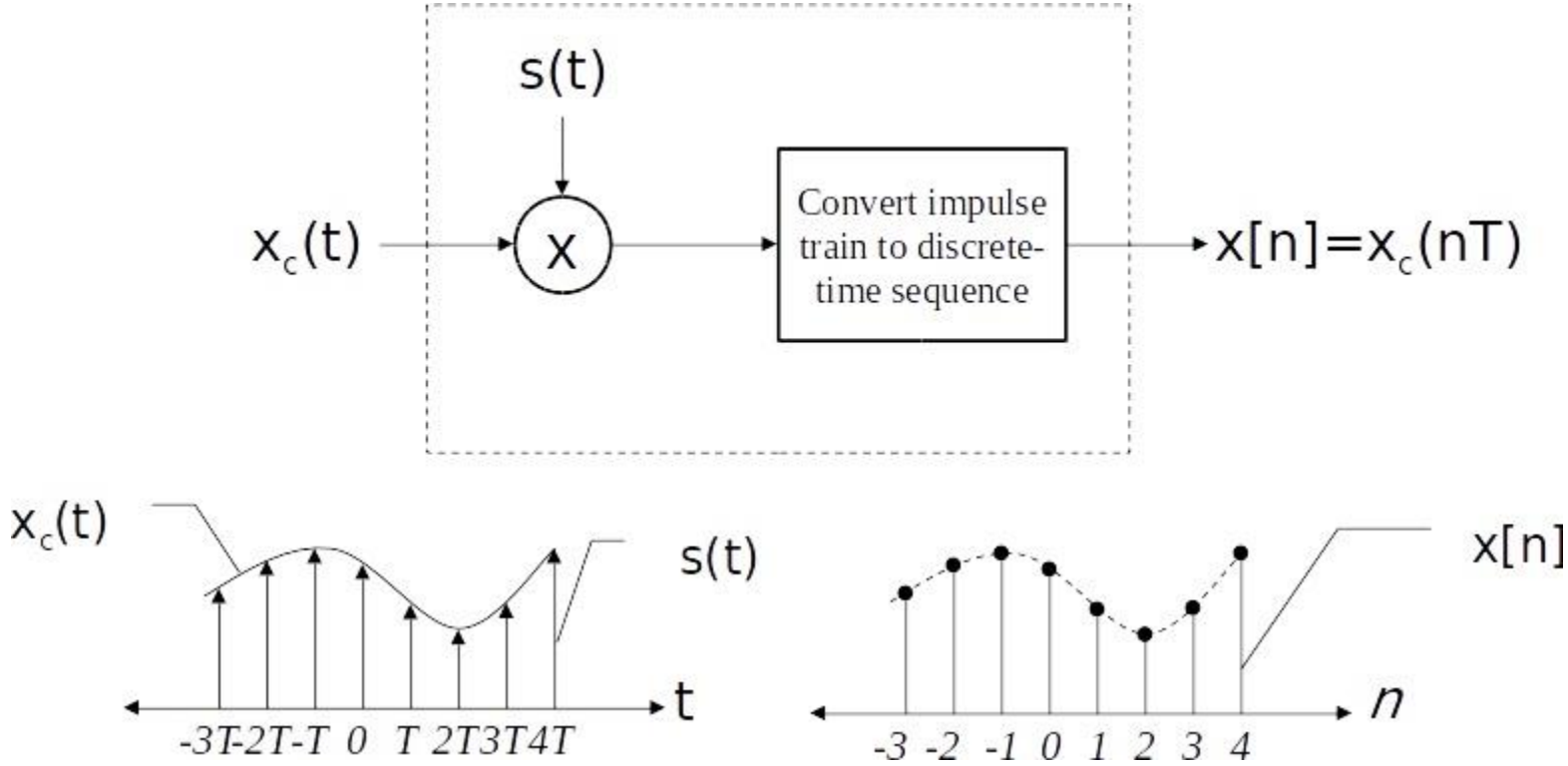
CAN WE GO FROM DIGITAL TO ANALOG?

