ABDULLAH ALMOSALAMI ECE461 HW10 (REALLY LATE) 12/16/2020 Wednesday

$T = 0.5s$. $G(s) = \dfrac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}$ 18.9707 23.2351 17.8070 2.3091

1) Design an I controller $C(z) = K\dfrac{z}{z-1}$ ⇒ 20% OS

Using pole-zero matching in MATLAB, (c2d)

$$G(z) = 0.0016816\,\frac{(z+1)^3}{(z-0.9223)(z-0.5945)(z-0.2568)(z-0.08006)}$$

To get 20% OS, $\zeta = 0.4560$ contour in z-plane /w rlocus, I get intersection at $0.9679 + j\,0.0582$, which corresponds to $K = 0.0947$. So,

$$C(z) = 0.0947\left(\frac{z}{z-1}\right)$$

Checking in MATLAB using stepinfo() ... Yup! 19.4549% OS. Other info:

2% $T_s \approx 63s$
Step Error: 0%
Max u(t) value = 2.0
Final u(t) value: ~1.6

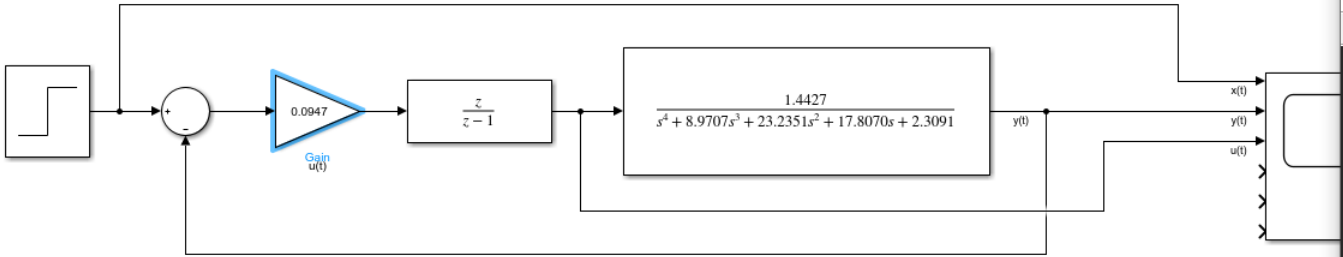
System is STRONGLY low pass looking at Bode.

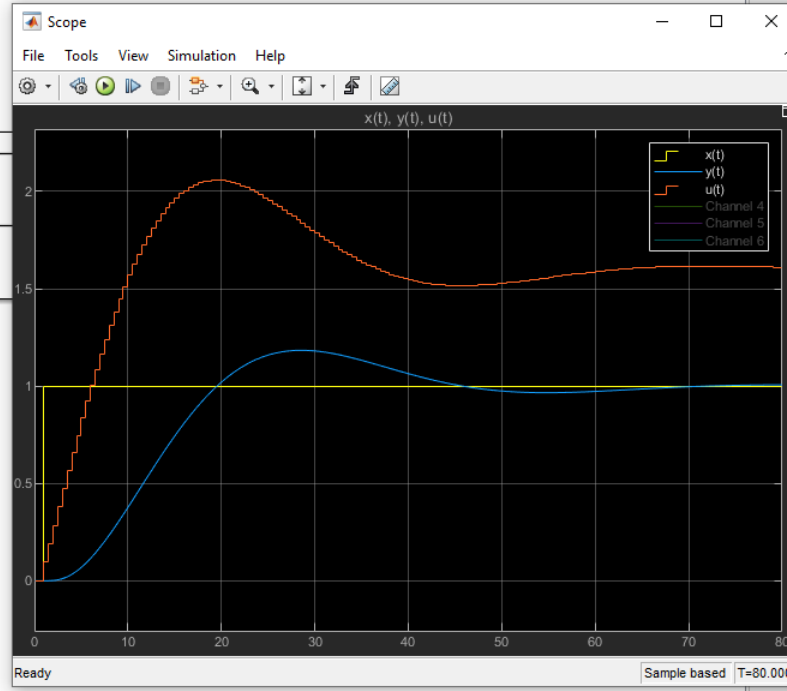
On Simulink (next page):

