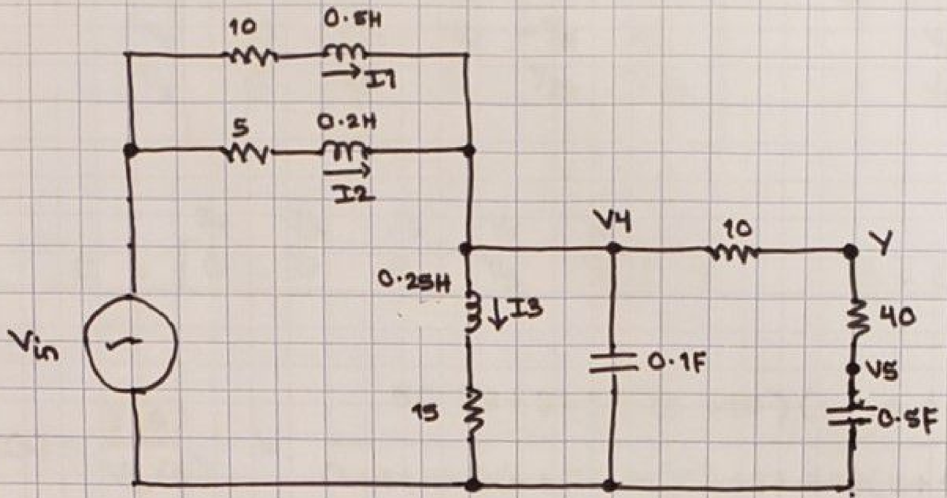
ABDULLAH ALMOSALAMI | ECE463 HW # 2 | 01/24/2021 SUNDAY



$V_L = L\frac{di_L}{dt} \Rightarrow V_L = sLI_L$

$i_c = C\frac{dv_c}{dt} \Rightarrow I_c = sCV_c$

1) • Specify dynamics
• Express in state-space form
• $T: V_{in} \to Y$

Let the state variables be: I1, I2, I3, V4, V5.

$I_1 + I_2 - I_3 - s(0.1)V_4 + -0.5sV_5 = 0$

$s0.5V_5 = \frac{V_4 - V_5}{50}$

Hmm...

$V_4 = V_{in} - 5I_2 - 0.2sI_2$

$= V_{in} - 10I_1 - 0.5sI_1$

$= 0.25sI_3 + 10I_3$

$Y = 40\frac{V_4 - V_5}{50} + V_5$

$Y = \frac{4}{5}V_4 + \frac{1}{5}V_5$

Ok, so:

$sI_1 = \cancel{20I_1} + \cancel{2V_4} - \cancel{2V_{in}}$

$sI_2 = \cancel{25I_2} + \cancel{V_4} - \cancel{V_{in}}$

$sI_3 = \cancel{-40I_3}$

$sV_5 = \frac{1}{25}V_4 - \frac{1}{25}V_5$

$sI_1 = -20I_1 - 2V_4 + 2V_{in}$

$sI_2 = -25I_2 - 5V_4 + 5V_{in}$

$sI_3 = -40I_3 + 4V_4$

$sV_4 = 10I_1 + 10I_2 - 10I_3 - \frac{1}{5}V_4 + \frac{1}{5}V_5$

$$
A = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \end{array}
\begin{array}{c}
\begin{array}{ccccc} I_1 & I_2 & I_3 & V_4 & V_5 \end{array} \\
\begin{bmatrix}
-20 & 0 & 0 & -2 & 0 \\
0 & -25 & 0 & -5 & 0 \\
0 & 0 & -40 & 4 & 0 \\
10 & 10 & -10 & -1/5 & 1/5 \\
0 & 0 & 0 & 1/25 & -1/25
\end{bmatrix}
\end{array}
\qquad
B = \begin{array}{c} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \end{array}
\begin{bmatrix} 2 \\ 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
C = \begin{array}{c}
\begin{array}{ccccc} I_1 & I_2 & I_3 & V_4 & V_5 \end{array} \\
\begin{bmatrix} 0 & 0 & 0 & 4/5 & -1/5 \end{bmatrix}
\end{array}
\qquad D = 0
$$

$$
G(s) = \frac{Y(s)}{V_{in}(s)} = \frac{56\,(s+21.43)(s+40)(s+0.03)}{(s+39.08)(s+23.14)(s+17.74)(s+5.237)(s+0.03808)}
$$

