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ECE461
HW#9

z-Transforms

1)
$$Y(z) = \left(\frac{0.005z}{\underbrace{(z-0.95)(z-0.9)(z-0.5)}} \right) X(z)$$

⇓ using poly() for

$$(z^3 - 2.35z^2 + 1.78z - 0.4275)\,Y(z) = (0.005z)\,X(z)$$

$$y[n+3] - 2.35y[n+2] + 1.78y[n+1] + 0.4275y[n] = 0.005x[n+1]$$

$$y[n] - 2.35y[n-1] + 1.78y[n-2] + 0.4275y[n-3] = 0.005x[n-2]$$

To implement,

$$\boxed{y[n] = 0.005x[n-2] + 2.35y[n-1] - 1.78y[n-2] - 0.4275y[n-3]}$$

2) $H(z) =$ " " , $x[n] = u[n]$

$$X(z) = \frac{z}{z-1} \Rightarrow Y(z) = H(z)X(z)$$

$$Y(z) = \frac{0.005z^2}{(z-0.95)(z-0.9)(z-0.5)(z-1)}$$

$$\frac{Y(z)}{z} = \frac{0.005z}{\cdots} = \frac{A}{z-1} + \frac{B}{z-0.95} + \frac{C}{z-0.9} + \frac{D}{z-0.5}$$

$$A = (z-1)\left(\frac{Y}{z}\right)\Big|_{z=1} = \frac{0.005}{\cancel{(\ldots)}\,(0.05)(0.1)(0.5)} = 2$$

$$B = (z-0.95)\frac{Y}{z}\Big|_{z=0.95} = \frac{(0.005)(0.95)}{(0.05)(0.45)(-0.05)} = -4.222$$

$$C = (z-0.9)\frac{Y}{z}\Big|_{z=0.9} = \frac{(0.005)(0.9)}{(-0.05)(0.4)(-0.10)} = -2.25$$

$$D = (z-0.5)\frac{Y}{z}\Big|_{z=0.5} = \frac{(0.005)(0.5)}{(-0.45)(-0.4)(-0.5)} = -0.0278$$

So,

$$Y(z) = 2\frac{z}{z-1} + -4.222\frac{z}{z-0.95} + -2.25\frac{z}{z-0.9} + -0.0278\frac{z}{z-0.5}$$

$$\boxed{y[n] = 2u[n] - 4.222(0.95)^n u[n] - 2.25(0.9)^n u[n] - 0.0278(0.5)^n u[n]}$$

verified w/ residue

3) $$H(z) = \frac{0.005z}{\underbrace{(z^2-1.6z+0.68)}_{0.8\pm j0.2}(z-0.5)} \qquad x[n] = u[n]$$

$$Y(z) = \left(\frac{z}{z-1}\right)\left(\frac{0.005z}{(z-0.8+j0.2)(z-0.8-j0.2)(z-0.5)}\right)$$

$$\frac{Y(z)}{z} = \frac{0.005z}{(z-1)(z-0.8\pm j0.2)(z-0.5)} = \frac{A}{z-1} + \frac{B}{z-0.8+j0.2} + \frac{C}{z-0.8-j0.2} + \frac{D}{z-0.5}$$

$B = C^*$

Using residue command, $A = 0.125$, $B = C^* = 0.1011 \angle 2.0132\,rad$, $D = -0.0385$

$$Y(z) = 0.125\frac{z}{z-1} + 0.1011\angle 2.0132\frac{z}{z-0.8\pm j0.2} + -0.385\frac{z}{z-0.5}$$

$\underbrace{0.8 \pm j0.2}_{0.8246 \angle 0.2450\,rad}$

$$\boxed{y[n] = 0.125u[n] + 0.2022(0.8246)^n \cos[0.245n + 2.0132]u[n] - 0.385(0.5)^n u[n]}$$

<u>s to z conversion</u>

$$G(s) = \frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}$$

$0.1617 \to 0.9223$, $1.04 \to 0.5945$, $2.719 \to 0.2568$, $5.05 \to 0.0801$

4) Use $T = 0.5s$

Using pole-zero matching:

$$G(z) = \frac{\alpha\, 1.4427(z+1)^3}{(z-0.9223)(z-0.5945)(z-0.2568)(z-0.0801)}$$

(To match DC-gain)

$G(0)$ for the analog filter: 0.6248

$G(1)$ for dig. fil. : 536.0223

So $\boxed{\alpha = 0.0012}$

$$\boxed{G(z) = 0.0012\frac{1.4427(z+1)^3}{(z-0.9223)(z-0.5945)(z-0.2568)(z-0.0801)}}$$