Part1

To create a connection, click a port, terminator, or line segment, and then click a compatible, highlighted model element. More information. Do not show again

I Compensation





2) Now PI.

$$C(z) = K\left(\frac{z-9}{z-1}\right)$$

$$G(z) = 0.001618\,\frac{(z+1)^3}{(z-0.9223)(z-0.5945)(z-0.2568)(z-0.08006)}$$

Let's cancel pole at 0.9223 :

$$C(z) = K\left(\frac{z-0.9223}{z-1}\right)$$

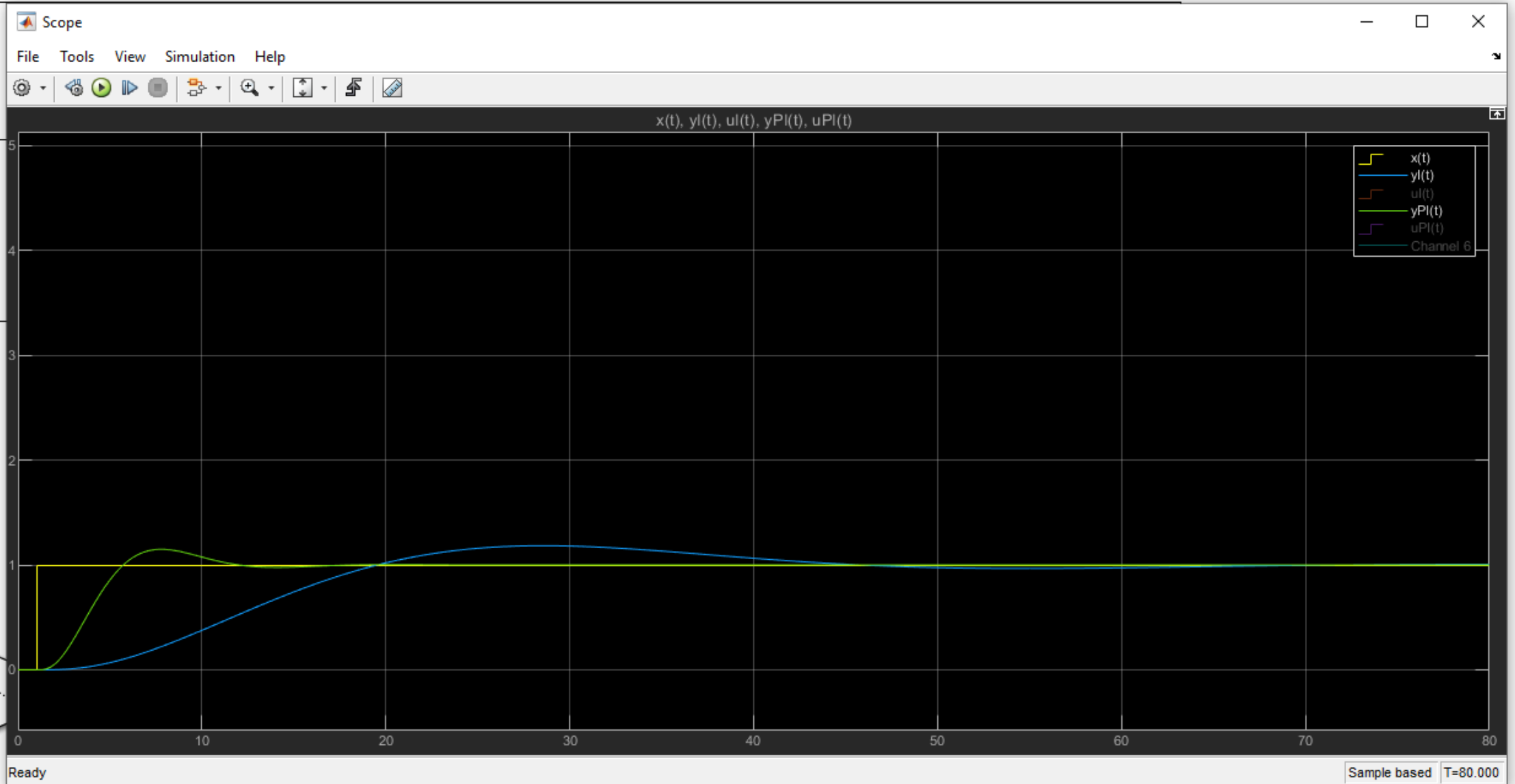
Using rlocus for $\zeta = 0.4560$ contour : $0.8525 \pm j\,0.2175 \Rightarrow K = 4.1357$