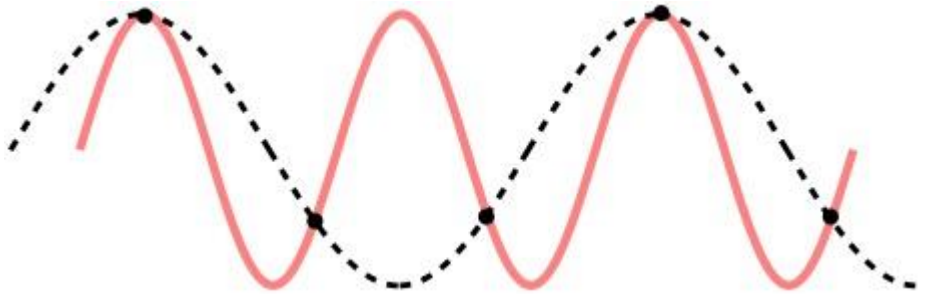
USUALLY NOT...




s(t)
$x_c(t)$
x
Convert impulse train to discrete-time sequence
$x[n]=x_c(nT)$
$x_c(t)$
s(t)
t
-3T-2T-T 0 T 2T3T4T
x[n]
n
-3 -2 -1 0 1 2 3 4

NYQUIST-SHANNON THEOREM

- The sampling frequency must be at least twice the bandwidth of the signal.



THEN...AN IMAGE IS A SPECIAL CASE OF A 2D SIGNAL

- An image can take a continuous value within an interval.
- An image is discrete so it can take values from an accounting set, for example in a finite set of $\{0, 1, 2, \ldots, 255\}$ for an image depicted in 8-bits.

THEN...AN IMAGE IS A SPECIAL CASE OF A 2D SIGNAL

- Values always come from continuous quantization, for example a digital analog conversion or a count as in nuclear images.
- A binary image only takes values {0,1}.

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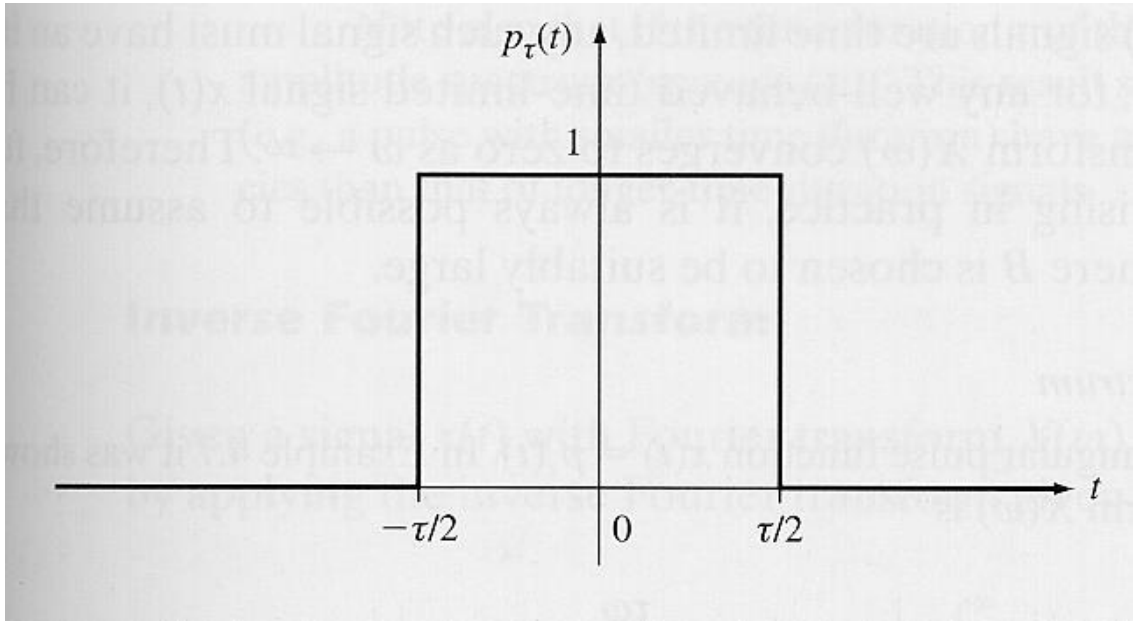
SO WE MUST LEARN EVERYTHING AGAIN FOR A 2D SIGNAL?

FOURIER TRANSFORM

$$\mathcal{F}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$\mathrm{f}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

