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ECE463
MODERN CONTROL
HW #10

05/08/2021
SATURDAY

(1) servo-comp w/ LQR for the C&P system of HW#5 so that

- You track constant setpoints
- 2% $T_s = G_s$
- You reject constant disturbances
- $\leq 5\%$ OS

1) Give control laws and explain how you chose Q & R.
2) Plot step response
3) Check w/ nonlinear

From $m_{CART} = 1kg$, $m_{LOAD} = 4kg$, $L = 1m$ ⟹ Using my linearizedCartPend.m function:

For $T_s = G_s$, $OS \leq 5\%$.

⇓

$\sigma = -2/3$, $\omega_d \leq 0.6991$

$(\zeta \geq 0.6901)$

des_dom_poles $= -2/3 \pm j0.6991$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -39.2 & 0 & 0 \\ 0 & 49 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

I wrote a err_from_Q_a_b.m function to give me a norm-squared error $\|Y - Y_d\|^2$ (where Y is step response of system and Y_d is desired step response) and so I can use fminsearch w/ (a, b), where $Q = aQ_x + bQ_z$. From that...

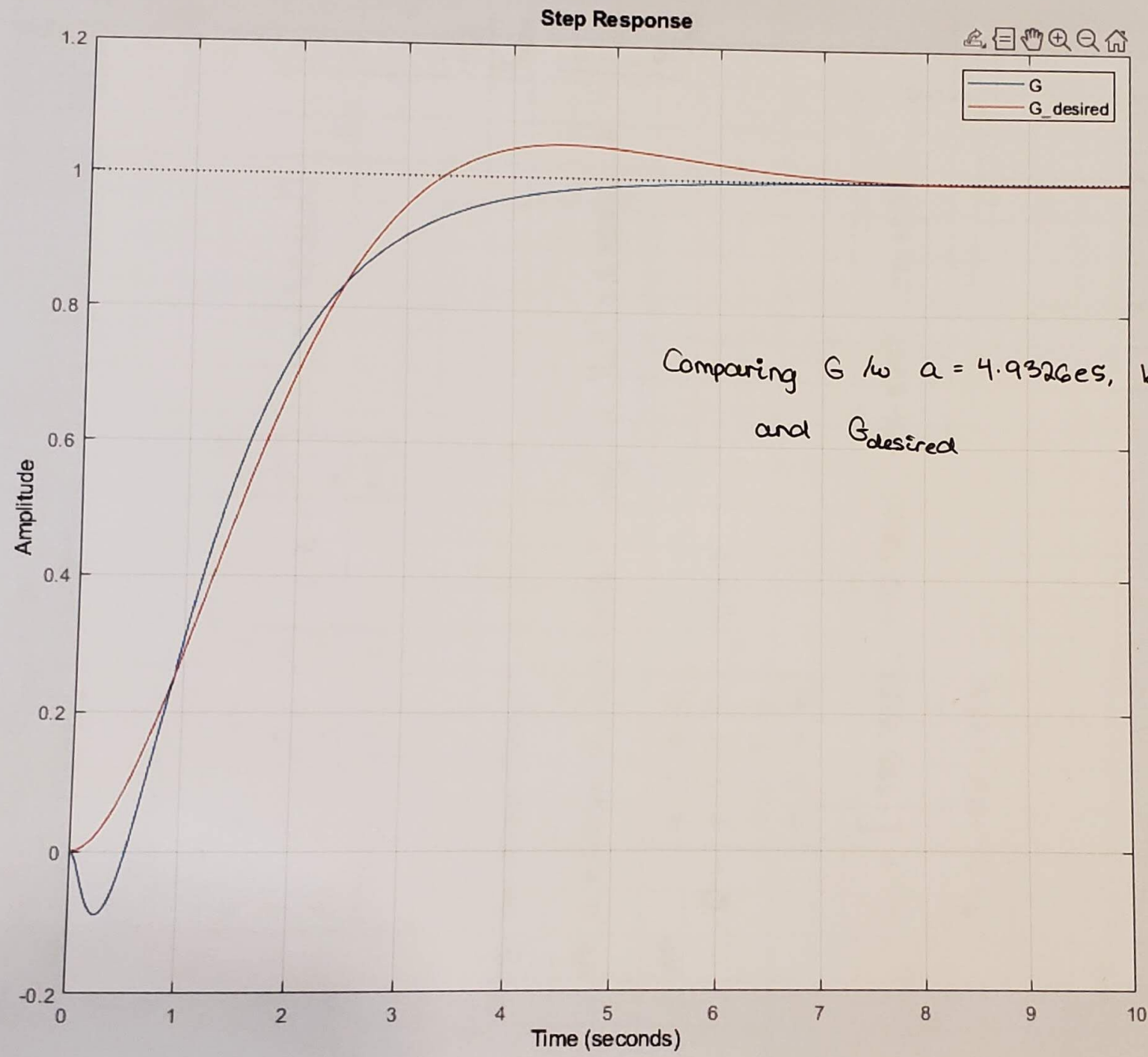
$a = 4.9326e5$, $b = 5.5260e5$ ⟹ $\|Y - Y_d\|^2 \cong 2.0379$