2) First or Second order Approx.
Plot

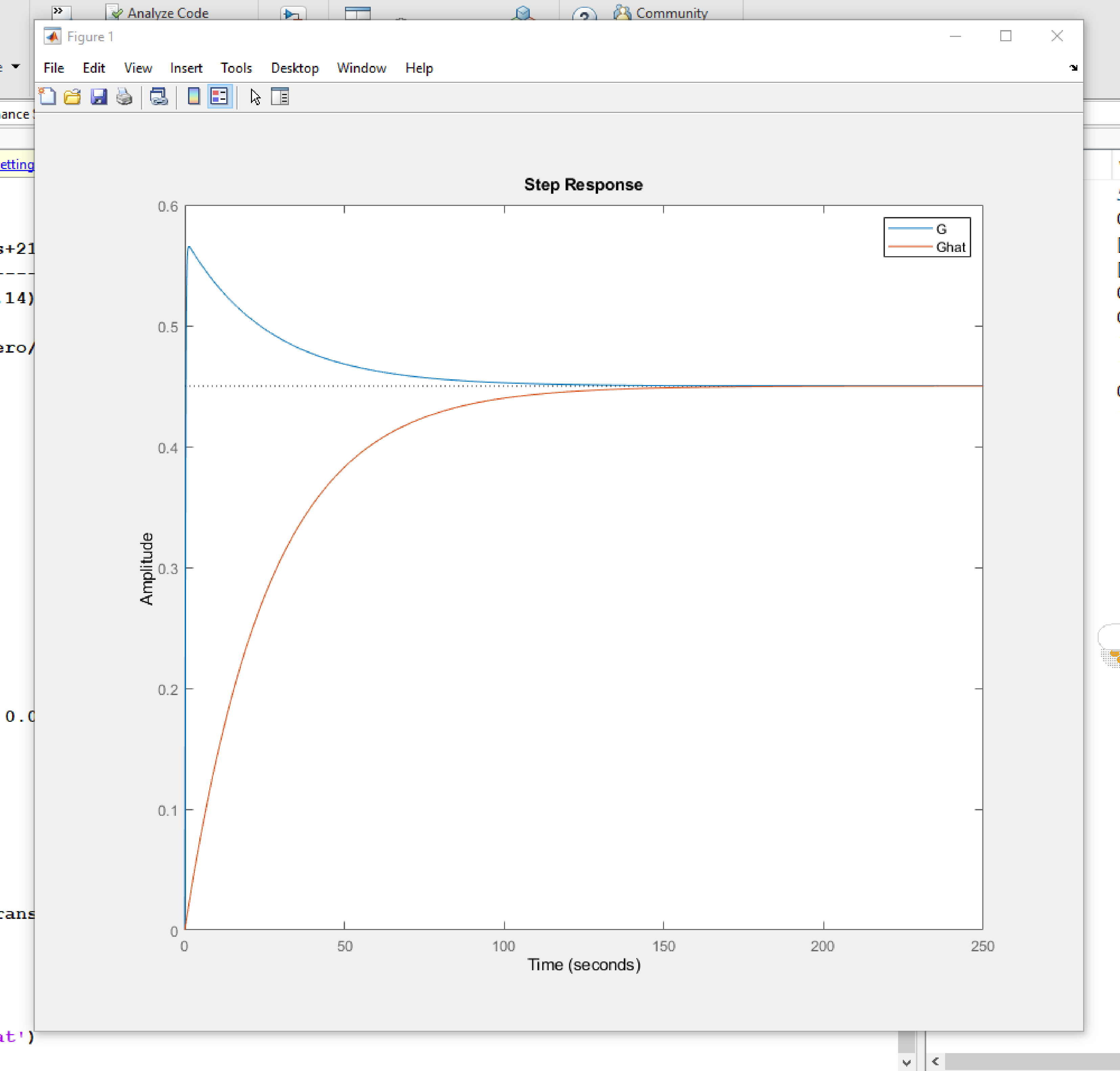
DC Gain : 0.45 $\qquad \hat{G} = \dfrac{k}{s+0.03808}$, $k = DC * 0.03803$

$= 0.0171$

$$
\boxed{\hat{G}(s) = \frac{0.0171}{s+0.03808}}
$$

Eh… Not great. Better as 2nd order approx:

$$
\boxed{\hat{G}(s) = \frac{0.074784\,(s+40)(s+0.03)}{(s+5.237)(s+0.03808)}}
$$



Analyze Code
Community
Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Step Response
G
Ghat
Amplitude
0
0.1
0.2
0.3
0.4
0.5
0.6
0
50
100
150
200
250
Time (seconds)

