Assessment on the Three Options

The Kariba Dam stands 128 metres tall and 579 metres long in the Kariba Gorge of the Zambezi river basin between Zambia and Zimbabwe. It forms Lake Kariba which extends for 280 kilometres and holds 185 cubic kilometres of water. Opened in 1959, the Kariba Dam has been heavily eroded by torrents for years and now bears precarious foundations. It’s warned that without urgent repairs the whole dam will collapse. If that happened, catactrophic flooding would cause 3.5 million life losses as well as knock out 40% of southern Africa's hydroelectric capacity, seasonal drought would arouse devastating reduction, and electricity deficiency on a large scale would become everlasting[1]. Therefore, efficient measures have to be taken under this critical situation.

To estimate the three options of maintenance of the Kariba Dam, we’d like to take both the costs and benefit into account[2]. The specific factors we consider are listed as follows:

1. Costs:

- Engineering cost - dam construction cost or dam repair cost;
- Security risks - flood damage and casualties;
- Environment disruption - the ecological balance breakage and water quality deterioration.

2. Benefits:

- Water transfer - flood control, drought resisting;
- Energy production - electricity production;
- Others - irrigation, tourism development and job creation.

At present, security level of the Kariba dam is extremely low. Since the factor of safety is the main cause of casualties, we respectively have the qualitative analysis for three different options.

For Option 1, to repair the dam has the following effects:

- Benefit can be basically maintained in terms of the three aspects we consider;
- As for costs, repair is undoubtedly cheaper in comparison with the other two options;
- Although the service life of the Kaliba dam will be increased and its safety level will be partially improved, there is still a great potential risk. In case of flood, the possibility and harm of the dam will be great.

For Option 2, to build the dam has the following effects:

- Benefit can be basically maintained in terms of the three aspects we consider;
- As for costs, reconstruction ranks moderate among all three options;
- The service life and the safety level of the Kaliba dam will be increased relatively high, but there still exists potential risks since the flood control capacity is not

enough.

For Option 3, to build 10 to 20 smaller dams in the substitute of the Kaliba Dam has the following effects:

- All three aspects of the benefits are significantly improved;
- As for costs, construction of a new multiple dam system is the most expensive;
- The new dam system has the highest safety level. Thus the dam system can reduce flood damages by method of multi-level regulation.

Based on the above analysis of the three options, we visualize our assessment in the following table:

Table 1 Analysis Table

	Costs			Benefits		
	Engineering Cost	Security Risks	Environment Disruption	Water Transfer	Energy Production	Others
Option 1	low	high	moderate	low	low	moderate
Option 2	moderate	moderate	moderate	low	low	moderate
Option 3	high	low	low	high	high	high

Due to the lack of weight factors, we could not make an arbitrary decision on which option to adopt.

From both the analysis and the table presented above, we may infer that:

- When funds are sufficient, we can consider Option 3 for a maximum of benefits and a minimum of security risks.
- When funds are not that sufficient, Option 1 and Option 2 can be considered, which are inferior due to a relative high security risks, much environment disruption, low water transfer and low energy production.

Water Storage Regulation Model Based on POA

Content

1 Introduction

1.1 Background

The Kariba Dam, located in the Kariba Gorge of the Zambezi river basin between Zambia and Zimbabwe, impounds the 185 cubic kilometres Lake Kariba which is the world's largest man-made lake and reservoir by volume, lying nearly 1300 kilometres upstream from the Indian Ocean, along the border between Zambia and Zimbabwe.

It is warned by Institute of Risk Management of South Africa that the dam faces a dire need for repairing. The way to solve the problems of Kariba Dam can be simply divided into three categories: repairing dams, rebuilding dams and removing dams and replacing it with a series of dams. Three options feasible for us to address is quite different for the fact that their impacts on the ecology, economics and other aspects varies.

Generally, The Zambezi River can be separated into three sub-sections according to its different physical and biological characteristics: the Lower Zambezi, the Middle Zambezi, and the Upper Zambezi. The Middle Zambezi, where the Kariba Dam lies, is 600 miles long in low-lying country, and is divided from the Victoria Falls to the Lake Cahora Bassa. An overview of the Zambezi River generated from the Zambezi River Authority[3] is as follows:

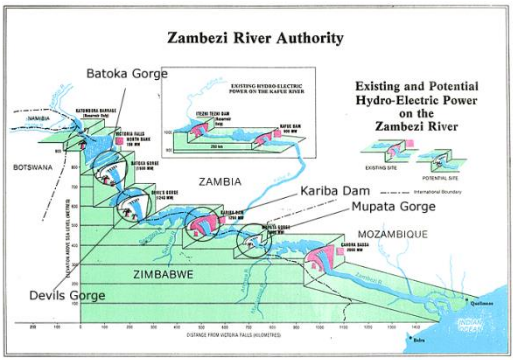


Figure 1.1 An Overview of the Zambezi River Generated from the Zambezi River Authority

1.2 Our Work

We evaluated three possible solutions to the current Kariba Dam problem and discussed in detail the advantages and disadvantages of each option. To remove the existing dam and create a new series of dams, we developed a model to determine the best choice of the number and placement of the new dams along the Zambezi River. Then, a discrete dynamic programming model of series dams is established, which provides detailed guidance for the action ought to be taken to minimize negative influences under different geographical, climatic and environmental conditions.

We generate the elevation map of the whole Kariba River. Combining the map with the potential dam sites ZAR proposed, we select the feasible range of Kariba River for the construction of the dam. Since not all the locations of a river are suitable for the construction of a dam, we first use the topography information to establish a rule to select the feasible dam site, and then we use the established rules to select the feasible address of the dam. At last, we establish a general optimization model for the selection of the optimal combination of dams, and then a local optimal solution using Hybrid PSO Algorithm. On the basis of this, in order to adjust the parallel-dam system to adapt to the dynamic changes of various environments, we establish a discrete dynamic programming model, and use the Progress Optimality Algorithm to relieve the computational complexity of the model.

We simulate the whole calculation process via computer. The number and the placement of dams selected by our site selection model has a high average safety level and competitive total construction costs. On the basis of the dynamic control model, we design the optimization strategy to adapt to the extreme conditions and local adverse conditions, and the algorithm we use can quickly converge to a local optimal solution that is good enough for scheduling. In order to accelerate the convergence rate of the algorithm, we discretize the height of the dam according to a certain interval to make it an integer variable. Thus, we choose the discrete height interval of the dam for sensitivity analysis. And the results show that our model is very robust. Also, in the sensitivity analysis of the risk assessment method, we find the possible problems and point out the corresponding solutions.

