

① Take the C&P of HW #6 ($m_c = 1\text{kg}$, $m_L = 4\text{kg}$, $L = 1\text{m}$) w/ input disturbance $F = B$ and $n_u \sim N(0, 0.02^2)$ and sensor noise $n_y \sim N(0, 0.01^2)$.

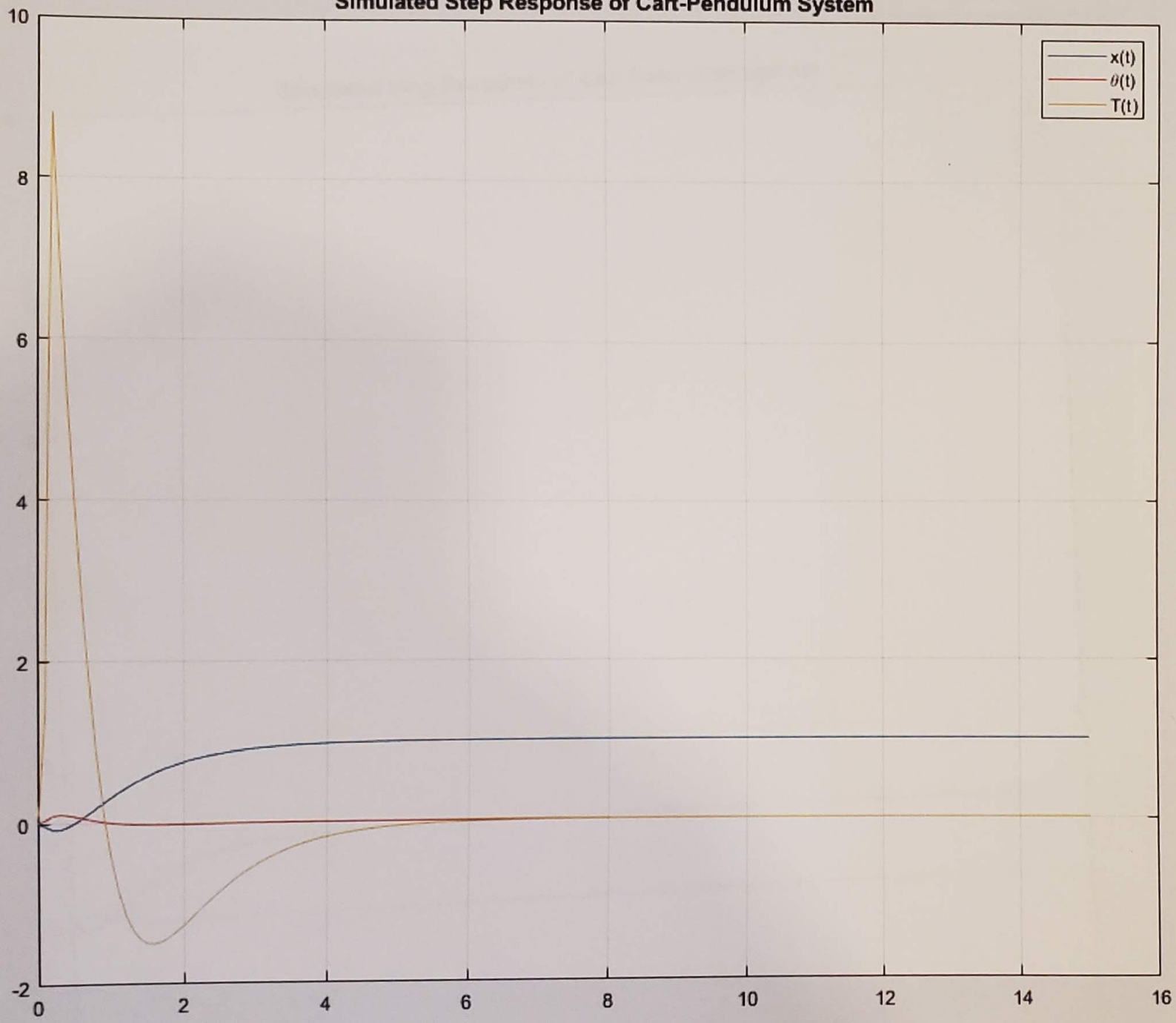
(1) Use a servo-comp to force DC gain $\rightarrow 1$ and observe response (a) w/o noise and (b) w/ noise.

