IMAGE PROCESSING PART 1

- Professor: Rubén Alvarez

CONTENT

- Digital filters
- Filter analysis
- Filter design
- Filters comparison
- Model colors

REMEMBER

- Convolution theorem

$$\mathcal{F}(f * g) = F(\omega)G(\omega)$$

2D CONVOLUTION

$$y_{k,l} = \sum_{m=-1}^{1} \sum_{n-1}^{1} x_{k-m,l-n} g_{m,n}$$

DIGITAL FILTER

WHAT IS A DIGITAL FILTER AND WHAT IS IT FOR IMAGES?

DIGITAL FILTER

- Objectives:
  - Image smoothing
  - Noise removal
  - Improve the image
  - Edge detection?

WHAT IS NOISE?

- In images is the unwanted information that contaminates the image.
- Defined as

$$g(x, y) = f(x, y) + r(x, y)$$

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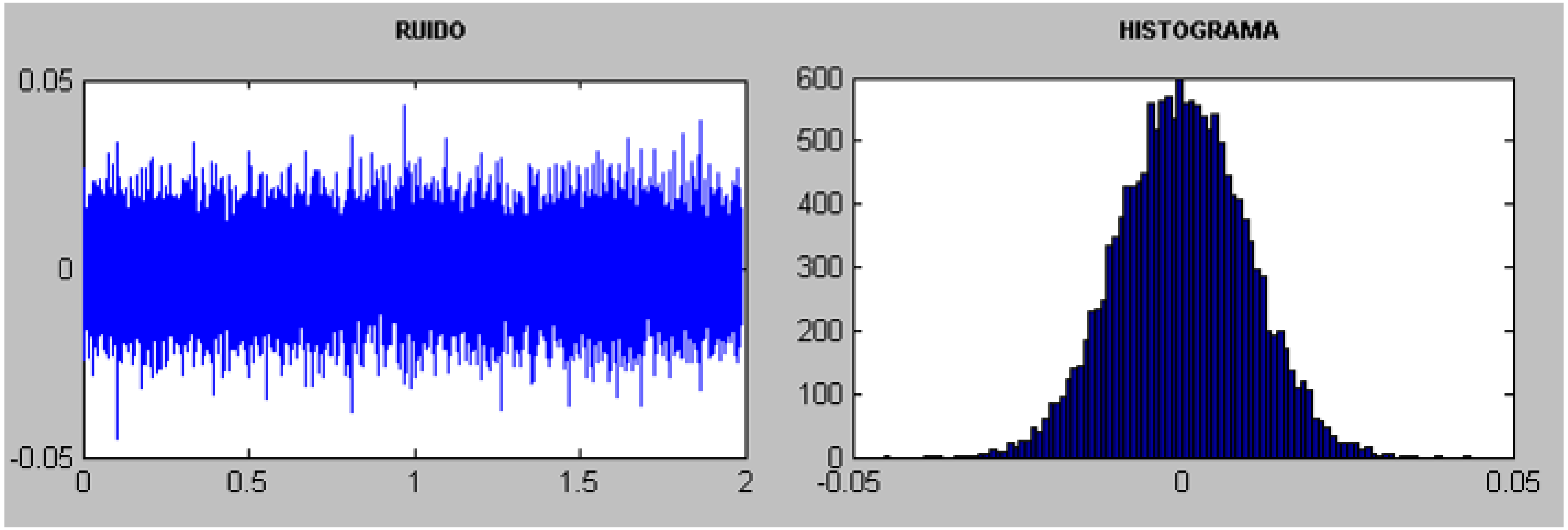
TYPES OF NOISE

- Gaussian Noise
- Uniform noise ... (Bias?)
- Impulsive noise (Salt and pepper)
- Etc.

GAUSSIAN NOISE



SALT AND PEPPER NOISE

KERNEL?

KERNEL OR CONVOLUTION MASK?

AVERAGE FILTER

$$f'(x,y) = \frac{1}{MN} \sum_{i=-\frac{M}{2}}^{\frac{M}{2}} \sum_{j=-\frac{N}{2}}^{\frac{N}{2}} f(x+i, y+j)$$

- Where M and N sun the size of the kernel or filter convolution mask.

AVERAGE FILTER

- Then for a neighborhood of 3…

$$\frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

AND IN PICTURES...