1.3 Symbol Description

We represent the symbols employed in this paper as follows:

Table 1.1 Notations

Symbol	Meaning
$(SR_n)_i$	Risk score of the n th factor in the i th dam construction
m	Number of risk factors in dam construction
l_R	distance between two adjacent selected dams
x_i	distance between two adjacent potential dams
H_R^{water}	height of water stored in the Rth reservoir
λ	scale factor of H_R^{water} and H_R^{dam}
V_R	volume of the Rth selected dam
B_R	base length of the Rth selected dam
n	number of dams connected in series
$S_i(t)$	amount of water stored in the i th dam before the regulation
$E_i(t)$	0-1 variable that determines whether a significant water flow apart fom the upstream exists in the i th dam
IN_i	total inflow increment between the $(i-1)$ and the i th dam
φ_i	the emergency parameter of the i th dam

1.4 General Assumptions

- When selecting the dam construction sites, the height of the water surface is assumed to be proportional to the height of the dam under safe conditions.
- The cross-sectional shapes of dams and rivers are assumed to be isosceles trapezoidal.
- Water storage capacity of the dam is proportional to the spacing between two adjacent dams and the width of the river, irrespective of river spillage.
- Along the river flow, the precipitation per unit length of river is the same as that required for drought.
- When using the dam system to control the water storage capacity, the dynamic changes in the storage volume is not taken into account. That is to consider only the initial and final storage volume.

2 Dam Sites Selection

The option 3 require us to remove the Kariba Dam and substitute it with ten to twenty smaller dams. Water in the Lake Kariba should also be shunted to avoid catastrophic impact. As the Zambezi River flows through a series of countries with ever-changing environment, not all locations are suitable for dam sites. In this Section, we will represent the process of our determination of alternative dam sites.

2.1 Factors Affecting Dam Site Selection

We take into account the factors affecting selection of dam sites[4] as follows:

- Topography
- Geology
- Geography
- Foundation Conditions
- Earthquake zone

In consideration of topography, dams can be constructed in narrow U-shaped valleys, low plain countries or narrow V-shaped valleys which are out of the seismic zone. Since the geology conditions in the neighborhood of the Kariba Dam do not vary significantly, we can ignore its effect on our selection. We will discuss in details the geographical conditions of alternative dam sites and the foundation conditions of dams in the following sections.

2.1.1 Geographical Conditions of Alternative Dam Sites

As for Geographical Conditions, we confine our alternative dam sites within the Middle Zambezi. Due to the fact that dams cannot be constructed in neither the Lake Kariba nor the Lake Cahora Bassa, we further divide their addresses into the following two categories:

- Upstream of Lake Kariba - from the Victoria Falls to the Lake Kariba
- Downstream of Lake Kariba - from the Kariba Dam to the Lake Cahora Bassa

For the sake of clarity, we gather the elevation of the Zambezi River in Google Earth and generate a cross sectional sketch map via Mathematica:

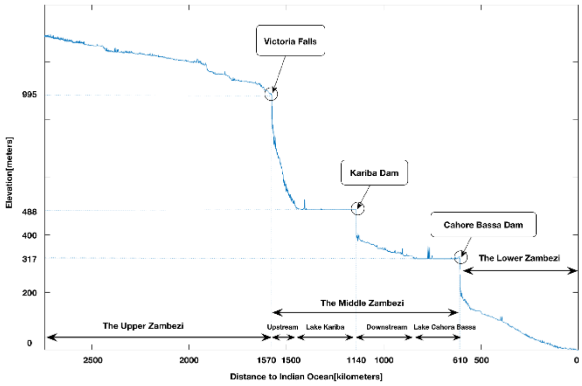


Figure 2.1 Cross Sectional Sketch Map of the Zambezi River

From Fig. 2.1, conclusions can be summarized as follows:

- In the upstream of the Lake Kariba, the terrain provides favorable conditions for the construction of dams and the diversion of water.
- In the downstream, water is diverted due to the closure of a dam at the Lake Cahora Bassa.

Both conclusions perfectly surpport the reliability and feasibility of our selection.

2.1.2 Foundation Conditions of Dams

In accordance with common sense, risk is the primary factor in terms of constructing dams. Therefore, we analyze the foundation conditions of dams mainly on risks by means of risk score method[5].

Ⅰ. Original Risk Score Method

The United States Bureau of Reclamation (USBR) recommends using the site rating method to measure the risk of reservoir dams, which was developed under the inspiration of the US Army Engineer Group, Hagen[6]. Therefore, the formula of our dam risks analysis[7] is:

$$SR_i = \sum_{n=1}^{m} (SR_n)_i \tag{2.1}$$

Where:

SR_i	risk score of the i th dam construction
$(SR_n)_i$	risk score of the n th factor in the i th dam construction
m	number of risk factors in dam construction

The factors of risks we actually take into account contain two categories:

- Potential Dangers - including storage capacity, water head, hidden dangers, floods, earthquakes and so forth.
- Dam Dangers - including the age of the project, construction quality, seepage

situation, structural safety and other factors.

Due to the loss of reliable data, it's not easy to quantitatively describe these factors of risks. We need to simplify our model to avoid consequences of reliable data loss, while maintaining the correctness of our model. The modified formula of our dam risks analysis is to be discussed in the next subsection.

Ⅱ. Modified Risk Score Method

The International Commission on Large Dams (ICOLD) classifies dam risks into potential energy risk, life safety risk, economic risk, environmental risk and social crisis[8]. And all these risks are relevant to the size of each dam. The specific potential hazards classification (HPC) of dams ICOLD put forward is as follows:

Table 2.1 HPC for Dams

Item	Potential Hazards Classification		
	Low	Medium	High
$H^2\sqrt{V}$	<20	$20\sim200$	>200

In Tab. 2.1, H represents the height of each dam, and V represents the volume of each dam. Since V is corresponding to H, we then further simplify that dam risks only relate to the dam height.

As shown in Fig. 2.1, the valley elevation difference among the existing and potential dam construction sites ranges from 20m to 150m. In order to achieve higher accuracy, we divide H into four subintervals as:

$$[20m,30m),[30m,45m),[45m,70m),[70m,150m)$$

We respectively classify the degree of dam risks into low, medium, high and extremely high, four levels in total. Then, we assign each level a risk value (SR) accordingly. Thus the higher the SR is, the more dangerous the project is. The relationship between H and SR is as follows:

$$\begin{aligned}
&20\le H_i<30 \quad, SR=1\\
&30\le H_i<45 \quad, SR=2\\
&45\le H_i<70 \quad, SR=3\\
&70\le H_i<150 \quad, SR=4
\end{aligned} \tag{2.2}$$

2.2 Determination of Alternative Dam Sites

Due to the uncertainties in dam sites selection, fieldwork is of great significance, but we obviously could not realize it. In order to render our determination of alternative dam sites more feasible, we also take the ZAR's original dam construction plan into consideration, which plays a guiding role.