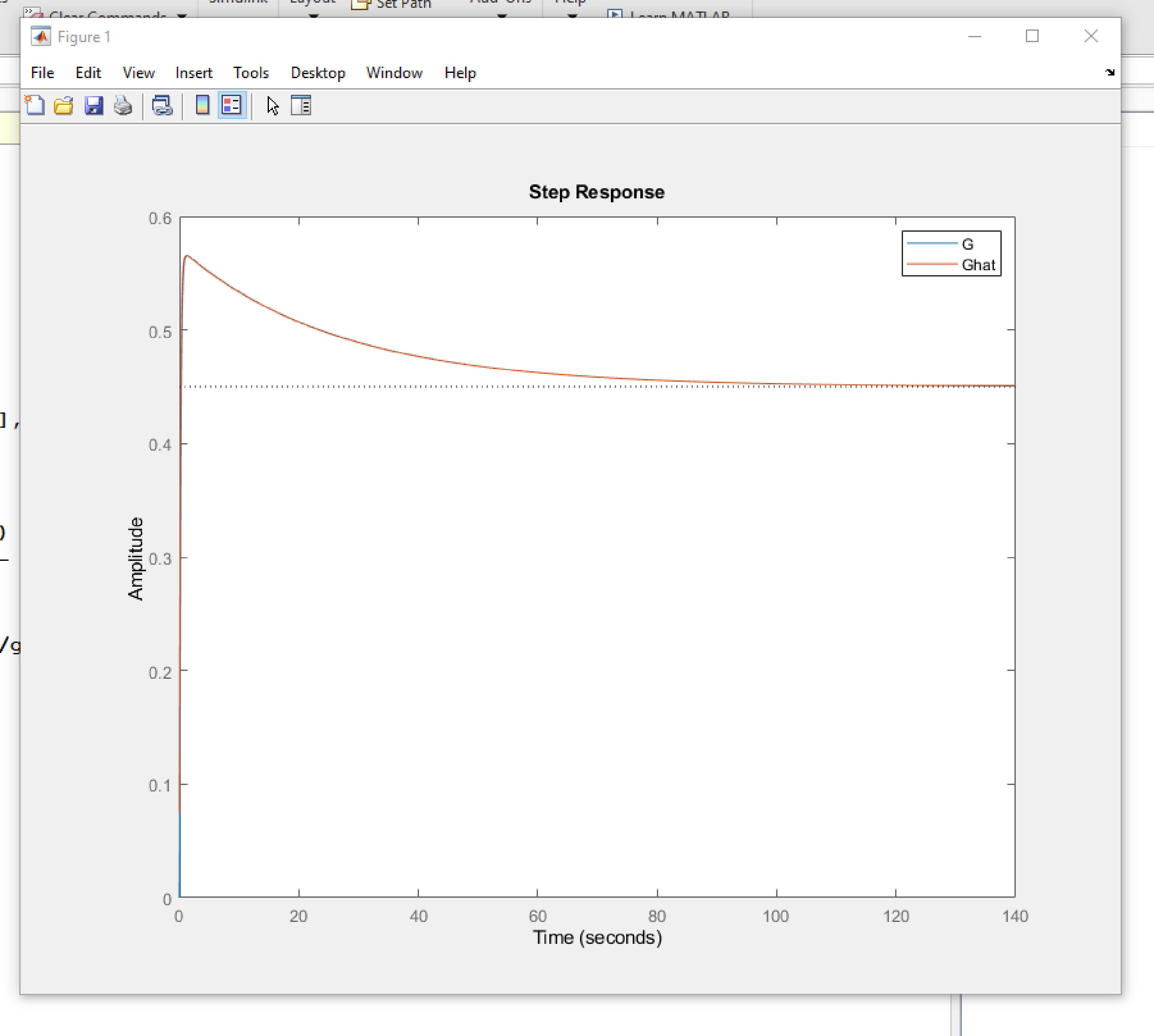


Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Step Response
G
Ghat
Amplitude
0
0.1
0.2
0.3
0.4
0.5
0.6
0
20
40
60
80
100
120
140
Time (seconds)

3) For this circuit,

(i) What ICs decay slowest?
(ii) " " fastest?

Any scalar multiple of

$$\begin{bmatrix} -0.0048 \text{ A} \\ -0.0096 \text{ A} \\ 0.0048 \text{ A} \\ 0.0479 \text{ V} \\ 0.9988 \text{ V} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \end{bmatrix}$$

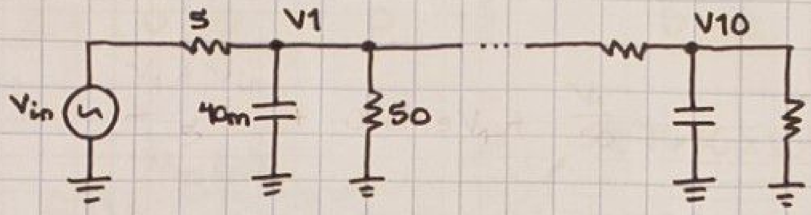
Basis for eigenspace of $\lambda = -0.0381$

will be slowest (2% $T_s \doteq 105$ seconds).

