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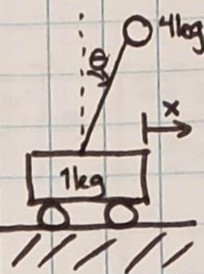
ECE463
HW#6

03/01/2021
MONDAY

1) Use the dynamics of the cart & pendulum system from HW#4:

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -39.2 & 0 & 0 \\ 0 & 49 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} F$$



Design a feedback control law so that

- $2\% \; T_s = 8s$
- 5% OS

and check the response of the linearized and non-linearized system:

$2\% \; T_s = 8s \longrightarrow \tau = 2s \longrightarrow \sigma = -1/2$

5% OS $\longrightarrow \zeta = 0.6901 \longrightarrow \theta = 46.3620° \longrightarrow \omega \doteq 0.5244$

$\%OS = \exp\left(-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}\right)$ $\theta = \cos^{-1}(\zeta)$ $\omega = |\sigma| \tan\theta$

So, dominant poles at $-0.5 \pm j0.5244$

Currently, open loop poles are: $0, 0, \pm 7$

well, can't really keep any poles except -7 so... I'll just change all poles:

des_poles = [$-0.5 \pm j0.5244$, ~~-4~~ 2.5, ~~-5~~ 3.5]

Using my ObtainCartFeedback function...

Hmm... just realized that A here is NOT full rank! But still controllable T_1...?

All positive feedback

$K_x = [-1.0714, -79.5964, -2.5230, -12.5230]$
$K_r = -1.0714$

Fairly high gain on θ feedback

- Even l/w the high gains though, F(t) only goes as high as ~1.6N
  - Furthermore, θ only goes as much as ~0.03rad ≈ 2°.
- Linearized system step response looks good.
- For non-linear system response, recall, $\cancel{\begin{bmatrix} 3 & 2\cos\theta \\ \cos\theta & 1 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 2\dot{\theta}^2\sin\theta \\ g\sin\theta \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F}$

I modified Glover's function CartDynamics to be more general:

$$\begin{bmatrix} (m_1+m_2) & m_2 R\cos\theta \\ m_2 R\cos\theta & m_2 R^2 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} m_2 R\dot{\theta}^2 \sin\theta \\ m_2 R g \sin\theta \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F$$

and now running the Cart.m file to observe more accurate system...

It worked as expected. We didn't push the limits yet.