Based on the analysis above, the starting point of our selection lies in the Victoria Falls (15°35'12.88"S, 30°24'25.85"E), and the end point lies in Mpazangwe (17°57'04.46"S, 25°51'59.24"E).

To figure out the number and specific location of alternative dam sites, we take a series of measures as the flow chart shows below:

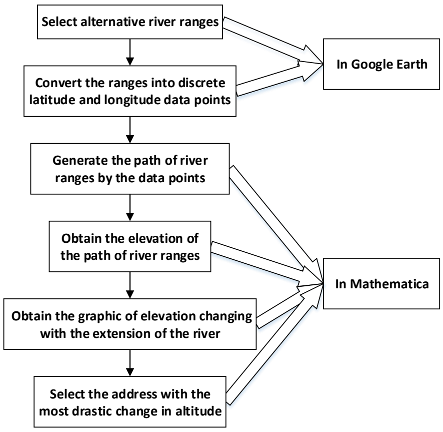


Figure 2.2 Flow Chart of Determination of Alternative Dam Sites

2.2.1 Alternative River Ranges Selection

After selecting alternative river ranges and converting them into discrete latitude and longitude data points in Google Earth, we then track the available reach of the Zambezi River for dam constructions via Mathematica as follows:

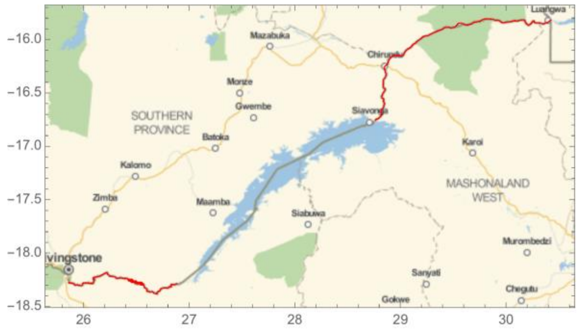


Figure 2.3 Available Reaches of the Zambezi River for Dam Constructions

As shown in Fig. 2-3, the available reaches of the Zambezi River for dam constructions are marked with red lines.

2.2.2 Alternative Dam Sites Selection

After generating the path of river ranges by the latitude and longitude data points along with obtaining the elevation of the path of river ranges, we acquire the graphics of elevation changing with the extension of the river in the alternative reaches:

With factors of topography, geography, and earthquake zone being fixed, the more drastic the elevation of the river is, the more suitable the location to be as a dam construction site. Based on the previous analysis, we select 30 alternative dam construction sites along the available reaches of the Zambezi River. And the relative data are shown in the appendices.

As shown in Appendix I, our alternative dam construction sites are all of the most drastic change in altitude within their respective neighborhood, which means these sites are the local optimum selections. We can therefore conclude that our alternative dam construction sites are reasonable and realistic.

We further map our alternative dam construction sites out via Mathematica as follows:

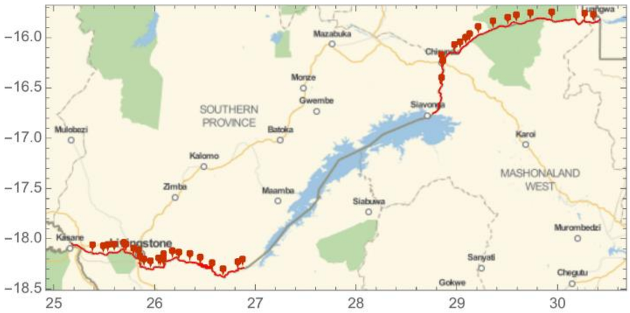


Figure 2.4 Alternative Dam Construction Sites

We can conclude from Fig. 2.4 that our alternative dam construction sites distribute rather evenly along the alternative river ranges, which conforms to common sense. This phenomenon indicates the reliability and feasibility of our alternative dam construction sites.

The results are shown in Appendix 1.

2.3 Determination of Finalized Dam Sites

2.3.1 Cost - Risk Bi-Objective Optimization Model

Based on the previous analysis, we depict the cross sectional sketch map of our alternative dam sites in Fig. 2.5:

As shown in Fig. 2.5, we can obtain the inequality below:

$$\sum_{R=1}^{n} l_R \leq x_{total} \tag{2.3}$$

Where:

l_R distance between two adjacent selected dams, and conforms to the following recursive formula:

$$l_R = \sum_{1}^{m} x_i - \sum_{1}^{R-1} l_R \quad (\sum_{1}^{m} place_m = R, R = 1, 2, \cdots, j) \tag{2.4}$$

x_{total} sum of the distances between two adjacent potential dams

n number of the selected dams

Due to the fact that dams cannot be constructed in the Lake Kariba nor the Lake Cahora Bassa, we then have the following restriction:

$$\sum_{1}^{i} x_i \leq 1423 \text{ or } \sum_{1}^{i} x_i \geq 1577 \text{ (i=1,2,}\cdots\text{,n)} \tag{2.5}$$

Where:

x_i distance between two adjacent potential dams

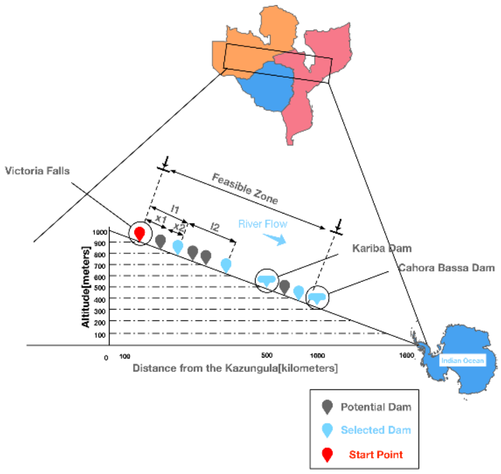


Figure 2.5 Cross Sectional Sketch Map of Alternative Dam Sites

For the sake of safety, the height of water stored in a reservoir is restricted to a certain value smaller than the height of a dam. Thus we simplify the relationship of the two parameters into a linear one:

$$H_R^{water} = \lambda \cdot H_R^{dam} \tag{2.6}$$

Where:

H_R^{water} height of water stored in the Rth reservoir

H_R^{dam} height of the Rth dam

λ scale factor of H_R^{water} and H_R^{dam}

Here λ is set up[5] as $\lambda = 0.75$, which will be further discussed latter.