For the fastest decay, any scalar multiple of (2% $T_s = 103$ ms)

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} -0.0231 \text{ A} \\ -0.0793 \text{ A} \\ -0.9712 \text{ A} \\ -0.2234 \text{ V} \\ 0.0002 \text{ V} \end{bmatrix}$$

Basis for eigenspace of $\lambda = -39.08$



4) Node 1: $\dfrac{V_1 - V_{in}}{5} + 0.04sV_1 + \dfrac{V_1}{50} + \dfrac{V_1 - V_2}{5} = 0$

Node 10: $\dfrac{V_{10} - V_9}{5} + 0.04sV_{10} + \dfrac{V_{10}}{50} = 0$

1: $\frac{10}{50}V_1 - \frac{1}{5}V_{in} + \frac{1}{25}sV_1 + \frac{1}{50}V_1 + \frac{10}{50}V_1 - \frac{1}{5}V_2 = 0$

$$\left(\frac{10}{50} + \frac{1}{50} + \frac{10}{50}\right)V_1 + \left(\frac{-1}{5}\right)V_2 + \left(\frac{-1}{5}\right)V_{in} = \frac{-1}{25}sV_1$$

$$-25\left(\frac{21}{50}\right)V_1 + 5V_2 + 5V_{in} = sV_1$$

$$-\frac{21}{2}V_1 + 5V_2 + 5V_{in} = sV_1$$

10: $5V_{10} - 5V_9 + sV_{10} + \frac{1}{2}V_{10} = 0$

$$sV_{10} = -\frac{11}{2}V_{10} + 5V_9$$

So,

$$A = \begin{array}{cc}
 & \begin{array}{cccccccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 & V_9 & V_{10} \end{array} \\
\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \end{array} &
\left[\begin{array}{cccccccccc}
-\frac{21}{2} & 5 & 0 & \cdots & & & & & & 0 \\
5 & -\frac{21}{2} & 5 & 0 & \cdots & & & & & 0 \\
0 & 5 & -\frac{21}{2} & 5 & 0 & \cdots & & & & \\
 & 0 & 5 & -\frac{21}{2} & 5 & 0 & \cdots & & & \vdots \\
\vdots & & 0 & 5 & -\frac{21}{2} & 5 & 0 & \cdots & & \\
 & & & 0 & 5 & -\frac{21}{2} & 5 & 0 & \cdots & \\
 & & & & 0 & 5 & -\frac{21}{2} & 5 & 0 & 0 \\
 & \ddots & & & & 0 & 5 & -\frac{21}{2} & 5 & 0 \\
 & & & & & & 0 & 5 & -\frac{21}{2} & 5 \\
0 & \cdots & & & & & & 0 & 5 & -\frac{11}{2}
\end{array}\right]
\end{array}
\qquad
B = \begin{bmatrix} 5 \\ 0 \\ 0 \\ \vdots \\ \\ 0 \end{bmatrix}$$

Command Window

New to MATLAB? See resources for Getting Started.

```
>> zpk(G)

ans =

                                    9.7656e+06
  ---------------------------------------------------------------------------
  (s+20.06) (s+18.76) (s+16.73) (s+14.15) (s+11.25) (s+8.275) (s+5.5) (s+3.169)
                                                              (s+1.49) (s+0.6117)

Continuous-time zero/pole/gain model.
```

