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ECE 463
MODERN CONTROL
HW #7

03/28/2021
SUNDAY
(LATE AF)

Ball & Beam system /w $m_{ball} = 0.5kg$, $J_{beam} = 2kg \cdot m^2$.

Full-State Feedback /w Constant Disturbances

1) Perform nonlinear simulation /w FSF

(a) $R = 1m$ and $m_{ball} = 0.5kg$

* I want to $\boxed{\text{NOTE!}}$ I have derived a slightly different model of a ball & beam system - this derivation is attached at the end along /w the corresponding MATLAB code. *

I'm going to assume $m_{beam} = \frac{8}{3} kg \approx 2.667kg$, $L_{beam} = L = 3m \Rightarrow J_{beam} = \frac{1}{12} m L^2 = 2 kg \cdot m^2$

$R_{ball} = 5cm$

This gives me the linearized model:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -9.8 & 0 & 0 \\ -2.45 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \end{bmatrix}$$

and since we're tracking $r(t)$, $C = [1 \ 0 \ 0 \ 0]$, $D = 0$.

Using $u = k_r R - K_x X$, place Poles (...) $\Rightarrow$ $K_x = [-5.8375, -30.5500, -2.42860, 14]$, $K_r = -0.9375$

des-poles $= -0.5 \pm j0.5244$, -2.5, -3.5

for 2% $T_s = 8s$, 5% OS

WEIRD! So I ran the nonlinear simulation and it was unstable! Screenshot showing this on next page. → Robust control?

So let me try des-poles $= [-2/3, -2, -3, -4] \Rightarrow$ $K_x = [-8.1653, 64, -8.4354, 19.\bar{3}]$, $K_r = -3.2653$

↳ worked fine!

Now try $m_{ball} = 0.6kg$ /w same gains...

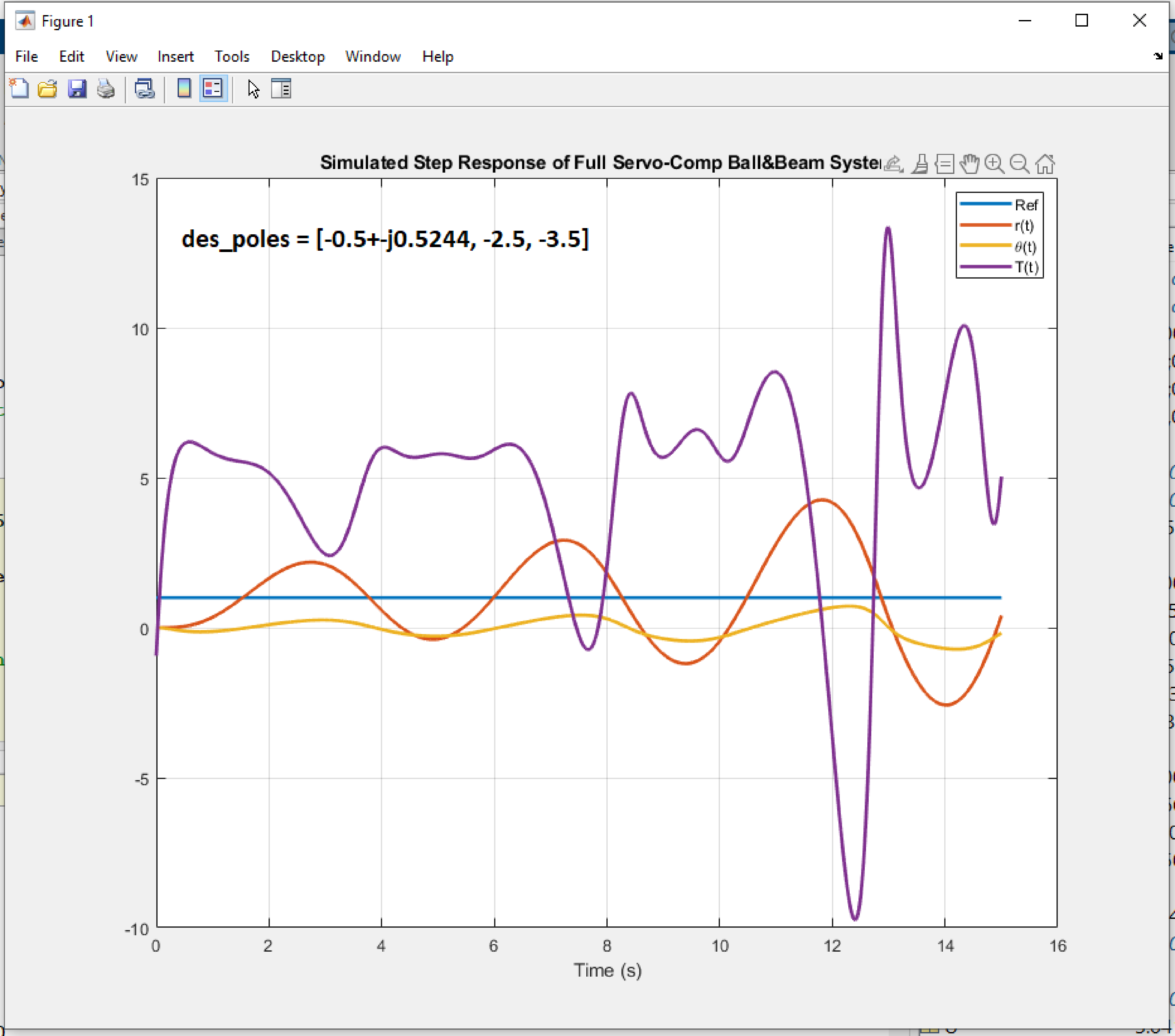
Figure 1
File Edit View Insert Tools Desktop Window Help
Simulated Step Response of Full Servo-Comp Ball&Beam Syste
des_poles = [-0.5+-j0.5244, -2.5, -3.5]
Ref
r(t)
θ(t)
T(t)
15
10
5
0
-5
-10
0
2
4
6
8
10
12
14
16
Time (s)

