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ECE 461
CONTROL SYSTEMS
HW #8

10/25/2020
SUNDAY

Meeting Design Specs

1) $$G(s) = \frac{1.4427}{(s+0.1617)(s+1.04)(s+5.05)}$$

Design a compensator K(s) that results in:

- No error for step input
- 2% settling time of 6s
- 20% OS

↓

Need to add integrator
Need Re{} dom. pole to be at −1.5
Need $\zeta \approx 0.456 \Rightarrow \pm 63^\circ \Rightarrow \pm j2.9279$

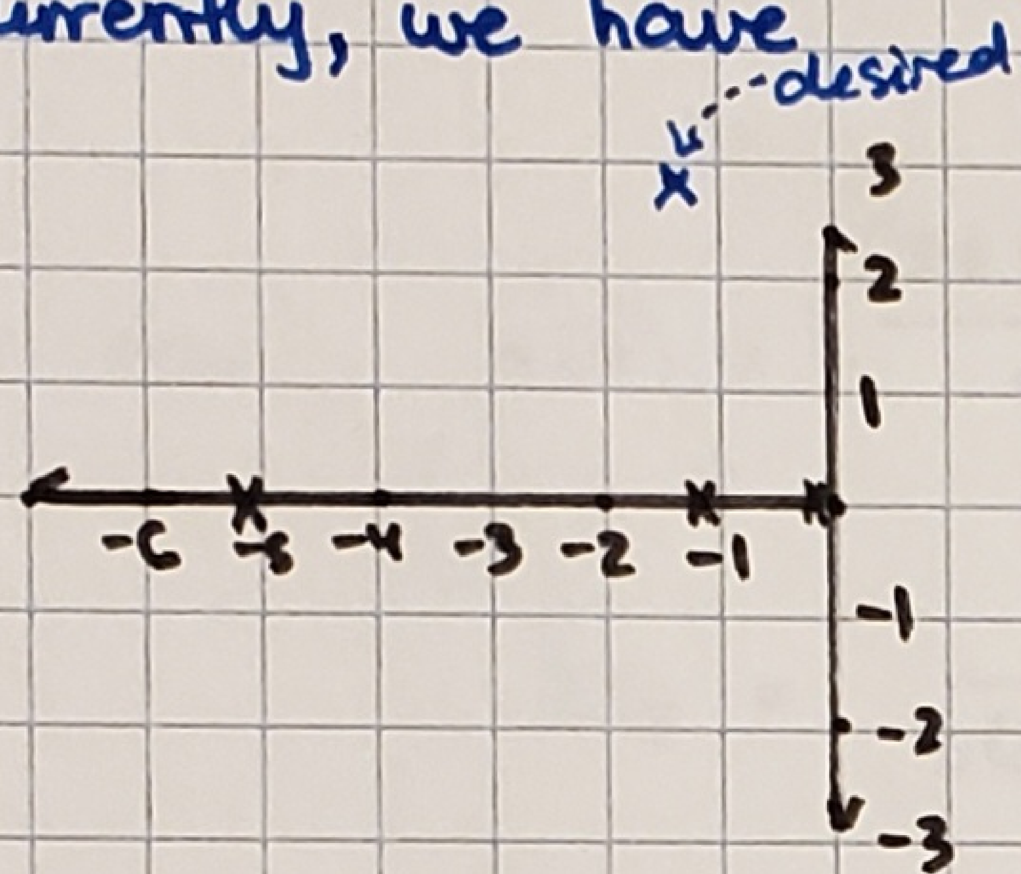
acos(ζ)

1.5tan(63°)

Dom. Pole @ $-1.5+j2.9279$

Check in sim, implement analog.

Currently, we have



- Adding the s=0 pole, it will meet up with −0.1617 pole and split, which is too slow. Add zero at −0.1617.
- Now s=0 pole meets −1.04, still no good. Add another zero @ s=−01.04
- Now s=0 pole meets −5.05, splits, and I THINK is too fast. As a quick check in MATLAB (next page): Yup, too fast.