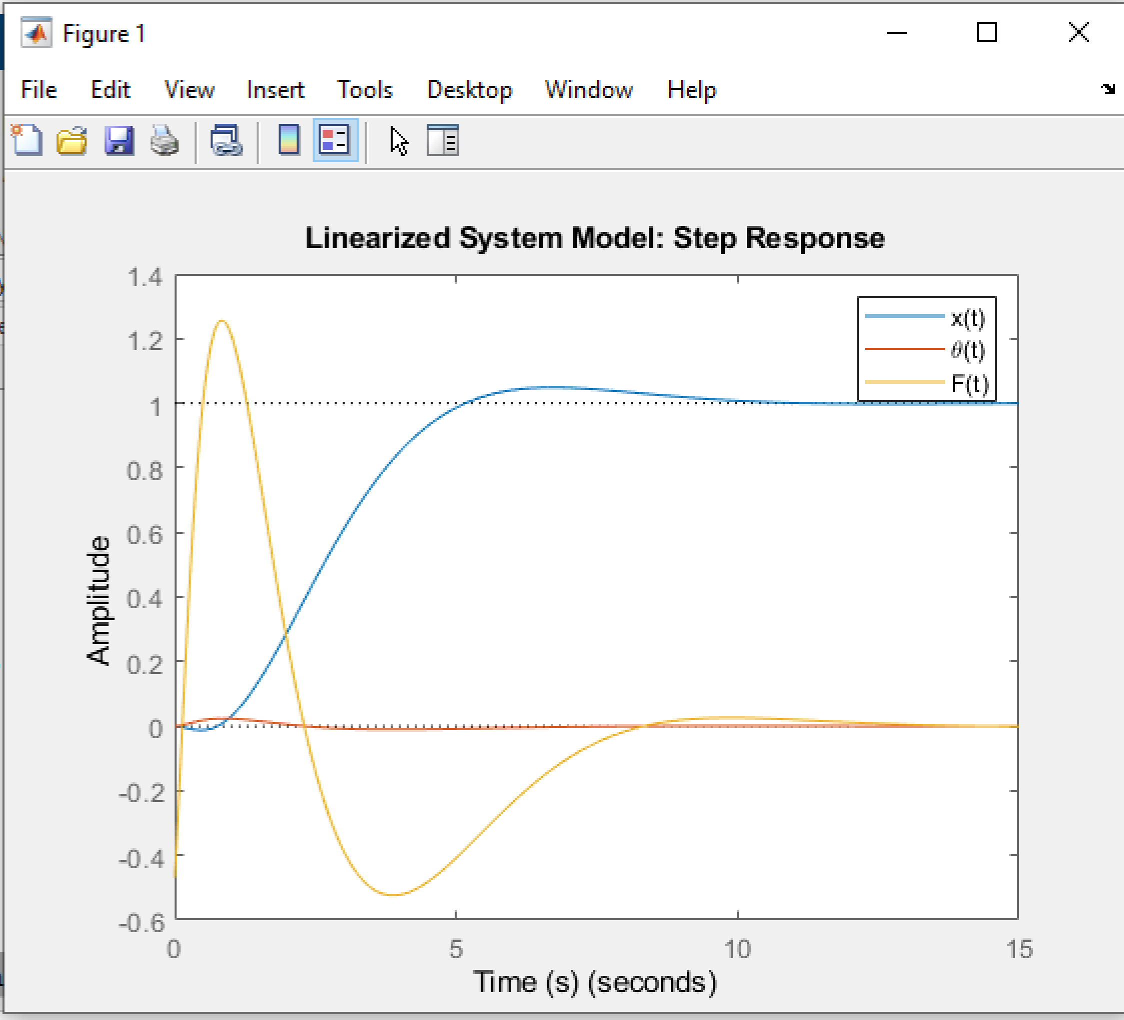


Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Linearized System Model: Step Response
x(t)
θ(t)
F(t)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Amplitude
0
5
10
15
Time (s) (seconds)