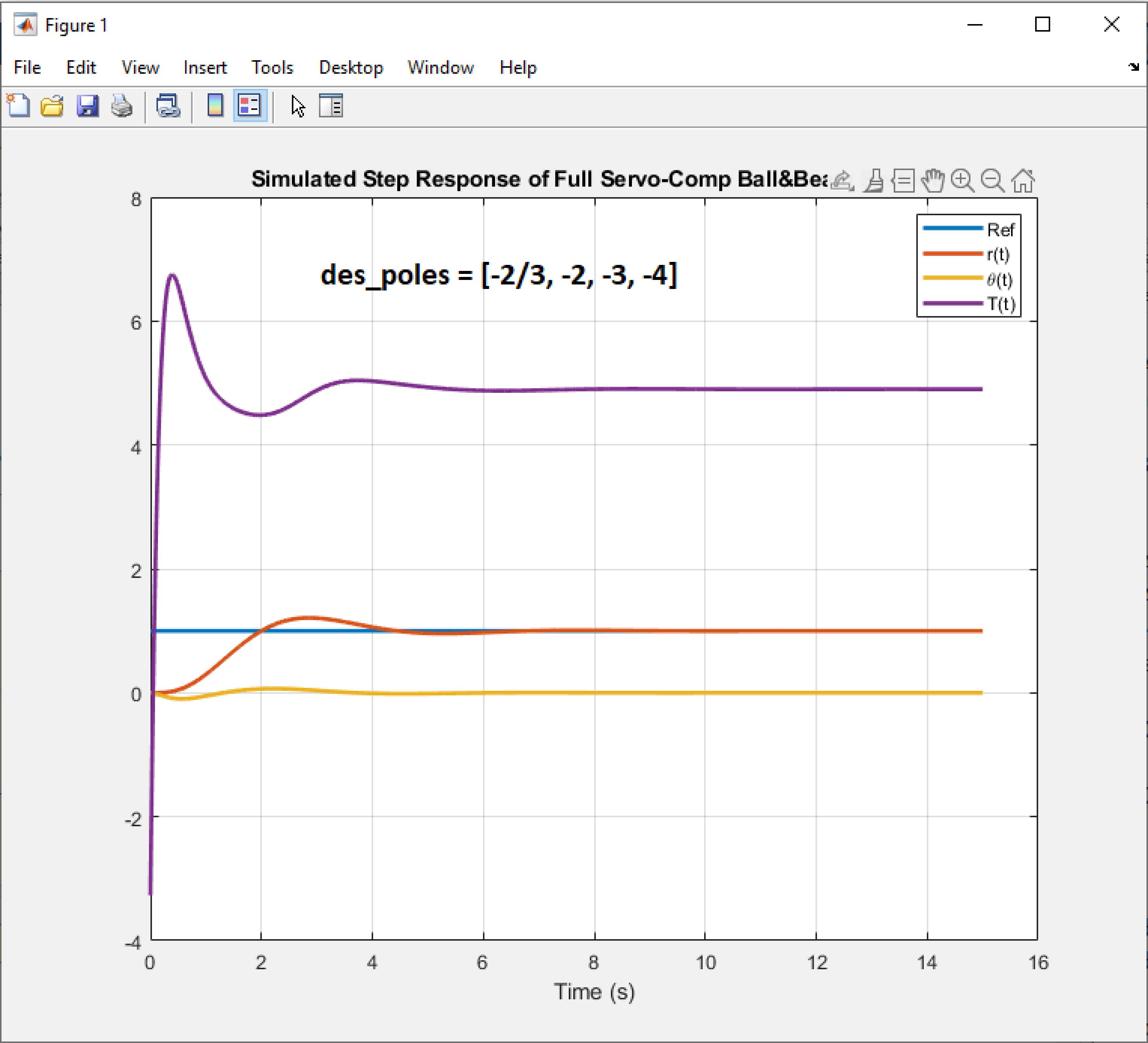
Figure 1
File Edit View Insert Tools Desktop Window Help
Simulated Step Response of Full Servo-Comp Ball&Be
des_poles = [-2/3, -2, -3, -4]
Ref
r(t)
θ(t)
T(t)
8
6
4
2
0
-2
-4
0 2 4 6 8 10 12 14 16
Time (s)

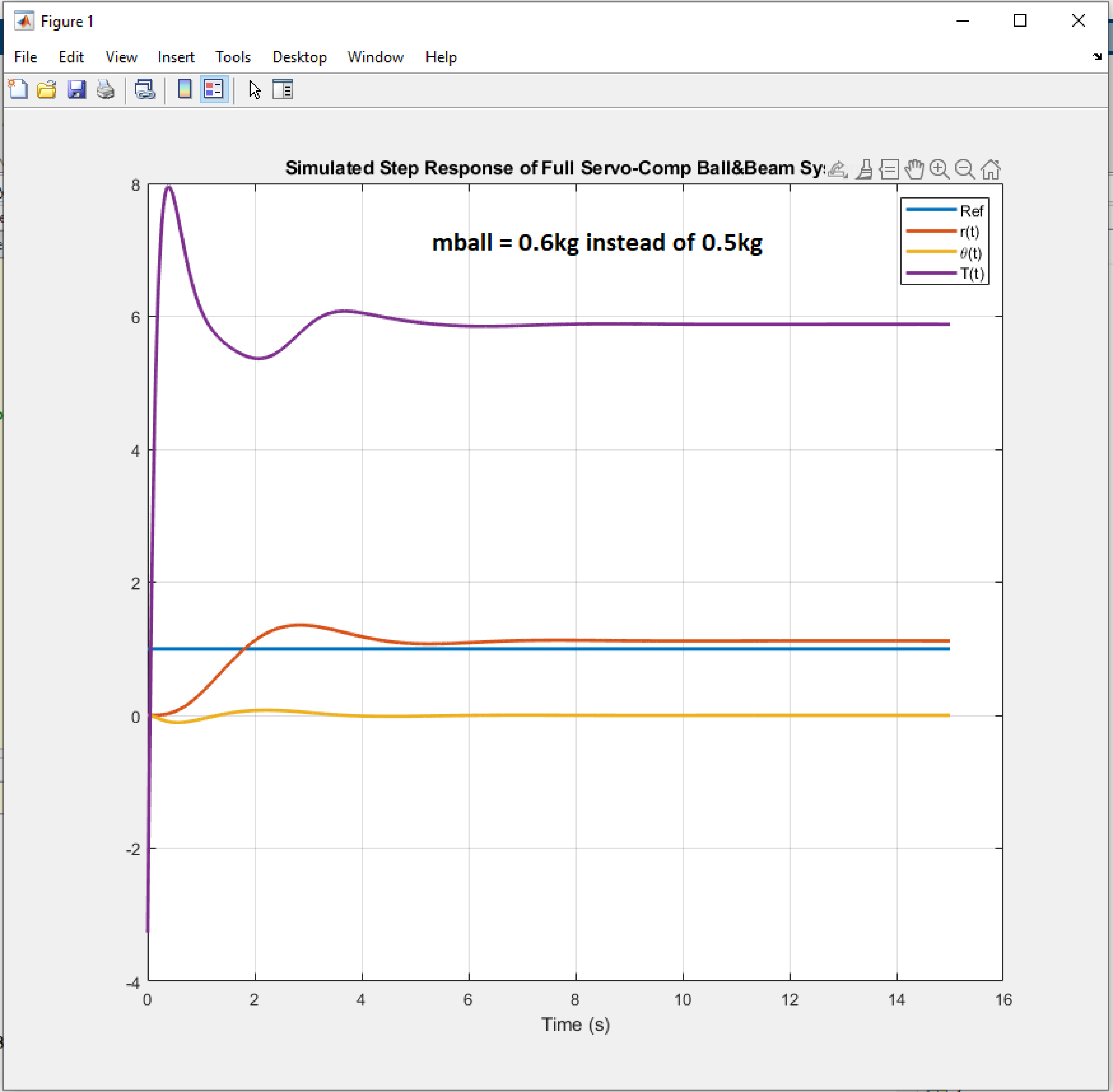

Figure 1
File
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Help
Simulated Step Response of Full Servo-Comp Ball&Beam Sy
mball = 0.6kg instead of 0.5kg
Ref
r(t)
θ(t)
T(t)
8
6
4
2
0
-2
-4
0
2
4
6
8
10
12
14
16
Time (s)

2) Assume a constant disturbance and/or setpoint. Design a feedback control law that

- Results in the ability to track a constant setpoint
- " " reject a " " disturbance
- $2\% \; T_s = 6s$
- No OS for step input

We will merely add a servo-comp. w/ $A_z = 0$, $B_z = 1$, so $sZ = CX - Ref$ and make the control law $U = -K_x X - K_z Z$.

