# Question 2: Goal Programming Problem

## Problem Statement

Xilinx, Inc. manufactures three types of main components for electric vehicles: - Traction battery pack - Power inverter - Controller

The production times, profits, and other constraints are as follows: - Production time: 1.5 hours for traction battery pack, 2 hours for power inverter, and 2.5 hours for controller - Profits: $28 for traction battery pack, $32 for power inverter, and $35 for controller - Total available production time: 240 hours per week - Forecasted demand: 60 units for each product

The manager has set goals in order of importance (from most to least important): 1. A total of 240 hours utilized for each week should not be exceeded. 2. At least 60 units for each component type should be produced. 3. A total profit of at least $3500 should be generated.

## Formulation of the Goal Programming Problem

### Decision Variables

Let's define our decision variables: - $x_1$ = Number of traction battery packs to produce - $x_2$ = Number of power inverters to produce - $x_3$ = Number of controllers to produce

### Deviation Variables

For each goal constraint, we need to define deviation variables: - $d_1^+$ = Overachievement of production hours (exceeding 240 hours) - $d_1^-$ = Underachievement of production hours (less than 240 hours) - $d_{21}^-$ = Underachievement of traction battery pack production (less than 60 units) - $d_{22}^-$ = Underachievement of power inverter production (less than 60 units) - $d_{23}^-$ = Underachievement of controller production (less than 60 units) - $d_3^-$ = Underachievement of profit goal (less than $3500)

### Deviation Function (Objective Function)

Since the goals are ranked in order of importance, we need to use preemptive goal programming. The objective function is to minimize:

$P_1(d_1^+) + P_2(d_{21}^- + d_{22}^- + d_{23}^-) + P_3(d_3^-)$

Where $P_1$, $P_2$, and $P_3$ represent the priority levels, with $P_1$ being the highest priority.

### Constraints/Goals

1. Production hours constraint (Priority 1): $1.5x_1 + 2x_2 + 2.5x_3 + d_1^- - d_1^+ = 240$
2. Production quantity constraints (Priority 2): $x_1 + d_{21}^- \geqslant 60$ $x_2 + d_{22}^- \geqslant 60$ $x_3 + d_{23}^- \geqslant 60$
3. Profit constraint (Priority 3): $28x_1 + 32x_2 + 35x_3 + d_3^- \geqslant 3500$
4. Non-negativity constraints: $x_1, x_2, x_3, d_1^+, d_1^-, d_{21}^-, d_{22}^-, d_{23}^-, d_3^- \geqslant 0$

## Excel Solver Setup

Since QM for Windows is not available, we'll use Excel Solver to solve this goal programming problem. Here's how to set it up:

### Step 1: Create the Excel spreadsheet

Create a spreadsheet with the following structure:

1. Decision variables section:
2. Cell B2: $x_1$ (Traction battery packs)
3. Cell C2: $x_2$ (Power inverters)
4. Cell D2: $x_3$ (Controllers)
5. Deviation variables section:
6. Cell F2: $d_1^+$ (Over hours)
7. Cell G2: $d_1^-$ (Under hours)
8. Cell H2: $d_{21}^-$ (Under traction battery)
9. Cell I2: $d_{22}^-$ (Under power inverter)
10. Cell J2: $d_{23}^-$ (Under controller)
11. Cell K2: $d_3^-$ (Under profit)
12. Initialize all variables to 0.
13. Constraints section:
14. Cell B5: =1.5B2+2C2+2.5*D2+G2-F2 (Production hours)

15. Cell B6: =B2+H2 (Traction battery production)
16. Cell B7: =C2+I2 (Power inverter production)
17. Cell B8: =D2+J2 (Controller production)

18. Cell B9: =28B2+32C2+35*D2+K2 (Profit)

19. Goals/Target values:

20. Cell C5: 240 (Production hours target)
21. Cell C6: 60 (Traction battery target)
22. Cell C7: 60 (Power inverter target)
23. Cell C8: 60 (Controller target)

24. Cell C9: 3500 (Profit target)

25. Priority weights section:

26. Cell F5: 1000000 (Priority 1 weight)
27. Cell F6: 1000 (Priority 2 weight)
28. Cell F7: 1000 (Priority 2 weight)
29. Cell F8: 1000 (Priority 2 weight)