From HW 10, I got $K_x = [-1218, -2267.9, -666, -711.8]$, $K_z = -743.3707$.

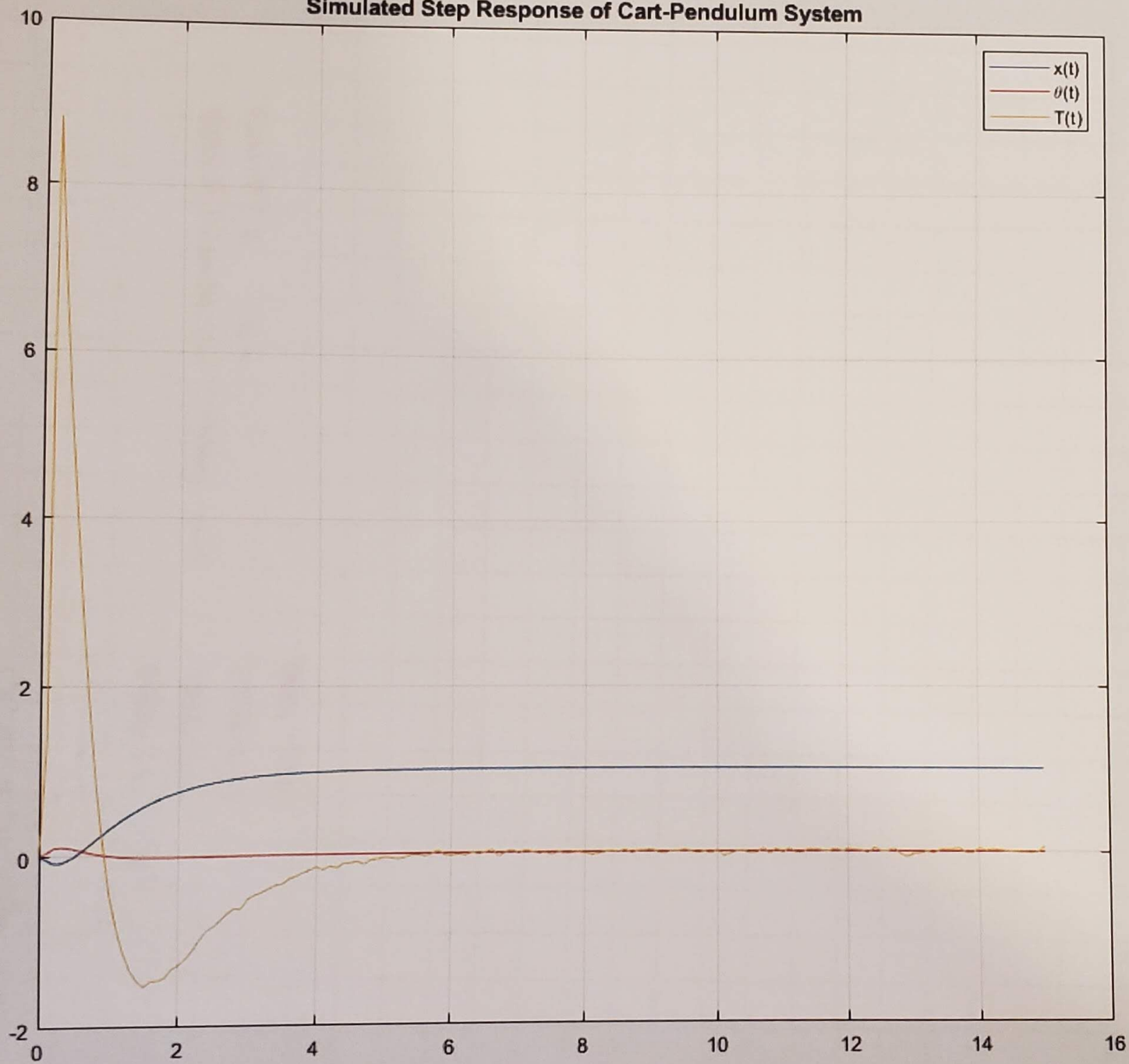
(a) Now w/o noise : (next page)