The augmented system is:

$$s\begin{bmatrix} X \\ Z \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}\begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

and like w/ 1), make the des_poles $= [-\tfrac{2}{3}, -2, -3, -4, -5]$. Now Bass-Gura:

$$\Rightarrow \boxed{\begin{aligned} K_x &= [-50.3423,\ 160.6667,\ -41.0885,\ 29.3333] \\ K_z &= -16.3266 \end{aligned}} \Rightarrow \text{Check } eig\left(\begin{bmatrix} A - BK_x & -BK_z \\ C & 0 \end{bmatrix}\right)$$

$\Rightarrow$ Yup!

3) Plot the linear-step-response:

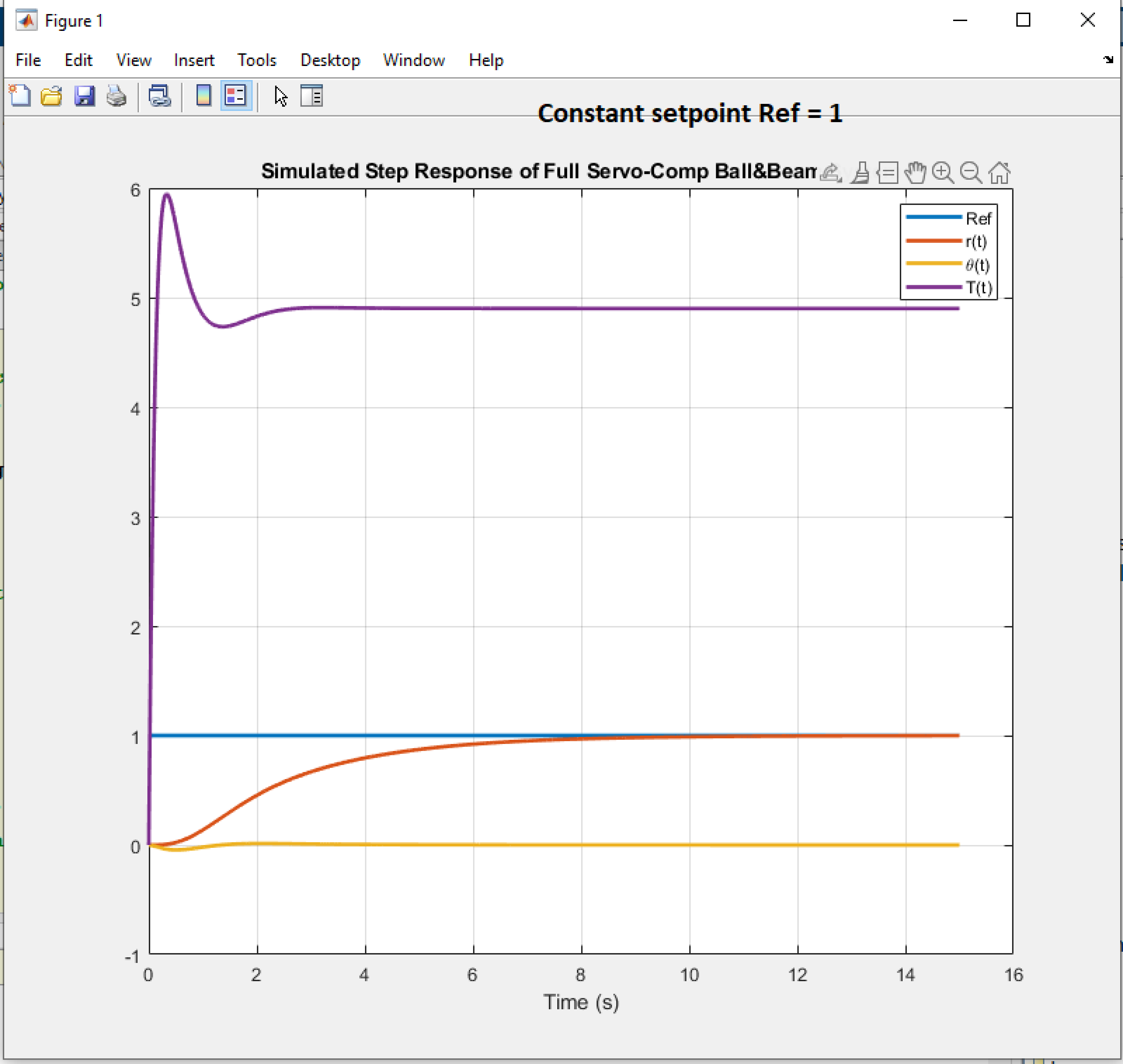

Figure 1
File Edit View Insert Tools Desktop Window Help
Constant setpoint Ref = 1
Simulated Step Response of Full Servo-Comp Ball&Beam
Ref
r(t)
θ(t)
T(t)
6
5
4
3
2
1
0
-1
0
2
4
6
8
10
12
14
16
Time (s)

