

We will use the following: (gantry system)

$$s \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 39.2 & 0 & 0 \\ 0 & -49 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} F + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} d$$

Feedback Controller Design

- 1) Design a servo compensator to track a sinusoidal setpoint $R(t) = \sin(1.5t)$ and reject a constant disturbance. $d=10$. Give the resulting CL matrices.

We will need our servo comp to have poles at $0, \pm j1.5$.

$$\Rightarrow A_z = \begin{bmatrix} 0 & 1.5 & 0 \\ -1.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_z = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Augmented system will be: $s \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} A & 0 \\ B_z C & A_z \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ -B_z \end{bmatrix} R$

Let's set the des_poles = $[-1, -2, -3, -4, -5, -6, -7] \Rightarrow \text{place Poles}(\dots)$

\Downarrow

$A_{\text{closed-loop}} \approx$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -624 & -314 & -80 & -52 & -414 & -462 & -238 \end{bmatrix}$$

$$K_x = [624.6481, 353.8981, 80.7256, 52.7256]$$

$$K_z = [414.6627, 462.6552, 228.5714]$$

Ac1 =

0	0	1.0000	0	0	0	0
0	0	0	1.0000	0	0	0
-624.6481	-314.6981	-80.7256	-52.7256	-414.6627	-462.6552	-228.5714
624.6481	304.8981	80.7256	52.7256	414.6627	462.6552	228.5714
1.0000	0	0	0	0	1.5000	0
1.0000	0	0	0	-1.5000	0	0
1.0000	0	0	0	0	0	0

```
>> Bcl
```

```
Bcl =
```

0

0

0

-1

-1

-1

2) Verify your control law on the linearized system.

- Plot when $R = \sin(t)$ and $d = 0$.

- " " $R = 0$ $d = 10$

- " " $R = \sin(1.5t)$ $d = 10$

(See attached \Rightarrow Code at end of submission)

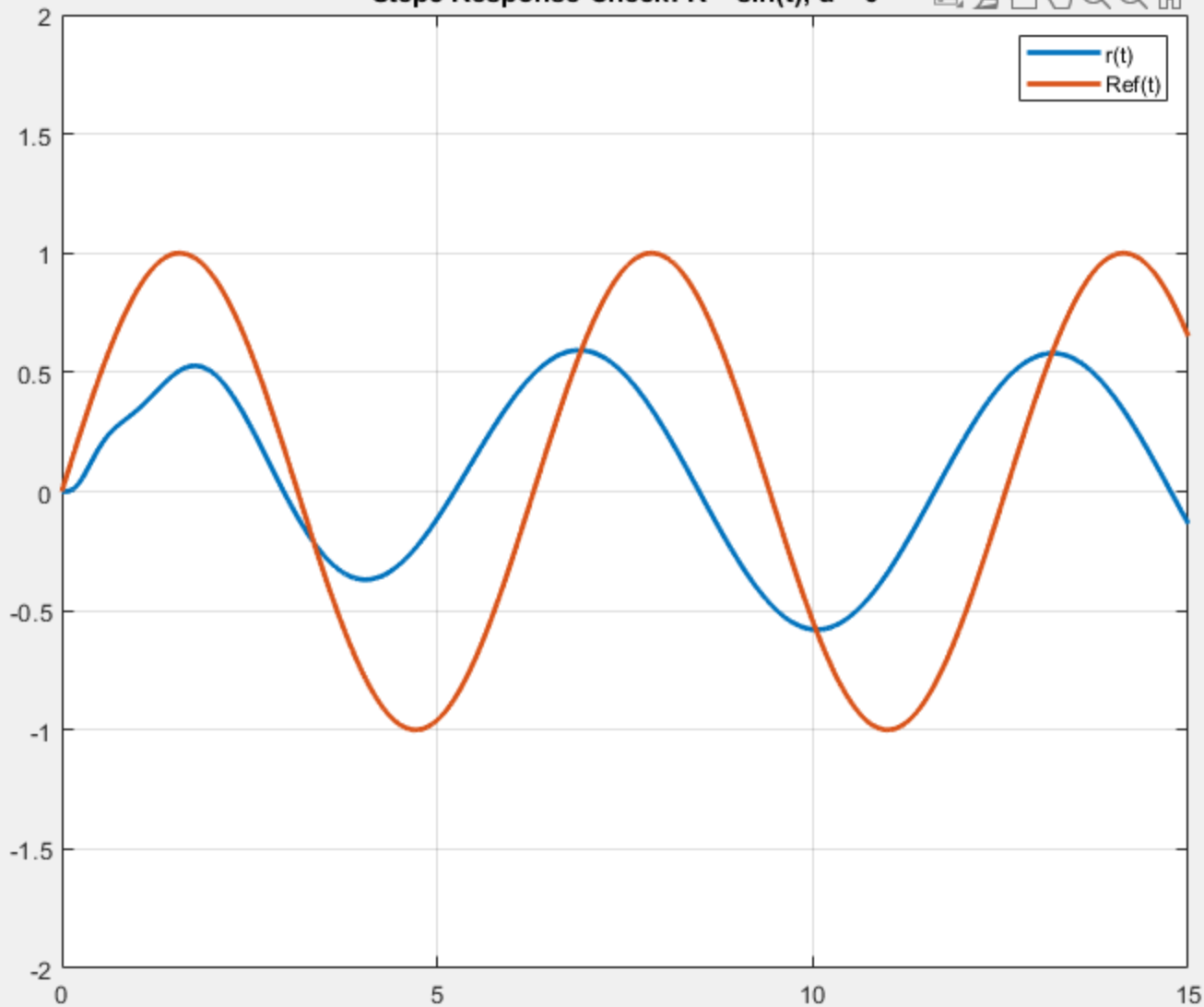
3) Verify in the nonlinear sim (w $R = \sin(1.5t)$ and $d = 10$).