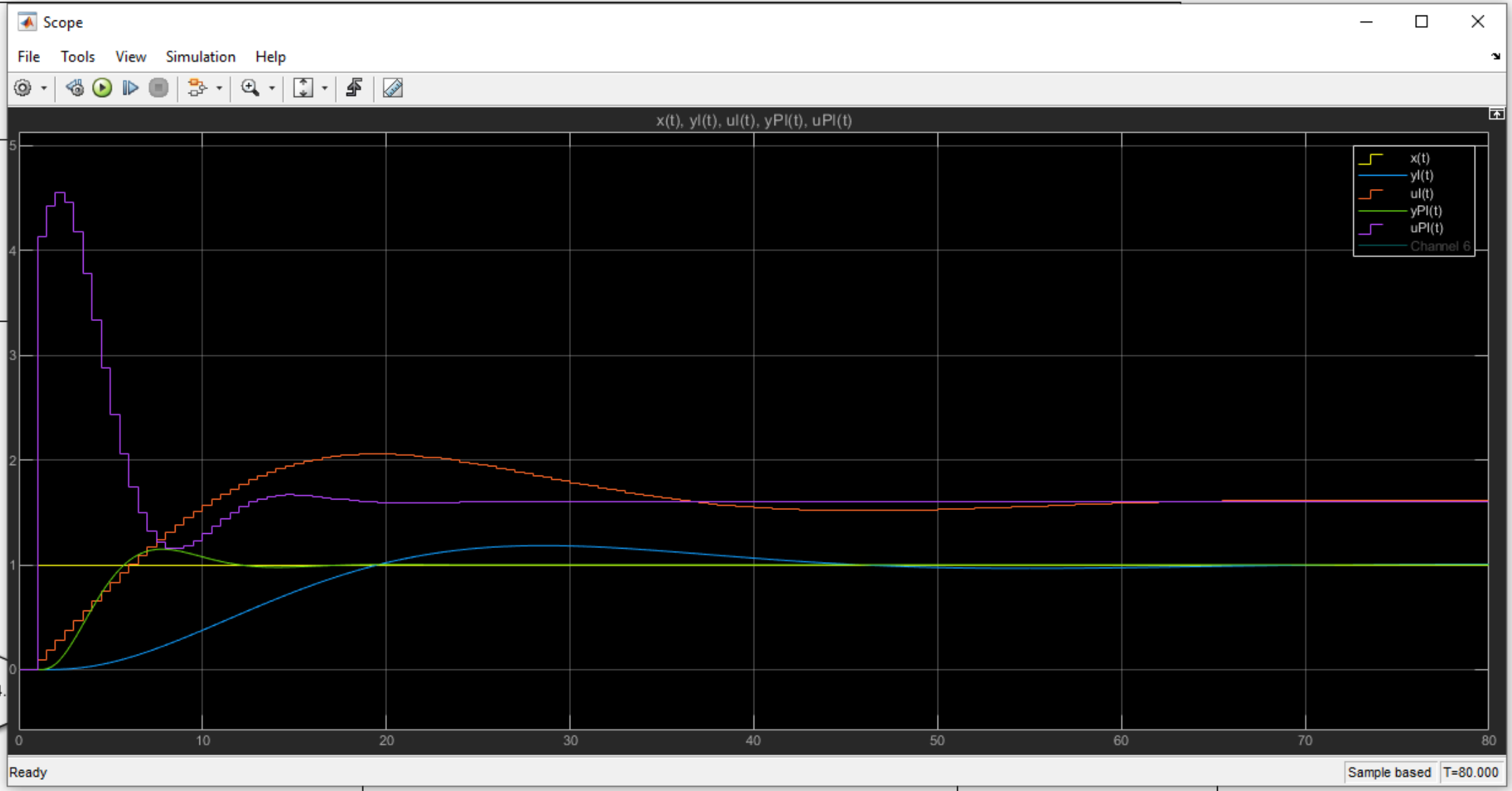
stepinfo: 2% T_s = 15.5s (Faster than I)
OS = 19.4243%.

u(t) max : = 4.5 (more than I)
Final u(t) : = 1.6 (same as I)

So PI slams on the gas harder and then settles down at same final value as I-comp.