5) Use $T = 0.1s$.

$$\boxed{G(z) = \alpha\frac{1.4427(z+1)^3}{(z-0.9840)(z-0.9012)(z-0.7619)(z-0.6035)}}$$

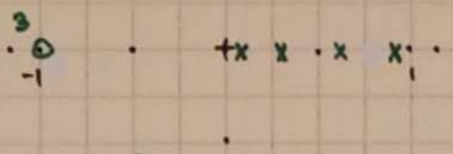
$G_a(0) = 0.6248$

$G_d(1) = 77,175$

$\boxed{\alpha = 8.0959 \times 10^{-6}}$

Root Locus in the z-domain

$T = 0.5s$ $\qquad G(s) =$ " "



5) rlocus of $G(z)$

$$G(z) = \underbrace{(0.0012)(1.4427)}_{0.0017312} \frac{(z+1)^3}{(z-0.9223)(z-0.5945)(z-0.2568)(z-0.0801)}$$

Well, we have four poles and three zeros at -1. Adjacent poles will come together then split, ~~&~~ three of whom to the zeros at -1, one off to ∞.

1) Real axis loci Will include real line between 0.5945 and 0.9223, 0.0801 and 0.2568

2) Asymptotes $\quad ang = \frac{\pm 180°(2k+1)}{N-M} \overset{!}{=} \boxed{180°}$

3) we will have breakaway points between poles (because of zeros, closer to left pole)

The characteristic eq: $(z-0.9223)(z-0.5945)(z-0.2568)(z-0.0801) + (0.0012)(1.4427)(z+1)^3 K = 0$

(Wolfram this...)

$$\boxed{K(z) = \frac{-(z-0.9223)\cdots(z-0.0801)}{(0.0012)(1.4427)(z+1)^3}}$$

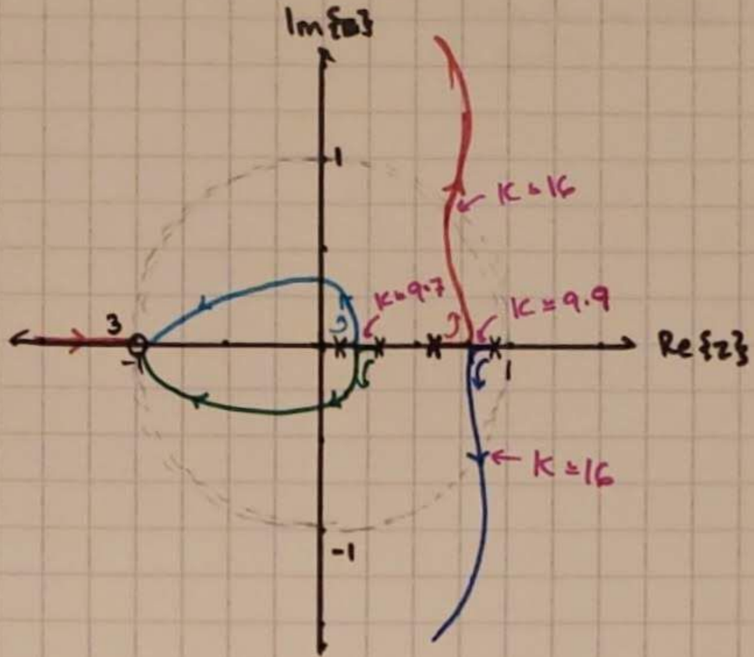
$\frac{dK}{dz} = 0$

$z = -5.3369$
0.7802
0.4107
0.1460 $\rightarrow$ ✓

$\Rightarrow$ At these points, K is...

$$\boxed{K(0.7802) = 0.9.9 \qquad K(0.1460) = 9.7568}$$

Using MATLAB to get a fuller picture...



7) K for no OS. At breakaway points, as I found earlier, K = ~~9.7~~ 0.97.

