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ECE463

MODERN CONTROL

HW #12

05/12/2021

TUESDAY

$m_{ball} = 0.5^{kg}$, $R_{ball} = 1^m$, $m_{beam} = 4.8990kg$

$L = 2.4746m$

① Design a control law so that the ball & beam system behaves like

$$Y = \frac{0.5}{s^2 + s + 0.5} R \longrightarrow \text{Controller Canonical:}$$

$b_0 = 0,\ b_1 = 0,\ b_2 = 0.5$

$a_1 = 1,\ a_2 = 0.5$

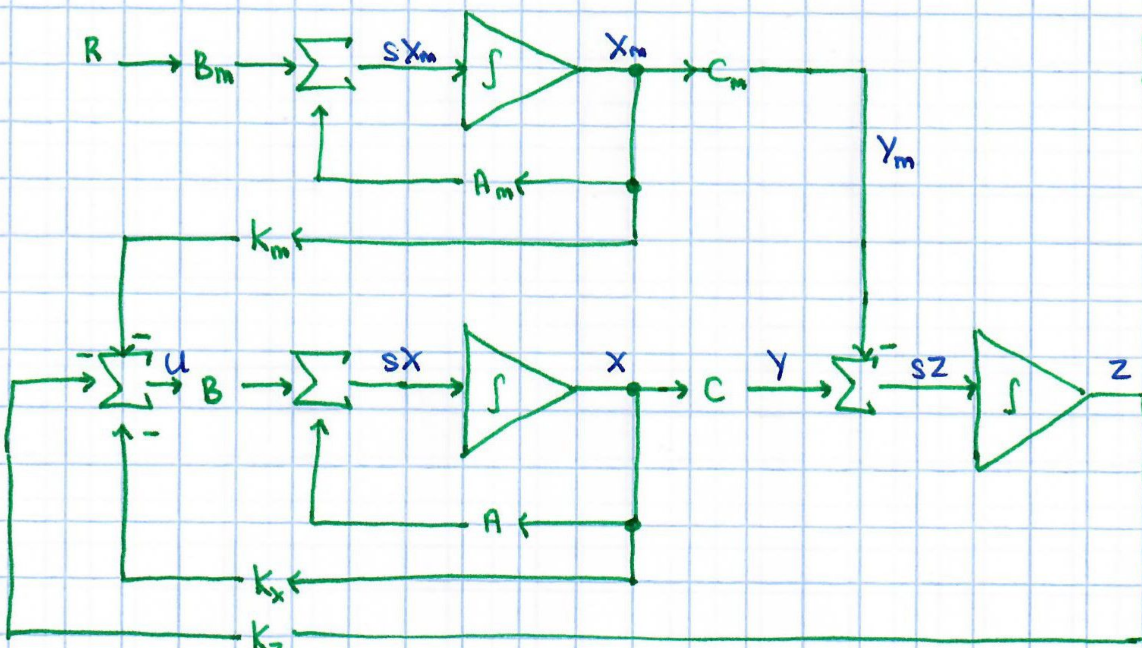
$$A_m = \begin{bmatrix} 0 & 1 \\ -0.5 & -1 \end{bmatrix},\quad B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

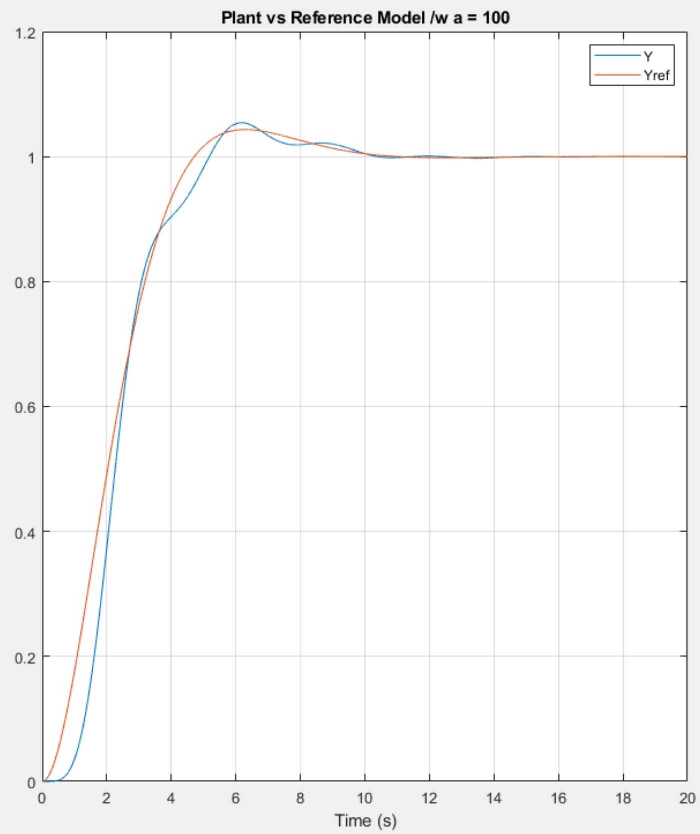
$$C_m = \begin{bmatrix} 0.5 & 0 \end{bmatrix}$$