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0								

AND IN PICTURES...

$$F[x, y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x, y]$$

	0	10							

AND IN PICTURES…

$$F[x,y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x,y]$$

	0	10	20						

AND IN PICTURES…

$$F[x,y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x,y]$$

	0	10	20	30					

AND IN PICTURES…

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30					

AND IN PICTURES…

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

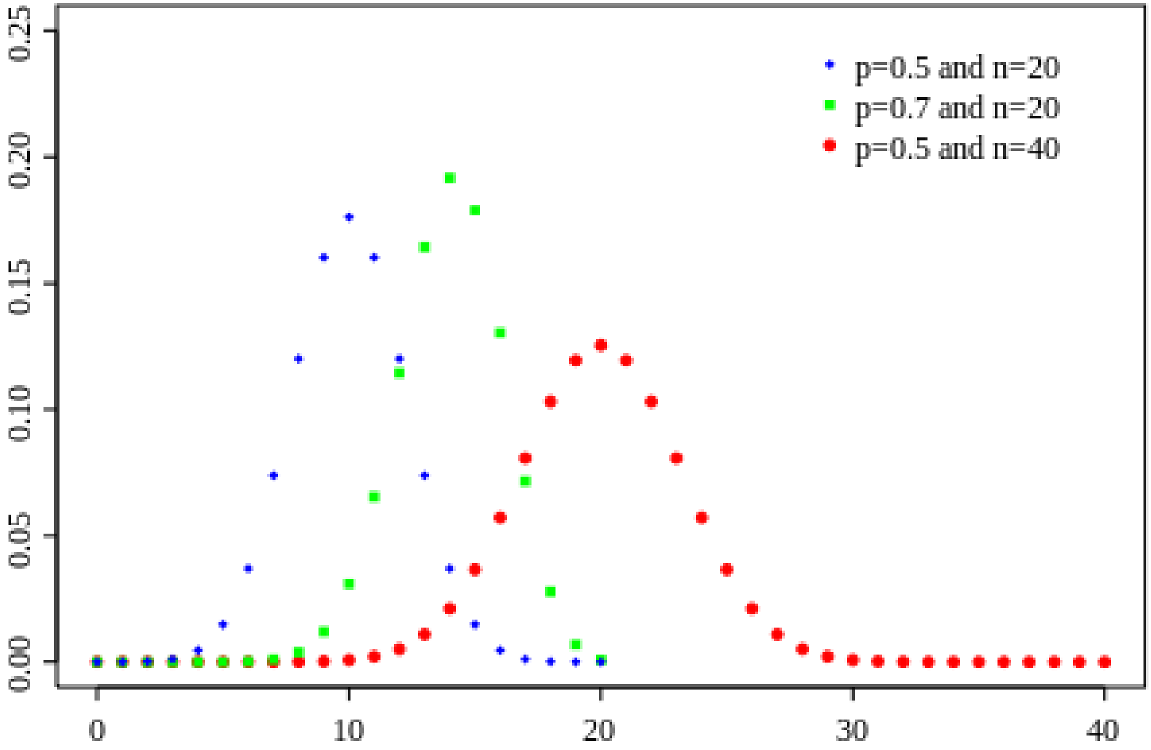
$G[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

BINOMIAL FILTER

- In statistics, binomial distribution is a discrete probability distribution that counts the number of successes in a sequence of Bernoulli independent n trials independent of each other, with a fixed probability p of success occurrence between trials.

BINOMIAL FILTER



BINOMIAL FILTER

- How do I build a binomial filter for images?

PASCAL TRIANGLE

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

WE USE CONVOLUTION

$$h_{2D}^{n} = h_{x}^{n} * h_{y}^{n}$$

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WE USE CONVOLUTION

$$h_{2D}^{n} = h_x^n * h_y^n$$

$$h_x^2 * h_y^2 = \frac{1}{4}\begin{pmatrix} 1 & 2 & 1 \end{pmatrix} * \frac{1}{4}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

WE USE CONVOLUTION

$$h_{2D}^n = h_x^n * h_y^n$$

$$h_x^2 * h_y^2 = \frac{1}{4}\begin{pmatrix}1 & 2 & 1\end{pmatrix} * \frac{1}{4}\begin{pmatrix}1\\2\\1\end{pmatrix}$$

$$= \frac{1}{16}\begin{pmatrix}1 & 2 & 1\\2 & 4 & 2\\1 & 2 & 1\end{pmatrix}$$

BUT HOW CAN WE KNOW THE BEHAVIOR OF THE FILTER BEFORE APPLYING IT TO THE IMAGE?

BUT HOW CAN WE KNOW THE BEHAVIOR OF THE FILTER BEFORE APPLYING IT TO THE IMAGE?

OR IS THE FILTER DESIGN EMPIRICAL?