RECTANGULAR PULSE



FOURIER TRANSFORM

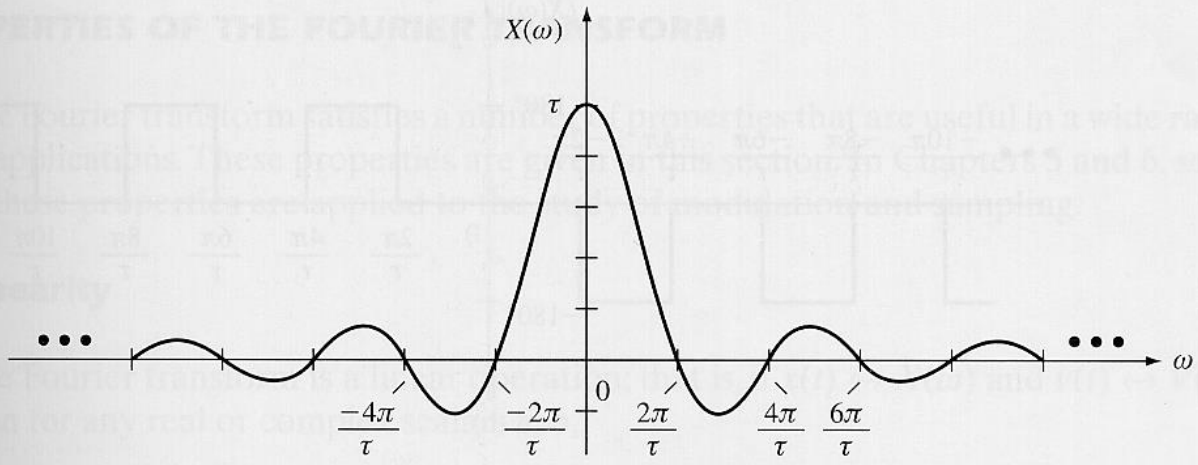


Figure 4.19 Fourier transform of the τ-second rectangular pulse.

SPECTRUM IN AMPLITUDE AND PHASE

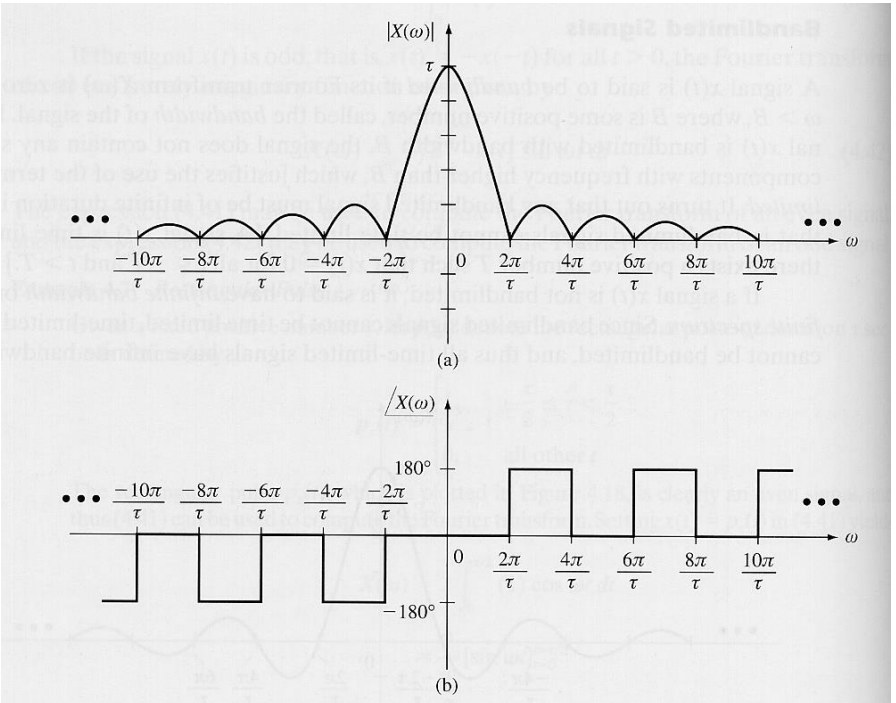


Figure 4.20 (a) Amplitude and (b) phase spectra of the rectangular pulse.

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AND WHAT DOES THE FREQUENCY OF AN IMAGE, TELL ME?

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WHAT IS THE CONVOLUTION?