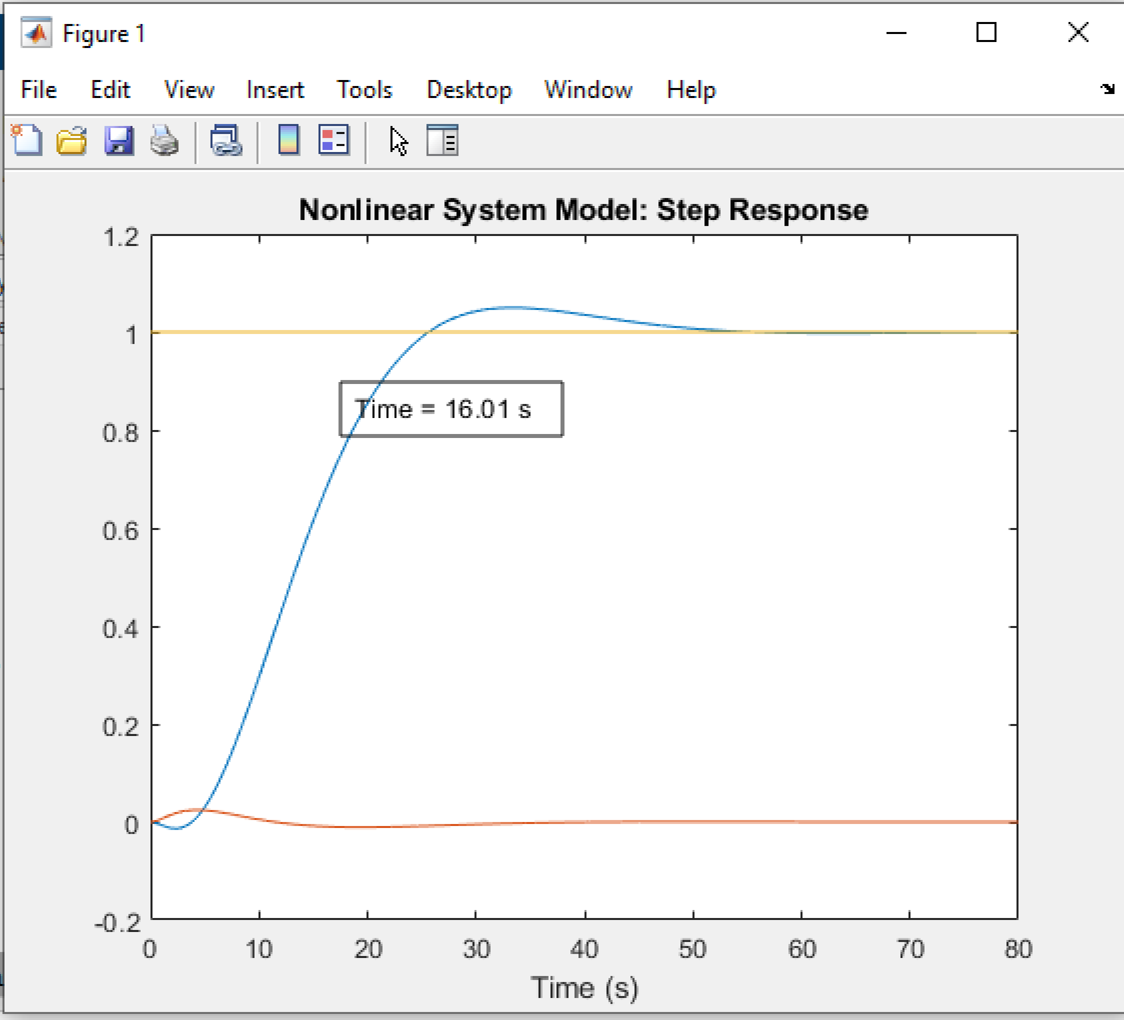


Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Nonlinear System Model: Step Response
Time = 16.01 s
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
10
20
30
40
50
60
70
80
Time (s)

```matlab
% Cart and Pendulum
% main calling routine

X = zeros(4,1);
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
t = 0;

m1 = 1;
m2 = 4;
R = 1;
Ts = 8; MOS = 0.05;
[Kx, Kr] = ObtainCartFeedback(m1,m2,R,Ts,MOS);
% Kx = [-1.0714, -78.5964, -2.5230, -12.5230];
% Kr = -1.0714;

pause;
U = 0;
y = [];
dim = [0.3 0.6 0.2 0.15];
an = annotation('textbox', dim, 'String', '', 'FitBoxToText',
'on');
while(t < Ts*2)
 U = Kr*Ref - Kx*X;
 dX = CartDynamics(X, U, m1, m2, R);
 X = X + dX * dt;
 t = t + dt;

 CartDisplay(X, Ref);
 y = [y ; X(1), X(2), Ref];

 str = sprintf('Time = %.2f s', t);
 an.String = str;
 drawnow;
end

t = linspace(0,Ts*10,length(y));
plot(t,y);
xlabel('Time (s)'); title('Nonlinear System Model: Step
Response');
```

```
function [Kx, Kr] = ObtainCartFeedback(m1,m2,R, Ts, MOS)

% m1 = 1; m2 = 4; R = 1;
[A,B] = linearizedCartPend(m1,m2,R) %#ok<NOPRT>
sig = getSigForTs(Ts);
[zeta, th, w] = getForOS(MOS,sig); %#ok<ASGLU>
dom_poles = [sig+1j*w, sig-1j*w];
des_poles = [dom_poles, 5*sig, 5*sig-1] %#ok<NOPRT>

C1 = [1 0 0 0]; % Position of cart
C2 = [0 1 0 0]; % Angle of bar
C = C1;
% olPoles = flip(eig(A));

[Kx,Kr] = placePoles(A,B,C,des_poles) %#ok<NOPRT>

G1 = ss(A-B*Kx, B*Kr, C1, 0);
G2 = ss(A-B*Kx, B*Kr, C2, 0);
G = G1;
U = ss(A-B*Kx, B*Kr, -Kx, Kr);

step(G);
hold on;
step(G2);
step(U);
legend('x(t)','\theta(t)','F(t)');
xlabel('Time (s)'); title('Linearized System Model: Step
Response');
hold off;

end
```

```
