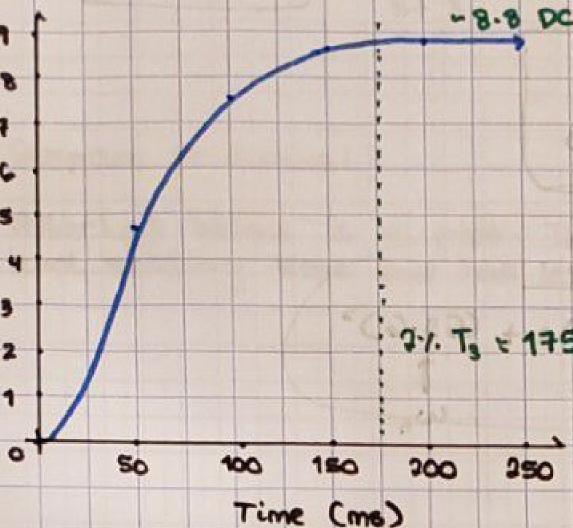
ABDULLAH ALMOSALAMI

ECE463 HW #1

01/15/2021 FRIDAY

1) Name that System! Give the TF for a system w/ the following step response:



Dominant Pole at $\boxed{\doteq -22.9}$

Approximately 1ST order system (probably higher w/ that delay in the beginning)

2% $T_s \approx 175\,ms \Rightarrow \tau = 43.75\,m$

$\Rightarrow \sigma \doteq \frac{-1}{\tau} \doteq -22.857$

$G(s) = \frac{A}{s + 22.9} \Rightarrow G(0) = \text{DC Gain}$

$= \frac{A}{22.9} = 8.8$

$\Rightarrow \boxed{A \approx 201}$

So, $\boxed{G(s) \approx \frac{201}{s + 22.9}}$

2) Name that System!

DC Gain ≈ 1.3, Underdamped System (two dom. conj. pair poles)

Damped Freq: 4 oscillations in 400ms ⇒ 10Hz ⇒ $\boxed{\omega_d \approx 62.8\ rad/s}$

2% Settling Time: ≈ 400ms ⇒ $\tau = 100\,ms$ ⇒ $\boxed{\sigma \approx -10\ rad/s}$

So dom. poles @ $-10 \pm j62.8$ ⇒ Quick check: $\theta = \tan^{-1}\left(\frac{62.8}{10}\right)$

$\theta \approx 80.9°$

⇓

$\zeta \approx 0.1573$

⇓

Yup. Checks out.

$\frac{2.05}{1.3} \approx 1.5769$ (peak / DC Gain) ⇐ $OS\% \approx 60\%$ ⇐ $OS\% \approx \exp\left(-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}\right)$

$$G(s) = \frac{A}{(s+10+j62.8)(s+10-j62.8)} \Rightarrow G(0) = 1.3 = \frac{A}{\|10+j62.8\|^2}$$

$$\Downarrow$$

$$\boxed{A = 5257}$$

So,

$$\boxed{G(s) = \frac{5257}{(s+10+j62.8)(s+10+j62.8)}}$$

$$= \frac{(1.3)(63.6)^2}{s^2 + 2\underbrace{(0.1573)}_{\zeta}\underbrace{(63.6)}_{\omega_n}s + \underbrace{(63.6)^2}_{\omega_n}}$$

P3 → 6)

$$Y = \left(\frac{300}{(s+3)(s+9)(s+10)}\right)X$$

3) Differential equation?

$$(s+3)(s+9)(s+10) = (s+3)(s^2+19s+90) = s^3 + 19s^2 + 90s + 3s^2 + 57s + 270$$

$$= s^3 + 22s^2 + 147s + 270$$

$$\boxed{\frac{d^3y}{dt^3} + 22\frac{d^2y}{dt^2} + 147\frac{dy}{dt} + 270y(t) = 300x(t)}$$

4) $y(t)$ assuming $x(t) = 2\cos(3t) + 4\sin(3t)$

Recall:

Through a LTIC system,

$e^{st} \to H(s)e^{st}$

$= |H(s)| \angle H(s) e^{st}$

$$x(t) = \cancel{2}\left(\frac{1}{\cancel{2}}(e^{j3t} + e^{-j3t})\right) + 4\left(\frac{1}{2j}(e^{j3t} - e^{-j3t})\right)$$

$$H(s) = \frac{300}{(s+3)(s+9)(s+10)} \Rightarrow H(j3) = 0.1223 - j0.7034$$

$$\Rightarrow = 0.7139 \angle -80^\circ$$

$$y(t) = 0.7139\left(e^{j3t} + e^{-j3t} + -j2e^{j3t} + j2e^{-j3t}\right)e^{j(-80^\circ)}$$

$$\boxed{y(t) = 1.4278\cos(3t - 80^\circ) + 2.8556\sin(3t - 80^\circ)}$$

5) Step response.