WITH FOURIER'S TRANSFORM IN FREQUENCY!

$$G_{k,l} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} g(m,n) e^{-j\frac{2\pi k}{K}m} e^{-j\frac{2\pi l}{L}n}$$

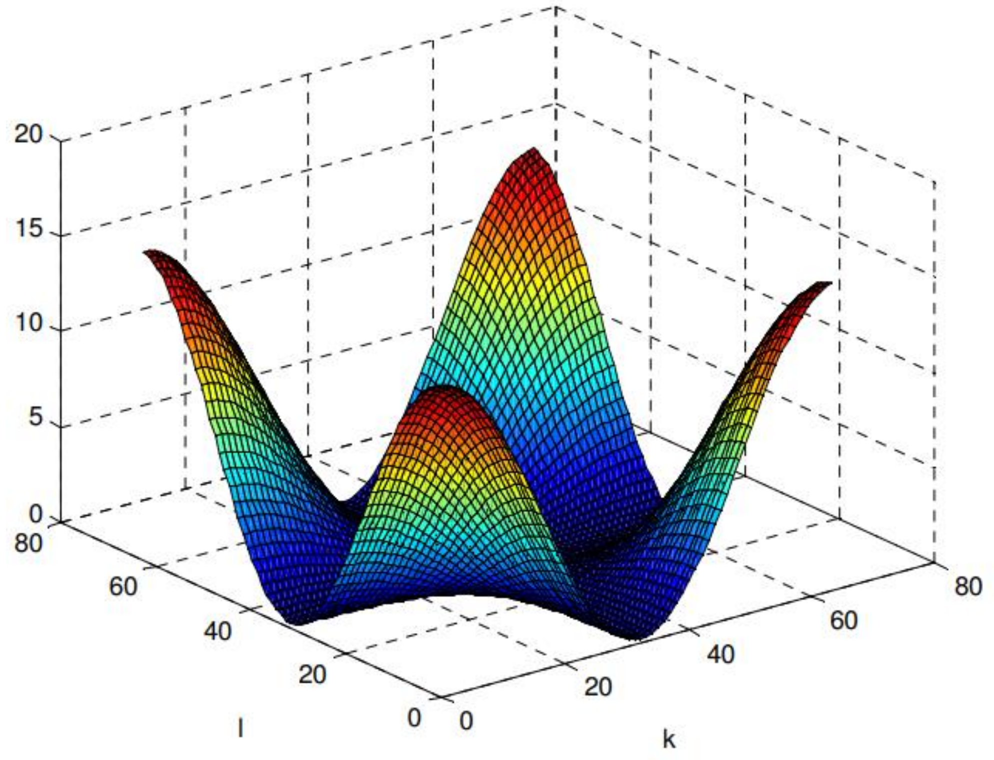
- With $k = 0,1,2, \dots, K-1$ & $l = 0,1,2, \dots, L-1$ for 2 dimensions.

EXERCISE

- We have:

$$\{g_n\} = \frac{1}{4}\{-1\,, \qquad 0, \qquad 2, \qquad 0, \qquad -1\}$$

VIEWING IN FREQUENCY EXAMPLE



GAUSSIAN FILTER

QUALITY VS COMPLEXITY?

QUALITY VS COMPLEXITY?

IS THERE A BETTER FILTER FOR SALT AND PEPPER NOISE?

GAUSSIAN FILTER

- By definition

$$h_{\sigma^2}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

HOW IS THE GAUSSIAN FILTER BUILT?

- An approximation is used to determine the size of the kernel in 2D

$$w \geq 3c$$

- We look for radial symmetry
- Where

$$c = 2\sqrt{2\sigma^2}$$

EXAMPLE

- If we want a Gaussian filter with variance of ¼

$$c = 2\sqrt{\frac{2}{4}} \; then \; w \approx 3$$

Then…

- If we want a Gaussian filter with variance of ¼

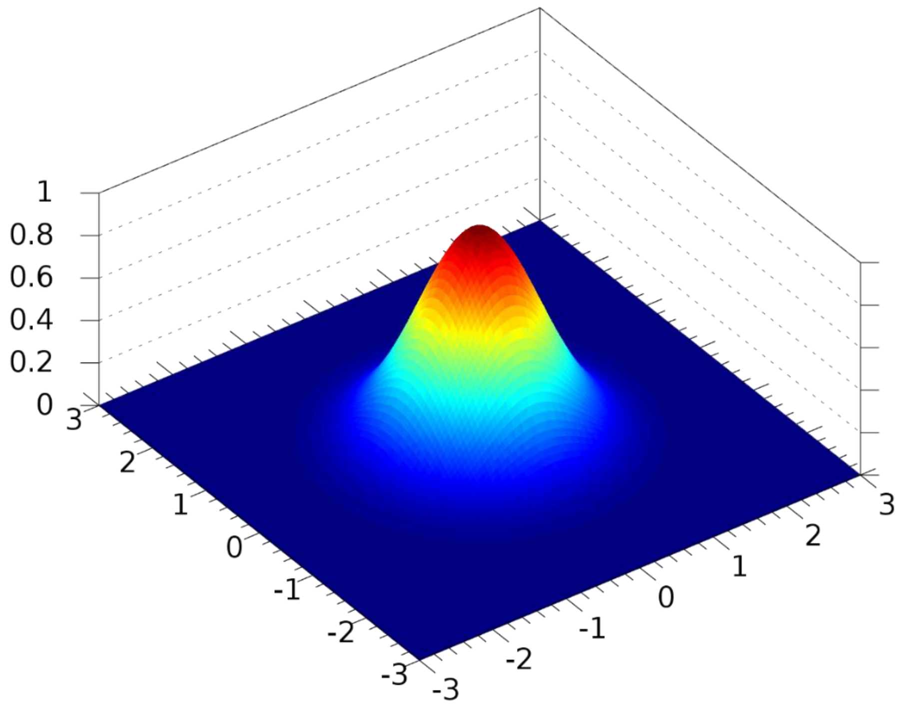
$$c = 2\sqrt{\frac{2}{4}} \ then \ w \approx 3$$