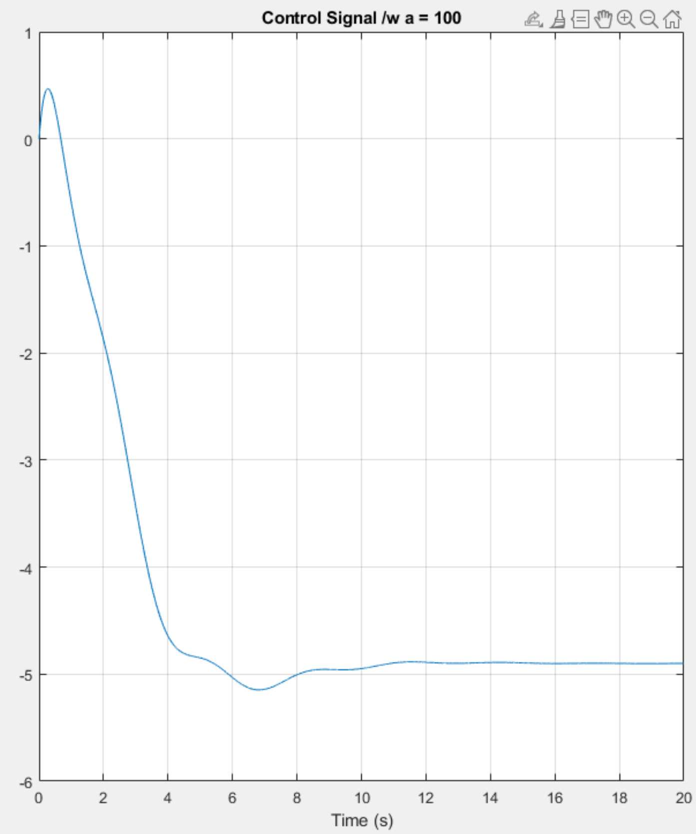
(1) Give a block diagram

(2) Plot the step response w/ $a = 100, 1000, 10{,}000$

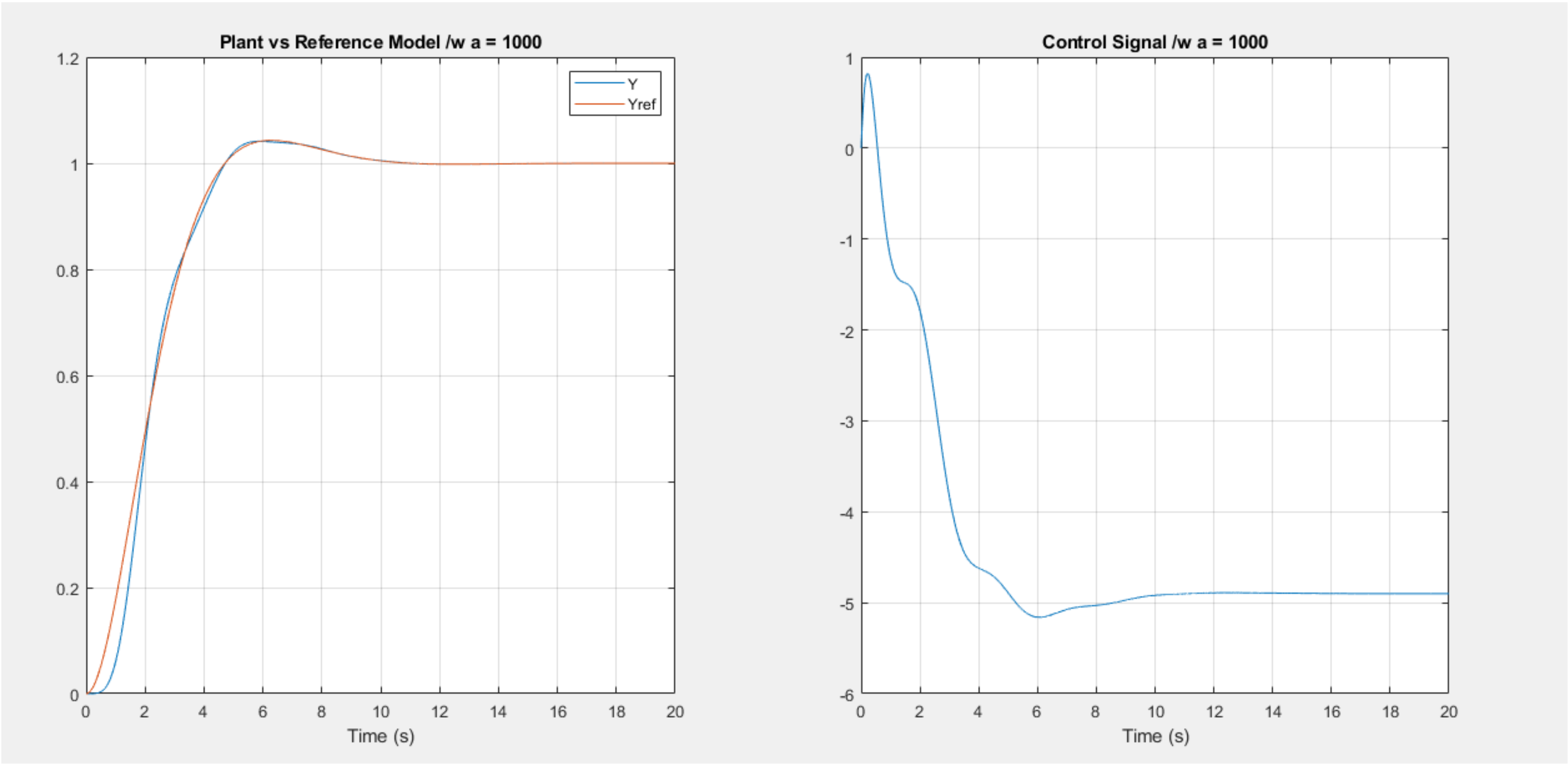
I'll use LQR/LQT w/ servo-comp :