(b) w/ noise : (page after that)

Simulated Step Response of Cart-Pendulum System



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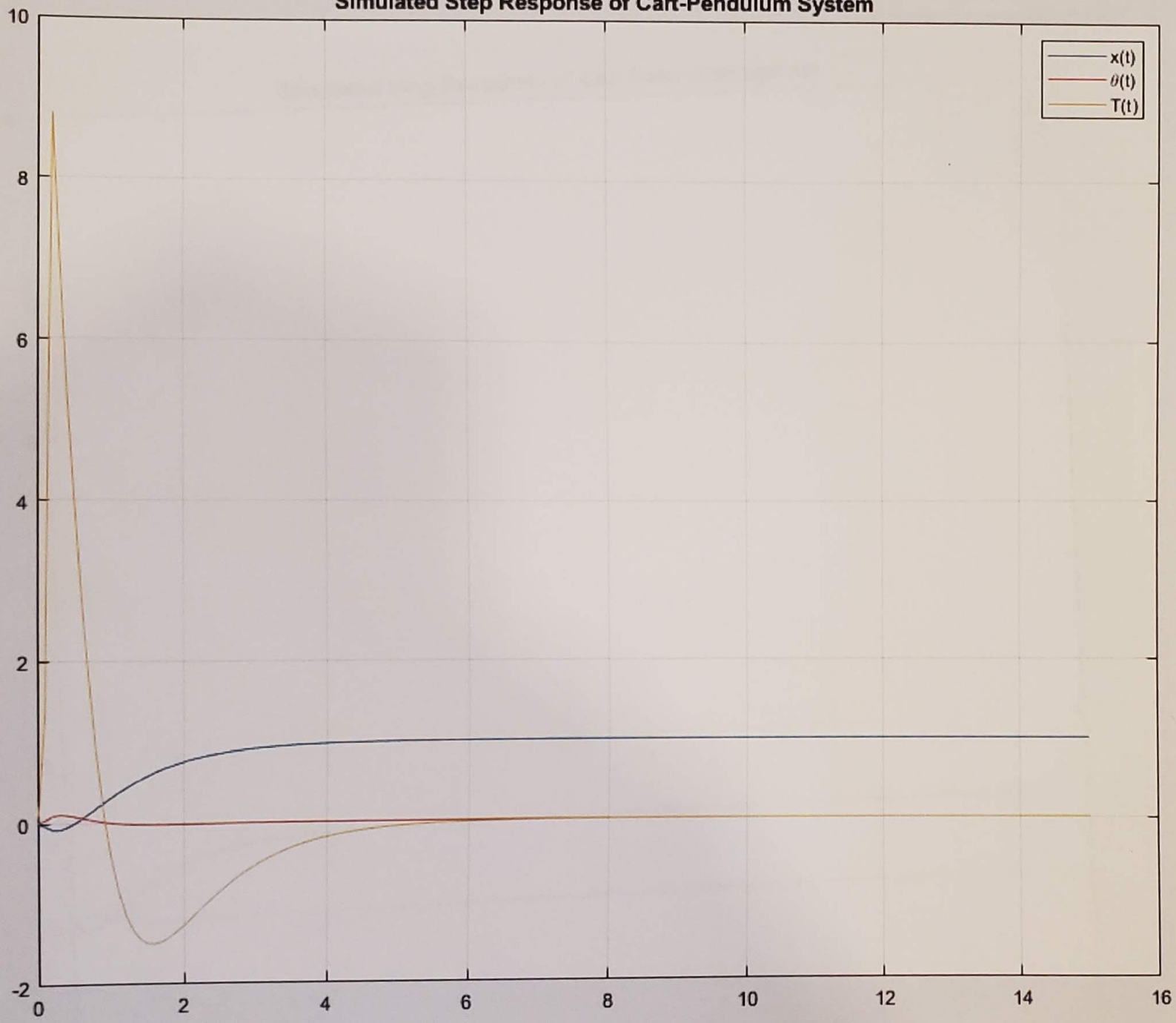