$$Q = \begin{bmatrix} 493,260 & 0 & \cdots \\ \vdots & \ddots & \\ 0 & & 552600 \end{bmatrix}$$

$R = 1$

$K_x = [-1218, -2267.9, -666, -711.8]$

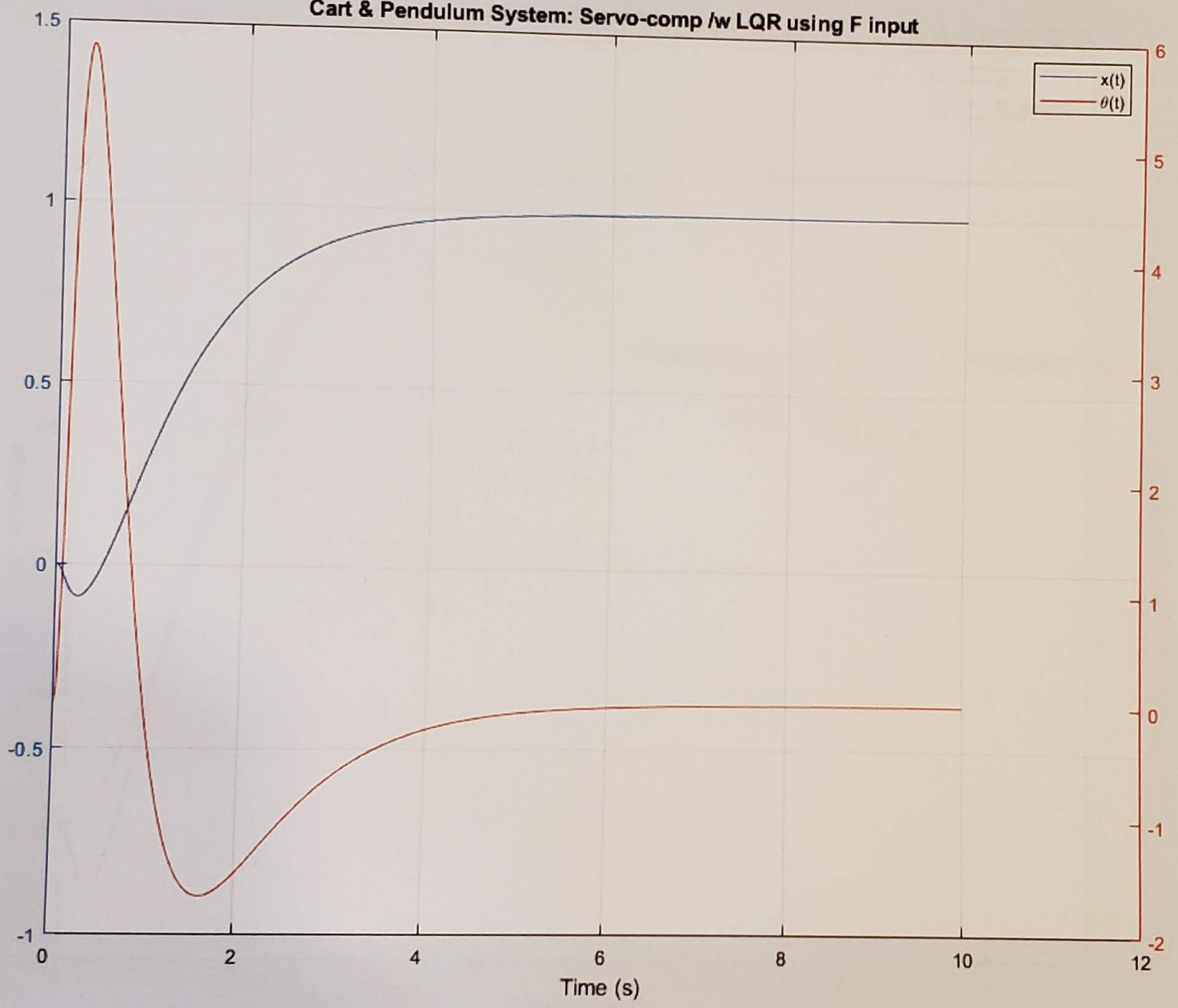
$K_z = -743.3707$



Comparing G & a = 4.9326e5, b = 5.5260e5 → LQR

and $G_{desired}$

NONLINEAR SIM



(2) Same thing but now lw Ball & Beam from HW#6. $m_{ball} = 0.5kg$, $J_{beam} = 2\,kg\,m^2$

$\Rightarrow m_{ball} = 0.5kg$, $R_{ball} = 1m$

$m_{Beam} = 4.8990kg$, $L = 2.4746m$.

$J_{beam} = \frac{1}{12} m_{beam} L^2$