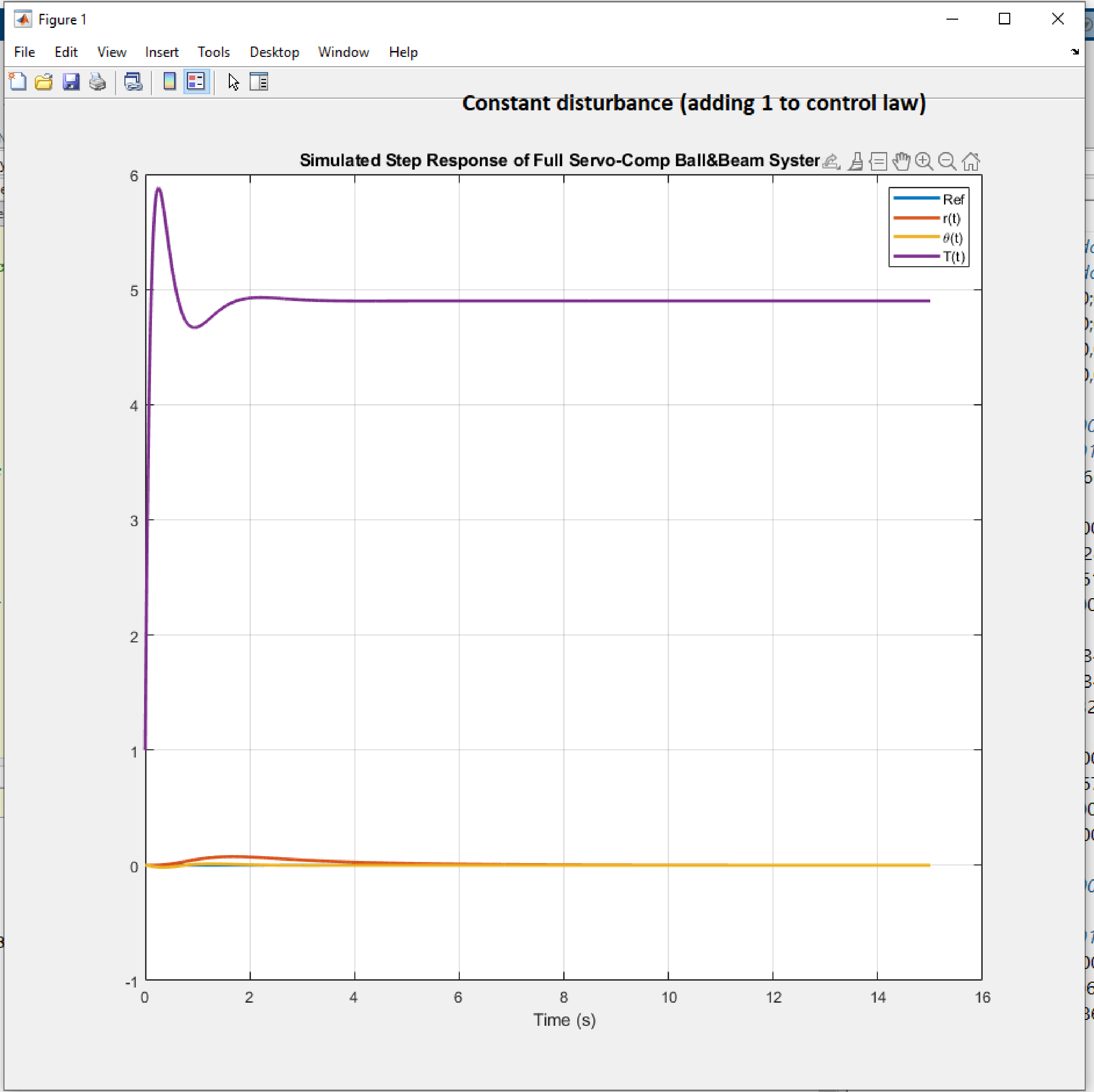
Figure 1
File Edit View Insert Tools Desktop Window Help
Constant disturbance (adding 1 to control law)
Simulated Step Response of Full Servo-Comp Ball&Beam Syster
Ref
r(t)
θ(t)
T(t)
6
5
4
3
2
1
0
-1
0 2 4 6 8 10 12 14 16
Time (s)

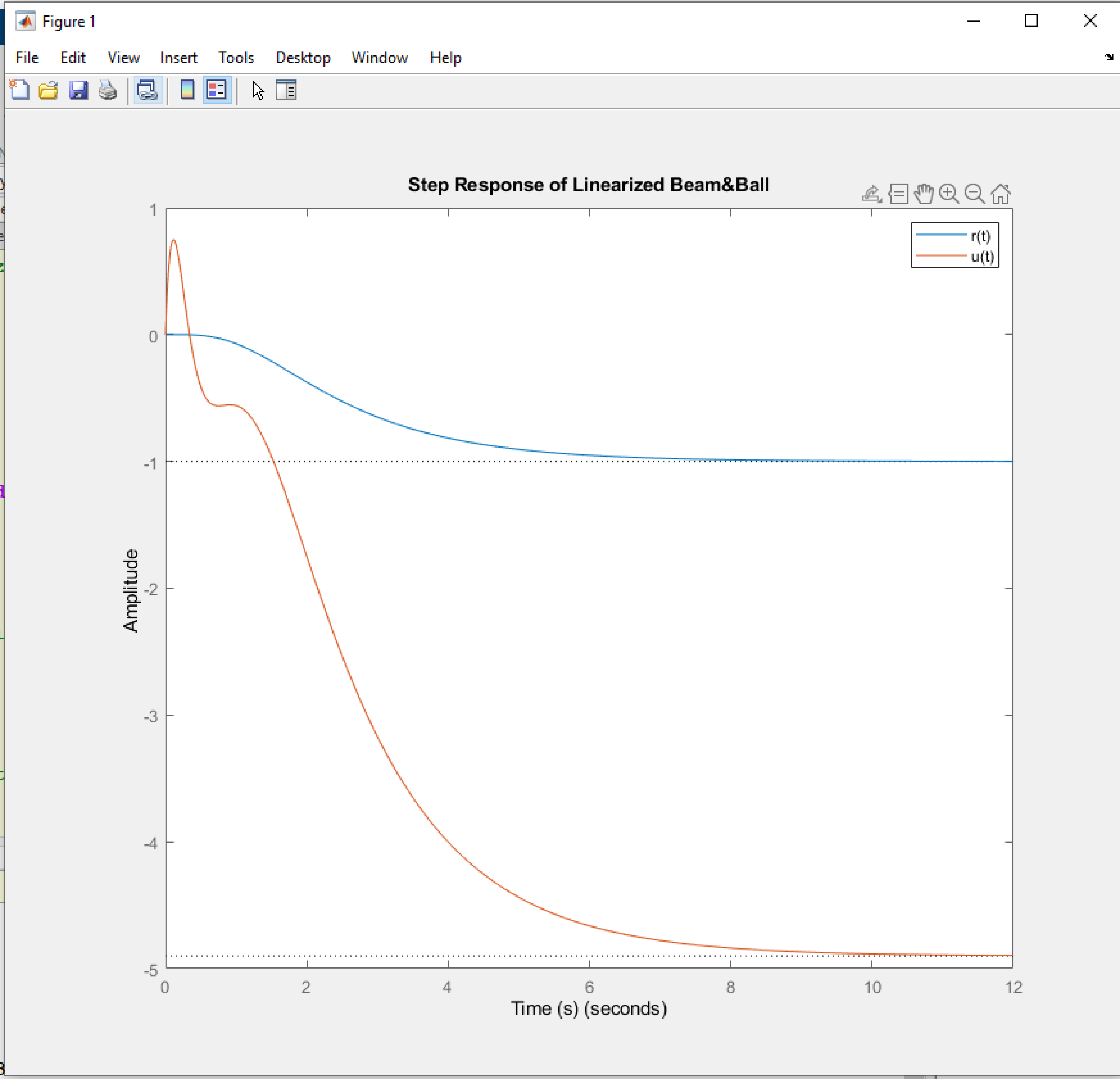
Figure 1
File Edit View Insert Tools Desktop Window Help
Step Response of Linearized Beam&Ball
r(t)
u(t)
1
0
-1
-2
-3
-4
-5
Amplitude
0
2
4
6
8
10
12
Time (s) (seconds)