So,

$$KG(z) = \frac{\cancel{0.016793}\ 0.0016793\,(z+1)^3}{(z-0.9223)\cdots(z-0.0801)}$$

↓

Using simulink, (next page)

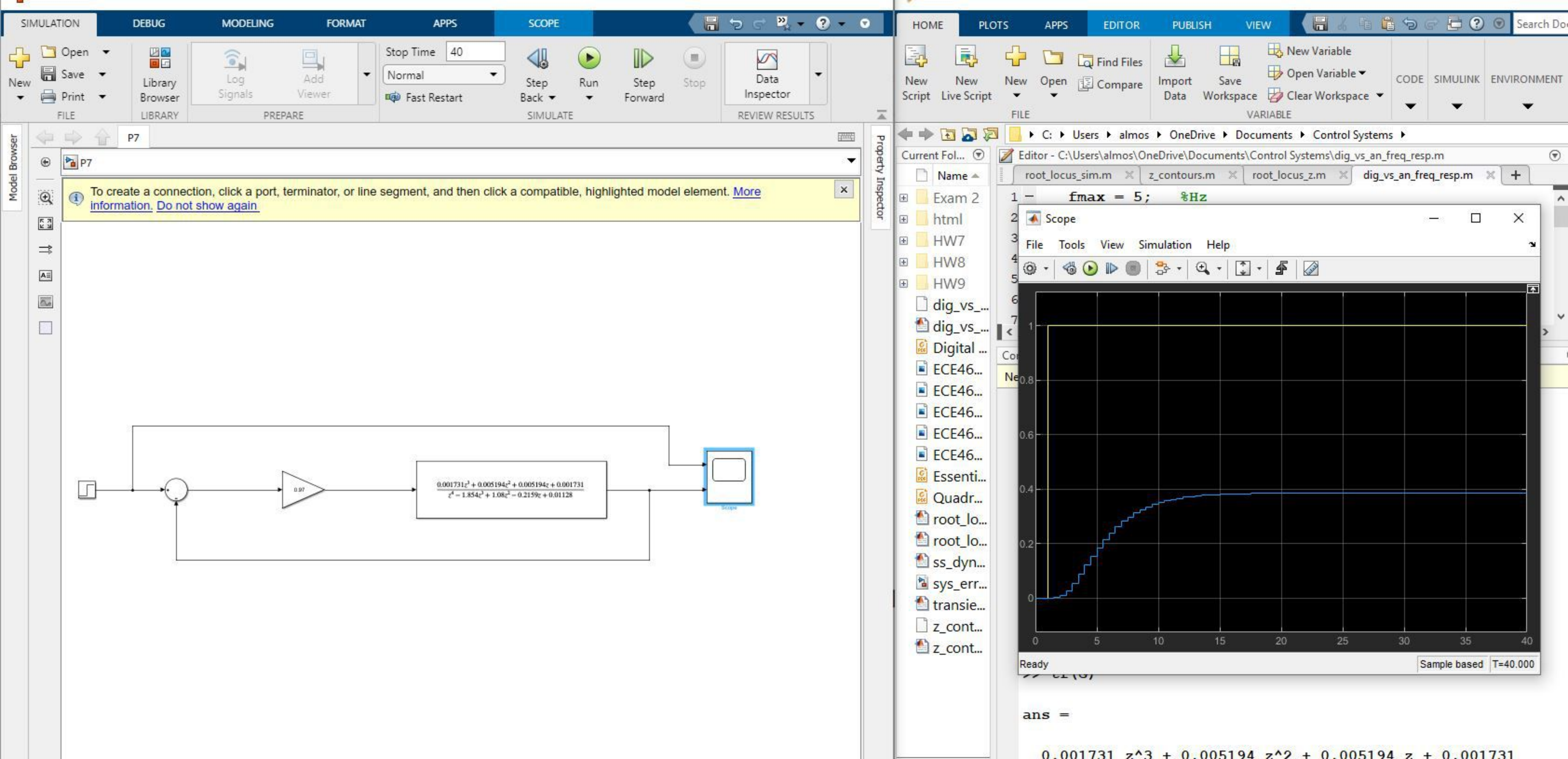
8) K for 20% OS $\Rightarrow \zeta = 0.4559$

The dominant poles are the ones along the rlocus from 0.9 and 0.5, so I'll look where they intersect the $\zeta = 0.4559$ spiral on MATLAB w/ help of zgrid.

I see intersection at $\boxed{z = 0.819 \pm j0.253}$ $\Rightarrow$ At this point,

$\boxed{K(z) = 4.1504}$

So, let's try it on Simulink...