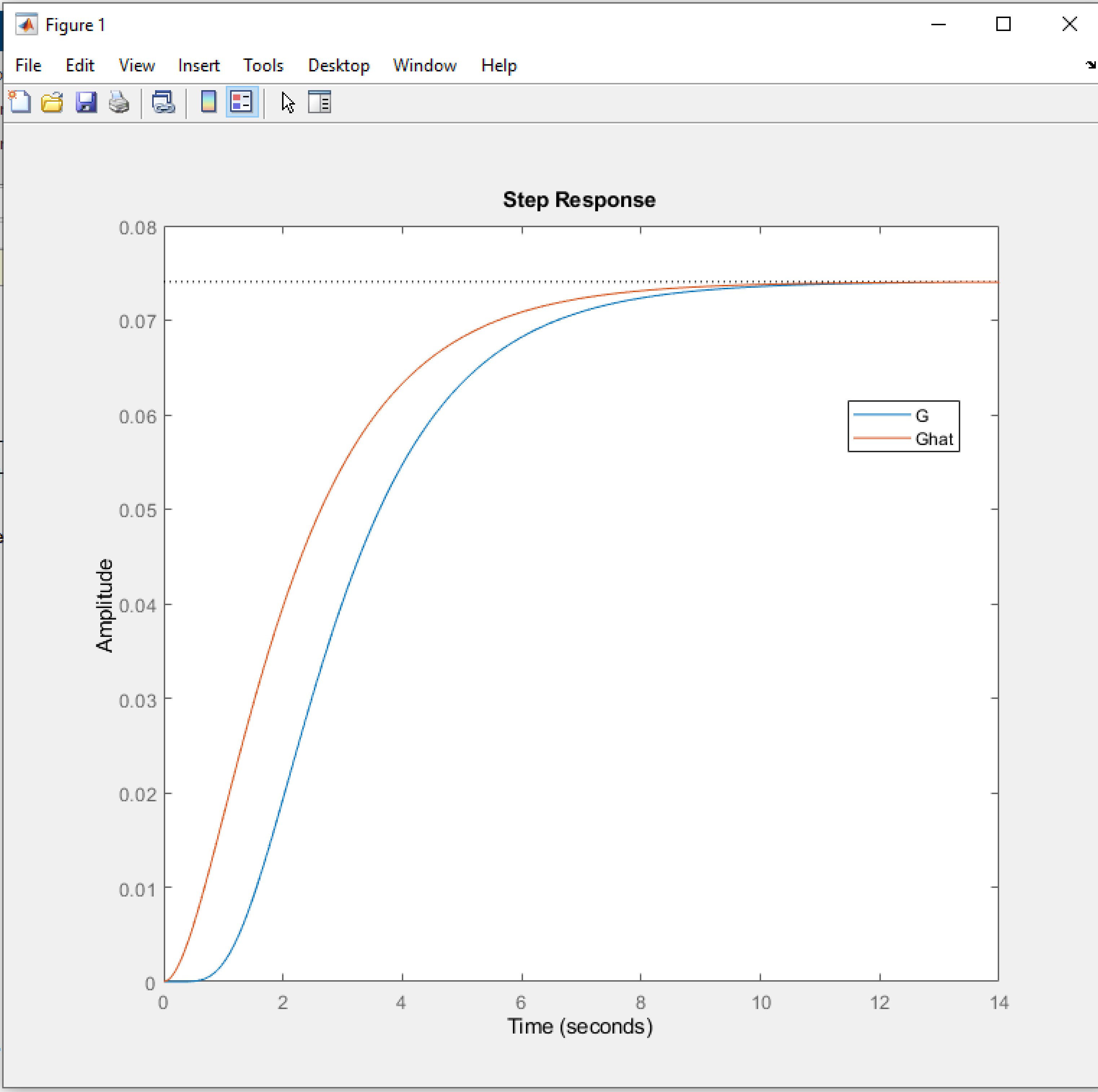


Figure 1
File Edit View Insert Tools Desktop Window Help
Step Response
Amplitude
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
2
4
6
8
10
12
14
Time (seconds)
G
Ghat

$$C = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \qquad D = 0$$

$$G(s) = \frac{V_{10}(s)}{V_{in}(s)} = \frac{9.7656 \times 10^{6}}{(s+20.06)(s+18.76)(s+16.73)\cdots(s+5.5)(s+3.169)(s+1.49)(s+0.6117)}$$

5)

$\underline{DC}$: 0.0741 $\Rightarrow k = DC * P_1 * P_2 = 0.0675$

$$\Rightarrow \hat{G}(s) = \frac{0.0675}{(s+1.49)(s+0.6117)}$$

(Plot is attached next page)

6) Slowest initial condition would be any scale of

$$\underline{V} = \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \end{array} \begin{bmatrix} 0.0680\ V \\ 0.1286\ V \\ 0.1894\ V \\ 0.2459\ V \\ 0.2969\ V \\ 0.3412\ V \\ 0.3780\ V \\ 0.4063\ V \\ 0.4255\ V \\ 0.4352\ V \end{bmatrix} \Rightarrow$$