$$Y = \left(\frac{1}{s}\right)\left(\frac{300}{(s+3)(s+9)(s+10)}\right) = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+9} + \frac{D}{s+10}$$

$$F(s) = \frac{300}{s(s+3)(s+9)(s+10)}$$

$$A = F(s)(s)\Big|_{s=0} = \frac{300}{(3)(9)(10)} = \frac{30}{27} = \frac{10}{9} = 1.11$$

$$B = F(s)(s+3)\Big|_{s=-3} = \frac{300}{(-3)(6)(7)} = \frac{-100}{42} = \frac{-50}{21} = -2.38$$

$$C = F(s)(s+9)\Big|_{s=-9} = \frac{300}{(-9)(-6)(1)} = \frac{300}{54} = \frac{150}{27} = 5.56$$

$$D = F(s)(s+10)\Big|_{s=-10} = \frac{300}{(-10)(-7)(-1)} = \frac{-30}{7} = -4.29$$

So,

$$Y = 1.11\,\frac{1}{s} + -2.38\,\frac{1}{s+3} + 5.56\,\frac{1}{s+9} + -4.29\,\frac{1}{s+10}$$

$$\boxed{y(t) = \left(1.11 - 2.38e^{-3t} + 5.56e^{-9t} - 4.29e^{-10t}\right)u(t)}$$

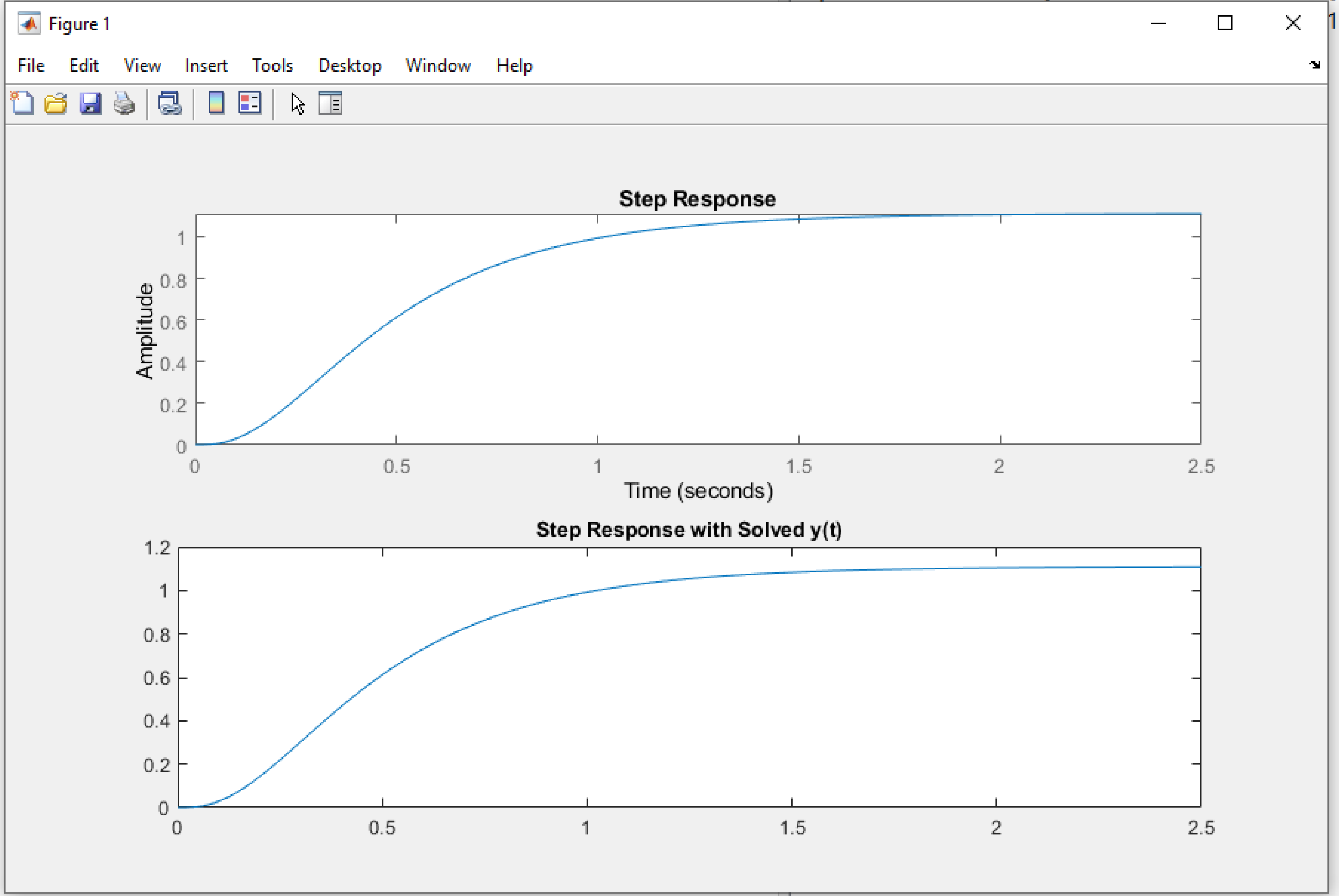
Confirm using [R, P, K] = residue(B, A), and step(H), (below)

```
>> [r,p,k] = residue([300],[1 22 147 270 0])

r =

   -4.2857
    5.5556
   -2.3810
    1.1111


p =

  -10.0000
   -9.0000
   -3.0000
         0


k =

     []
```