Simulink:


Scope
File
Tools
View
Simulation
Help
x(t), yI(t), uI(t), yPI(t), uPI(t)
x(t)
yI(t)
uI(t)
yPI(t)
uPI(t)
Channel 6
5
4
3
2
1
0
0
10
20
30
40
50
60
70
80
Ready
Sample based
T=80.000


Scope
File
Tools
View
Simulation
Help
x(t), yl(t), ul(t), yPI(t), uPI(t)
x(t)
yl(t)
ul(t)
yPI(t)
uPI(t)
Channel 6
0
1
2
3
4
5
0
10
20
30
40
50
60
70
80
Ready
Sample based
T=80.000

$$G(z) = 0.001618 \frac{(z+1)^3}{(z-0.9223)(z-0.5945)} \cdots$$

3) Now PID:

$$C(z) = K \frac{(z-a)(z-b)}{z(z-1)}$$

Cancelling two slowest poles and proceeding as before...

$$C(z) = K \frac{(z-0.9223)(z-0.5945)}{z(z-1)} \rightarrow 1 \quad -1.5168 \quad 0.5483$$

Intersection: $0.744425 \pm j\,0.322222 \Rightarrow K_c = 15.3375$

0.70204 ± j0.35506

stepinfo:

2% T_s : 9.5s (faster than PI)

OS : 19.0351 %

$u(t)_{mx}$: ~15 (way more than PI)

$u(t)_{end}$: ~1.6 (same)