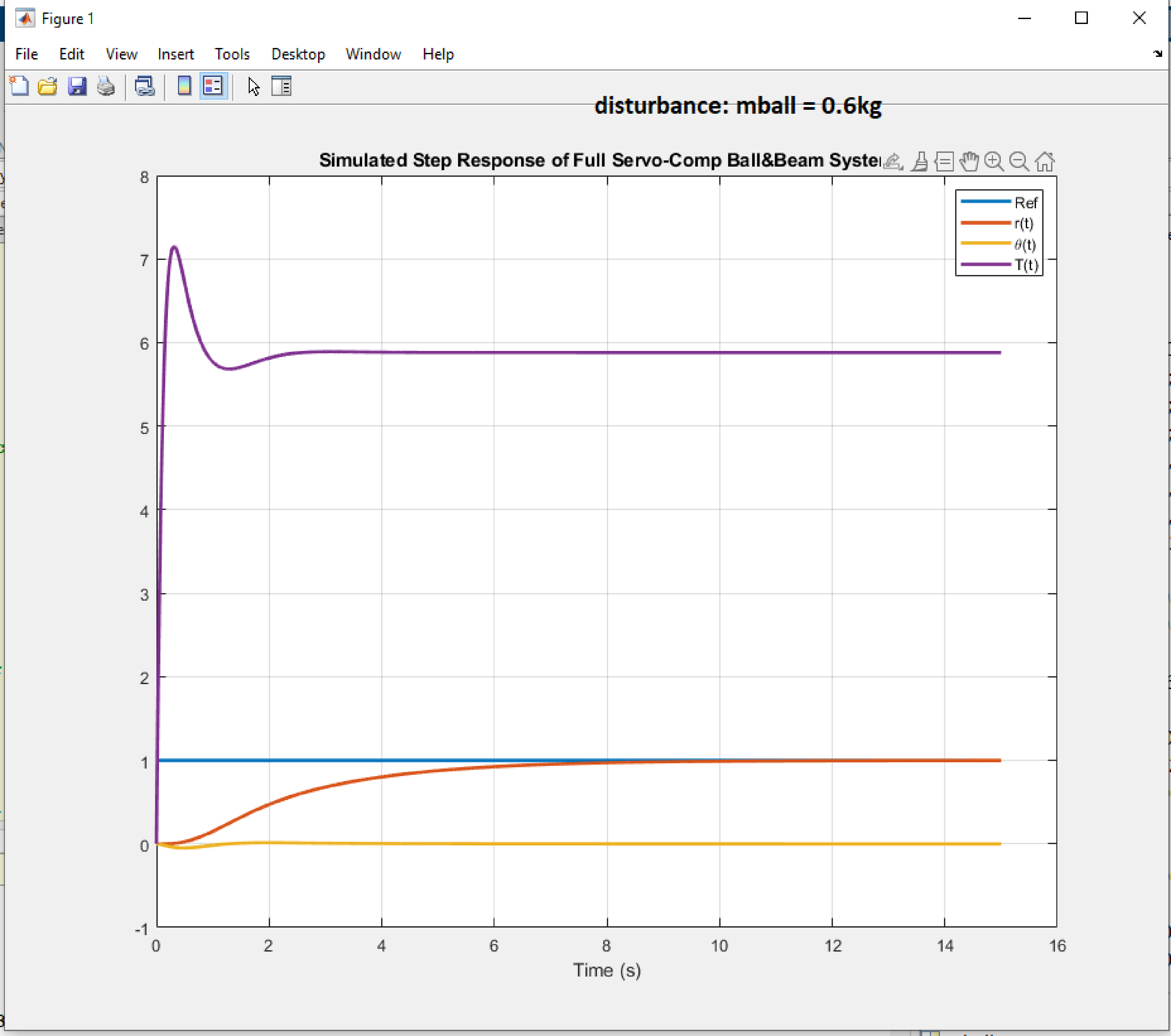
Figure 1
File Edit View Insert Tools Desktop Window Help
disturbance: mball = 0.6kg
Simulated Step Response of Full Servo-Comp Ball&Beam Syste
Ref
r(t)
θ(t)
T(t)
8
7
6
5
4
3
2
1
0
-1
0
2
4
6
8
10
12
14
16
Time (s)

4) Now the nonlinear one :

5) Now sinusoidal setpoint & dist @ 1 rad/s. First design:

We'll have our servo-comp: $A_z = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $B_z = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$,

$$\Rightarrow \quad A_{aug} = \begin{bmatrix} A & 0 \\ B_z C & A_z \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ -B_z \end{bmatrix} R$$

$$\Rightarrow \boxed{\begin{aligned} K_x &= [-300.8191, 348.0000, -145.7146, 42.0000] \\ K_z &= [-32.6581, -181.6331] \end{aligned}}$$

↳ Check: $\text{eig}\left(\begin{bmatrix} A - BK_x & -BK_z \\ B_z C & A_z \end{bmatrix}\right) \Rightarrow$ ✓

6) Now show response to $R(t) = \sin(t)$ and $d(t) = \sin(t)$

7) Now nonlinear, and $m = 0.6kg$ ...