SIMULATION DEBUG MODELING FORMAT APPS SCOPE
Stop Time 40
Normal
Fast Restart
P7
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HOME PLOTS APPS EDITOR PUBLISH VIEW
C: ▸ Users ▸ almos ▸ OneDrive ▸ Documents ▸ Control Systems ▸
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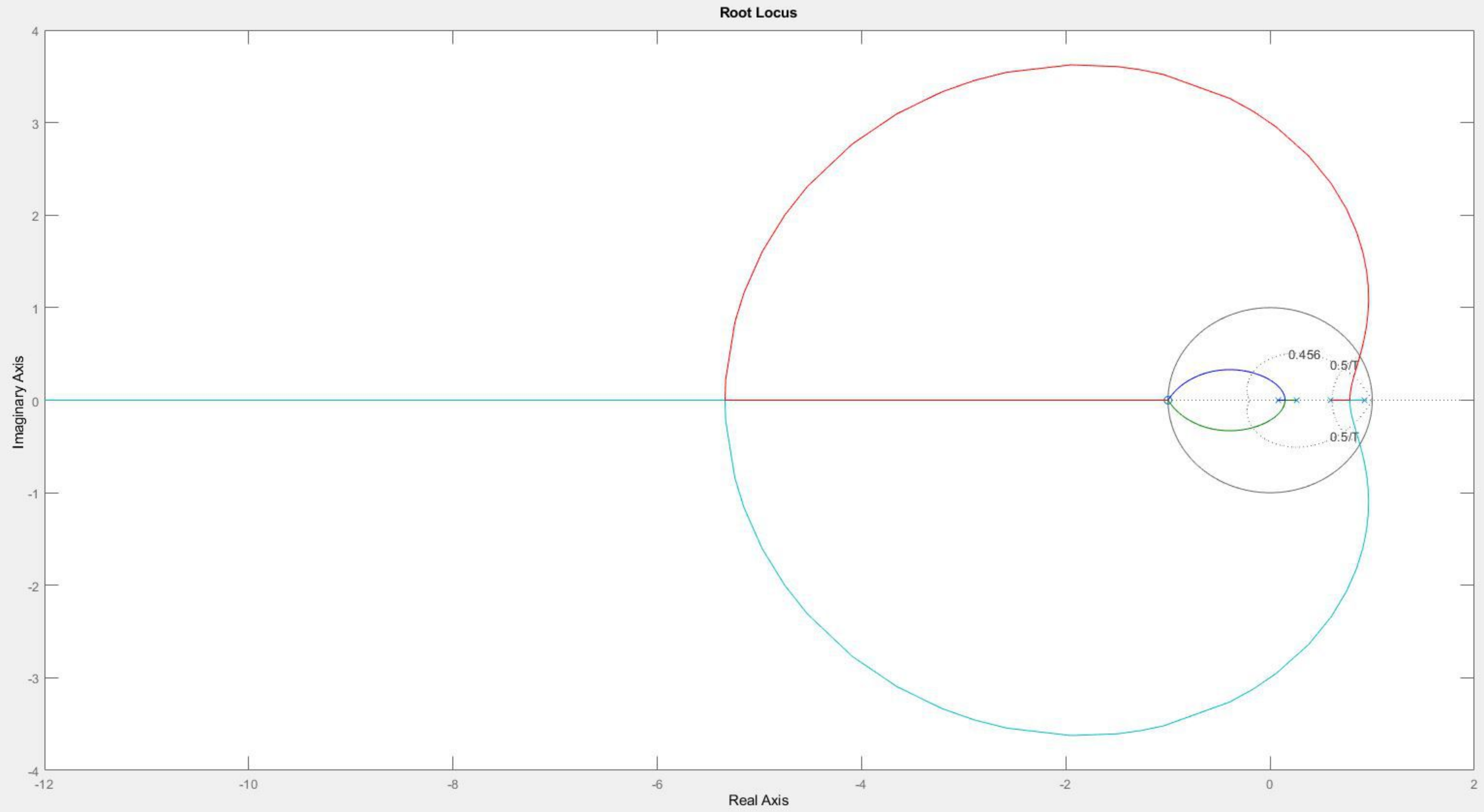