function [A,B] = linearizedCartPend(m1, m2, R)
g = 9.8;
A = [0 0 1 0; 0 0 0 1; 0 (-m2*g/m1) 0 0; 0 ((m1+m2)*g/m1) 0 0];
B = [0;0;1/m1;-1/(m1*R)];
end

function [sig] = getSigForTs(Ts)
tau = Ts/4;
sig = -1/tau;
end

function [zeta, th, w] = getForOS(desired_OS, sig)

MOS = @(z) exp(-pi * (z./sqrt(1-z.^2)));
E = @(z) abs(MOS(z) - desired_OS);
[zeta,err] = fminsearch(E, 0.5);
th = acosd(zeta);
w = abs(sig)*tand(th);

end
```

```matlab
function [ dX ] = CartDynamics( X, F, m1, m2, R )

%cart dynamics  (Sp21 version)
% X = [x, q, dx, dq]
g = 9.8;

x = X(1); %#ok<NASGU>
q = X(2);
dx = X(3);
dq = X(4);

M = [(m1+m2) (m2*R*cos(q)); (m2*R*cos(q)) (m2*R^2)];
A = [m2*R*dq*dq*sin(q); m2*R*g*sin(q)];
B = [1;0];
d2X = inv(M) * (A + B*F); %#ok<MINV>
dX = [dx; dq; d2X];

end

function [] = CartDisplay(X, Ref)
% Cart Display
% ECE 463 Lecture #7

x = X(1);
q = X(2);
% cart
xc = [-0.2,0.2,0.2,-0.2,-0.2] + x;
yc = [0,0,0.2,0.2,0];
xm = x + sin(q);
ym = 0 + cos(q);
% ball
q = [0:0.1:1]' * 2*pi;
xb = 0.05*cos(q) + xm;
yb = 0.05*sin(q) + ym;
plot([-3,3],[0,0],'b-',xc,yc,'r-',[x,xm],[0,ym]+0.2,'r-',xb,
0.2+yb, 'r-',[Ref, Ref],[-0.1,0.1],'b')
ylim([-0.5,1.5]);
pause(0.01);

end
```

2) Use the dynamics of the Ball & Beam system:

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -1.96 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix} T$$

Design a feedback control law so that 2% $T_s = 8s$, 5% OS.
Check linearized system response.
nonlinear system response.

Same poles as 1): $[-0.5 + j0.5244, -0.5 - j0.5244, -2.5, -3.5]$

$\hookrightarrow$ Running ObtainBeamBallFeedback ...

$$\boxed{\begin{aligned} K_x &= [-6.5406, 38.1875, -4.25, 17.5] \\ K_r &= -1.6406 \end{aligned}}$$

This also worked in the nonlinear model.