Then…

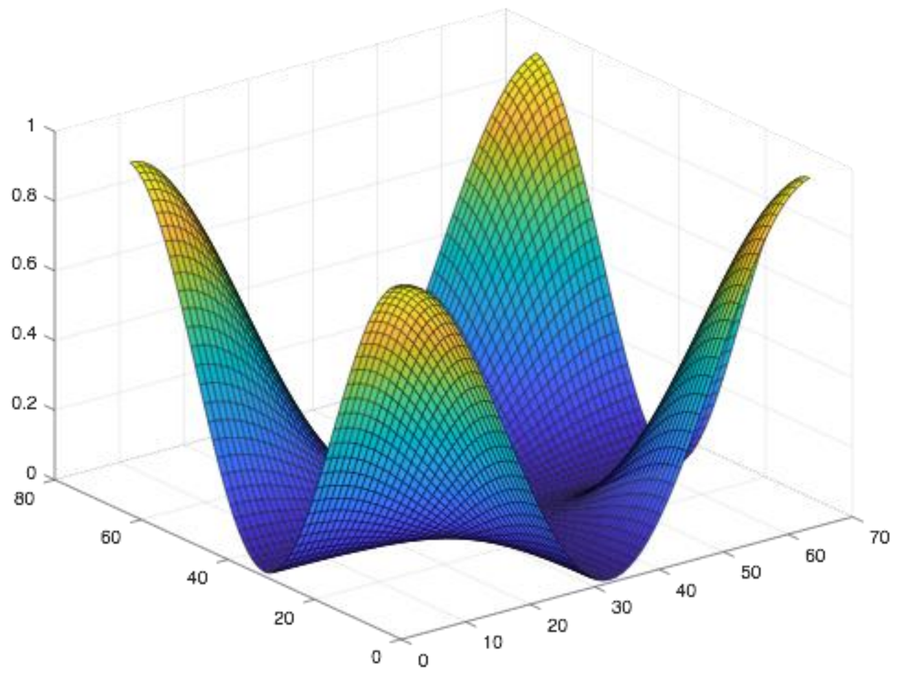
- h_{σ^2}

$$= \frac{\pi}{2}\begin{pmatrix} e^{-4} & e^{-2} & e^{-4} \\ e^{-2} & 1 & e^{-2} \\ e^{-4} & e^{-2} & e^{-4} \end{pmatrix}$$