New to MATLAB? See resources for Getting Started.

```
>> t = linspace(0,2.5,1001);
>> y = 1.11 - 2.38*exp(-3*t) + 5.56*exp(-9*t) - 4.29*exp(-10*t);
>> subplot(2,1,1);
>> step(H)
>> subplot(2,1,2);
>> plot(t,y)
>> title('Step Response with Solved y(t)');
fx >>
```

Name	Value
ans	[42.8571;-50.0000;7.1429]
H	1x1 zpk
hh	0.1223 - 0.7034i
k	[]
p	[-10.0000;-9.0000;-3.0000;0]



6) a) 1^{st} Order Approx. :

DC Gain : $H(0) = 1.11$

$$\hat{H}(s) = \frac{A}{s+3} \Rightarrow A = 3.33 \Rightarrow \boxed{\hat{H}(s) = \frac{3.33}{s+3}}$$

b) Compare to actual?

Attached below is a plot. The approximation matches the DC gain and settling time but has less delay in the beginning.

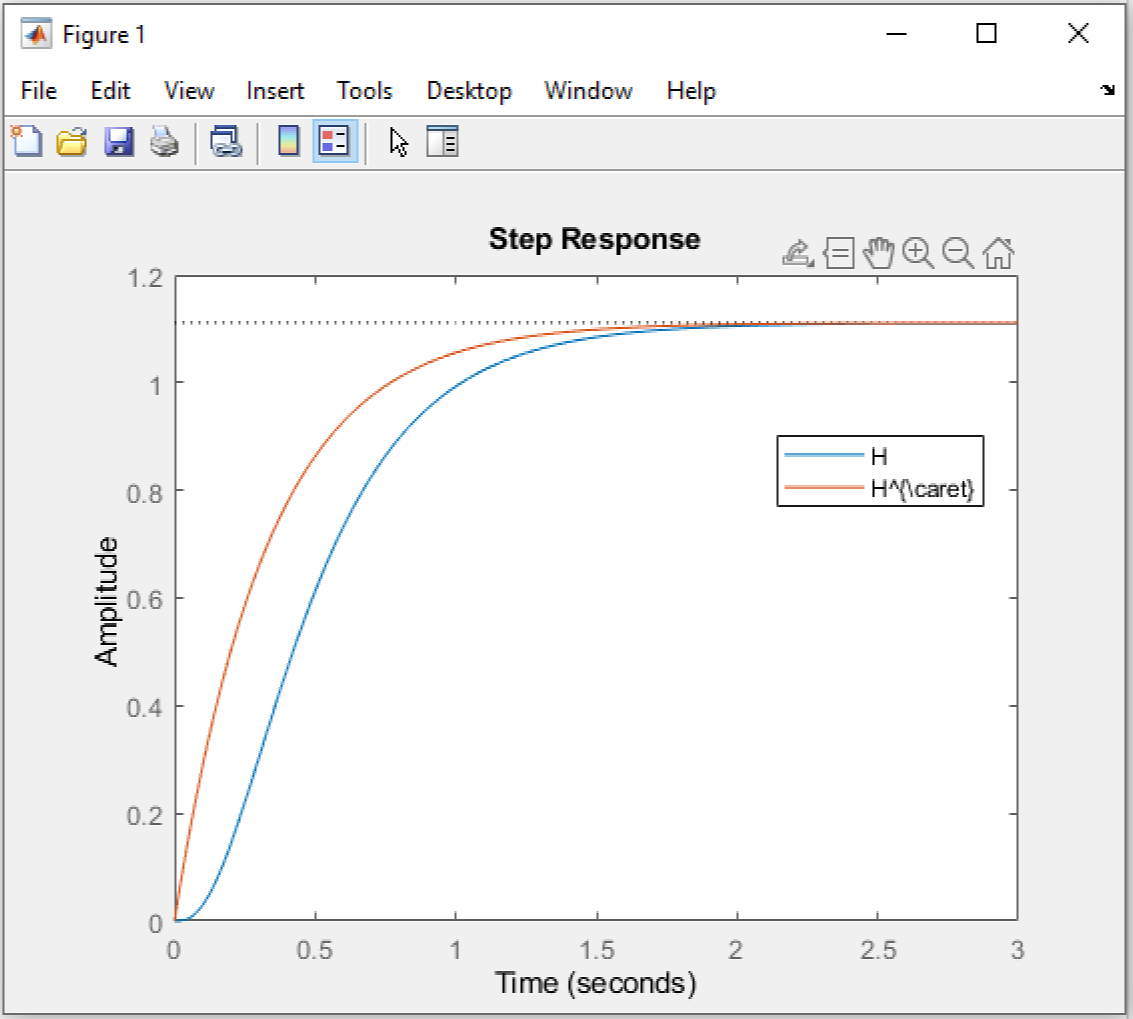


Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Step Response
Amplitude
Time (seconds)
H
H^{\caret}
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5
3