30. Cell F9: 1 (Priority 3 weight)

31. Weighted deviations:

32. Cell H5: =F5*F2 (Weighted over hours)
33. Cell I5: =F5*G2 (Weighted under hours)
34. Cell H6: =F6*H2 (Weighted under traction battery)
35. Cell H7: =F7*I2 (Weighted under power inverter)
36. Cell H8: =F8*J2 (Weighted under controller)

37. Cell H9: =F9*K2 (Weighted under profit)

38. Objective function:

39. Cell B12: =H5+I5+H6+H7+H8+H9 (Sum of weighted deviations)

### Step 2: Set up Excel Solver

1. Go to Data tab and click on Solver
2. Set the objective to minimize cell B12 (sum of weighted deviations)
3. By changing variable cells: B2:D2,F2:K2 (all decision and deviation variables)
4. Subject to the constraints:
5. B5 = C5 (Production hours constraint)

6. B6 >= C6 (Traction battery constraint)
7. B7 >= C7 (Power inverter constraint)
8. B8 >= C8 (Controller constraint)
9. B9 >= C9 (Profit constraint)
10. B2:D2 >= 0 (Non-negativity for decision variables)
11. F2:K2 >= 0 (Non-negativity for deviation variables)
12. Select "Simplex LP" as the solving method
13. Click Solve

### Step 3: Interpret the results

After running Solver, examine the values of the decision variables and deviation variables to determine: 1. How many of each component to produce 2. Which goals have been met or not met 3. The extent of any underachievement or overachievement

## Expected Solution and Interpretation

Based on the problem constraints and priorities, we would expect the following solution:

### Decision Variables

- $x_1 = 60$ (Produce 60 traction battery packs)
- $x_2 = 60$ (Produce 60 power inverters)
- $x_3 = 24$ (Produce 24 controllers)

### Production Hours Used

$1.5(60) + 2(60) + 2.5(24) = 90 + 120 + 60 = 270$ hours

### Deviation Variables

- $d_1^+ = 30$ (Exceeding production hours by 30)
- $d_1^- = 0$ (No underachievement of production hours)
- $d_{21}^- = 0$ (No underachievement of traction battery production)
- $d_{22}^- = 0$ (No underachievement of power inverter production)
- $d_{23}^- = 36$ (Underachievement of controller production by 36 units)
- $d_3^- = 0$ (No underachievement of profit goal)

### Total Profit

$28(60) + 32(60) + 35(24) = 1680 + 1920 + 840 = \$4440$

## Goal Achievement Analysis

1. **Priority 1: Production hours constraint**
2. Goal: Not exceed 240 hours
3. Result: 270 hours used ($d_1^+ = 30$)
4. Status: **Not satisfied** - This highest priority goal is not met as we exceed the production hours by 30 hours.
5. **Priority 2: Production quantity constraints**
6. Goal: At least 60 units of each component
7. Result:
    - Traction battery packs: 60 units ($d_{21}^- = 0$)
    - Power inverters: 60 units ($d_{22}^- = 0$)
    - Controllers: 24 units ($d_{23}^- = 36$)
8. Status: **Partially satisfied** - We meet the production targets for traction battery packs and power inverters, but we're 36 units short on controllers.
9. **Priority 3: Profit constraint**
10. Goal: At least $3500 profit
11. Result: $4440 profit ($d_3^- = 0$)
12. Status: **Satisfied** - We exceed the profit goal by $940.

## Findings

The solution reveals a conflict between the goals. The highest priority goal (not exceeding 240 production hours) cannot be met while simultaneously satisfying the second priority goal (producing at least 60 units of each component). This is because producing 60 units of each component would require: $1.5(60) + 2(60) + 2.5(60) = 90 + 120 + 150 = 360$ hours

This exceeds the available 240 hours by 120 hours. The model attempts to balance these conflicting goals by: 1. Producing the full 60 units of traction battery packs and power inverters 2. Reducing the production of controllers to only 24 units 3. Exceeding the production hours constraint by 30 hours

Despite not meeting all goals, the solution still generates a profit of $4440, which exceeds the third priority goal of $3500.

This analysis suggests that Xilinx, Inc. should either: 1. Increase production capacity beyond 240 hours 2. Adjust the production targets to more realistic levels 3. Consider outsourcing some production, particularly for controllers

The solution demonstrates the classic trade-offs in goal programming, where lower priority goals may need to be sacrificed to satisfy higher priority goals as much as possible.