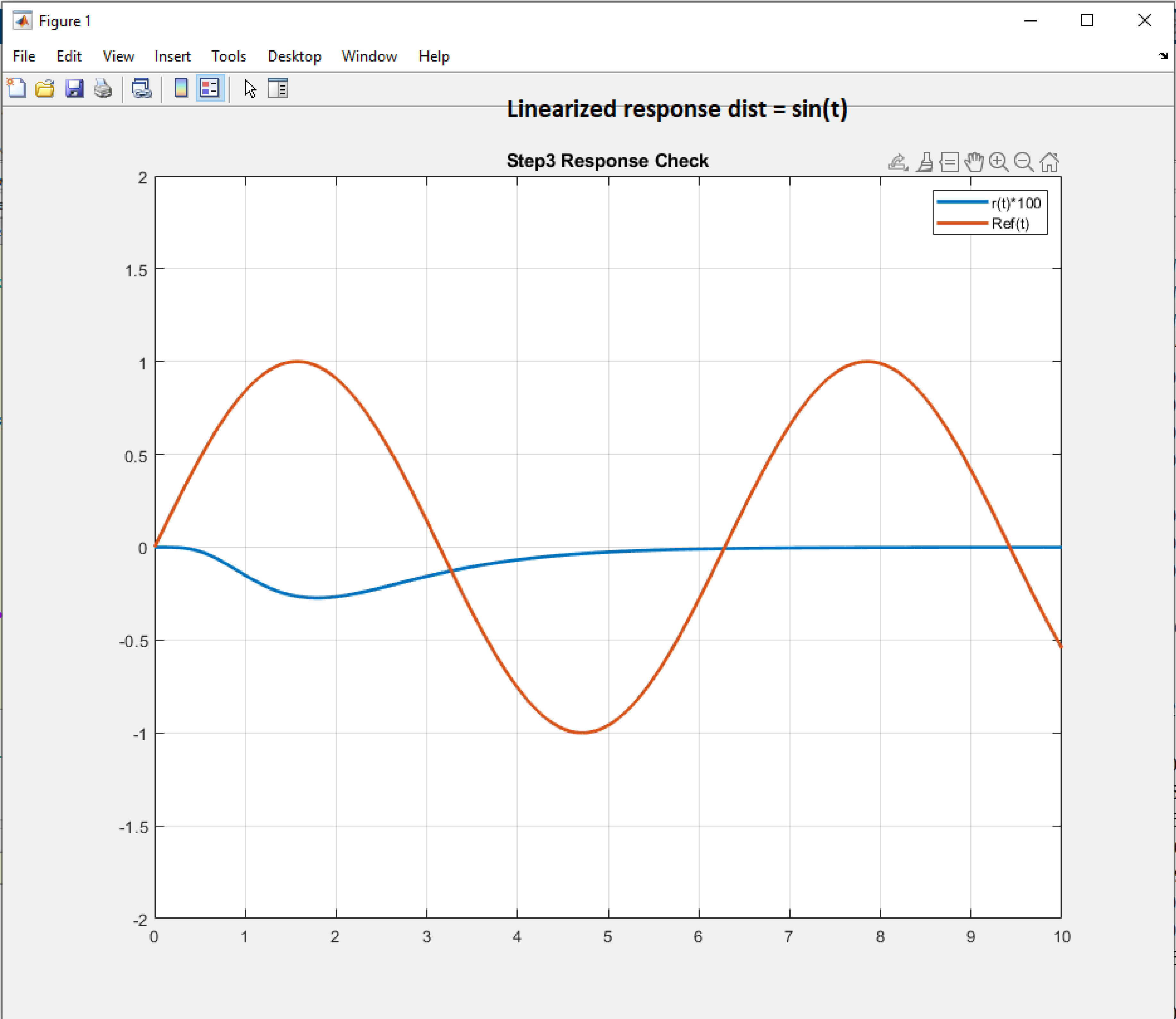
Figure 1
File Edit View Insert Tools Desktop Window Help
Linearized response dist = sin(t)
Step3 Response Check
r(t)*100
Ref(t)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0 1 2 3 4 5 6 7 8 9 10

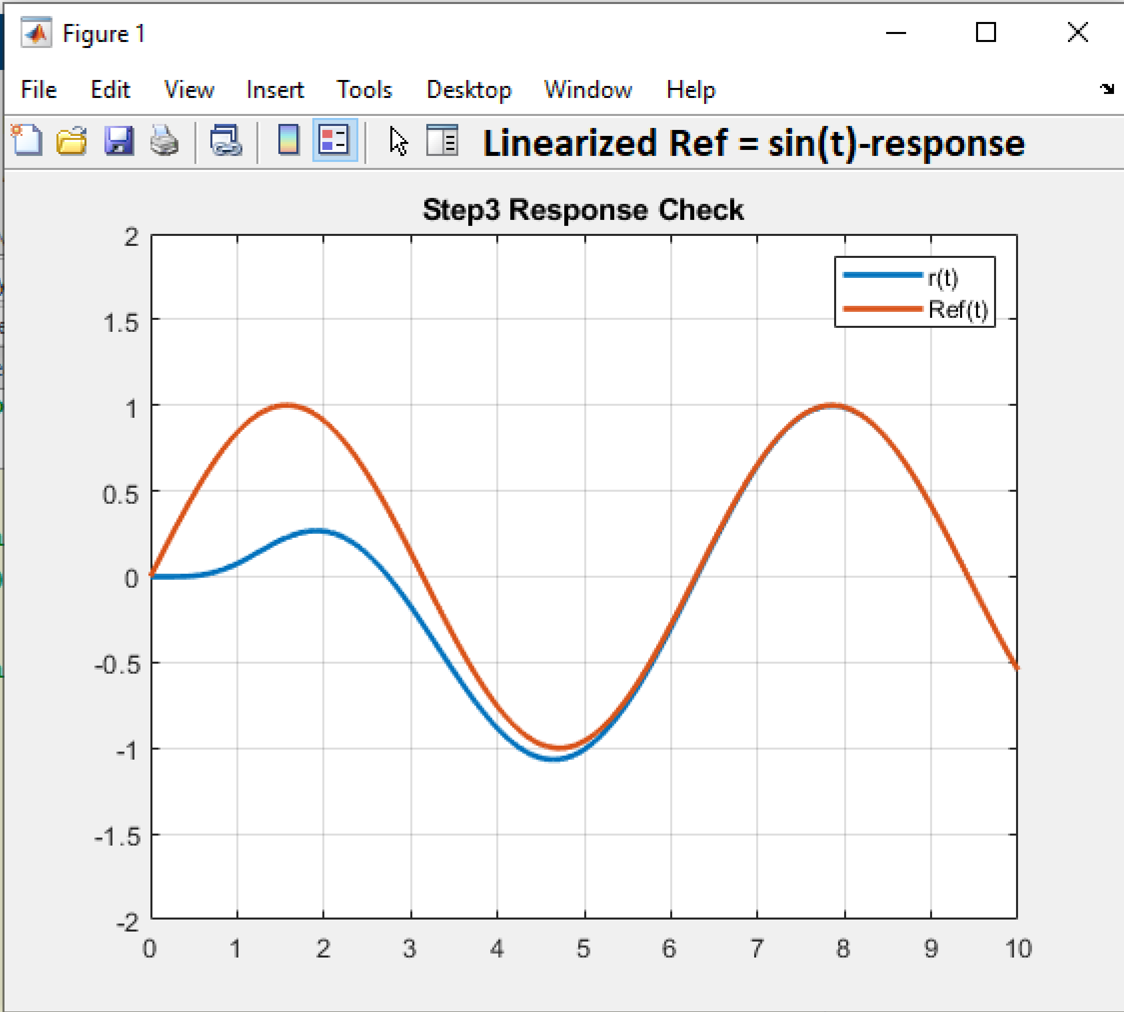
Figure 1
File
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Help
Linearized Ref = sin(t)-response
Step3 Response Check
r(t)
Ref(t)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
1
2
3
4
5
6
7
8
9
10