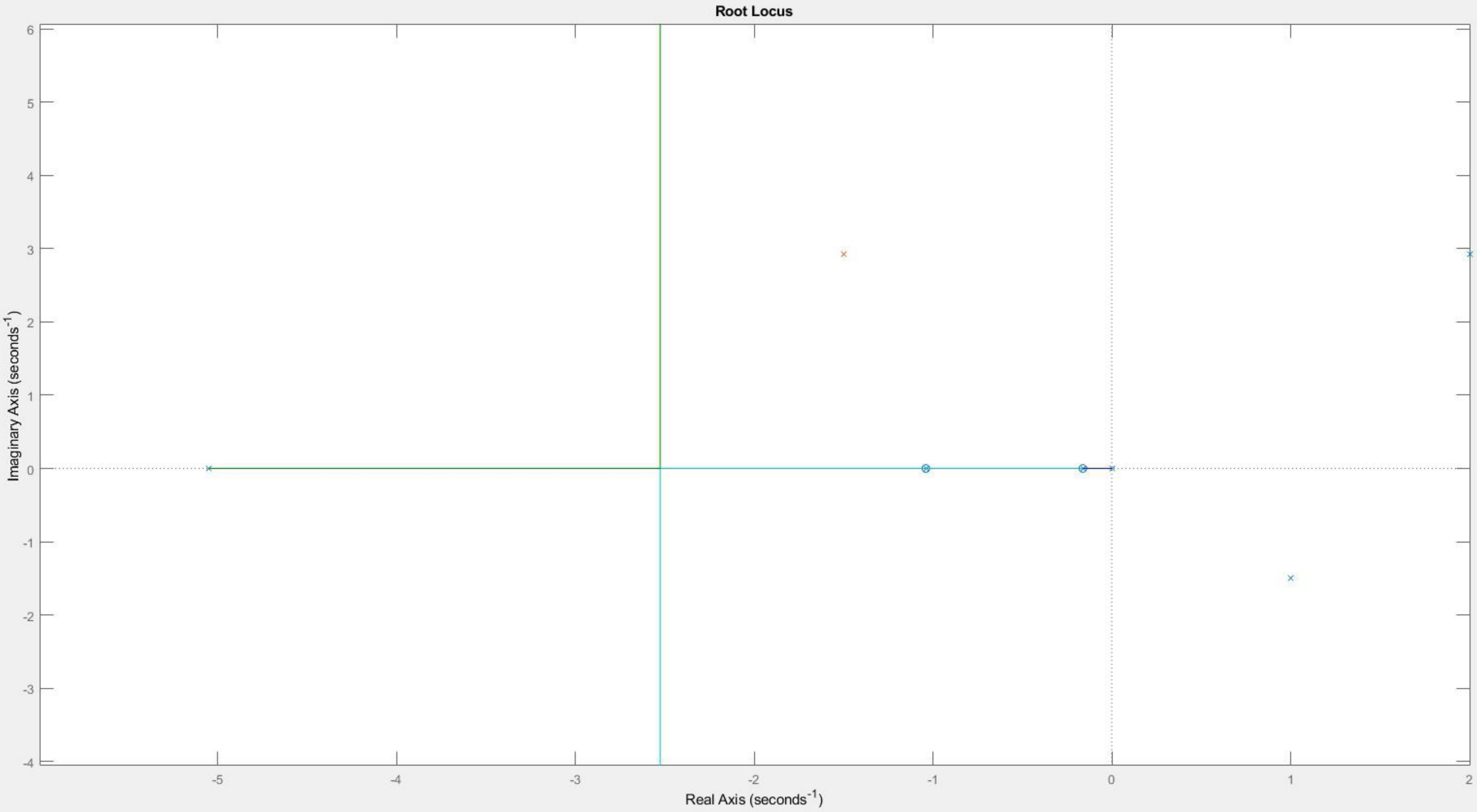
Now need to add a pole so that rlocus includes target. So,