If I increase T to 800ms, and do the same process, I get

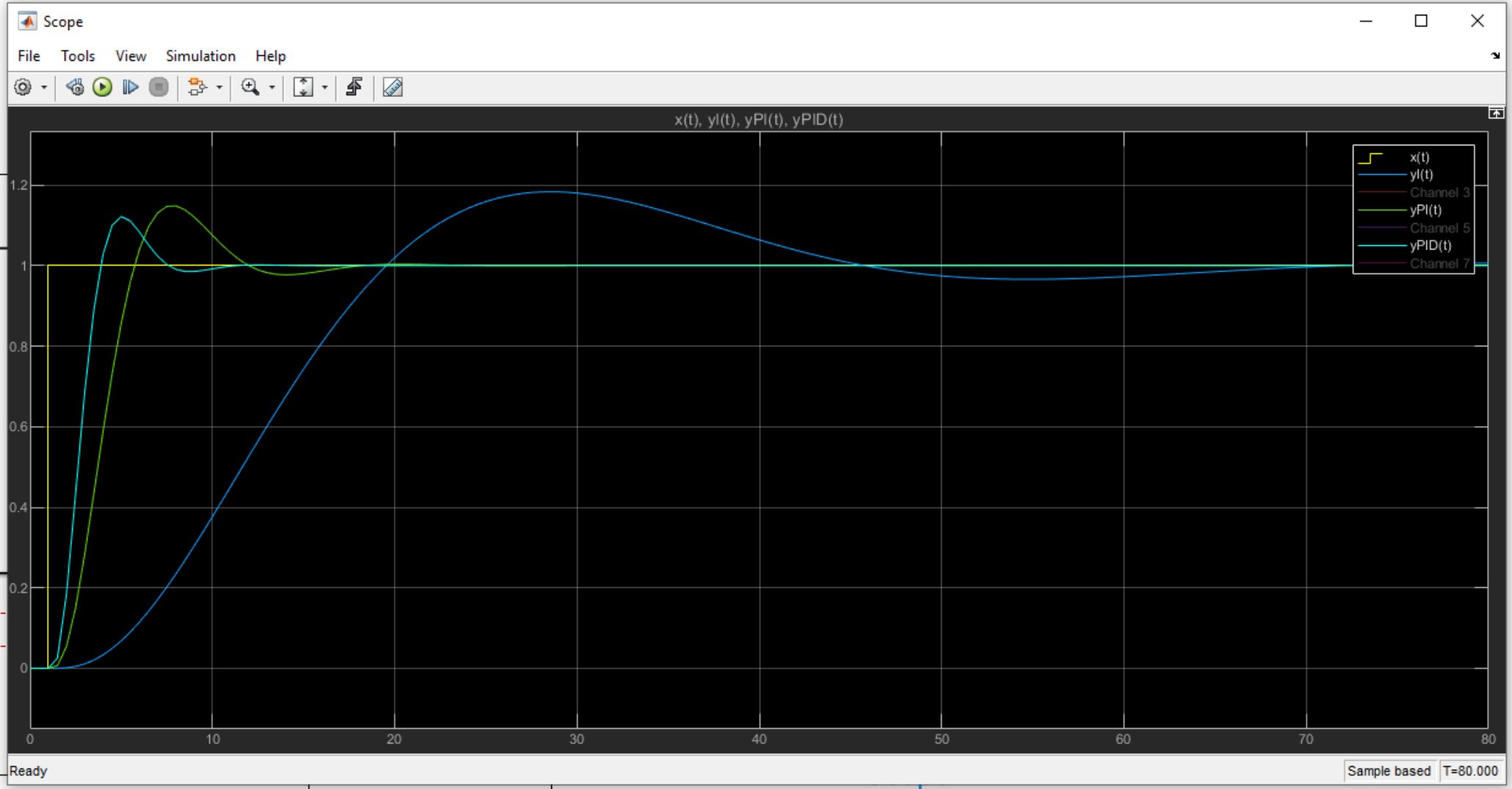
2% T_s : 13.6s

OS : 18.9117 %

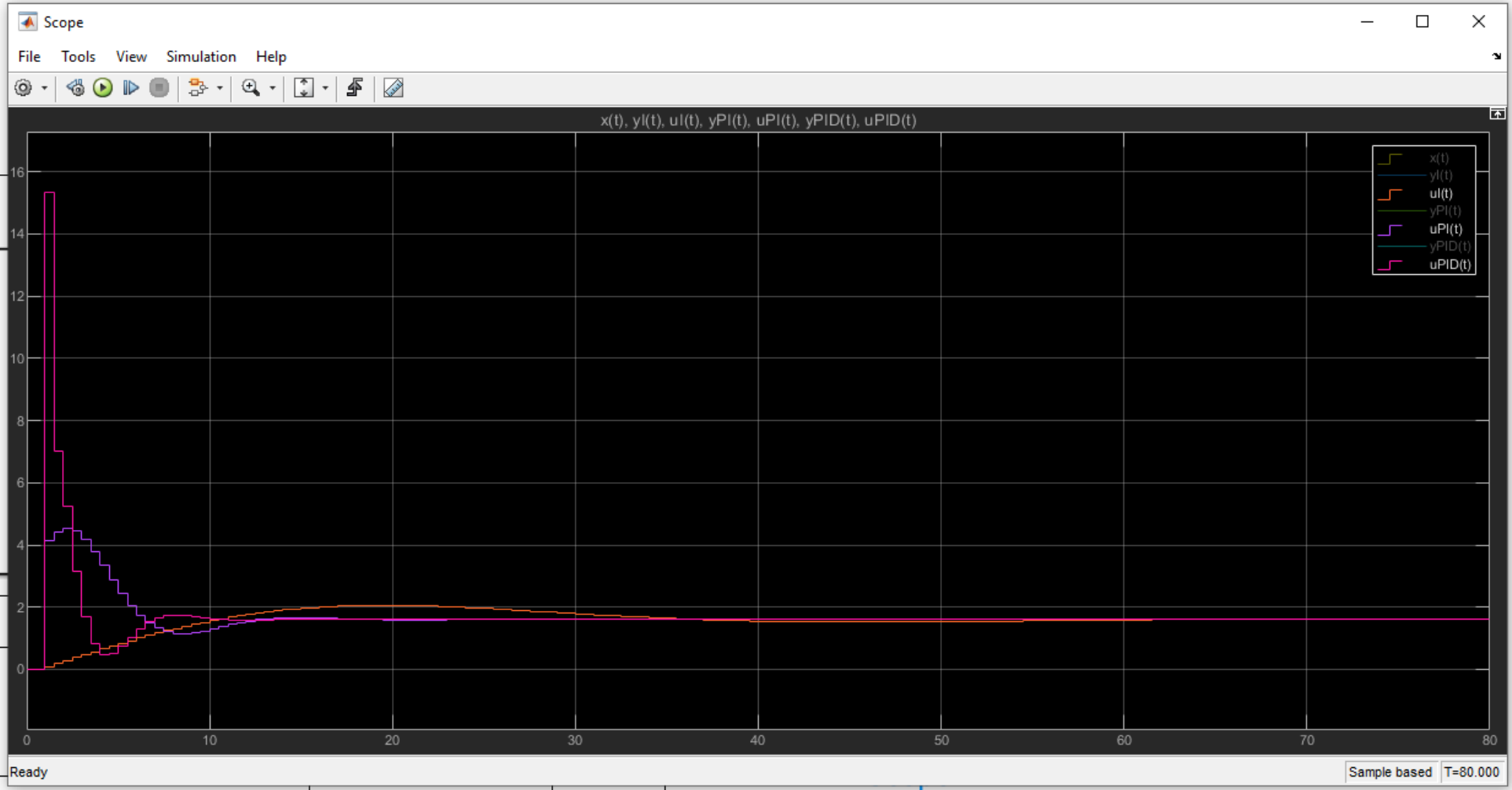
u_{MAX} : ~8

So, slower but less u_{MAX}.

Simulation: (I have also attached u(t) signals)



line segment, and then click a compatible, highlighted model element. More information. Do not show again



.3091

MEETING DESIGN SPEC

4) Now design a comp. for ($\omega T = 0.5$)

- No step err
- 20% OS
- 2% $T_s = 10s$

Recall,

$$G(z) = 0.0016816 \frac{(z+1)^3}{(z-0.9223)(z-0.5945)(z-0.2568)(z-0.08006)}$$