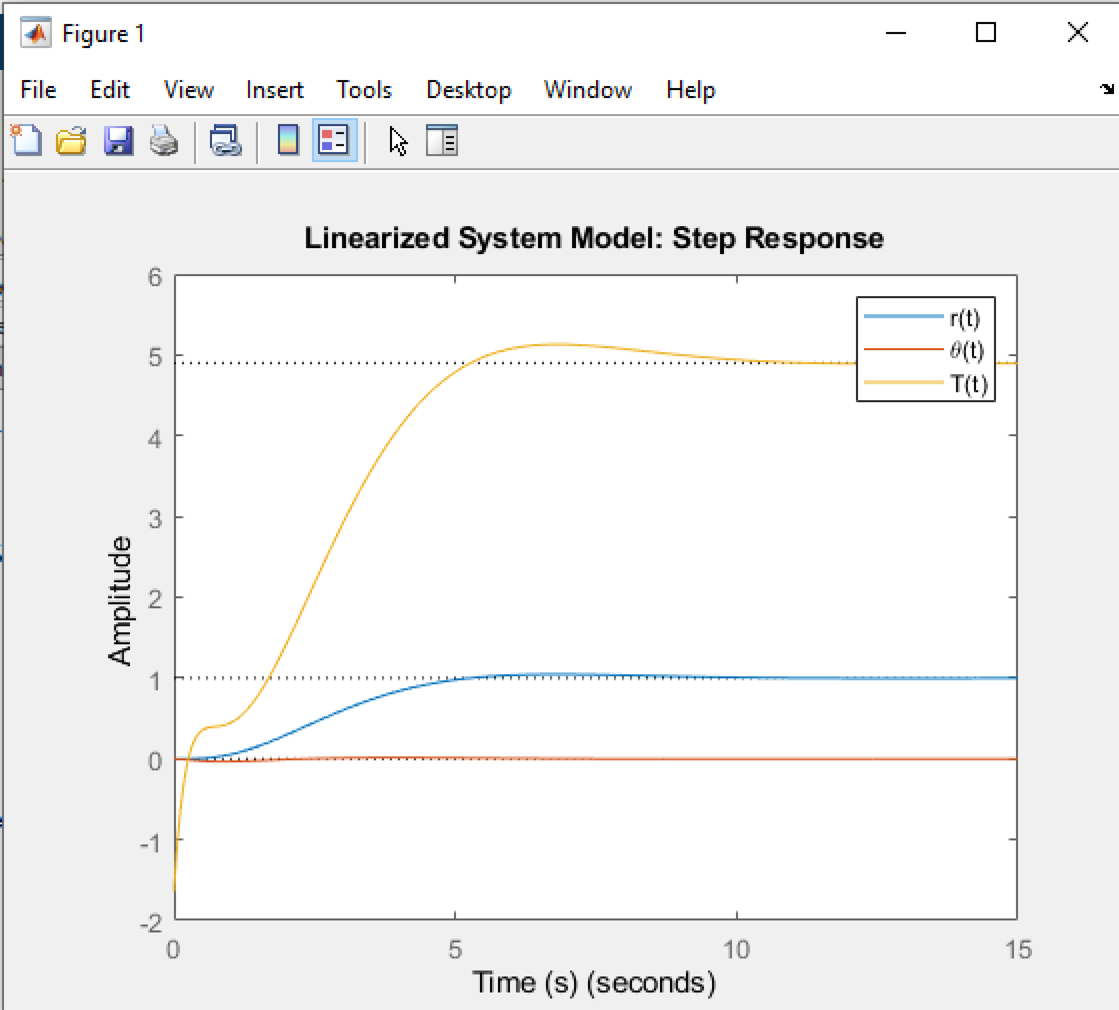


Figure 1
File Edit View Insert Tools Desktop Window Help
Linearized System Model: Step Response
r(t)
θ(t)
T(t)
Amplitude
6
5
4
3
2
1
0
-1
-2
0
5
10
15
Time (s) (seconds)