$$K(s) = k\frac{(s+0.1617)(s+1.04)}{s(s+a)}$$

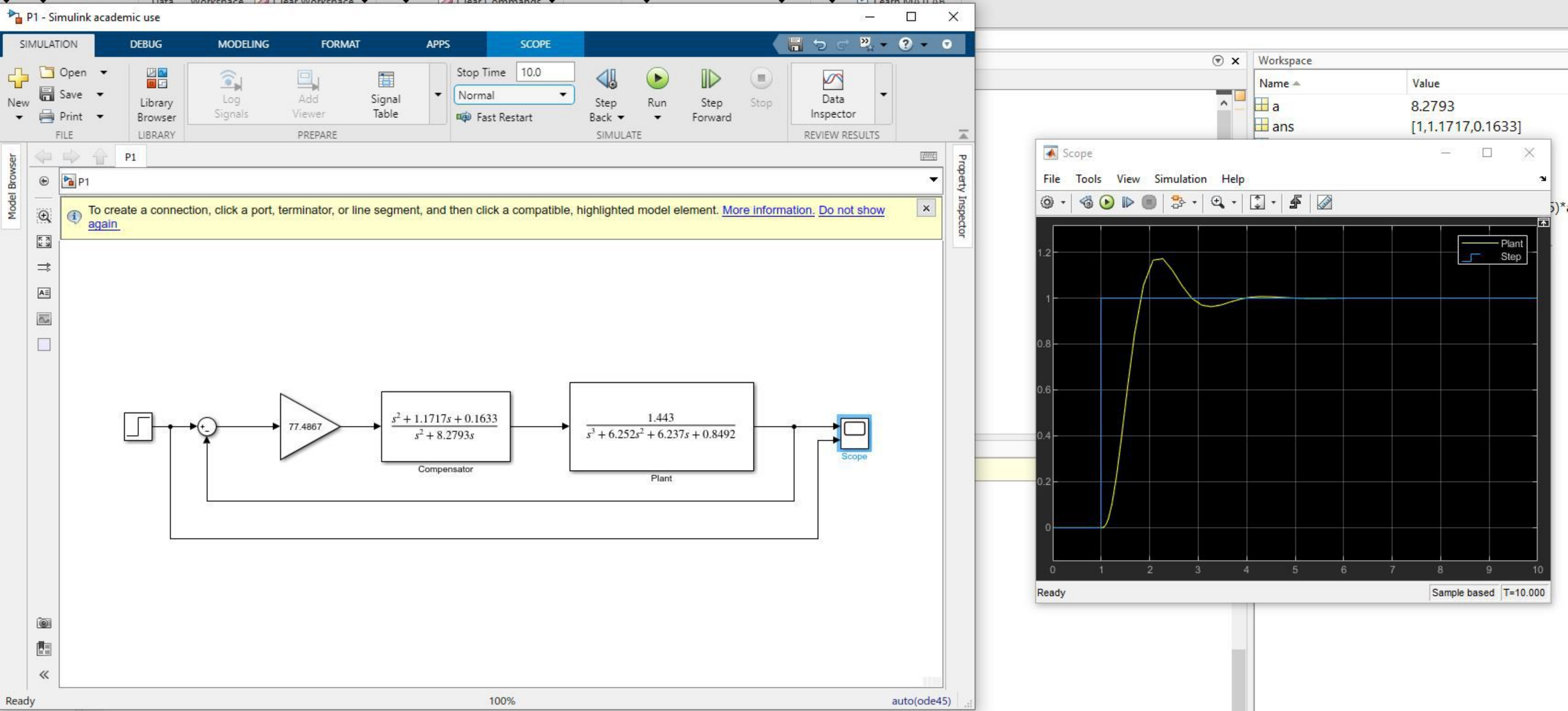
$$K(s)G(s) = \frac{1.4427k}{s(s+5.05)(s+a)}$$

$$-(\angle s + \angle s+5.05 + \angle s+a)\Big|_{s=-1.5+j2.9279} = 180^\circ$$

$$\angle s+a = -180^\circ - \angle s - \angle s+5.05$$



Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-4
-3
-2
-1
0
1
2


P1 - Simulink academic use
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Log Signals
Add Viewer
Signal Table
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Stop Time 10.0
Normal
Fast Restart
Step Back
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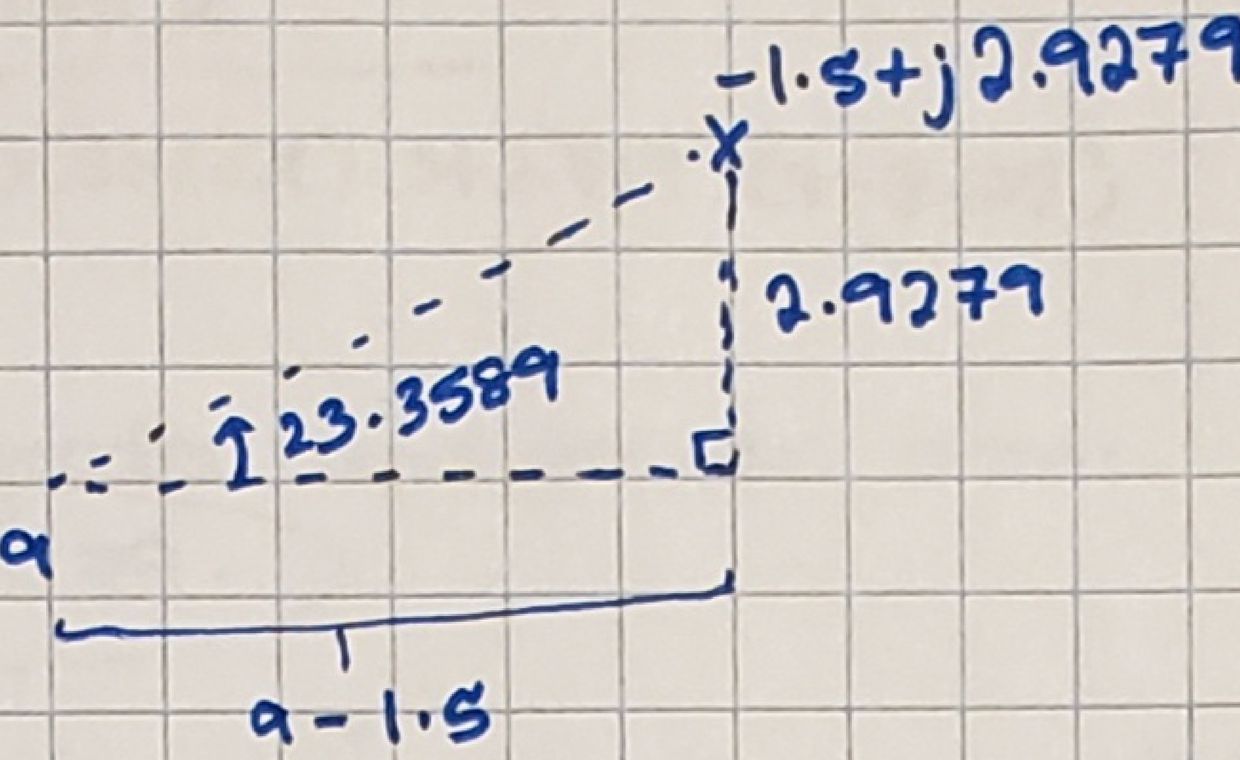
Step Forward
Stop
SIMULATE
Data Inspector
REVIEW RESULTS
P1
To create a connection, click a port, terminator, or line segment, and then click a compatible, highlighted model element. More information. Do not show again
77.4867
s^2 + 1.1717s + 0.1633
s^2 + 8.2793s
Compensator
1.443
s^3 + 6.252s^2 + 6.237s + 0.8492
Plant
Scope
Ready
100%
auto(ode45)
Workspace
Name
Value
a
8.2793
ans
[1,1.1717,0.1633]
Scope
File
Tools
View
Simulation
Help
Plant
Step
Ready
Sample based
T=10.000

