Matrix Multiplication Basics and Neural Networks

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I Summary

Matrix Multiplication is a **binary operation** in which the product of two $m{\cdot}n$ matricies $(A \times B)$ results in a third matrix C. The resultant matrix will have a row size equal to the row size in matrix A, and a column size equal to that of the column size of matrix B.

I.1 Definition of Matrix Multiplication

For an $m \cdot n$ matrix A, and an $n \cdot p$ matrix B:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}, \; B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1p} \\ b_{21} & b_{22} & \dots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{np} \end{pmatrix} \tag{1}$$

there is a resultant matrix $\boldsymbol{C}$ such that $C = \boldsymbol{AB}$:

$$C = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1p} \\ c_{21} & c_{22} & \dots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \dots & c_{mp} \end{pmatrix} \tag{2}$$

where

$$C_{ij} = a_{i1}b_{j1} + a_{i2}b_{j2} + \ldots + a_{in}b_{jn} = \sum_{k=1}^{n} a_{ik}b_{kj}$$

for $i = 1 \ldots m$ and $j = 1 \ldots p$.

Importantly, c_{ij} is the ***dot product*** of the i^{th} row of $\boldsymbol{A}$ and the j^{th} column $\boldsymbol{B}$.

II Computational Complexity