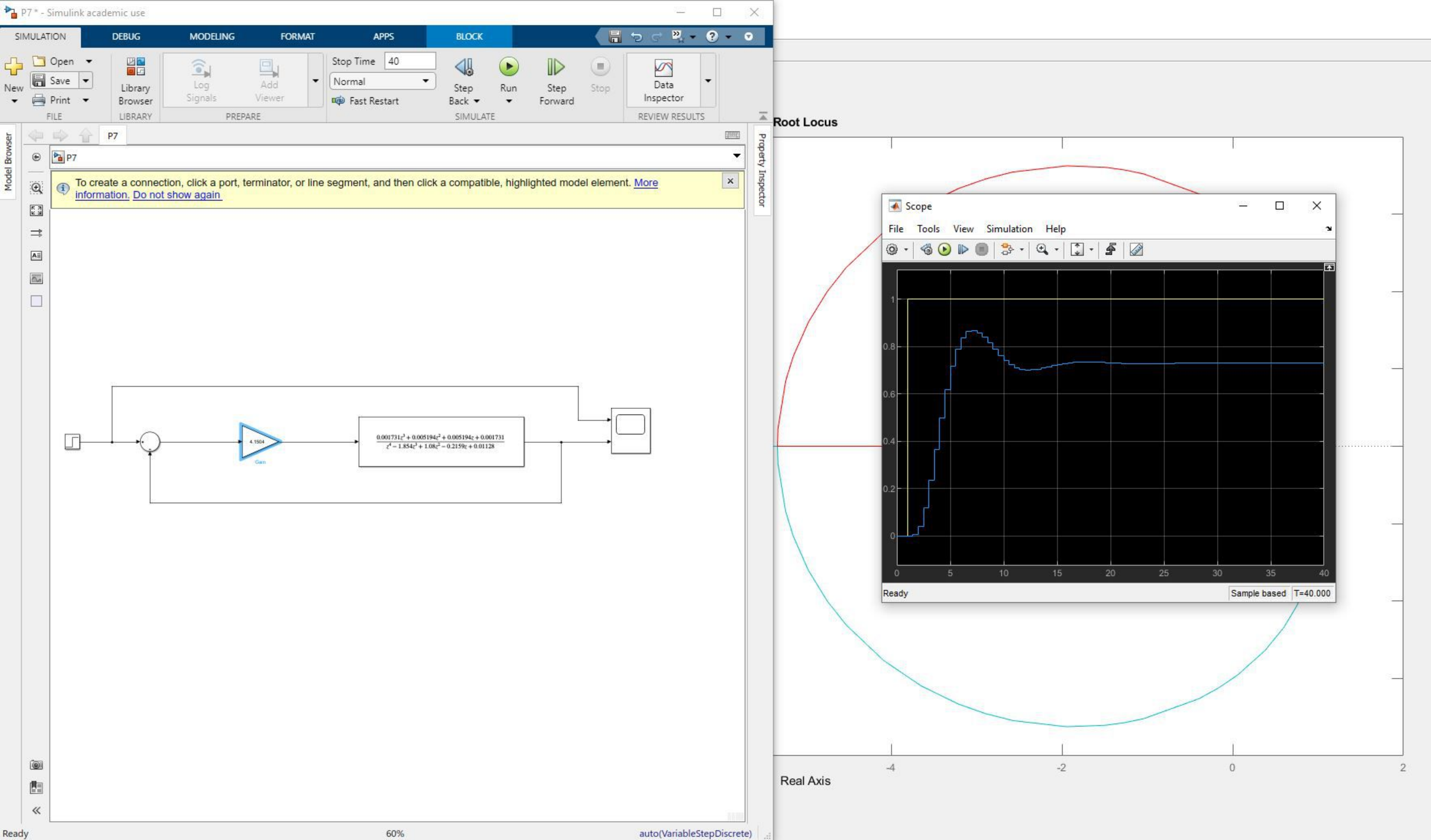
root_locus_sim.m z_contours.m root_locus_z.m dig_vs_an_freq_resp.m
fmax = 5; %Hz
Scope
File Tools View Simulation Help
Ready
Sample based T=40.000
ans =
0.001731 z^3 + 0.005194 z^2 + 0.005194 z + 0.001731