6.252

6.237 0.8492

$\Rightarrow \angle s+a = 23.3589°$



$$\tan(23.3589°) = \frac{2.9279}{a-1.5} \Rightarrow \boxed{a} = \frac{2.9279}{\tan(\dots)} + 1.5 = \boxed{8.2793}$$

So, now $K(s) = k\dfrac{(s+0.1617)(s+1.01)}{s(s+8.2793)}$

$$KG = \frac{1.4427k}{s(s+5.05)(s+8.2793)} \Rightarrow k = \left.\frac{|s|\,|s+5.05|\,|s+a|}{1.4427}\right|_{s=\text{target}}$$

$$\boxed{k = 77.4867}$$

So finally:

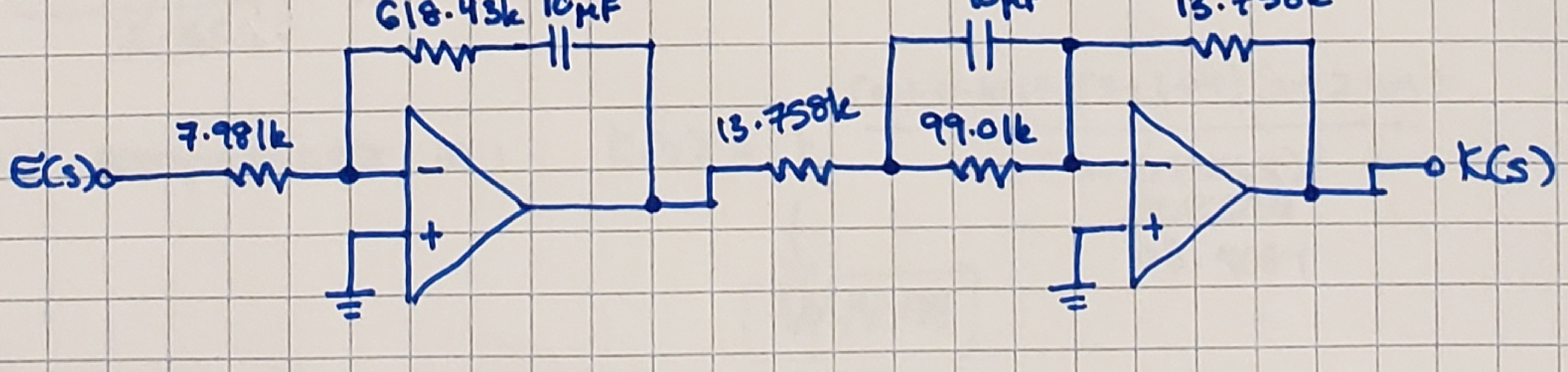
$$\boxed{K(s) = \frac{77.4867(s+0.1617)(s+1.01)}{s(s+8.2793)}} \qquad \frac{77.4867(s^2+1.1717s+0.1633)}{s^2+8.2793s}$$

DC: 0.1220

HF: 1

Simulation checks out! (:

To implement, I'll write $K(s) = \left(77.4867\dfrac{s+0.1617}{s}\right)\left(\dfrac{s+1.01}{s+8.2793}\right)$



$$\frac{1}{(10\mu)(0.1617)} \approx 618.43k$$

$$\frac{R_2}{R_1} = 1 \Rightarrow R_2 = R_1 \qquad \frac{R_2}{R_1+99k} = 0.122 \rightarrow \qquad R_2 = 0.122R_2 + 0.122(99k)$$

$$R_2 = \frac{0.122(99k)}{1-0.122}$$

1	8.971	23.24	17.81	2.309
1	3.921	3.436	0.4572	
1	104.8	2747	0	

Systems with Delays

$$G(s) = \frac{1.4427\, e^{-0.1s}}{(s+0.1617)(s+1.04)(s+2.719)(s+5.08)}$$

Design a compensator:

- No error for step
- 2% Ts
- 20% OS

Will need to add integrator and ensure dom. poles at $s = -1.5 \pm j2.9279$.