$J_{ball} = \frac{2}{5} m_{ball} R_{ball}^2$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -1.96 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix}, \quad C = [1, 0, 0, 0]$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m_b g}{m_b + J_b R_b^2} & 0 & 0 \\ \frac{-m_b g}{J_{Beam}} & 0 & 0 & 0 \end{bmatrix}$$

Doing the same thing ...

$\Rightarrow a = 6098.8$, $b = 6826.7$ $\Rightarrow \|Y - Y_d\|^2 \leq 1.8686$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/J_{Beam} \end{bmatrix}$$

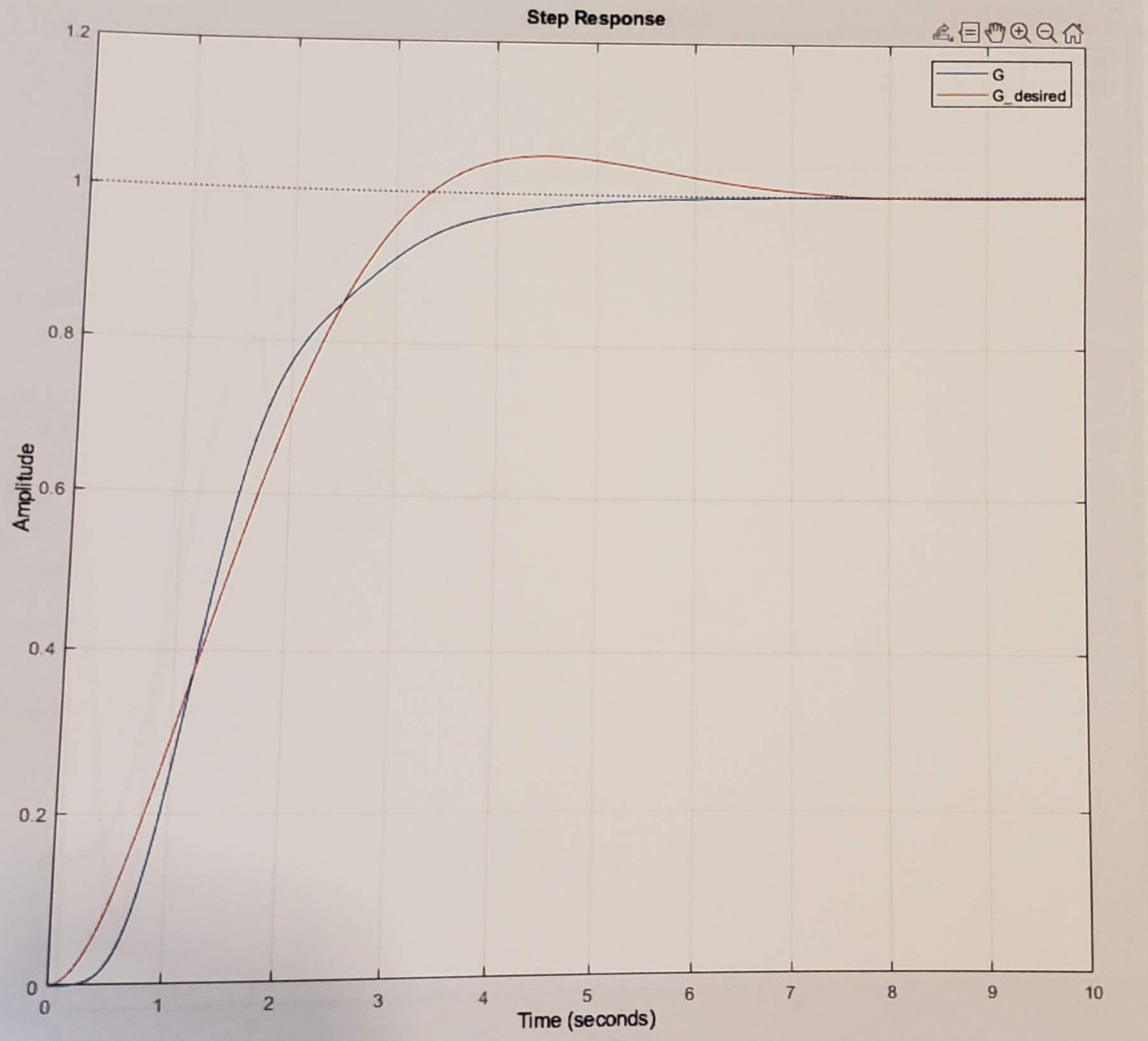
$$\Rightarrow Q = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b \end{bmatrix}, \quad R = 1$$