Figure 1

File Edit View Insert Tools Desktop Window Help




P7 * - Simulink academic use
SIMULATION
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PREPARE
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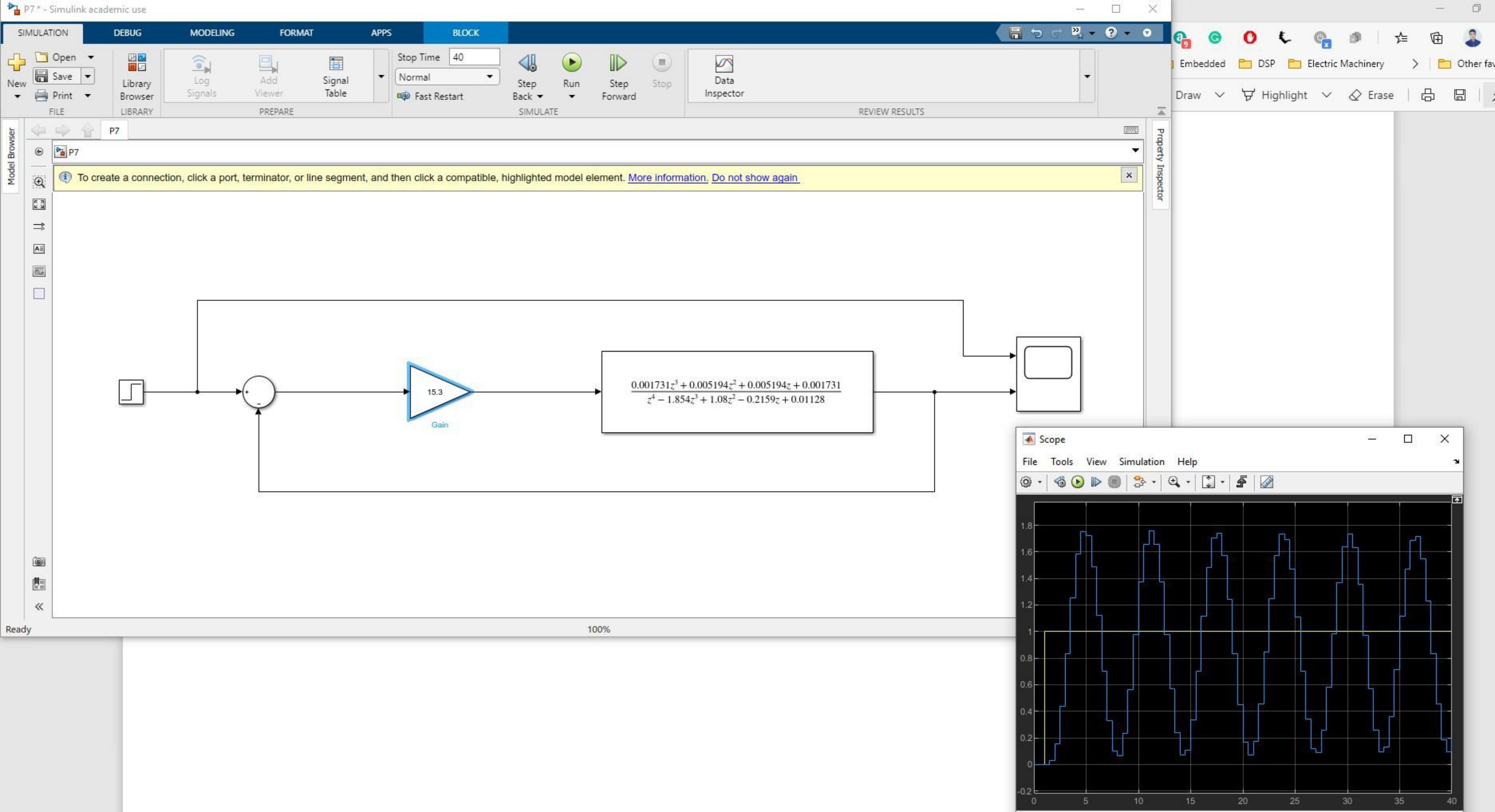
Stop
SIMULATE
Data Inspector
REVIEW RESULTS
Model Browser
Property Inspector
P7
To create a connection, click a port, terminator, or line segment, and then click a compatible, highlighted model element. More information. Do not show again
4.1504
Gain
0.001731z^3 + 0.005194z^2 + 0.005194z + 0.001731
z^4 − 1.854z^3 + 1.08z^2 − 0.2159z + 0.01128
Ready
60%
auto(VariableStepDiscrete)
Root Locus
Real Axis
-4
-2
0
2
Scope
File
Tools
View
Simulation
Help
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
35
40
Ready
Sample based
T=40.000

9) k for 0 damping. Well that's just the max allowable k for stability, which is 16 (from earlier).

(maybe a little less)


P7 * - Simulink academic use
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Erase
Model Browser
Property Inspector
P7
To create a connection, click a port, terminator, or line segment, and then click a compatible, highlighted model element. More information. Do not show again
15.3
Gain
$\frac{0.001731z^3+0.005194z^2+0.005194z+0.001731}{z^4-1.854z^3+1.08z^2-0.2159z+0.01128}$
Ready
100%
Scope
File
Tools
View
Simulation
Help
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20
25
30
35
40