Using 4th order approx. for $e^{-0.1s}$, the plant overall is:

Poles WAY out

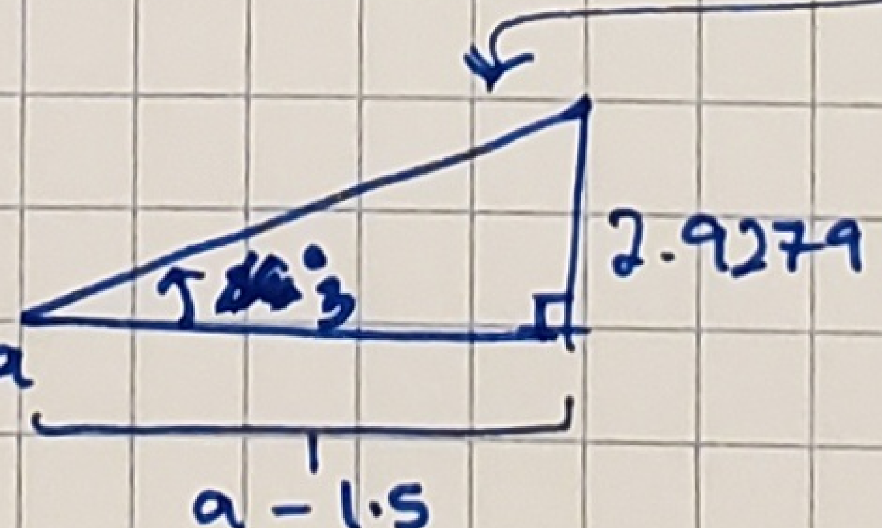
$$G(s) \approx \frac{1.4427(s^2 - 115.8s + 3656)(s^2 - 84.15s + 4595)}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)(s^2+115.8s+\cdots)(\cdots)}$$

Looking at resulting rlocus, I think I def. need to cancel out -0.1617 and -1.04. and -2.719. Hopefully I can keep 5.08. So,

Nope. Num. Let's keep the $e^{-0.1s}$. Easier to work with and more accurate.

$$K(s)G(s)(s+a)^2\Big|_{s=-1.5+j2.9279} =$$

Need $2\angle s+a = 6°$ ⇒ $\angle s+a = 3°$



$$a = \frac{2.9279}{\tan(3°)} + 1.5$$

$\boxed{a \approx 57.4084}$

So, our compensator is: $K(s) = k\,\dfrac{(s+0.1617)(s+1.04)(s+2.719)}{s(s+57.4084)^2}$