CONVOLUTION

- Continuous

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\eta) g(t - \eta)\, d\eta$$

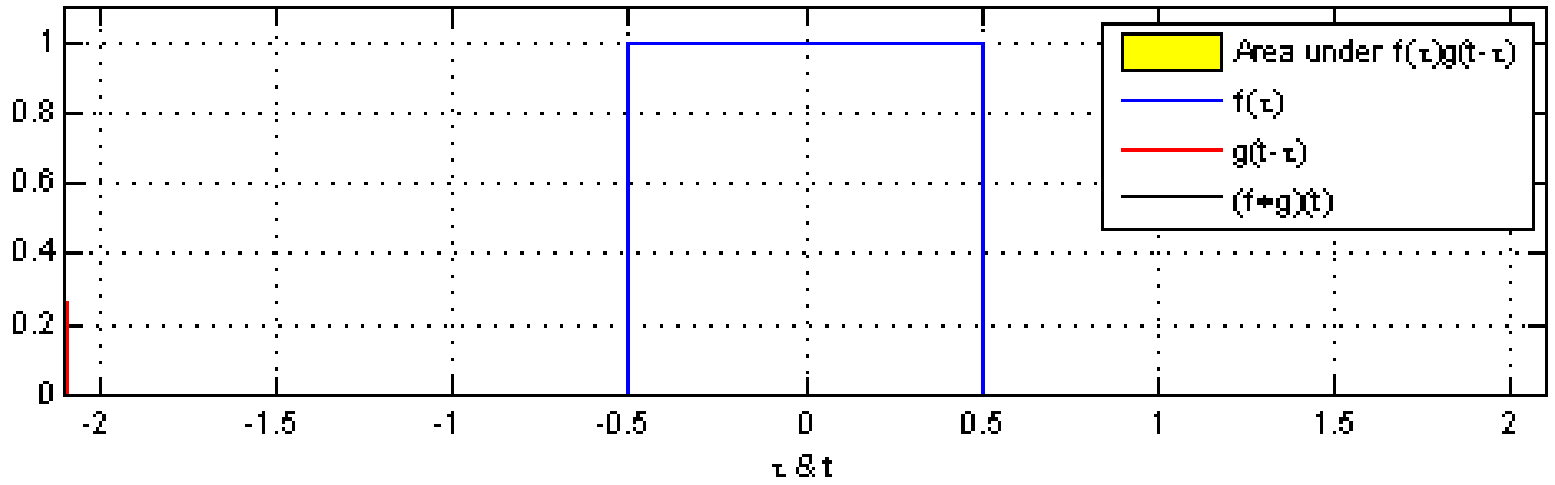
- Discrete

$$(f * g)[\eta] = \sum_{-\infty}^{\infty} f[m] g[n - m]$$

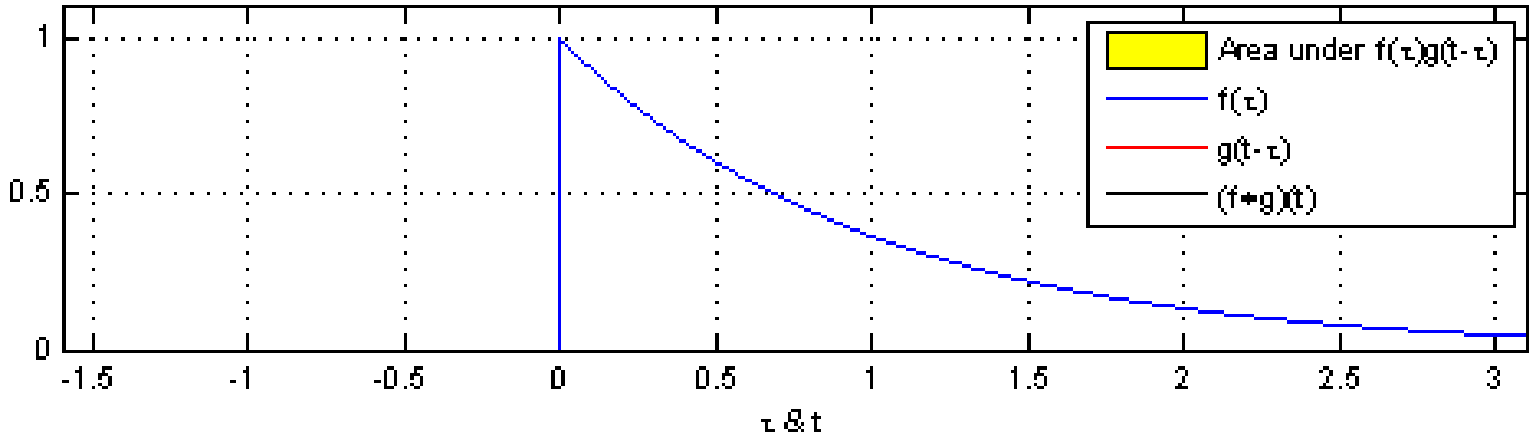
2D CONVOLUTION

$$y_{k,l} = \sum_{m=-1}^{1} \sum_{n-1}^{1} x_{k-m,l-n} g_{m,n}$$

WHAT IS CONVOLUTION?



WHAT IS CONVOLUTION?



CONVOLUTION THEOREM

$$\mathcal{F}(f * g) = F(\omega)G(\omega)$$

PRACTICE 1.2

- Deliver the results of the following in a .pdf file:

- Calculate the Fourier Transform of an image.
  - Calculate the Fourier transform of all frames in a video input.

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Questions?