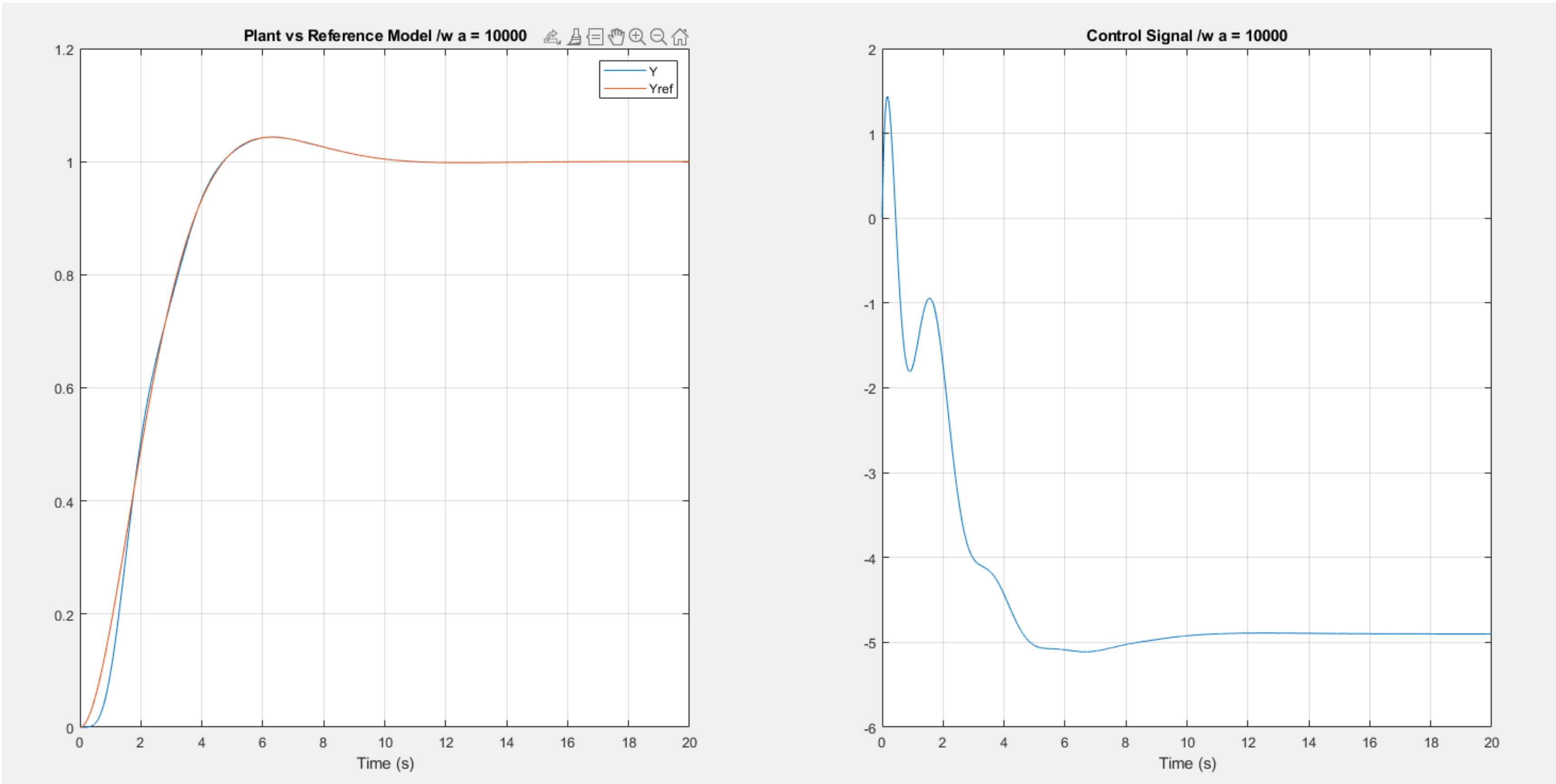

Plant vs Reference Model /w a = 100
Y
Yref
1.2
1
0.8
0.6
0.4
0.2
0
0 2 4 6 8 10 12 14 16 18 20
Time (s)


Control Signal /w a = 100
1
0
-1
-2
-3
-4
-5
-6
0 2 4 6 8 10 12 14 16 18 20
Time (s)

```
>> hw12

Kx =

  -21.8366   44.4984  -13.1420   14.9161


Kz =

  -10.0000


Kxm =

    6.9855    3.9188


ans =

  -0.3614 + 2.0983i
  -0.3614 - 2.0983i
  -2.2604 + 0.0000i
  -1.4916 + 0.7123i
  -1.4916 - 0.7123i
  -0.5000 + 0.5000i
  -0.5000 - 0.5000i

>>
```


Plant vs Reference Model /w a = 1000
Y
Yref
Time (s)
Control Signal /w a = 1000
Time (s)

```
>> hw12

Kx =

  -46.1684   74.5725  -26.5484   19.3096


Kz =

  -31.6228


Kxm =

   18.4937    8.6785


ans =

  -0.6186 + 2.4483i
  -0.6186 - 2.4483i
  -2.6247 + 0.0000i
  -1.9310 + 1.2497i
  -1.9310 - 1.2497i
  -0.5000 + 0.5000i
  -0.5000 - 0.5000i
```


Plant vs Reference Model /w a = 10000
Y
Yref
Time (s)
Control Signal /w a = 10000
Time (s)

```
>> hw12

Kx =

 -108.9701  121.9114  -54.0329   24.6891


Kz =

 -100.0000


Kxm =

   48.4817   19.2179


ans =

  -0.8805 + 2.9917i
  -0.8805 - 2.9917i
  -3.1769 + 0.0000i
  -2.4689 + 1.7225i