k = 23,484

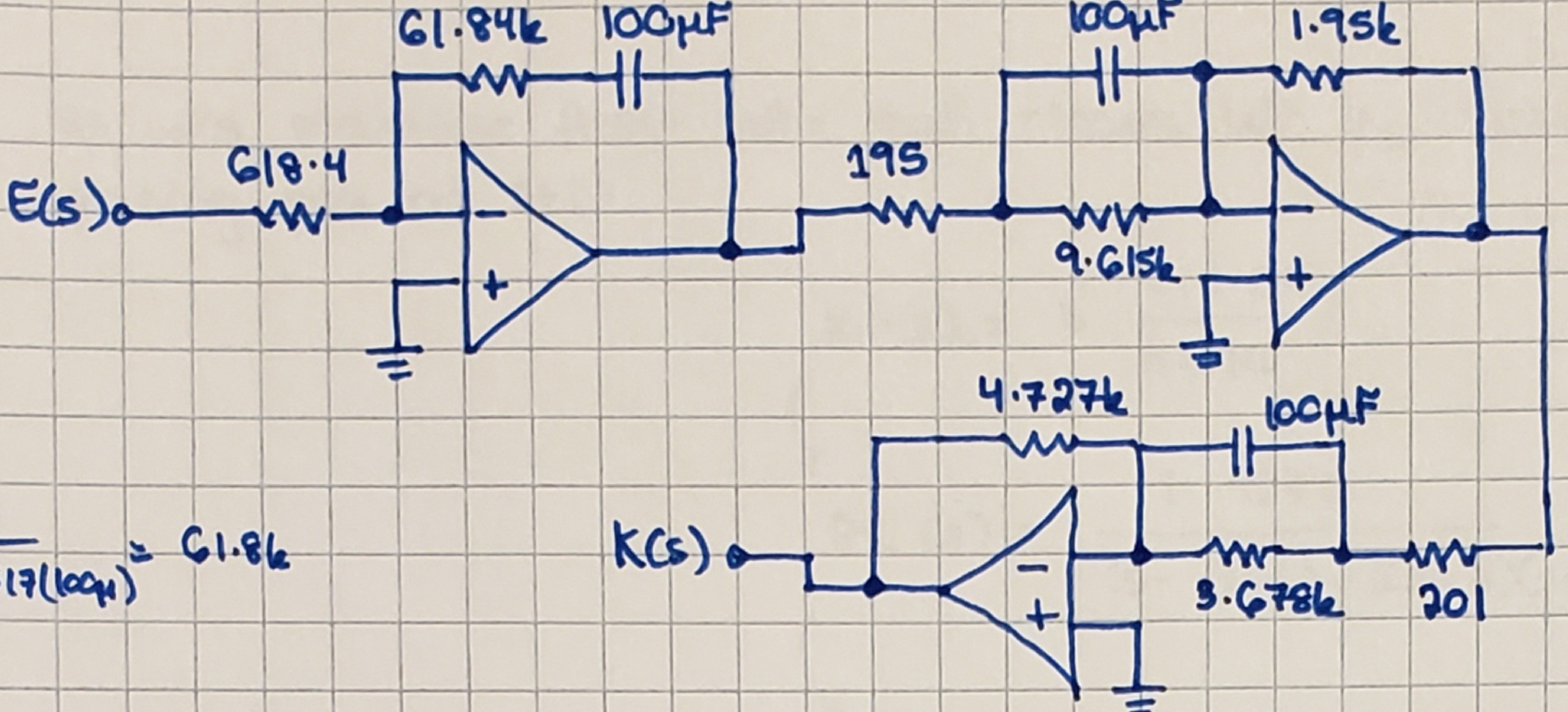
~~so for some reason~~

DC : 0.1985
HF : 10

DC : 1.2185
HF : 23.484

Op Amp:

$$K(s) = \underbrace{\left(100\,\frac{s+0.1617}{s}\right)}_{(PI)}\underbrace{\left(10\,\frac{s+1.04}{s+52.4048}\right)}_{(Lead)}\underbrace{\left(23.484\,\frac{s+2.719}{s+52.4048}\right)}_{(Lead)}$$