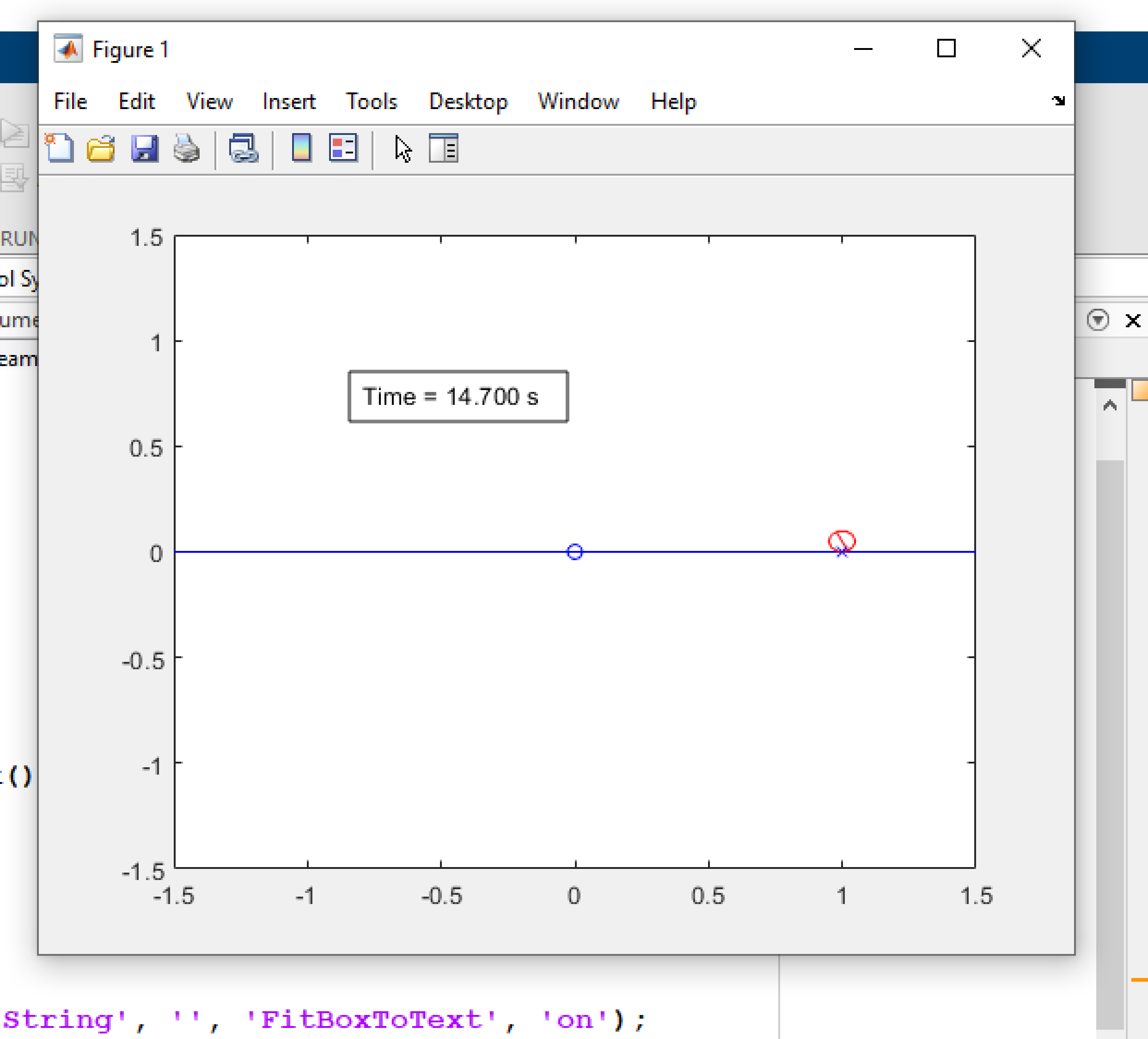


Figure 1
File Edit View Insert Tools Desktop Window Help
Time = 14.700 s
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
String', '', 'FitBoxToText', 'on');