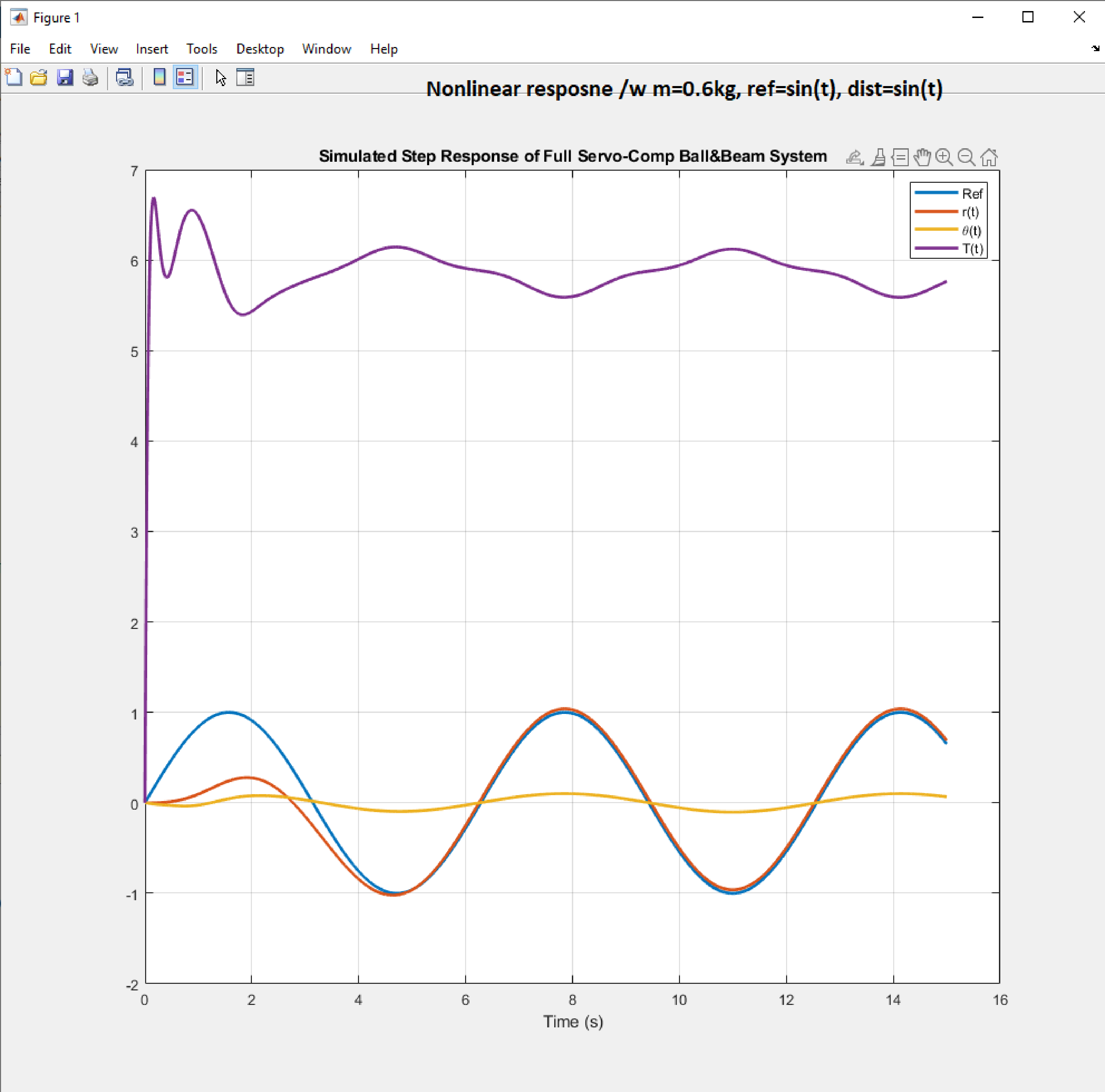
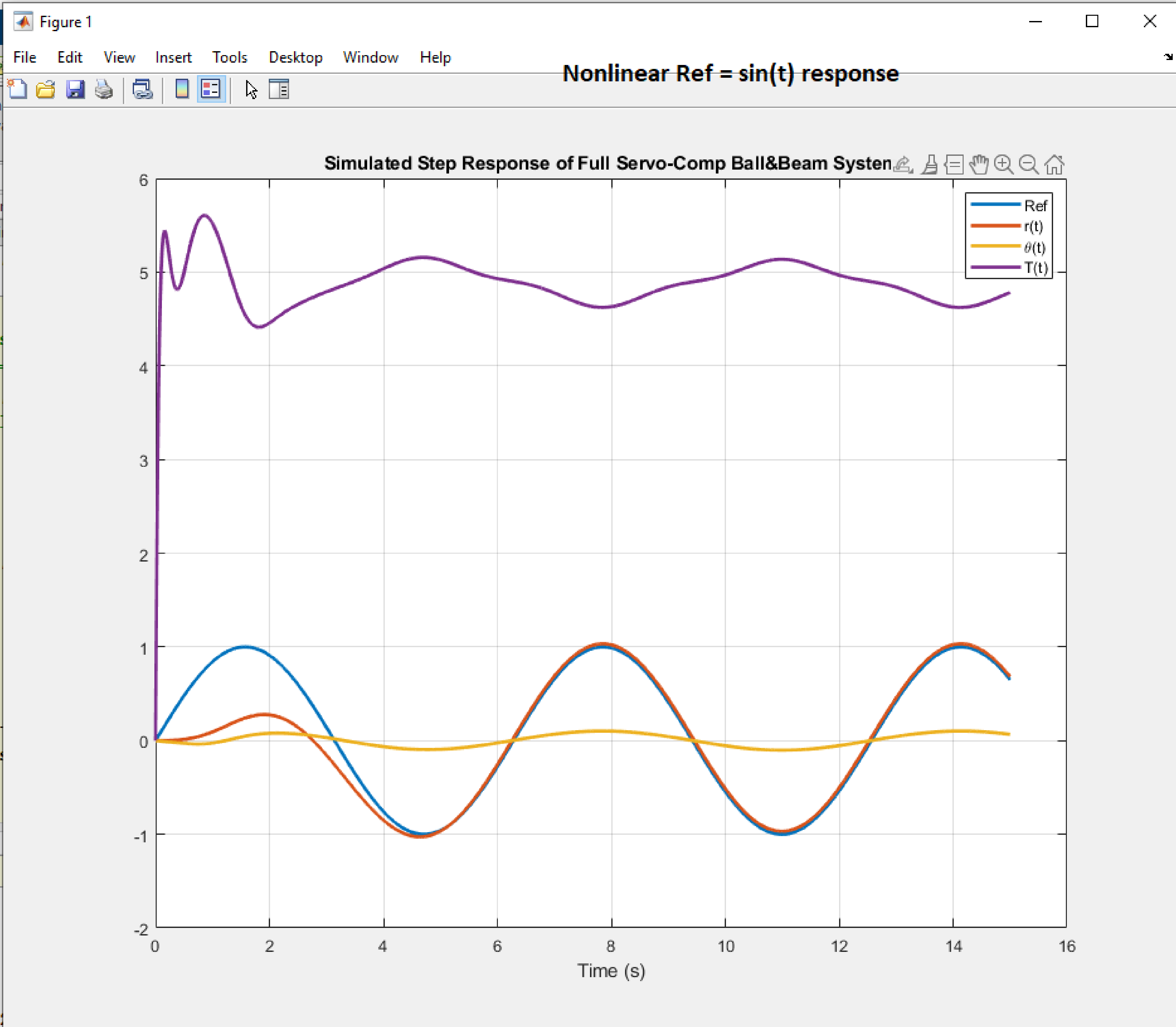