$\frac{1}{0.1617(100\mu)} = 61.8k$

$R_2 = 10R_1$

$\frac{10R_2}{R_1+9.6k} = 0.1985$

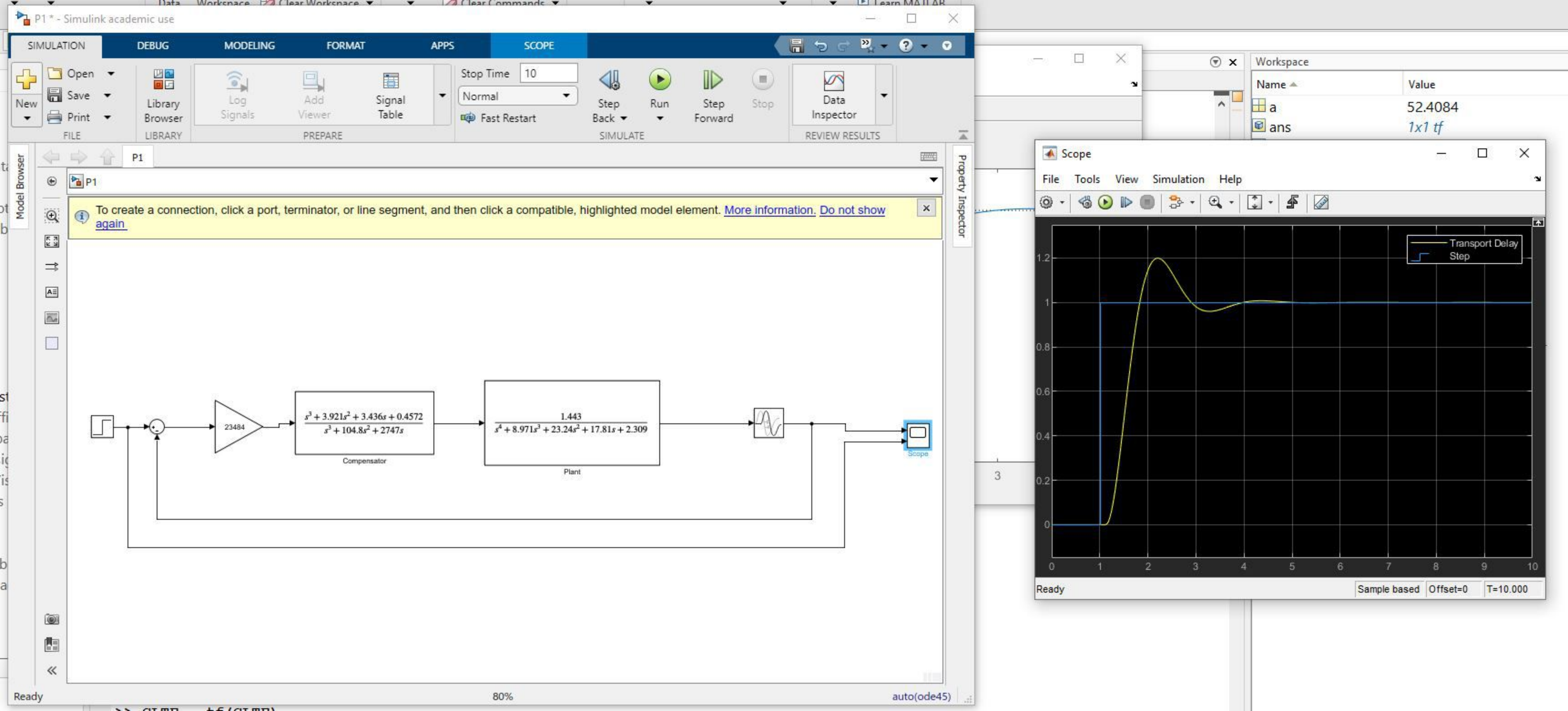
$\downarrow$

$R_1 = \frac{0.19\;9.6k}{10-0.1985} = 194.7$

$\frac{23\cdot R_1}{R_1+3.6k} = 1.21$

$\downarrow$

$R_1 = \frac{1.21\;3.6k}{23-1.21} = 201$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Property Inspector
P1
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23484
$\frac{s^3 + 3.921s^2 + 3.436s + 0.4572}{s^3 + 104.8s^2 + 2747s}$
Compensator
$\frac{1.443}{s^4 + 8.971s^3 + 23.24s^2 + 17.81s + 2.309}$
Plant
Scope
Ready
80%
auto(ode45)
Workspace
Name
Value
a
52.4084
ans
1x1 tf
Scope
File
Tools
View
Simulation
Help
Transport Delay
Step
1.2
1
0.8
0.6
0.4
0.2
0
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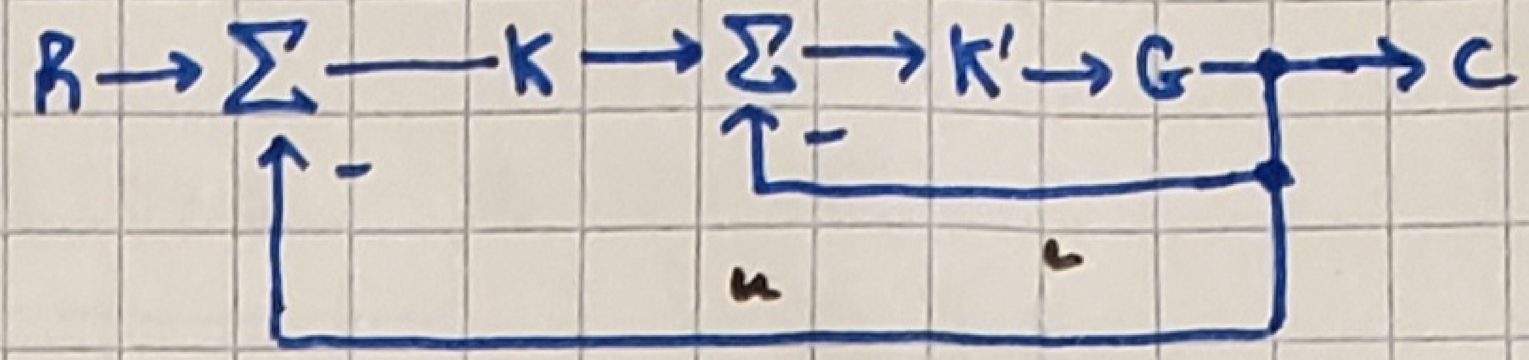
9
10
Ready
Sample based
Offset=0
T=10.000

Unstable Systems



3)

$$G(s) = \frac{1.4427}{(s-0.1617)(s+1.04)(s+2.719)(s+5.05)}$$

So, let's stabilize first. Let's pull rlocus left by cancelling s+1.04 and placing pole at s+10.

$$K'(s) = k' \frac{s+1.04}{s+10}$$

$$GK'(s) = \frac{1.4427k'}{(s-\cdots)(s+2.\cdots)(s+5)(s+10)}$$

Looking at corresponding root locus...
I see that 0.1617 pole meets with -2.719 pole and splits. Breakaway is around $s = -0.9333$, and corresponding k' value is $\approx 50.5877 \Rightarrow$ I'll go with k' = 50

So,

$$\boxed{K'(s) = 50 \frac{s+1.04}{s+10}}$$

Closed-loop TF!