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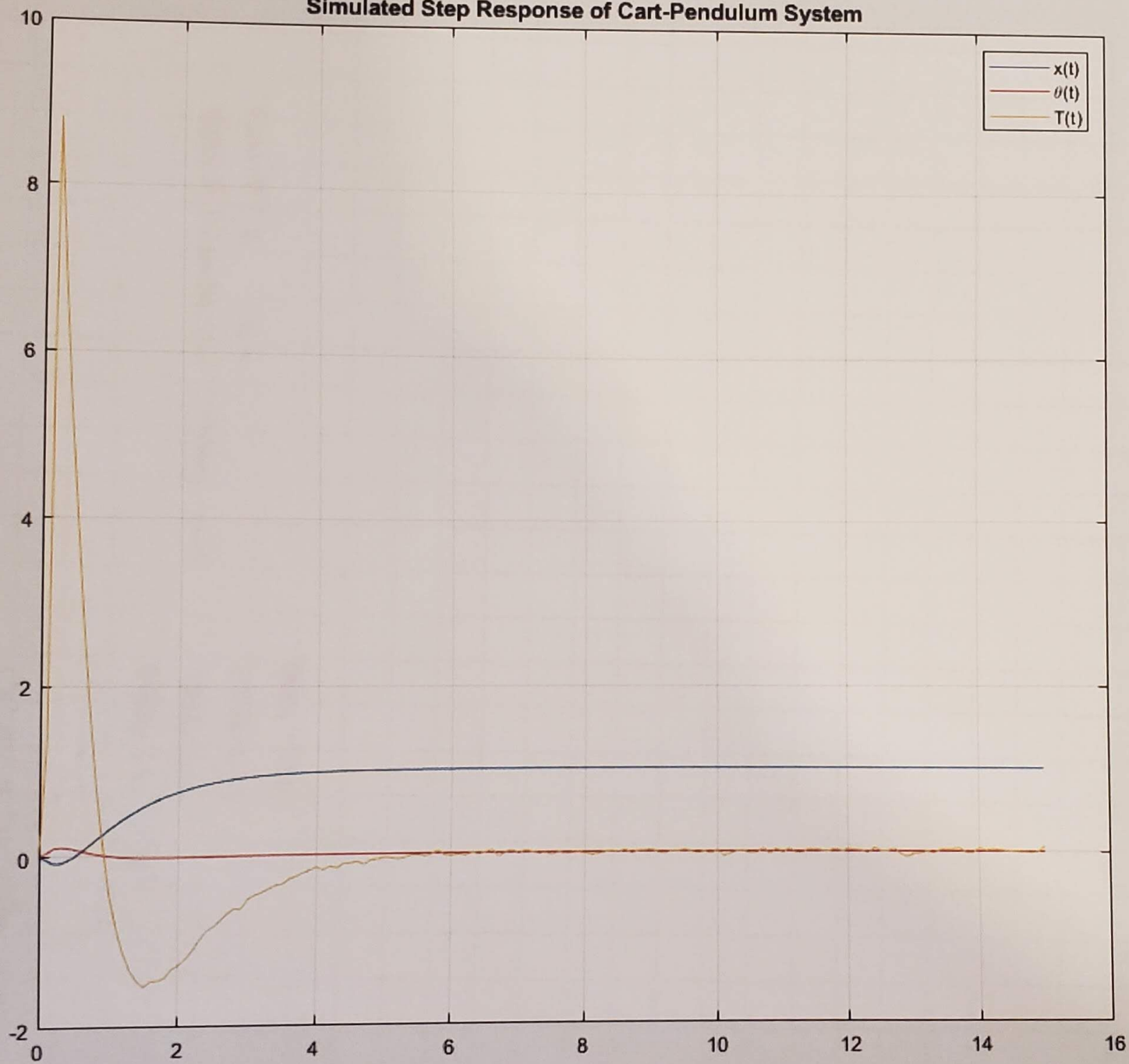
Now design a full-order observer to place the observer poles at $[-3, -4, -5, -6]$, simulate, and plot, using observer estimate.