To reduce computation, we further simplify the shape of dams to similar trapezoids, of which the waist inclination remains a constant. In accordance to the reality, the altitude variation between two adjacent dams is relatively small. We can therefore approximate l_R as the length of one reservoir. Thus, the volume of the reservoir in each selected dam can be calculated as follows:

$$V_R = l_R \cdot (B_R + \frac{H_R^{water}}{\tan\theta}) \cdot H_R^{water} \tag{2.7}$$

Where:

V_R volume of the Rth selected dam

B_R base length of the Rth selected dam

θ waist inclination of each trapezoidal dam, set up[5] as $\theta \approx 45^\circ$

Ⅰ. Derivation of the Cost of a Dam

Because the construction cost is mainly related to the amount of material, namely the volume of the dam, we suggest the following relationship:

$$\mathrm{C}_i = k \times B_R^{\alpha} (H_R^{dam})^{\beta} \tag{2.8}$$

Where:

C_i construction cost of the ith alternative dam site

α, β, k coefficients to be determined

Table 2.2 Relative Data of Arch Dams

Name	h/m	l/m	Real Cost /million dollar	Anticipated Cost /million dollar	Percentage of Error
Hoover Dam	221.4	379	700	723.973	-3.425%
Deriner Dam	249	720	2000	2004.52	-0.226%
Glen Canyon	220	480	997	967.34	2.975%
Kariba Dam	128	579	480	484.693	-0.978%

Table 2.2 is generated from Wikipedia[9].

Using the data in Tab. 2.2 and the least squares method, we calculate the value of each parameter as:

$$\alpha = 1.27274, \beta = 1.71658, k = 0.0000356525$$

So we obtain the final cost formula as follow:

$$\mathrm{C}_i = 0.0000356525 \times B_R^{1.27274} \times (H_R^{dam})^{1.71658} \tag{2.9}$$

Using these coefficients to verify the original data, we can find that the relative error is within 5% for the four groups of dams, and it can calculate the true price of the dam in a certain extent.

Ⅱ. Cost - Risk Bi-Objective Optimization Model

Therefore, we put forward our cost - risk bi-objective optimization model:

$$\begin{aligned} &\min \quad \sum_{i=1}^{30} place_i \cdot C_i \\ &\min \quad SR = \sum_{i=1}^{30} place_i \cdot SR_i \\ &\text{st.} \quad \begin{cases} l_R \geq l_{\min} \\ \sum_{1}^{R} V_R \geq V_{total} \end{cases} \end{aligned} \tag{2.10}$$

Where:

$place_i$ 0-1 variable that determines whether the ith alternative dam site will be taken, and conforms to the following equation:

$$\sum_{i=1}^{30} place_i = j(place_i = 0 \ or \ 1; j = 10, 11, 12, \cdots, 20) \tag{2.11}$$

$l_{\min}$ minimum distance between two adjacent selected dams for the sake of safety

R number of the selected dams ($10 \leq R \leq 20$)

V_{total} volume of the Lake Kariba ($V_{total} = 180.6 \times 10^9 m^3$)

2.3.2 Hybrid PSO Algorithm

As for the implementation of our cost - risk bi-objective optimization model, we apply the hybrid PSO algorithm[10].

The flow chart of this algorithm is as follows:

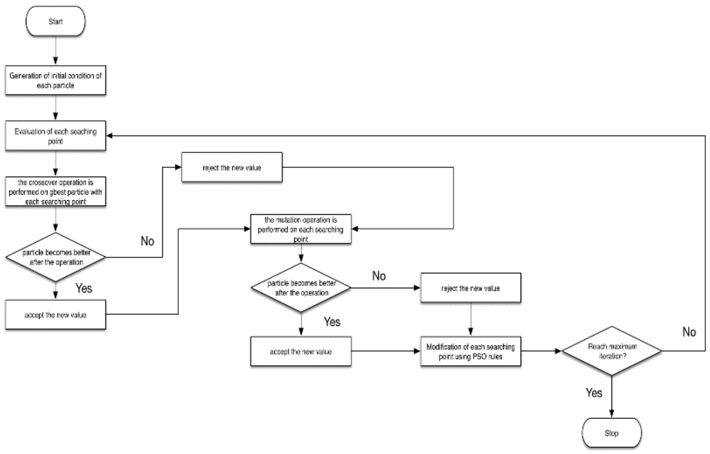
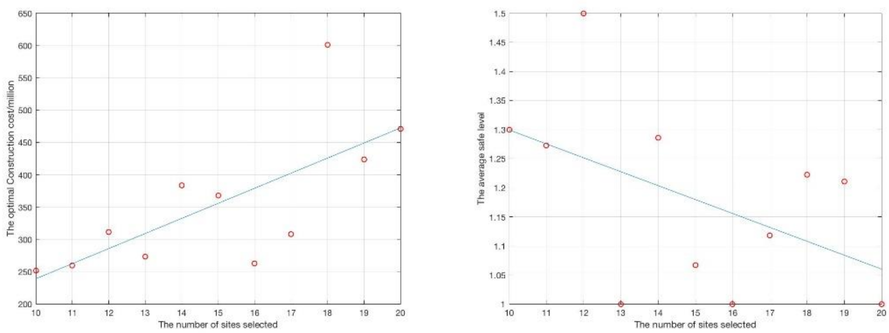


Figure 2.6 Flow Chart of the Hybrid PSO Algorithm

We function this algorithm in Matlab to get the optimum solution of our model.

2.3.3 Implementation in Computer

First, we set the average safety level objective as a constraint. Then, we discretize the height of the dam and set the interval as a constant number to speed up the convergence of the algorithm. When the number of sites varies from 10-20, we can get the corresponding value of the minimum cost and the average safety level using the hybrid PSO algorithm as follows:



(a) (b)

Figure 2-7 (a) Optimal Construction Cost Changing with the Number of Sites Selected

(b) Average Safe Level Changing with the Number of Sites Selected

It can be seen from Fig. 2-7 (a) that, in general, with the increase of the select quantity, the minimum dam construction cost increases continuously. Besides, there is a sudden decline in the minimum cost of the dam and then a sudden increase. This is caused by all impacts in every respect arising from the select quantity.

From Fig. 2-7 (b), the average safe level of the overall multi-dam system continues to decline as the select quantity increases, contrary to the steadily increasing construction costs. This phenomenon means that the safe level and the construction cost conflict each other to a certain degree. Therefore, we have to accept a relatively high construction cost to get desired security.