$$\Rightarrow \boxed{K_x = [-138.7254,\ 149.9580,\ -71.3257,\ 27.3822]}$$

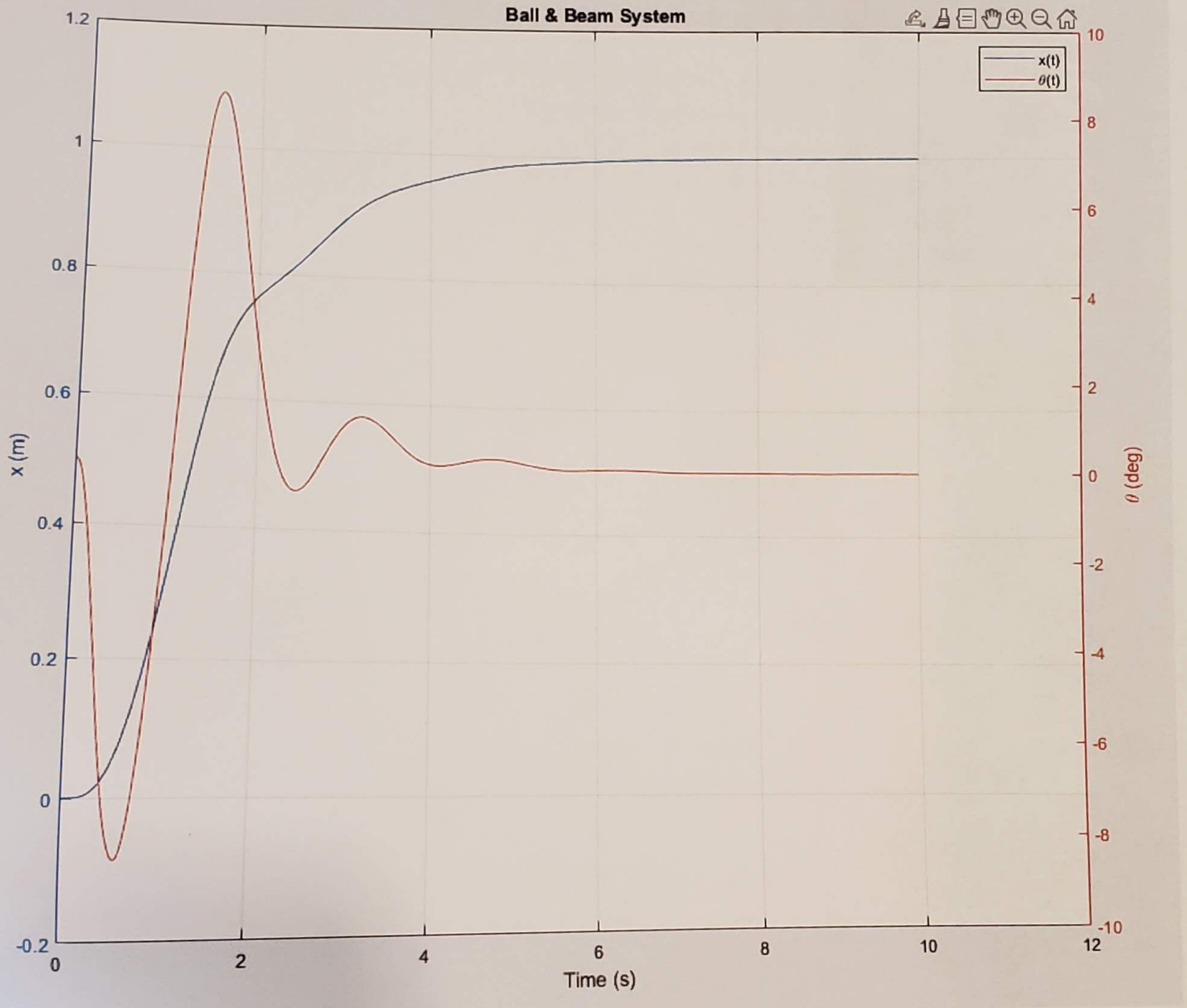
$$\boxed{K_z = -82.6238}$$

$\Rightarrow$

LIN.


Step Response
G
G_desired
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
0
1
2
3
4
5
6
7
8
9
10

NONLIN.


Ball & Beam System
x(t)
θ(t)
x (m)
θ (deg)
Time (s)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
2
4
6
8
10
12

lqr test script.m

```
%% Desired response
des_dom_poles = [-2/3+0.6991j, -2/3-0.6991j];
num = abs(des_dom_poles(1))^2;
den = poly(des_dom_poles);
Gd = tf(num,den);

%% Current system
% Kr example
% [A,B] = linearizedCartPend2(1,1,1);
% C = [1 0 0 0;
%      0 1 0 0];
%
% Q = 100*C'*C;
% R = eye(2);

% [K,S,CLP] = lqr(A,B,Q,R)
% Kr = 1 ./ (-C*inv(A-B*K)*B)


% Servo-comp example
% mc = 1; ml = 4; L = 1;
% [A,B] = linearizedCartPend(mc,ml,L);
mball = 0.5; Rball=1; mbeam = 4.8990; L = 2.4746;
[A,B] = linearizedBeamBall(mball,Rball,mbeam,L);
C = [1 0 0 0];
Aaug = [A, zeros(4,1); C, 0]; Baug = [B;0]; Caug = [C,0];
Cz = [0 0 0 0 1];

Qx = Caug'*Caug;
Qz = Cz'*Cz;
a = 6.0988e3; b = 6.8267e3;
% a = 10e3; b = 50e3;
Q = a*Qx + b*Qz
R = 1;

[K,S,CLP] = lqr(Aaug,Baug,Q,R);

Kx = K(1:4); Kz = K(5);