Corresponds to $\lambda = -0.6117$

$\Downarrow$

$2\%\ T_s =$ ~~[illegible]~~ s

6.5392s

Fastest:

$$\underline{V} = \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \end{array} \begin{bmatrix} -0.1286 \\ 0.2459 \\ -0.3412 \\ 0.4063 \\ -0.4352 \\ 0.4255 \\ -0.3780 \\ 0.2969 \\ -0.1894 \\ 0.0680 \end{bmatrix} \Rightarrow$$

Corresponds to $\lambda = -20.0557$

$\Downarrow$

$2\%\ T_s = 199.4\,ms$

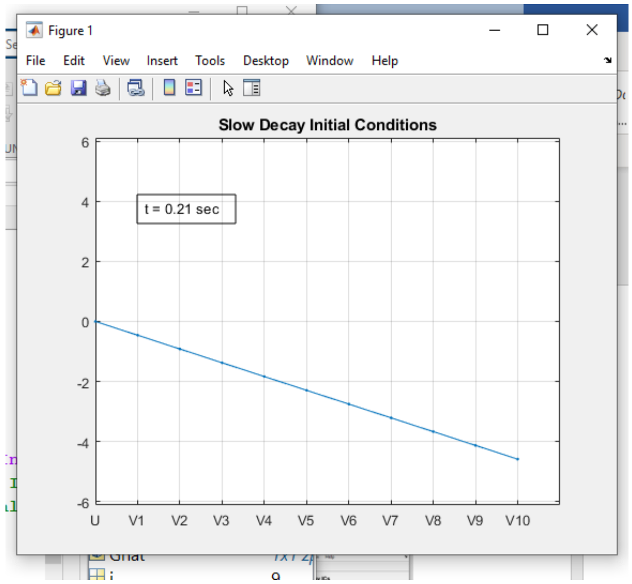


Figure 1
File Edit View Insert Tools Desktop Window Help
Slow Decay Initial Conditions
t = 0.21 sec
6
4
2
0
-2
-4
-6
U V1 V2 V3 V4 V5 V6 V7 V8 V9 V10

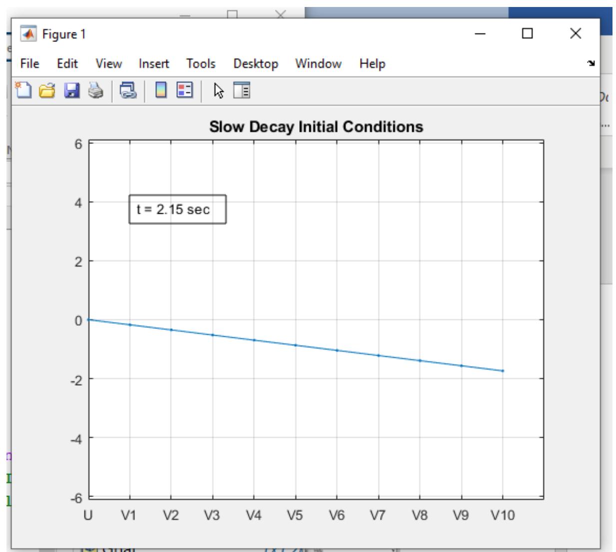


Figure 1
File Edit View Insert Tools Desktop Window Help
Slow Decay Initial Conditions
t = 2.15 sec
6
4
2
0
-2
-4
-6
U V1 V2 V3 V4 V5 V6 V7 V8 V9 V10

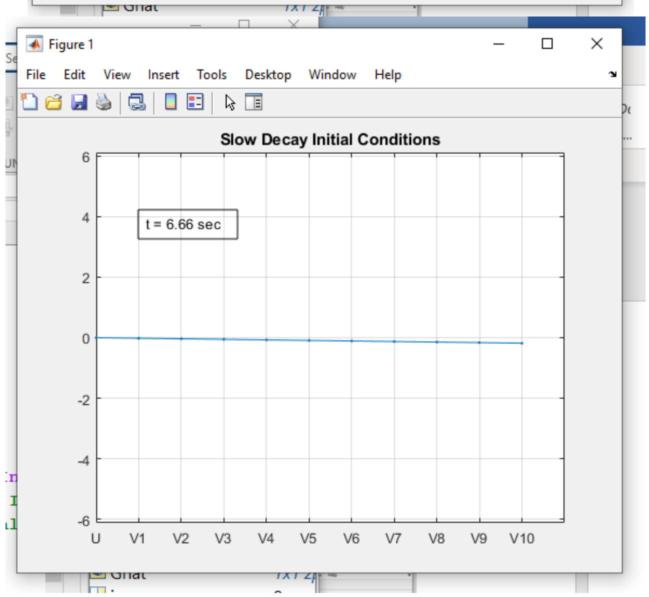


Figure 1
File Edit View Insert Tools Desktop Window Help
Slow Decay Initial Conditions
t = 6.66 sec
6
4
2
0
-2
-4
-6
U V1 V2 V3 V4 V5 V6 V7 V8 V9 V10