2.3.4 Normalization Processing

In this problem, we have two objective functions where we want to get a smaller construction cost along with a higher average security level. We then have to make a trade-off between the two aspects. As their dimensions are inconsistent, we need to normalize the original data instead of simply adding the two parameters. Here we apply Min-Max Normalization, which can be formulated as $x^* = (x - min) / (max - min)$, and obtain the normalized data as follows:

Table 2.3 Data after Min-Max Normalization

Select Quantities	Optimal Construction Cost/10^6\$	Normalized	Average Safe Level	Normalized	Sum of the Normalized
10	251.7307	0	1.3000	0.6000	0.6000
11	259.5766	0.0225	1.2727	0.5454	0.5679
12	311.5482	0.1712	1.5000	1.0000	1.1712
13	273.4285	0.0621	1.0000	0	0.0621
14	383.3909	0.3769	1.2857	0.5714	0.9483
15	367.6834	0.3320	1.0667	0.1334	0.4654
16	262.7261	0.0315	1.0000	0	0.0315
17	307.8411	0.1606	1.1176	0.2352	0.3958
18	601.0350	1.0000	1.2222	0.4444	1.4444
19	423.6269	0.4921	1.2105	0.4210	0.9131
20	470.7284	0.6270	1.0000	0	0.6270

We can conclude from Table 2.3 that the select quantities corresponding to the smallest normalization result is 16. Namely, taking both security and cost into consideration, the optimal choice is to actually construct 16 d am sites as below:

Figure 2.8 16 Selected Dam Construction Sites

2.4 Results Analysis and Processing

We use Google Earth to further process the elevation and topography of which points we actually calculate out. By the use of analytic hierarchy process, we can obtain the safety level indicator on the actual construction dam sites.

The results are shown in Appendix 2.

3. Regulation Strategy on Water Flow

When given a system of dams in series-parallel connection, the sites of the dams and their corresponding storage capacity, installed capacity, along with local precipitation can be quantified. In order to cope with emergencies (such as floods and minimum water flow), to ensure the overall safety of the system, to protect water resources and to derive benefits from it, we therefore focus on the scheduling of water resources between dams. The scheduling program will be different in different scenarios (e.g. Water cycle in different periods of different reservoirs, etc.). In order to reasonably cope with different situations, we put forward our discrete dynamic programming model.

3.1 Discrete Dynamic Programming Model

3.1.1 Model Establishment

Ⅰ. Determination of the Constraints

Since the existing dams on the Zambezi River are simply in series, we simplify the dam system as shown in Fig. 3.1.

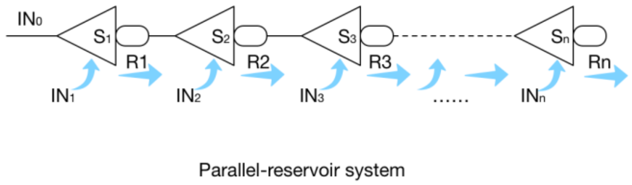


Figure 3.1 Parallel Reservoir System

In unit time, the amount of water in each reservoir is equal to the difference of two sums. Sum as the minuend consists of the original amount of water, the natural increment of the reservoir (including the precipitation) and the upstream reservoir drainage (including power generation and flood discharge). Sum as the subtrahend consists of water for other uses, power generation water and flood discharge.

For the tandem dam system, we have the following difference equation:

$$\begin{cases} S_1(t+1) = S_1(t) - R_i(t) + E_i(t) \times IN_i + IN_0 \\ S_i(t+1) = S_i(t) - R_i(t) + E_i(t) \times IN_i \end{cases}, i = 2, 3, ..., n \tag{3.1}$$

Where:

n number of dams connected in series

$S_i(t)$ amount of water stored in the i th dam before the regulation

$S_i(t+1)$ amount of water stored in the i th dam after the regulation

$R_i(t)$ total amount of water flow from the i th dam to the $(i+1)$ th dam

$E_i(t)$ 0-1 variable that determines whether a significant water flow apart fom the upstream exists in the i th dam. Its distribution is as follows:

$$E_i(t) = \begin{cases} 1 & flooding \\ 0 & normal \\ -1 & dry \end{cases} \quad (i = 1, 2, \cdots, n)$$

IN_i total inflow increment between the $(i-1)$ and the i th dam (considering rainfall, evaporation and other factors), except for the outflow of the $(i-1)$ th dam

In the tandem dam system, the safety factor and construction cost varies among different dams. There is a need to evaluate their overall impact on the system. In the case of normal dam water storage, the water level is proportional to the height of the dam:

$$H_i^{water} = \lambda \cdot H_i^{dam} \tag{3.2}$$

There are four different states corresponding to the value of λ - water shortage state, normal state (in a certain range), slight alert status (slightly exceeding the normal level) and emergent alert status (when an overflow occurs). In the Section 2, we assume problem 1 lies in the normal state. The change in water volume for each dam can be expressed in terms of the emergency parameter φ_i. According to the water level and storage capacity of the Yangtze River Three Gorges Dam during dry season, during normal period, during post-flood period and during post-catastrophic flood, we develop the following relationship[11,12]:

$$\varphi_i(t) = \begin{cases} 1 & 0.42 \le \dfrac{V_i(t+1)}{V_i(t)} \le 1.7 \\ 1.2 & 1.7 \le \dfrac{V_i(t+1)}{V_i(t)} \le 3.2 \\ 1.8 & 3.2 \le \dfrac{V_i(t+1)}{V_i(t)} \le 8.3 \\ 3 & others \end{cases} \tag{3.3}$$

This piecewise function implies the relationship between the emergency parameter and the volumn of the ith reservoir changing with time.

Ⅱ. Determination of the Objective Function

There are two kinds of widely used flood control optimization scheduling rules, one is to maximize the flood control benefit of reservoirs or to minimize the cost, the other is to minimize the peak discharge of reservoir.

The flood control dispatching target of reservoirs can be divided into three types: Flood control safety of hydro junction, upstream and reservoir flood control security, and downstream and reservation flood control security.

Considering the flood control regulation above, we take the highest water level and the lowest water level as the target and corresponding optimization objective function. We then take the minimum of the maximum discharge of reservoirs in dispatching as the optimization objective function of corresponding object. Therefore, we establish the multi-objective flood control optimal operation model[11].

Suppose the time length we are to optimize is *step*.