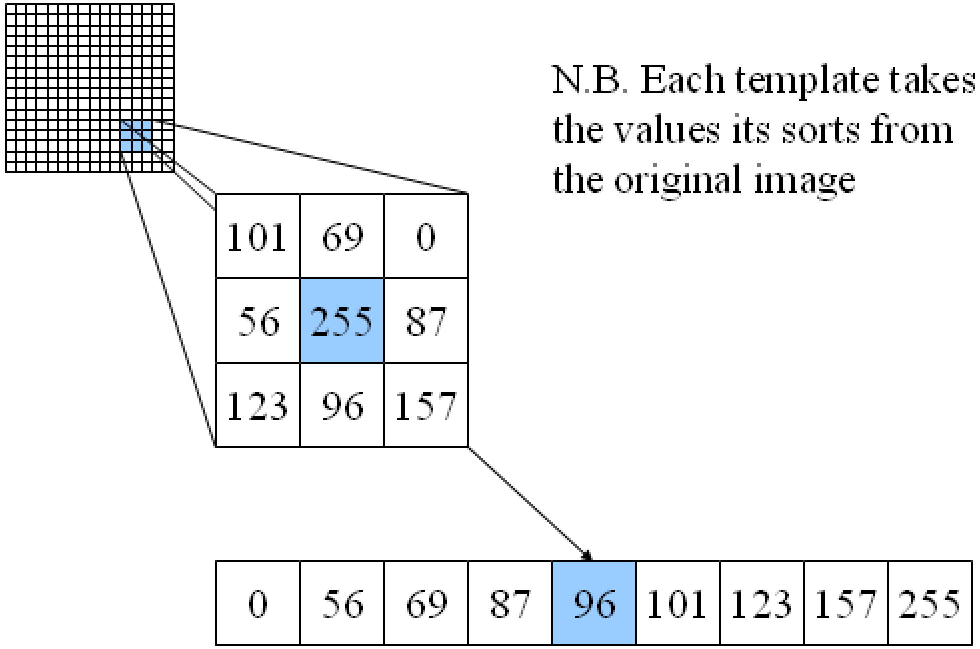
GAUSSIAN FILTER IN TIME



GAUSSIAN FILTER IN FREQUENCY

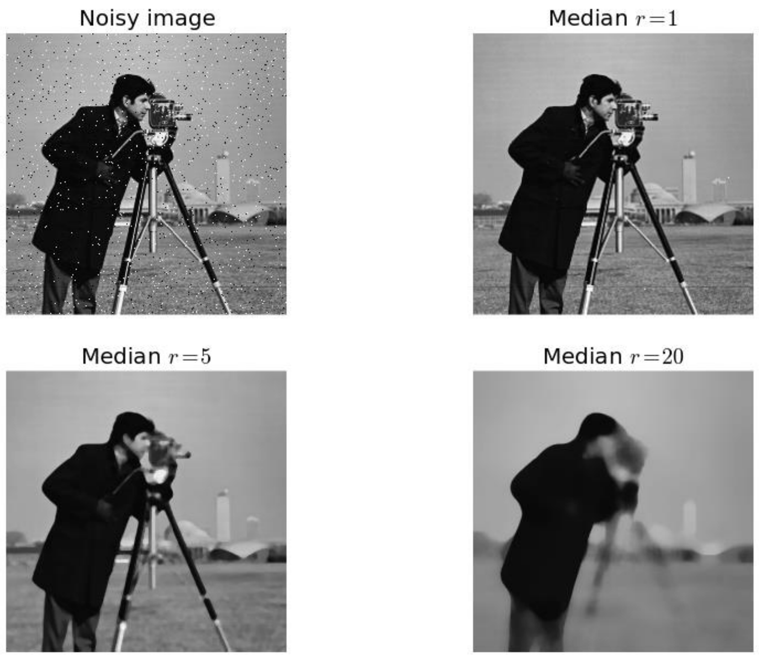


MEDIAN FILTER

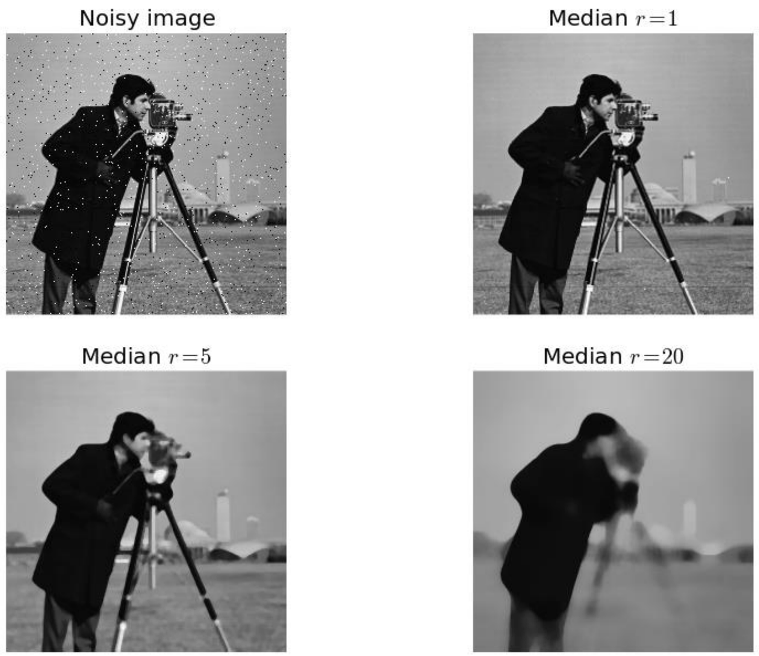


N.B. Each template takes the values its sorts from the original image

MEDIAN FILTER



MEDIAN FILTER, COMPLEXITY?