Figure 1
File Edit View Insert Tools Desktop Window Help
Nonlinear resposne /w m=0.6kg, ref=sin(t), dist=sin(t)
Simulated Step Response of Full Servo-Comp Ball&Beam System
Ref
r(t)
θ(t)
T(t)
7
6
5
4
3
2
1
0
-1
-2
0
2
4
6
8
10
12
14
16
Time (s)

Nonlinear Ref = sin(t) response



BeamServoComp.m

```
% Ball & Beam System
% Lecture #16
% Servo Compensators at DC

%% System
% System setup
mball = 0.5; mbeam = 8/3;
Rball = 5e-2; L = 3;

% Obtain matrices
[A,B] = linearizedBeamBall(mball,Rball,mbeam,L);
C = [1 0 0 0];  % We are looking at the position of the ball
D = 0;

% %% Simple Compensation
% % des_poles = [-0.5+0.5244j, -0.5-0.5244j, -2.5, -3.5];
% des_poles = [-2/3, -2, -3, -4];
% [Kx,Kr] = placePoles(A,B,C,des_poles)

%% Servo-compensation
% % For constant setpoint/disturbance
% Aaug = [A B*0; C 0]; Baug = [B;0]; Caug = [C, 0];
% des_poles = [-2/3,-2,-3,-4,-5];
% [Kx_aug, Kr_aug] = placePoles(Aaug, Baug, Caug, des_poles);
% Kx = Kx_aug(1:4)
% Kz = Kx_aug(5)

% Let's have it track inputs with frequency 1rad/s
Az = [0 1; -1 0]; Bz = [1;1];

% Augmented system
Aaug = [A, zeros(4,2); Bz*C, Az];
Baug = [B; 0; 0];
Caug = [C, 0, 0];
des_poles = [-1 -2 -3 -4 -5 -6];
[Kx_aug, Kr_aug] = placePoles(Aaug,Baug,Caug, des_poles);
Kx = Kx_aug(1:4) %#ok<NOPTS>
Kz = Kx_aug(5:6) %#ok<NOPTS>


%% Check /w approximation
% AA = [A - B*Kx, -B*Kz; Bz*C, Az];
% eg1 = [eig(A); 1.2345; 1.2345];
% fprintf('[Eigenvalues of Aapprox, Eigenvalues of Servo-comp Approx System]:\n');
% [eg1, eig(AA)]

% % Check linearized response
% % Acl = [A - B*Kx, -B*Kz; C, 0];
% Acl = [A - B*Kx, -B*Kz; Bz*C, Az];
% % Bcl = [zeros(4,1); 1];
% Bcl = [zeros(4,1); -Bz];
% Bcl2 = [B;0;0];
% % Ccl1 = Caug; Dcl = D;
% % Ccl2 = [-Kx, -Kz];
% Ccl = Caug; Dcl = D;
% % G1 = ss(Acl,Bcl,Ccl1,Dcl);
% % step(G1);
% % title('Step Response of Linearized Beam&Ball'); xlabel('Time (s)');
% % hold on;
% % G2 = ss(Acl,Bcl,Ccl2,Dcl);
% % step(G2);
```

```matlab
% % legend('r(t)','u(t)');
% X = zeros(6,1); t = transpose(linspace(0,10,1001));
% R = sin(t);
% Y = step3(Acl,Bcl2,Ccl,Dcl,t,X,R);
% % plot(t,Y,t,R,'LineWidth',2);
% plot(t,100*Y,t,R,'LineWidth',2);
% % legend('r(t)','Ref(t)');
% legend('r(t)*100','Ref(t)');
% ylim([-2,2]); grid on; title('Step3 Response Check');
% pause
% clf

%% Simulate
% Setting ICs and simulation config
X = zeros(4,1);  % [r q dr dq]
% Z = 0;
Z = zeros(2,1);
dt = 100e-6; T_end = 15;
t = 0;
Ref = sin(t);
N = (T_end / dt) + 1;
DATA = zeros(N,4);

% Change system and see if servo-comp still manages /w disturbance
mball = 0.6;
disturb = 0;

% Simulate
i=1;
tic
while(t < T_end)
%     U = Kr*Ref - Kx*X + disturb;
%     disturb = sin(3*t);
    U = -Kz*Z - Kx*X + disturb;   % Here, U is torque T
    Ref = sin(t);

    dX = BeamDynamics(X, U, mball, Rball, mbeam, L);
%     dZ = C*X - Ref;
    dZ = Bz*(C*X - Ref) + Az*Z;

    X = X + dX * dt;
    Z = Z + dZ * dt;
    t = t + dt;

    DATA(i,:) = [Ref, X(1), X(2), U];

%     if(mod(i,5) == 0)
%         BeamDisplay(X, Ref);
%     end

    i = i+1;
end
toc

t = [1:length(DATA)]' * dt; %#ok<NBRAK>
DATAds = downsample(DATA,10); tds = downsample(t,10);
plot(t,DATA, 'LineWidth',2);
grid on;
legend('Ref','r(t)','\theta(t)','T(t)');
title('Simulated Step Response of Full Servo-Comp Ball&Beam System'); xlabel('Time
(s)');
```

```
function [A,B] = linearizedBeamBall(mball, Rball, mbeam, L)
% mball=0.5kg, Rball=10e-2, mbeam=8/3kg, L=3m
g = 9.8;
Jball = (2/5)*mball*Rball^2;
Jbeam = (1/12)*mbeam*L^2;
A = [0 0 1 0; 0 0 0 1; 0 (-mball*g/(mball+Jball*Rball^2)) 0 0;
(-mball*g/Jbeam) 0 0 0];
B = [0;0;0;1/Jbeam];
end
```

```
function [dX] = BeamDynamics( X, T, mball, Rball, mbeam, L )
% Ball and Beam:  Sp21 Version
% m = 0.5 kg
% J = 2.0 kg m^2

r = X(1);
q = X(2);
dr = X(3);
dq = X(4);
g = 9.8;
% mball = 0.5; Rball = 10e-2;
% mbeam = 1; L = 4;

Jball = (2/5)*mball*Rball^2;
Jbeam = (1/12)*mbeam*L^2;

M = [(mball + Jball*Rball^2), 0;
    0, ((mball + Jball*Rball^2)*r^2 + Jbeam)];
B1 = (mball + Jball*Rball^2)*dq^2 - mball*g*sin(q);
B2 = T - 2*(mball + Jball*Rball^2)*dq*r*dr - mball*g*cos(q);

ddX = inv(M)*[B1; B2];
dX = [dr; dq; ddX];

end
```