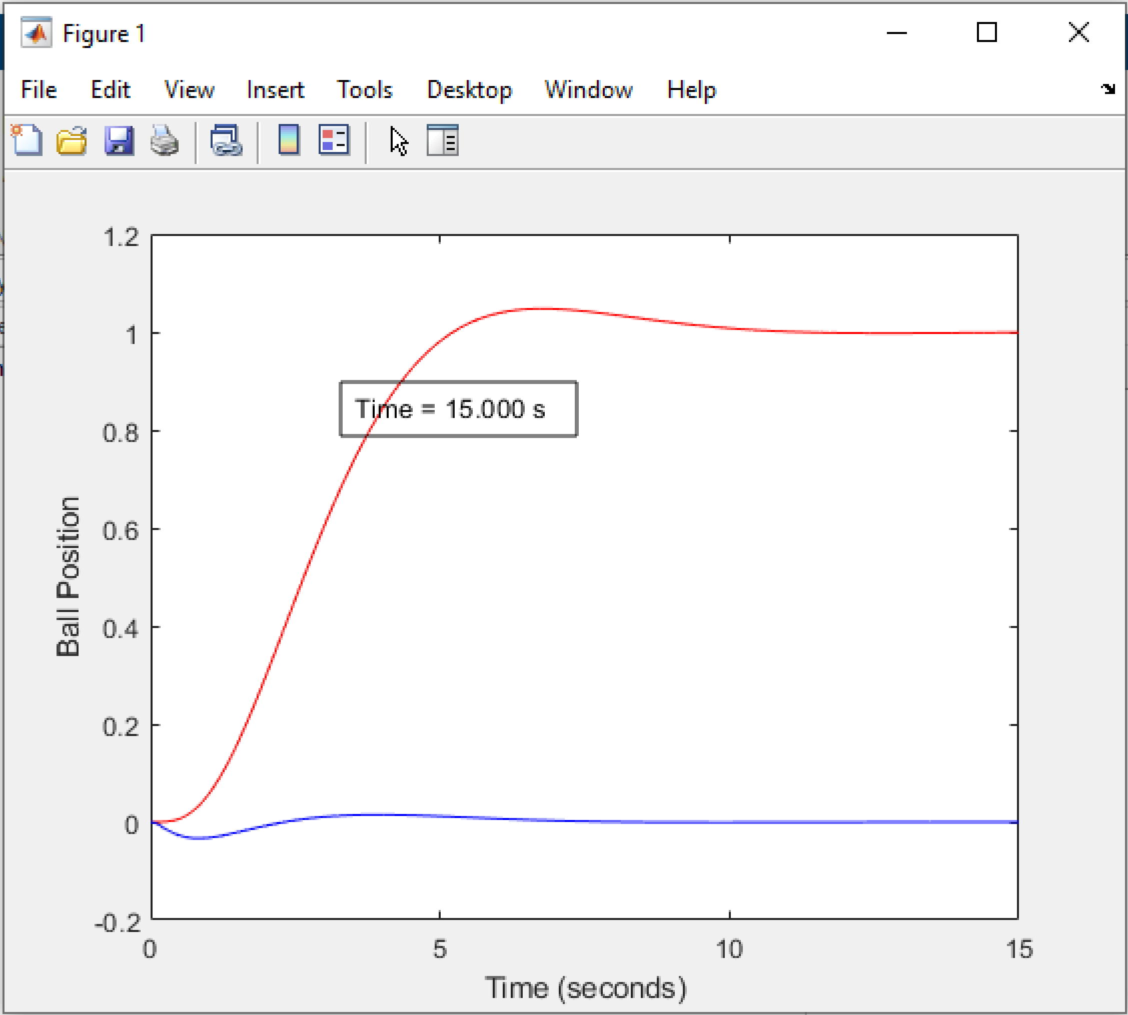
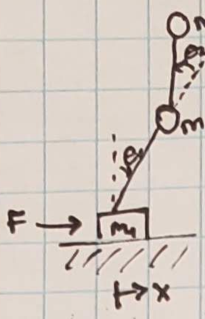