Well actually, the previously designed PID controller works!

5) Now $\omega T = 0.15$.

ok so now

$$G(z) = \quad \cdot 11.68 \times 10^{-6} \frac{(z+1)^3}{(z-0.984)(z-0.9012)(z-0.7619)(z-0.6035)}$$

To meet spec, I need rlocus to include intersection of $\zeta = 0.4560$ contour and $T_s = 10s$ circle, which occurs at $\sim 0.957858 \pm j\,0.074937$.

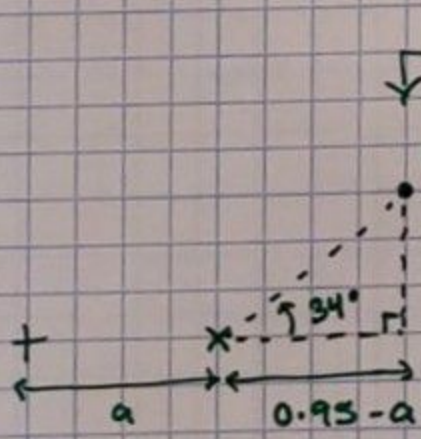
Let's ~~first~~ try

$$C(z) = K \frac{(z-0.984)(z-0.9012)}{(z-1)(z-a)}$$

To be on rlocus, $CG|_{z=0.95+j0.07}$ needs to have an angle of 180°.