(1) Reservoir and Upstream Flood Control Safety Objectives

As for flood control in the basin, reservoirs should be kept at low water level. Therefore, we make the minimum of the highest water level as our main optimization goal in order to reduce the inundation loss and the threats to flood control safety. The specific expression is as follows:

$$\min \sum_{t=1}^{step} \sum_{i=1}^{n} \varphi_i(t) \cdot SR \tag{3.4}$$

(2) Flood Control Targets of the Middle and Lower Reaches

We make the minimum of the maximum discharge flow as the optimization target in the protection of downstream flood reserves. The objective function is described as follows:

$$\min \sum_{t=0}^{step-1} \sum_{i=1}^{n} R_i \tag{3.5}$$

Now, we generate the following model:

$$\begin{cases} \min \sum_{t=1}^{step} \sum_{i=1}^{n} \varphi_i(t_i) \cdot SR_i \\ \min \sum_{1}^{step-1} \sum_{1}^{i-1} R_i(t) \end{cases} \tag{3.6}$$

$$s.t. \begin{cases} S_1(t+1) = S_1(t) - R_1(t) + E_1(t) \cdot IN_1 + IN_0 \\ S_i(t+1) = S_i(t) - R_i(t) + R_{i-1}(t) + E_i(t) \cdot IN_i, (i = 2,3,\cdots,n) \\ E_i(t) = -1, 01, (i = 1,2,\cdots,n) \end{cases}$$

3. 1. 2 Simulation and Analysis

Apply the above model into the data of 16 sets of dam addresses that we have obtained before. We can find that the value of $Rain_i$ and In_i differs under different conditions.

We classify the statuses of river based on the characteristics of E_i, and obtain the following two kinds of environments to which we will do the optimization according.

① Status during the floods or prolonged drought

②. Status when there is a sudden bad situation in normal flow

Ⅰ. During Floods or Prolonged Drought

We need to consider the effects of evaporation or precipitation on the water content of each dam in the event of flooding or prolonged drought, thus we assume that the influence spreads all over the river, and the character of the influence is the same. We have:

$$E_i(t) = \begin{cases} 1 & flooding \\ -1 & dry \end{cases} \quad (i = 1,2,\cdots,n) \tag{3.7}$$

Ⅱ. Water Flows Ranging from Maximum Discharges to Minimum Discharges

In the normal situation, we can assume that rains fall just above the Victoria Falls. Change the inflow IN_0 from maximum discharges to minimum discharges, we can then get the value of the objective function changing with it and obtain the regulatory strategies of each case.

Ⅲ. When There is a Sudden Bad Situation in Normal Flow

Effects of precipitation or evaporation distributed in the dam systems should be considered when a sudden bad situation occurs in normal flow. We should also note that the effects of precipitation or evaporation may be distributed anywhere in the basin of the dam system. Thus, we assume that the influence spreads all over the river. Now, the constraints become:

$$s.t.\begin{cases} S_1(t+1) = S_1(t) - R_1(t) + E_1(t)\cdot IN_1 + IN_0 \\ S_i(t+1) = S_i(t) - R_i(t) + R_{i-1}(t) + E_i(t)\cdot IN_i (i = 2,3,\cdots,n) \\ E_i(t) = -1,0,1,(i = 1,2,\cdots,n) \end{cases}$$

And the objective function is also the same.

3.1.3 Progressive Optimality Algorithm

The optimal scheduling model of reservoir group defined by Eq.3.6 is a complex multi-objective nonlinear optimization model, and its complexity is exponential with the number of reservoirs[13]. In order to overcome dimension disaster problems, we transfer the complex multi stage decision problem into a series of two-stage problems via POA. We process the optimization of only two stages of the multi-stage decision-making at one time, and take the previous optimization result as the initial optimization condition. Then, we continue to optimize until the convergence requirements is met.

The flow chart of the algorithm is listed as follow. We will use the method described above to give the optimal flow control strategy.

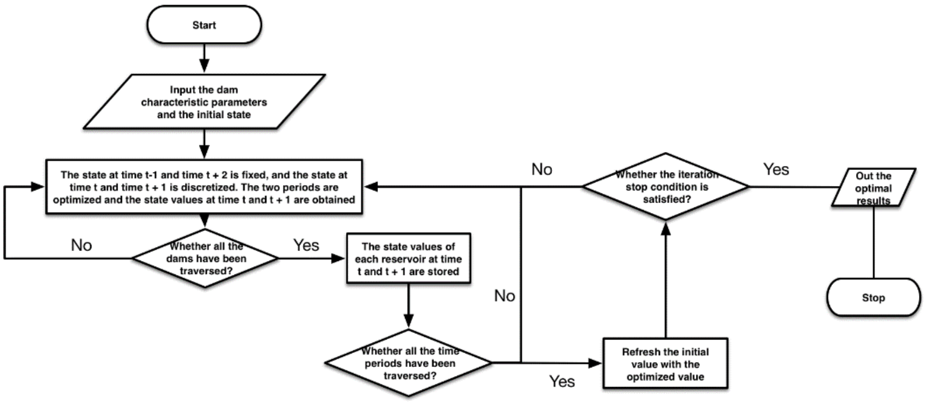


Figure 3.2 Flow Chart of the Algorithm

3.2 Model Test

We use the data of our 16 sites model to test our model, which is in Appendix.2. In addition, we make the number of steps to 5, so that the computational complexity can be controlled. Finally, we picked relative data from[14], transforming it into a data reflecting the changes to meet the requirements of calculation.

- **Test1: During Floods or Prolonged Drought**

Table 3.1[15] includes the initial data we use to test our model. In the case of rainstorms, the rainfall throughout the Zambezi river can be divided into the upstream of the Victoria Falls, which affects IN_0, and the downstream of the Victoria Falls. For the rainfalls downstream of the Victoria Falls, that is, within the range of the dams we built, we can further divide them into partial and overall rainfall. For the overall rainfall, we derive the amount of rainfall per unit length along the stream from all rainfall over the entire Zambezi River and its entire length. Since the distance between each dam is known, we can obtain approximately the value of the rainfall between each dam. Finally, we calculated the total volume of water enclosed in Lake Kariba, which is 60 units(the real value is 180 km^3), then we can obtain approximately the initial stock volume $S_i(t)$, precipitation and drought between each dam period relative to the amount of water.

Table 3.1 Parameters Table

$S_i(t)$	IN_i in a Drought	IN_i in a drastic flooding	$S_i(t)$	IN_i in a Drought	IN_i in a drastic flooding
0.8	0.2	4.2	1.4	0.5	3.2
0.8	0.2	3.5	1.8	0.6	6.6
3.4	1.1	13.7	1.9	0.6	3.8
0.3	0.1	1.1	3.4	1.4	14.8
0.5	0.2	1.8	60.0	15.5	258.0
2.5	1.1	10.9	13.3	3.4	42.5
1.0	0.6	4.3	36.1	8.4	111.5

We assume that $\mathrm{E_i} = 1(\mathrm{i} = 1..\mathrm{n})$ when flooding occurs, and $\mathrm{IN_0} = 4.3$. Then, we could find the optimal value of the decision variables and optimal objective value when flooding occurs, which is:

$$\sum_{t=1}^{5}\sum_{i=1}^{16}\varphi_i(t_i)\cdot SR_i = 421.8 \tag{3.8}$$

The results are shown in Appendix 3.