  -2.4689 - 1.7225i
  -0.5000 + 0.5000i
  -0.5000 - 0.5000i
```

```matlab
%% Reference model
Am = [0 1; -0.5 -1]; Bm = [0; 1]; Cm = [0.5 0];

%% System
mball = 0.5; Rball = 1; mbeam = 4.8990; L = 2.4746;
[A,B] = linearizedBeamBall(mball,Rball,mbeam,L);
C = [1 0 0 0]; D = 0;

%% Augmented system /w servo-compensation
Aaug = [A, zeros(4,3); C, 0, -Cm; zeros(2,5), Am]; Baug = [B;zeros(3,1)];
Caug = [zeros(1,4), 1, zeros(1,2)];
% return

%% LQR to find Kx and Kxm
a = 1e4;
Q = a*Caug'*Caug; R = 1;
K = lqr(Aaug,Baug,Q,R);
Kx = K([1:4])
Kz = K(5)
Kxm = K([6:7])

% % Check eigen-values of plant vs reference model
eig(Aaug - Baug*K)
% return

%% Linear simulation
Acl = Aaug - Baug*K; Bcl = [zeros(4,1); 0; Bm];
Ccl = [C,0,0,0; zeros(1,4),0,Cm]; Dcl = [0;0];

Tend = 20;
t = linspace(0,Tend,10001);
G = ss(Acl,Bcl,Ccl,Dcl);
Gu = ss(Acl,Bcl,K,0);
Y = step(G,t);
U = step(Gu,t);

subplot(1,2,1);
plot(t,Y);
grid on;
legend('Y','Yref');
title_txt = ['Plant vs Reference Model /w a = ',num2str(a)];
xlabel('Time (s)'); title(title_txt);

subplot(1,2,2);
plot(t,U);
grid on;
title_txt = ['Control Signal /w a = ',num2str(a)];
xlabel('Time (s)'); title(title_txt);
```