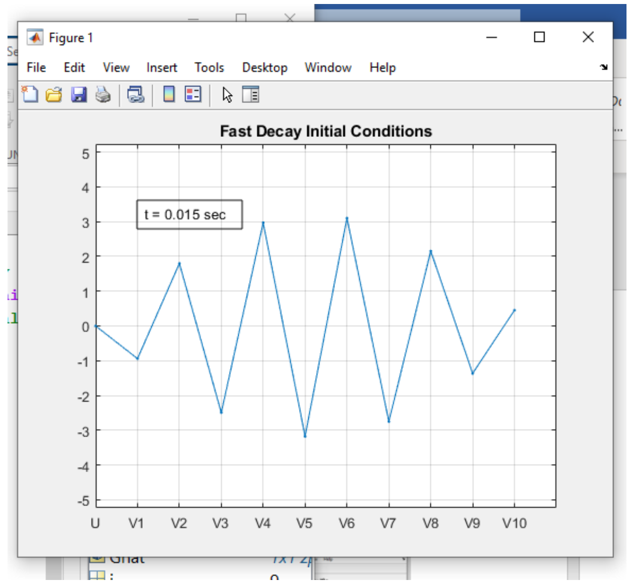


Figure 1
File Edit View Insert Tools Desktop Window Help
Fast Decay Initial Conditions
t = 0.015 sec
5 4 3 2 1 0 -1 -2 -3 -4 -5
U V1 V2 V3 V4 V5 V6 V7 V8 V9 V10

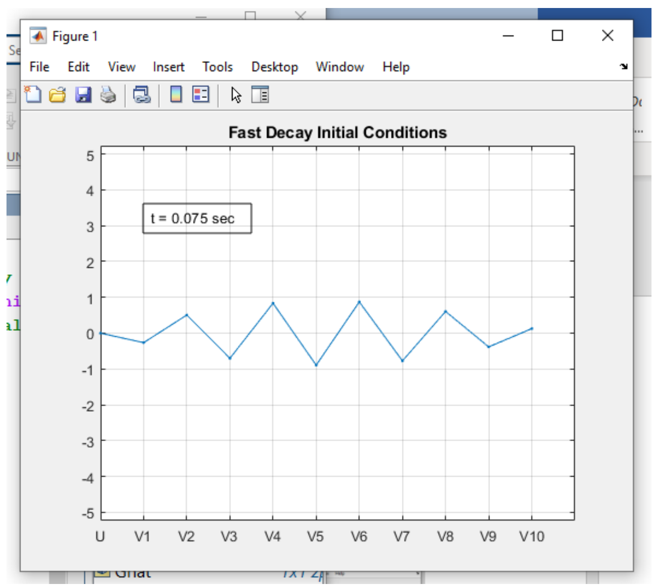


Figure 1
File Edit View Insert Tools Desktop Window Help
Fast Decay Initial Conditions
t = 0.075 sec
5 4 3 2 1 0 -1 -2 -3 -4 -5
U V1 V2 V3 V4 V5 V6 V7 V8 V9 V10

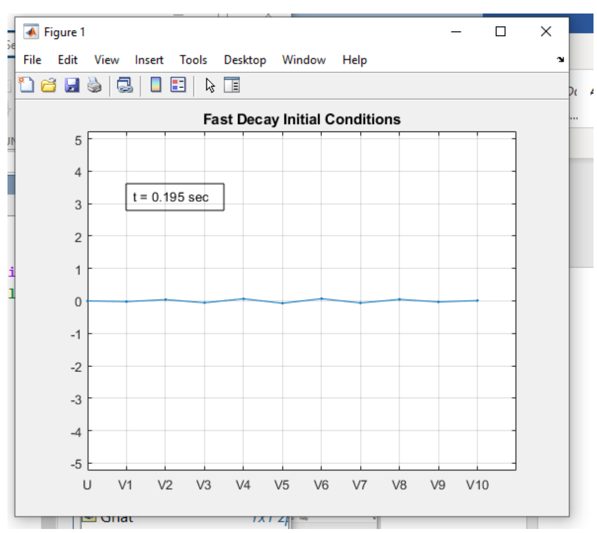


Figure 1
File Edit View Insert Tools Desktop Window Help
Fast Decay Initial Conditions
t = 0.195 sec
5 4 3 2 1 0 -1 -2 -3 -4 -5
U V1 V2 V3 V4 V5 V6 V7 V8 V9 V10