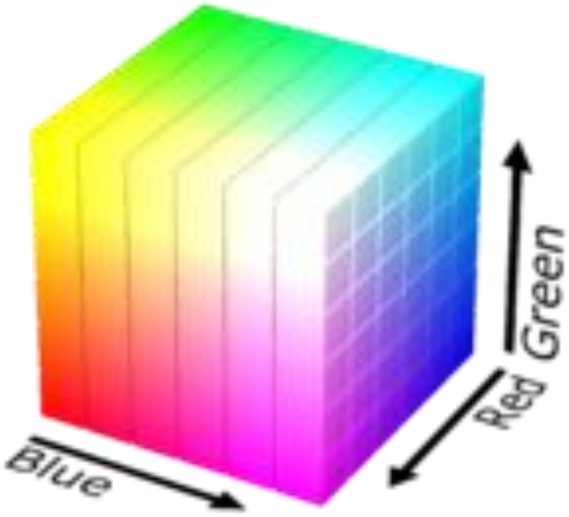
REMEMBERING GAUSSIAN FILTER

QUALITY VS COMPLEXITY?

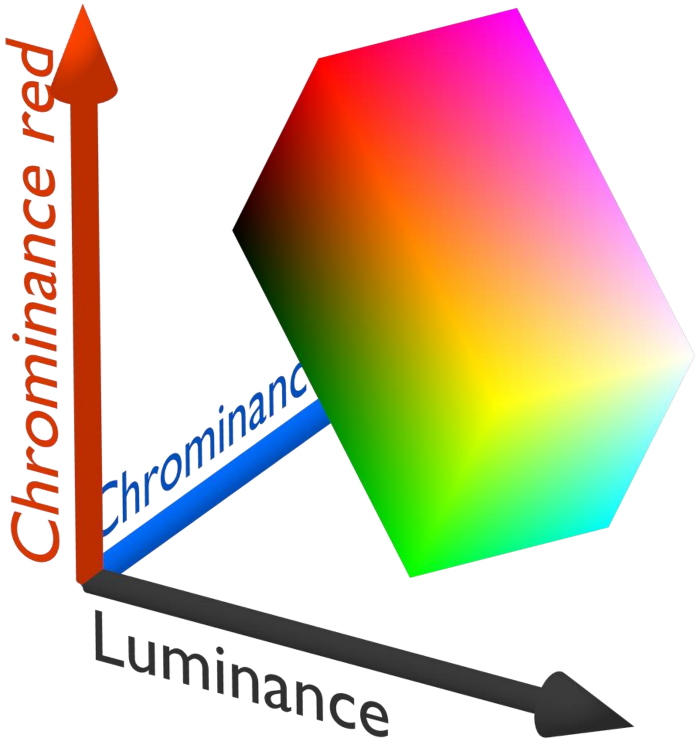
COLOUR MODELS

- RGB
- YCbCr
- HSV
- HSL

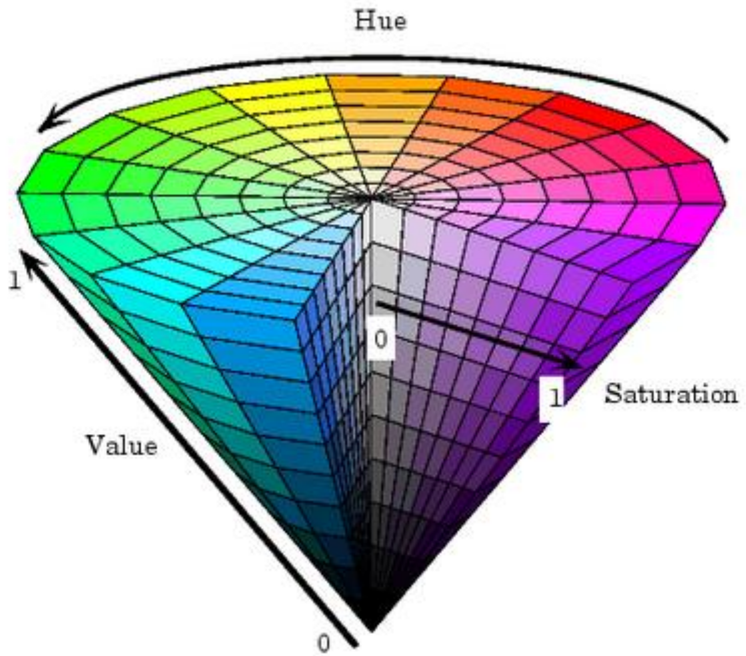
RGB



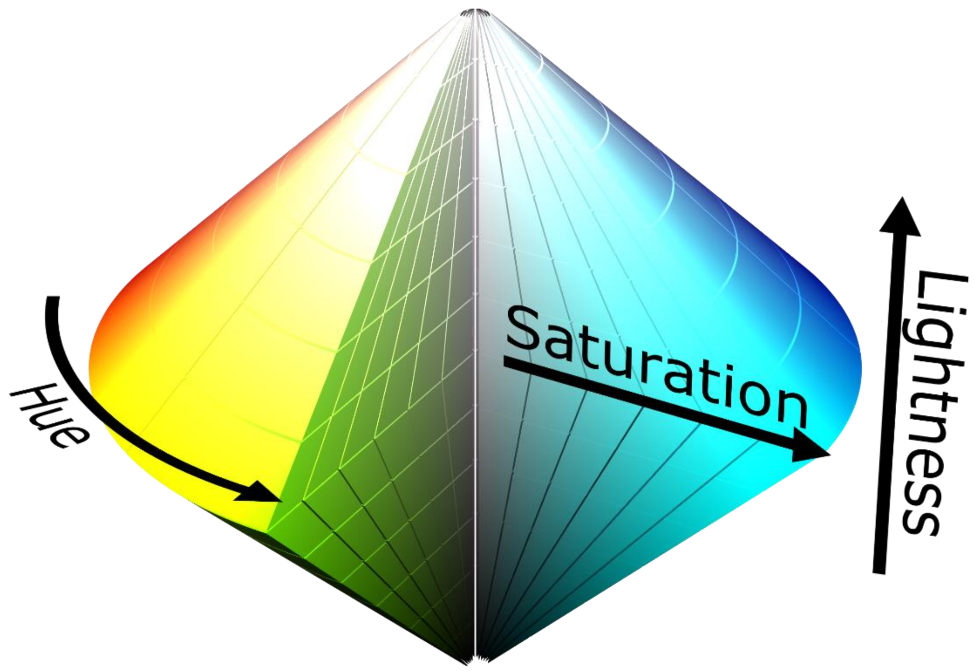
YCBCR



HSV



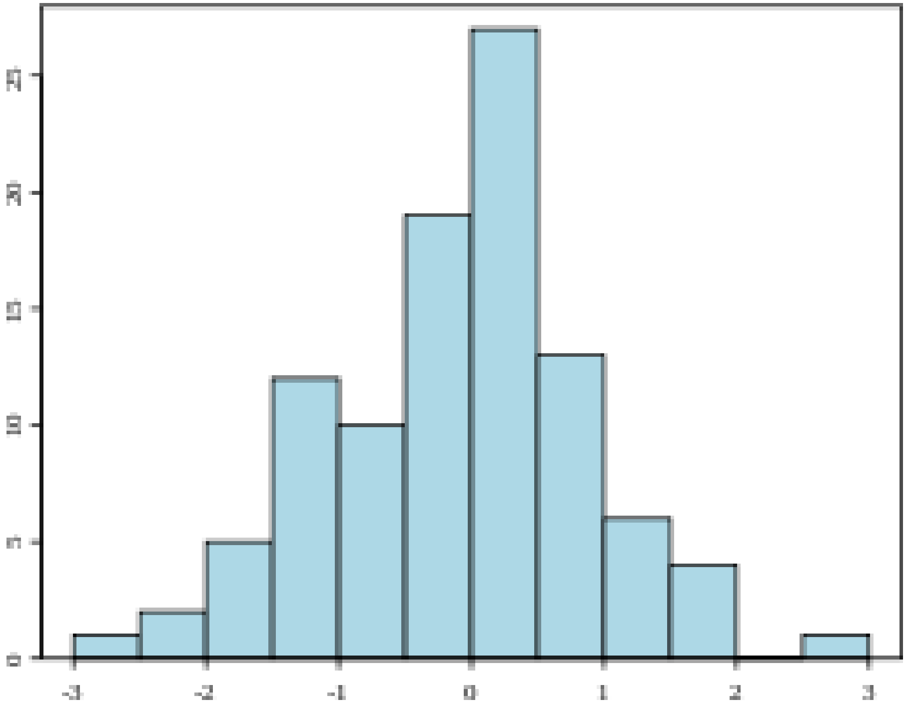
HSL



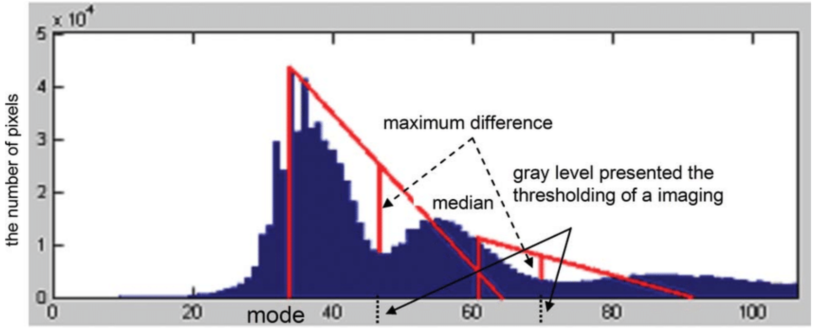
HISTOGRAM

- In statistics, a histogram is a graphical representation of a variable in the form of bars, where the surface of each bar is proportional to the frequency of the represented values. They serve to obtain a general "first view", or panorama, of the distribution of the population, or of the sample, with respect to a characteristic, quantitative and continuous (such as length or weight).