Figure 1
File Edit View Insert Tools Desktop Window Help
Time = 15.000 s
Ball Position
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15

```matlab
function [Kx, Kr] = ObtainCartFeedback(m1,m2,R, Ts, MOS)

% m1 = 1; m2 = 4; R = 1;
[A,B] = linearizedCartPend(m1,m2,R) %#ok<NOPRT>
sig = getSigForTs(Ts);
[zeta, th, w] = getForOS(MOS,sig); %#ok<ASGLU>
dom_poles = [sig+1j*w, sig-1j*w];
des_poles = [dom_poles, 5*sig, 5*sig-1] %#ok<NOPRT>

C1 = [1 0 0 0]; % Position of cart
C2 = [0 1 0 0]; % Angle of bar
C = C1;
% olPoles = flip(eig(A));

[Kx,Kr] = placePoles(A,B,C,des_poles) %#ok<NOPRT>

G1 = ss(A-B*Kx, B*Kr, C1, 0);
G2 = ss(A-B*Kx, B*Kr, C2, 0);
G = G1;
U = ss(A-B*Kx, B*Kr, -Kx, Kr);

step(G);
hold on;
step(G2);
step(U);
legend('x(t)','\theta(t)','F(t)');
xlabel('Time (s)'); title('Linearized System Model: Step
Response');
hold off;

end
```

```matlab
% Cart and Pendulum
% EC 463 Lecture 7
% main calling routine

X = zeros(4,1);
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
t = 0;

m1 = 1;
m2 = 4;
R = 1;
Ts = 8; MOS = 0.05;
[Kx, Kr] = ObtainCartFeedback(m1,m2,R,Ts,MOS);
% Kx = [-1.0714, -78.5964, -2.5230, -12.5230];
% Kr = -1.0714;

pause;
U = 0;
y = [];
dim = [0.3 0.6 0.2 0.15];
an = annotation('textbox', dim, 'String', '', 'FitBoxToText',
'on');
while(t < Ts*2)
    U = Kr*Ref - Kx*X;
    dX = CartDynamics(X, U, m1, m2, R);
    X = X + dX * dt;
    t = t + dt;

    CartDisplay(X, Ref);
    y = [y ; X(1), X(2), Ref];

    str = sprintf('Time = %.2f s', t);
    an.String = str;
    drawnow;
end

t = linspace(0,Ts*10,length(y));
plot(t,y);
xlabel('Time (s)'); title('Nonlinear System Model: Step
Response');
```

3) Now a double pendulum...



$$s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2g & 0 & 0 & 0 & 0 \\ 0 & 3g & -g & 0 & 0 & 0 \\ 0 & -3g & 3g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} F$$

Ok no matter what I do here, the reference gain K_r is just too high... can't proceed...

New to MATLAB? See resources for Getting Started.

```
>> ObtainDoublePendulumFeedback

Kx =

    0.0027  -28.9787   40.3435    0.0161    0.6496    5.6335


Kr =

  -Inf

```

```matlab
function [Kx, Kr] = ObtainDoublePendulumFeedback()

g = 9.8;
A = [0 0 0 1 0 0; 0 0 0 0 1 0; 0 0 0 0 0 1; 0 -2*g 0 0 0 0; 0
3*g -g 0 0 0; 0 -3*g 3*g 0 0 0];
B = [0;0;0;1;-1;1];

des_poles = [-0.5+0.5244j, -0.5-0.5244j, 0,-0.75,-0.8,-0.8];

C1 = [1 0 0 0 0 0]; % Position of cart
C2 = [0 1 0 0 0 0]; % Angle of first mass
C3 = [0 0 1 0 0 0]; % Angle of second mass
C = C2;

[Kx,Kr] = placePoles(A,B,C,des_poles) %#ok<NOPRT>

G1 = ss(A-B*Kx, B*Kr, C1, 0);
G2 = ss(A-B*Kx, B*Kr, C2, 0);
G3 = ss(A-B*Kx, B*Kr, C3, 0);
G = G1;
U = ss(A-B*Kx, B*Kr, -Kx, Kr);

step(G);
hold on;
step(G2);
step(U);
legend('x(t)','\theta_1(t)','\theta_2(t)');
xlabel('Time (s)'); title('Linearized System Model: Step
Response');
hold off;

end
```