We assume that $\mathrm{E_i} = 1(\mathrm{i} = 1..\mathrm{n})$ when drought occurs, and $\mathrm{IN_0} = 4.3$. Also, we could find the optimal value of the decision variables and optimal objective value when prolonged drought occurs, which is:

$$\sum_{t=1}^{5}\sum_{i=1}^{16}\varphi_i(t_i)\cdot SR_i = 323.6 \tag{3.9}$$

The results are shown in Appendix 4.

- **Test2: Water Flows Ranging from Maximum Discharges to Minimum Discharges**

We assume that $\mathrm{E_i} = 1(\mathrm{i} = 1..\mathrm{n})$ when flooding occurs, and $\mathrm{IN_0} = 4.3$ changes from 0.8 to 3.2. Then, we could find the optimal value of the decision variables and optimal objective value when flooding occurs, which is:

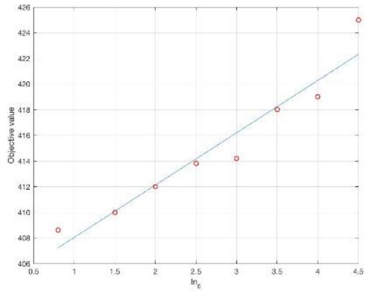


Figure 3.2 Graph of the Optimal Value

The results are shown in Appendix 5.

- **Test3: When There is a Sudden Bad Situation in Normal Flow.**

Using the same initial data above, and we set the $E_i(t)$ $\mathrm{E_i}$as follows:

Table 3.2 Assignment Table

E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}
1	0	0	−1	1	1	1	0	−1	−1	0	1	1	−1	0	1

Then, set $\mathrm{IN}_0 = 2.5$, we could find the optimal value of the decision variables, which is:

$$\sum_{t=1}^{5}\sum_{i=1}^{16}\varphi_i(t_i)\cdot SR_i = 245.2 \tag{3.10}$$

The results are shown in Appendix 6.

4. Sensitivity Analysis

4.1 Sensitivity Analysis of Height of the Dam

In the above discussion, we set that the height of the dam is discretized by 5 meters to simplify the model and speed up the process of searching for the optimal point. Thus, we take the 10 sites as an example. When the discrete interval changes from 2 to 20, the effect on the optimization target is shown in Fig. 4.1.

After the first-order polynomial fitting with Matlab, it can be seen that the model is very robust. After fixing the number of iterations and the number of particles, there is no significant relationship with the optimal value of the obtained objective function and the discrete interval. When the discretization guarantees the convergence rate and the speed of convergence, the discretization has little adverse effect on the convergence of the function value.

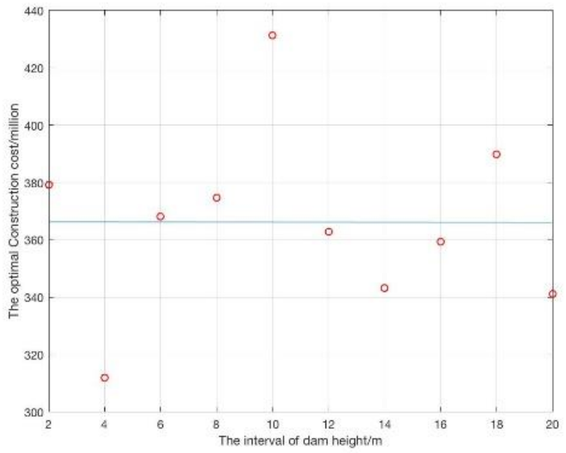


Figure 4.1 Graph of the Optimal Construction Cost

4.2 Sensitivity Analysis of Emergency Parameter

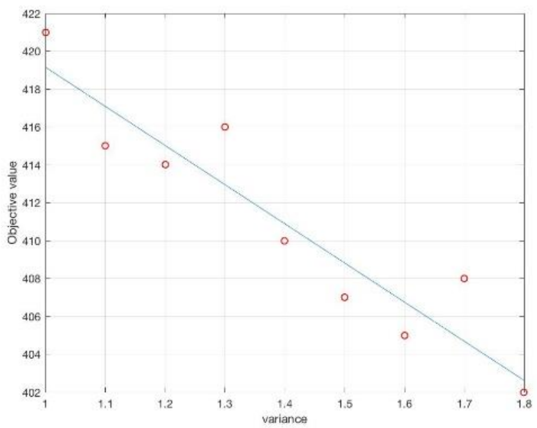


Figure 4.2 Graph of the Objective Value

When we change the boundary between the normal state and the slight alert state from 1 to 1.8 times, we can get the corresponding optimal objective function value in Fig. 4.2. As we can see, with the increase of the threshold, the value of the objective function will decrease , Which is due to the fact that with the rise of the limit, the dam lying in normal state will increase, thus contributing the decrease of the objective function. Thus, The effect of each boundary on the model results is relatively large. We need to refer to as much data as possible to get more accurate model results when determining the state boundaries.

5 .Strengths & Weaknesses

5.1 Strengths

- Our method of site selection meets both the basic requirements and the balance between cost and safety;
- Our model has a reasonable control strategy for rainfall (or prolonged drought) of various kinds, as well as sudden floods and so forth;
- Our control strategy well fits with the relevant data of the Zambezi River, because of which it can give more precise recommendations;
- Our PSO algorithm is easy to understand and the result is relatively reliable. The application of the POA dynamic algorithm in the multi-objective programming makes the results more feasible.

5.2 Weaknesses

- Due to the lack of the elevation of each terrain, we merely approximate the storage capacity of each dam rather than obtaining accurate results.
- Due to the lack of data, we neglect the actual shape of dams and the required installed capacity and construction thickness, and therefore ignore impacts of these factors on the construction costs.

6. Future Work

Improvement

Dynamic Optimization Model for Continuous Rainfall

$$\begin{cases} \min \quad \sum_{t=1}^{step}\sum_{i=1}^{n}\varphi_i(t_i)\cdot SR_i \\ \min \quad \sum_{1}^{step-1}\sum_{1}^{i-1}\left[Q_i^{in}(t)-Q_i^{out}(t)\right]\cdot\Delta t_i \end{cases}$$

$$s.t.\begin{cases} S_i(t+1)=S_i(t)+\left[Q_i^{in}(t)-Q_i^{out}(t)\right]\cdot\Delta t_i+E_i(t)\cdot IN_i(i=1,2,\cdots,n) \\ E_i(t)=-1,01,(i=1,2,\cdots,n) \end{cases}$$

Where

$Q_i^{in}(t)$ the influent flow of the ith dam ;

$Q_i^{out}(t)$ the effluent flow;

Δt_i the time needed for the water travel from the ith dam to the i+1th dam

.