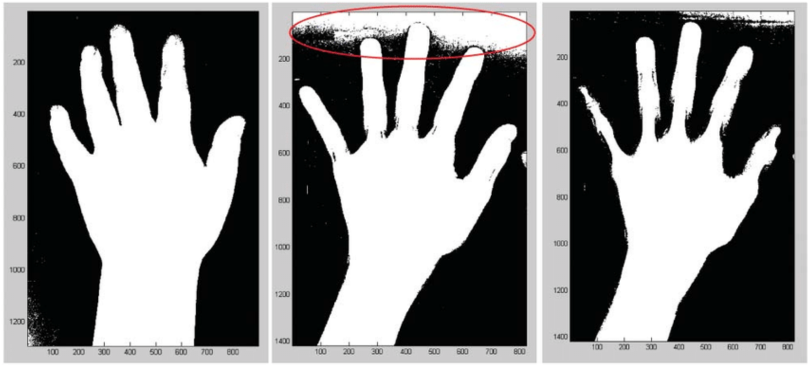
HISTOGRAM



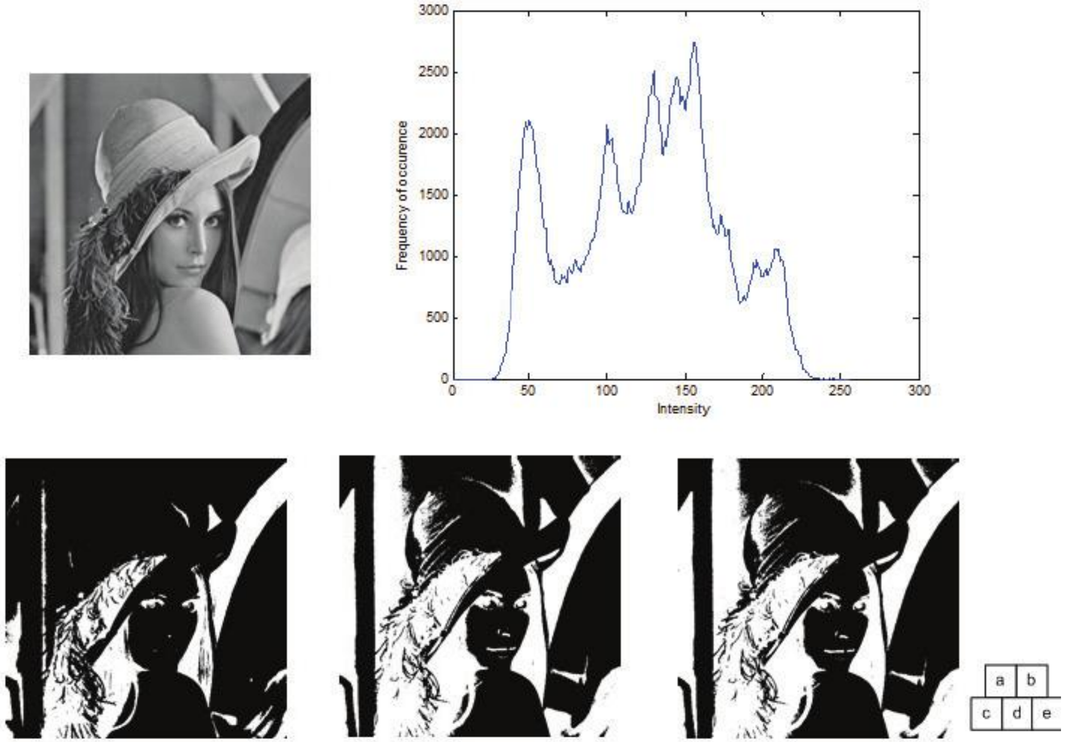
HISTOGRAM



(a)


(b) (c) (d)

BINARIZATION BY THRESHOLD



PRACTICE 1.3

- Deliver the results of the following to a .pdf file:
  - Calculate the convolution mask for a Gaussian Variance Filter "1".
  - Display the Gaussian filter convolution mask in the frequency spectrum.
  - Make a script that receives images, video, or streaming from a camera and do the following:
    - Have an option to display the histogram (grayscale and/or "n" layers in a color model).
    - Get the binary representation of the input data.
      - The binarization can be grayscale or some color model.
      - You can perform binarization by threshold.
      - Otsu model.
      - etc.

Questions?