Acl = [A-B*Kx, -B*Kz; C, 0]; Bcl = [zeros(4,1); -1]; Ccl = Caug; Dcl = 0;


%% Test
G = ss(Acl, Bcl, Ccl, Dcl);
step(G)
hold on;
step(Gd)
legend('G','G_{desired}');
grid on;

resp_err(G,Gd,10)
```

Functions:

```
function [err] = err_from_Q_a_b(X)
a = X(1); b = X(2);

if a<0 || b<0
    err = 1e3;
    return
end

% mc = 1; ml = 4; L = 1;
% [A,B] = linearizedCartPend(mc,ml,L);
mball = 0.5; Rball=1; mbeam = 4.8990; L = 2.4746;
[A,B] = linearizedBeamBall(mball,Rball,mbeam,L);
C = [1 0 0 0];
Aaug = [A, zeros(4,1); C, 0]; Baug = [B;0]; Caug = [C,0];
Cz = [0 0 0 0 1];
Qx = Caug'*Caug;
Qz = Cz'*Cz;

Q = a*Qx + b*Qz;
R = 1;

[K,S,CLP] = lqr(Aaug,Baug,Q,R);

Kx = K(1:4); Kz = K(5);
Acl = [A-B*Kx, -B*Kz; C, 0]; Bcl = [zeros(4,1); -1]; Ccl = Caug; Dcl = 0;
G = ss(Acl, Bcl, Ccl, Dcl);

des_dom_poles = [-2/3+0.6991j, -2/3-0.6991j];
num = abs(des_dom_poles(1))^2;
den = poly(des_dom_poles);
Gd = tf(num,den);

Tend = 10;
err = resp_err(G,Gd,Tend);

% dom_poles = eig(G);
% dom_poles = dom_poles([1,2]);
% err = norm(dom_poles - des_dom_poles)^2;
end
```

```
function [err] = resp_err(G,Gd,Tend)
t = linspace(0,Tend,1001);
y = step(G,t);
yd = step(Gd,t);
err = norm(y-yd)^2;
end
```