$$\boxed{G'(s) = \frac{72.135}{(s+9.783)(s+5.932)(s+1.075)(s+0.7981)}}$$

Now to meet specs, I need to add an integrator and rlocus needs to include $-1.5+j3$. That for sure means eliminate 1.075, 0.7981. So,

$$K(s) = k \frac{(s+1.075)(s+0.7981)}{s(s+a)}$$

So now $KG'(s+a)\big|_{s=-1.5+j3} \approx 0.4684\angle -169.9370° \Rightarrow \angle s+a = 10.0630°$

$$a = \frac{3}{\tan(\cdots)} + 1.5$$

$$\boxed{a \approx 17.9988}$$

Now to get k :

$$k = \frac{|s||s+9\ldots||s+3.\ldots|\cancel{\ldots}|s+a|}{72.135}\Bigg|_{s=-1.5+j3}$$

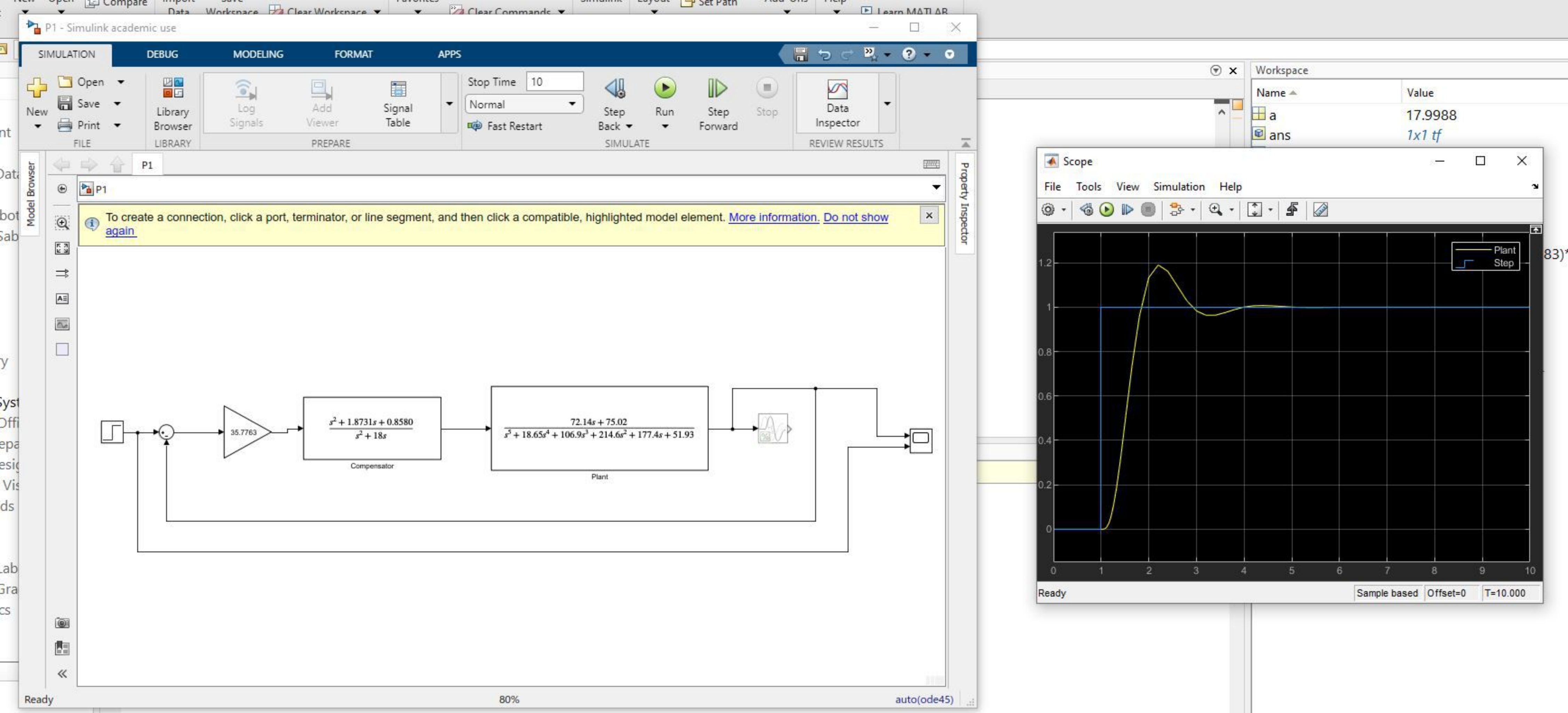
$$\boxed{k = 35.7763}$$

$$K(s) = \frac{\cancel{35.78s^2 + 67.91s + 30.69}}{} \quad \frac{s^2 + 1.8731s + 0.8580}{s^2 + 18s}$$

$$G'(s) = \frac{72.14s + 75.02}{s^5 + 18.65s^4 + 106.9s^3 + 214.6s^2 + 177.4s + 51.93}$$

Sim is good!

Loved this assignment!!



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35.7763
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s^2 + 18s
Compensator
72.14s + 75.02
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Plant
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