ω_0 $z-a$ term, $CG|_{z=\cdots} \approx -0.0111 - j0.0076 \Rightarrow \angle -145.6183°$

So $\angle(z-a)$ terms needs to be 34.3817°



$$a = 0.95 - \frac{0.07}{\tan(34°)} \approx 0.8484$$

So overall,

$$C(z) = K \frac{(z-0.984)(z-0.9017)}{(z-1)(z-0.8484)}$$

And K @ target point is 9.8597

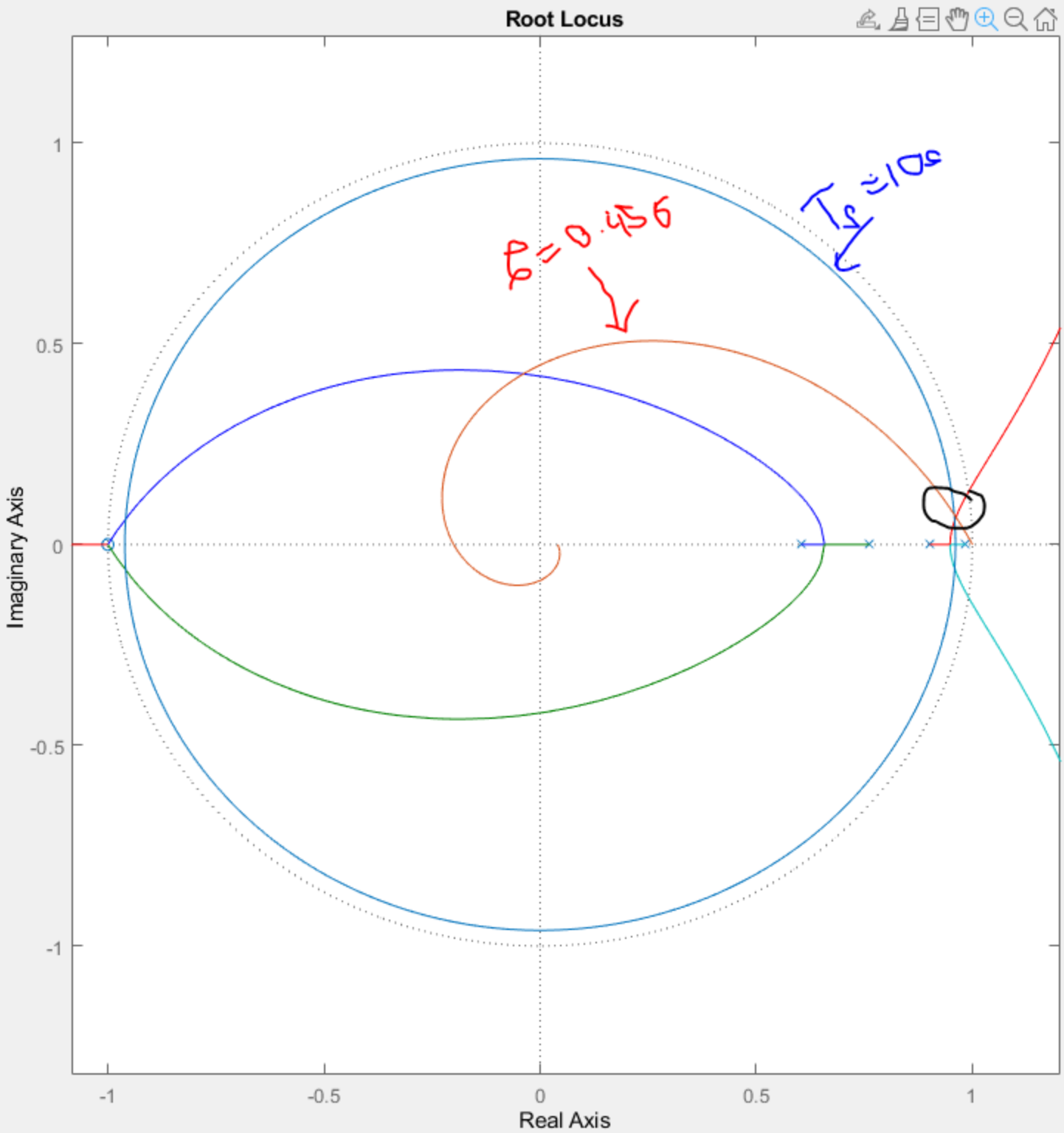
Results:

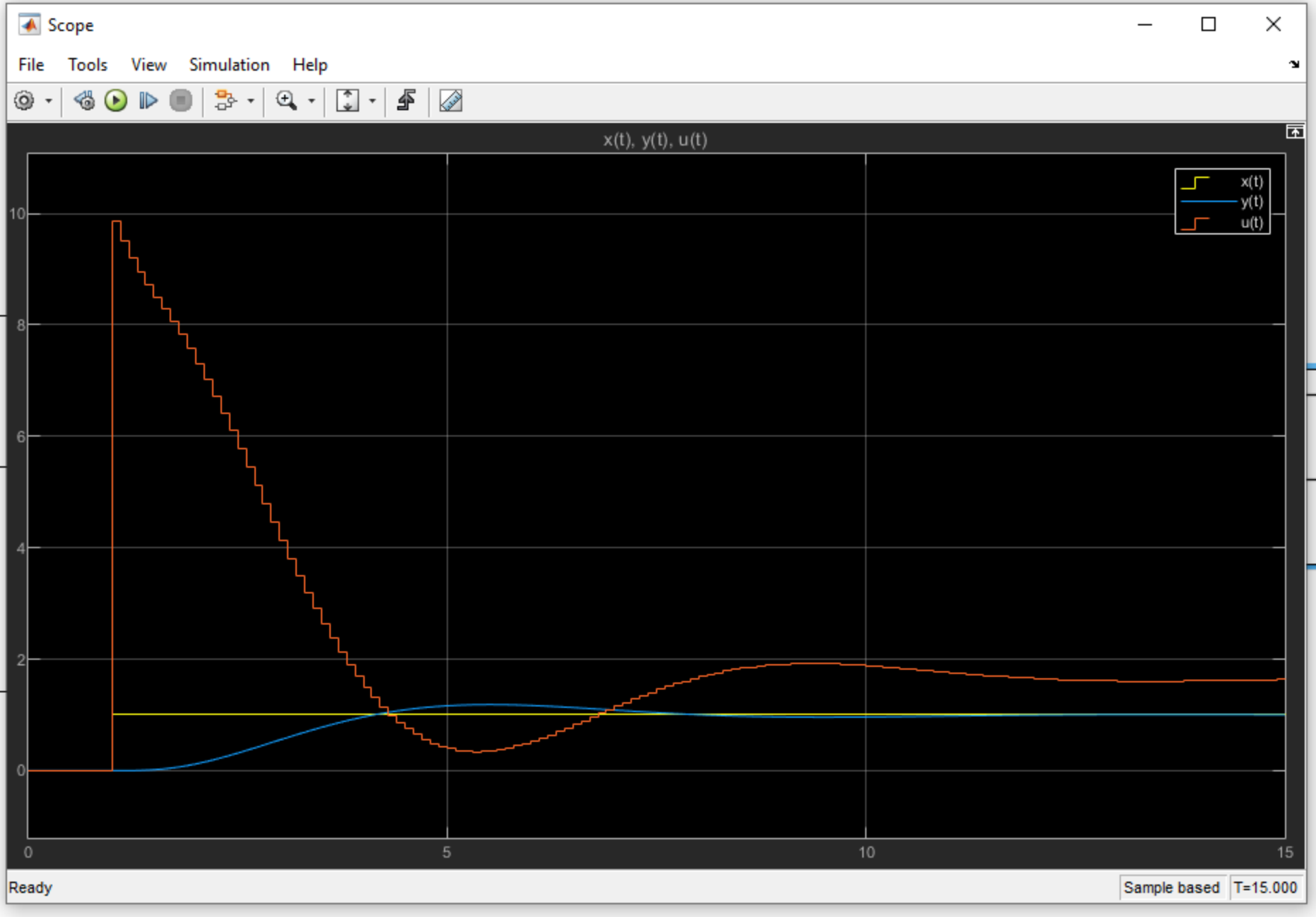
T_s = 10s (very close to PID design)

OS : 19.07%

u_{max} : 10 (actually less than PID!)

u_{final} : 1.6 (same as all)


Root Locus
Imaginary Axis
Real Axis
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
ζ = 0.456
Ts = 15s


Scope
File
Tools
View
Simulation
Help
x(t), y(t), u(t)
x(t)
y(t)
u(t)
10
8
6
4
2
0
0
5
10
15
Ready
Sample based
T=15.000
Scope