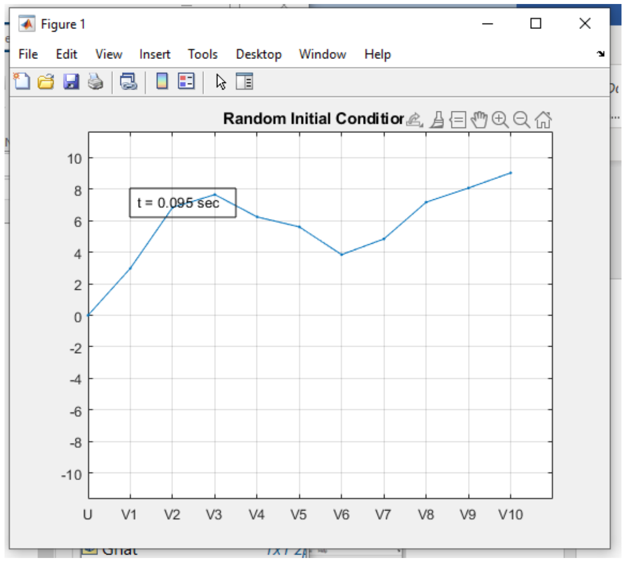


Figure 1
File Edit View Insert Tools Desktop Window Help
Random Initial Condition
t = 0.095 sec
10
8
6
4
2
0
-2
-4
-6
-8
-10
U V1 V2 V3 V4 V5 V6 V7 V8 V9 V10

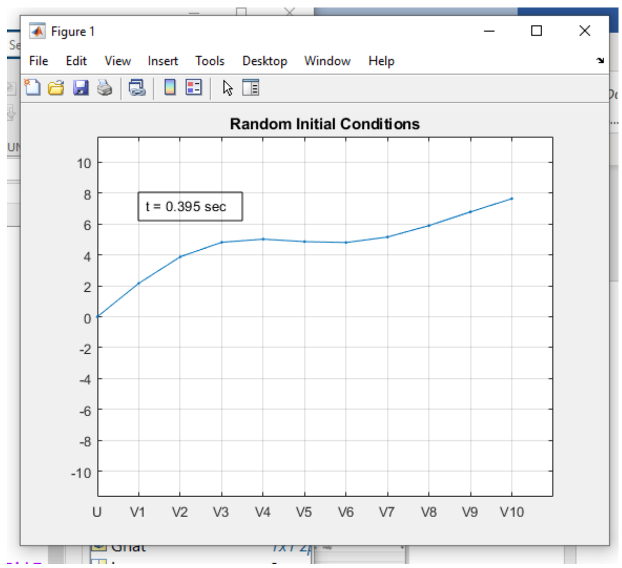


Figure 1
File Edit View Insert Tools Desktop Window Help
Random Initial Conditions
t = 0.395 sec
10
8
6
4
2
0
-2
-4
-6
-8
-10
U V1 V2 V3 V4 V5 V6 V7 V8 V9 V10

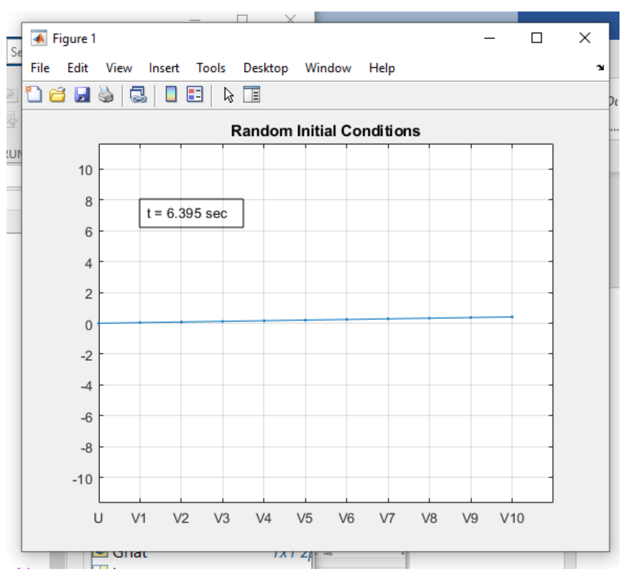


Figure 1
File Edit View Insert Tools Desktop Window Help
Random Initial Conditions
t = 6.395 sec
10
8
6
4
2
0
-2
-4
-6
-8
-10
U V1 V2 V3 V4 V5 V6 V7 V8 V9 V10