In our improved model, we can adjust the flow rate in real time to control the storage capacity of the dam in different time periods of rainfall when rainfall is a dynamic process so that hazards brought to the upstream and downstream of the dam are minimized. However, due to time limitations, we did not get the specific data needed to apply real-time control.

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Appendices

Appendix 1:

Tab. A1

Index	Latitude	Longitude	Index	Latitude	Longitude
1	-17.95429639	25.86001862	16	-16.25806428	28.85316048
2	-17.98185632	25.89936654	17	-16.09780434	28.86595697
3	-18.00023432	25.96391353	18	-16.03950673	28.8529397
4	-17.9729399	26.04504747	19	-15.94309125	28.98152802
5	-17.98077678	26.06237687	20	-15.91636531	29.03799959
6	-17.9811988	26.09209303	21	-15.87345469	29.09278958
7	-17.93370193	26.09502116	22	-15.83983952	29.13142281
8	-17.90436458	26.18154261	23	-15.77061989	29.21746982
9	-17.91802425	26.24549214	24	-15.71651528	29.36415062
10	-17.93072404	26.35505629	25	-15.68258206	29.51172709
11	-17.96519042	26.45792879	26	-15.65857082	29.59588383
12	-18.01840547	26.57676124	27	-15.63735344	29.73505655
13	-18.07516507	26.68457827	28	-15.63128574	29.94755332
14	-18.00889601	26.83070749	29	-15.64337934	30.27380023
15	-17.98417243	26.87804556	30	-15.65429104	30.36052369

Appendix 2:

Tab. A3

Serial number	H_i^{dam}	V	$H^2\sqrt{V}$	SR
1	20	37647840	1.51E+10	2
2	20	34041360	1.36E+10	2
3	20	1.53E+08	6.13E+10	3
4	20	12139200	4.86E+09	1
5	20	22818040	9.13E+09	1
6	20	1.14E+08	4.57E+10	3
7	20	43390720	1.74E+10	2
8	20	61756960	2.47E+10	2
9	20	80504520	3.22E+10	2
10	20	84650800	3.39E+10	2
11	20	1.53E+08	6.11E+10	3
12	20	2.72E+09	1.09E+12	4
13	20	6.01E+08	2.4E+11	4
14	20	1.63E+09	6.54E+11	4
15	20	2.12E+08	8.47E+10	3
16	20	6.54E+08	2.62E+11	4

Appendix 3:

Tab. A2

$R_i(t)$	t_0	t_1	t_2	t_3	t_4
S_1	0.253678	0.167742	0.066284	0.145165	1.124883
S_2	0.442927	0.194896	4.19723	0.12407	0.323058
S_3	0.046188	1.317897	0.167742	1.317897	13.62386
S_4	0.318879	0.416897	0.094933	0.06131	0.046188
S_5	0.001841	0.1573	0.049074	0.165615	1.839694
S_6	0.715616	0.715616	0.127202	0.416897	12.6853
S_7	0.323973	0.331876	0.413179	0.279307	0.088676
S_8	0.323058	0.282384	0.293696	0.013317	0.238636
S_9	0.164243	0.165615	0.319248	0.645223	6.699142
S_{10}	0.1573	0.1573	0.165615	0.526949	0.243713
S_{11}	1.227361	6.699142	1.124883	9.482194	12.6853
S_{12}	0.473477	4.758026	0.442927	27.18049	0.715616
S_{13}	1.855352	2.226402	1.523666	2.260582	25.93674
S_{14}	9.482194	6.699142	0.243713	0.013317	0.318879
S_{15}	1.622497	2.140284	1.523666	0.318879	12.63908
S_{16}	0.243713	6.699142	2.231884	0.145165	25.93674

Appendix 4:

Tab. A4

$R_i(t)$	t_0	t_1	t_2	t_3	t_4
S_1	13.12034	0.276803	0.193189	9.220603	0.269622
S_2	0.382539	0.079944	0.034448	0.046414	0.028256
S_3	0.60207	0.51241	0.927739	13.12034	0.306764
S_4	0.266758	0.007083	0.125772	0.32847	0.193189
S_5	0.145861	0.044553	0.166775	0.021829	2.640662
S_6	0.648595	6.432381	4.328838	0.293758	1.243197
S_7	0.130584	2.547788	0.306011	0.075162	0.513336
S_8	0.295248	2.547788	0.464098	0.121215	0.13911
S_9	0.034448	3.099231	11.74322	0.360493	2.06955
S_{10}	0.193189	0.079944	0.134453	0.535923	1.062608
S_{11}	0.296078	0.362727	0.911405	0.757652	0.007083
S_{12}	2.939173	0.075162	10.95067	13.28457	6.470555
S_{13}	0.044083	0.142426	2.640662	4.054622	2.640662
S_{14}	0.757652	0.513336	0.696506	0.276803	3.099231
S_{15}	2.06955	0.661643	6.432381	0.480884	1.821187
S_{16}	5.07181	3.513258	1.243761	2.547788	0.513336

Appendix 5:

Tab. A5

IN_0	0.8	1.5	2	2.5	3	3.5	4	4.5
Obj	408.6	410.0	412.0	413.8	414.2	418.0	419.0	425.0

Appendix 6:

Tab. A6

$R_i(t)$	t_0	t_1	t_2	t_3	t_4
S_1	0.116921	0.159951	0.352774	0.131328	0.330144
S_2	0.298847	0.307918	0.315758	0.299525	0.366954
S_3	1.165994	0.664536	0.352774	1.372774	0.182029
S_4	0.807607	0.141815	0.074222	0.008918	0.090889
S_5	2.016697	0.147185	0.021673	0.141815	0.315758
S_6	0.807607	4.366785	0.1102	0.459267	0.315758
S_7	0.233611	0.630807	0.135959	0.213807	0.807607
S_8	0.299525	0.592253	1.555712	0.400187	1.555712
S_9	0.552407	1.084771	0.861515	0.597299	0.928083
S_{10}	0.533499	0.502667	0.40453	1.705304	0.528265
S_{11}	0.075979	0.928083	0.807607	0.283497	0.910361
S_{12}	0.942991	20.27122	9.170862	4.341083	3.034552
S_{13}	4.490557	2.944287	4.366785	2.392773	4.904043
S_{14}	4.410904	1.555712	14.92403	5.99212	0.456788
S_{15}	0.008918	0.669015	2.016697	2.041949	0.557959
S_{16}	5.454342	5.96963	3.034552	2.077018	2.243818