$$H = \text{placePoles}(A', C', [-3, -4, -5, -6])' \Rightarrow H = \begin{matrix} 18.0 \\ -31.2245 \\ 168.0 \\ -219.1837 \end{matrix}$$

Simulated Step Response of Cart-Pendulum System



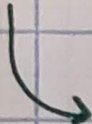
Simulated Step Response of Cart-Pendulum System



3) Now Kalman.

$$Q = B * B^T * 0.02^2; R = 0.012;$$

$$H = \text{lqr}(A', C', Q, R)'$$



$H :$ 14.9088

-19.9151

111.1360

-139.4200

\Rightarrow Poles: $-0.4544 \pm j0.4398$
 $-7 \quad \pm j0.1143$

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% Cart and Pendulum
% EC 463 Lecture 7
% main calling routine

% System
mcart = 1; mball = 4; L = 1;
[A,B] = linearizedCartPend(mcart,mball,L);
C = [1 0 0 0];

% Noise
F = B;
nu_mu = 0; nu_sig = 0.02;
ny_mu = 0; ny_sig = 0.01;

% Feedback gains
Kx = [-1218, -2267.9, -666, -711.8];
Kz = -743.3707;

% Observer gains
% H = [18; -31.2245; 168; -219.1837];
H = [14.9088; -19.9151; 111.1360; -139.4200];

% Initial conditions
% % Trial 1
% X = zeros(4,1); Z = 0;
% dX = zeros(4,1);

% Trial 2
X = zeros(4,1); Xe = zeros(4,1); Z = 0;
dX = zeros(4,1); dXe = zeros(4,1);

% Simulation setup
Ref = 1;
t = 0; dt = 100e-6; Tend = 20;
N = (Tend / dt) + 1;

% % Trial 1
% DATA = zeros(N,3); % x, th, T
% Trial 2
DATA = zeros(N,4); % x,th,x, the,T

i=1;
tic
while(t < Tend)

% % Trial 1
% % U = -Kz*Z - Kx*X;
% U = -Kz*Z - Kx*X + normrnd(nu_mu,nu_sig);
%
% dX = CartDynamics(X,U,mcart,mball,L);
% dZ = C*X + normrnd(ny_mu,ny_sig) - Ref;
%
% X = X + dX*dt;
% Z = Z + dZ*dt;
% t = t + dt;

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%      Trial 2
U = -Kz*Z - Kx*Xe + normrnd(nu_mu,nu_sig);

dX = CartDynamics(X, U, mcart, mball, L);
dXe = A*Xe + B*U + H*C*(X - Xe);
dZ = C*X + normrnd(ny_mu,ny_sig) - Ref;

X = X + dX*dt;
Xe = Xe + dXe*dt;
Z = Z + dZ*dt;
t = t + dt;

%      DATA(i,:) = [X(1), X(2), U];
DATA(i,:) = [X(1), X(2), Xe(1), Xe(2)];
i = i+1;

end
toc

kk = 10;
t = [1:length(DATA)]' * dt;
DATAds = downsample(DATA, kk);
tds = downsample(t, kk);

% plot(tds, DATAds(:,1), tds, DATAds(:,2), tds, DATAds(:,3));
% legend('x(t)', '\theta(t)', 'T(t)');
plot(tds, DATAds);
legend('x(t)', '\theta(t)', 'x_e(t)', '\theta_e(t)');
grid on;
title('Simulated Step Response of Cart-Pendulum System');

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