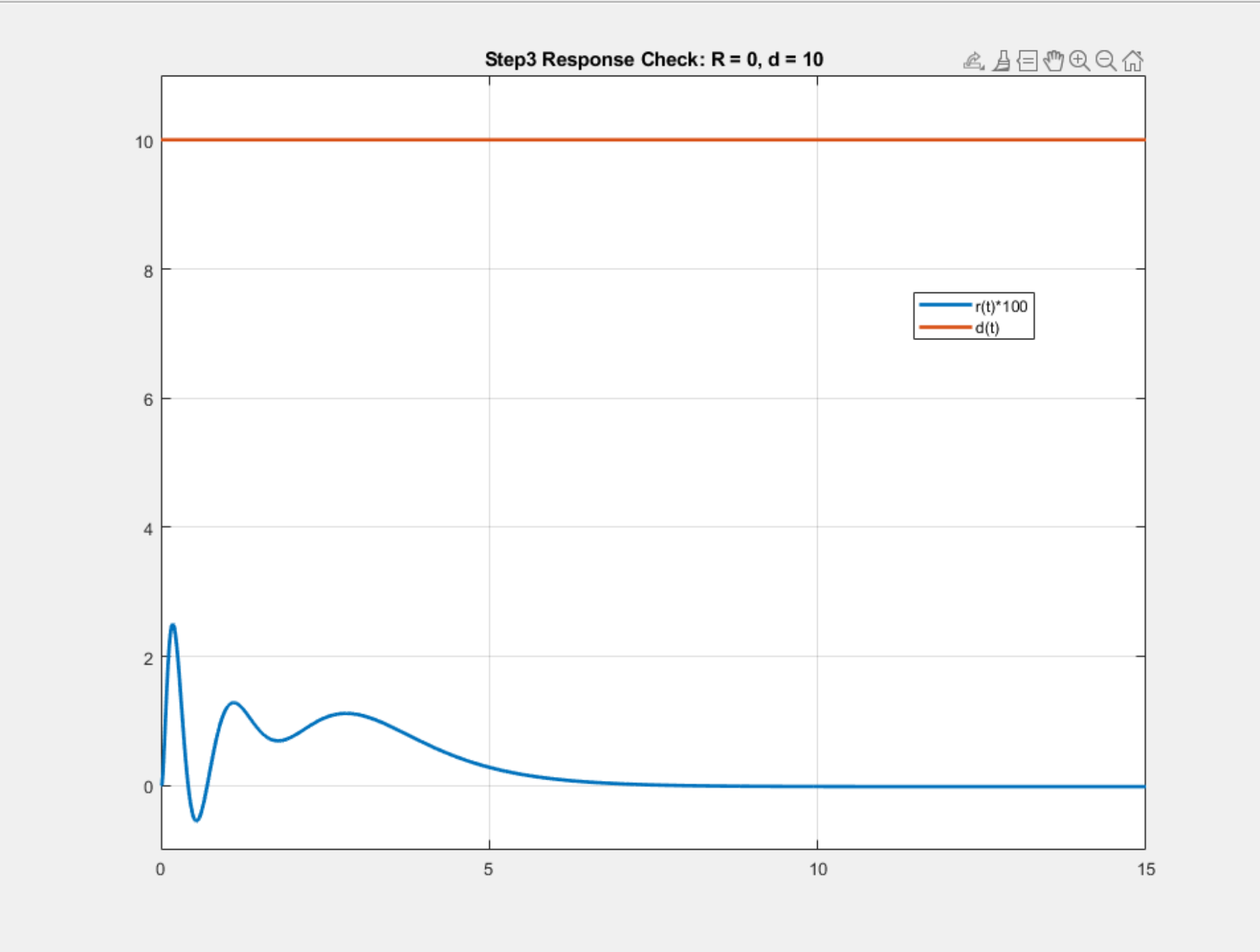
Figure 1

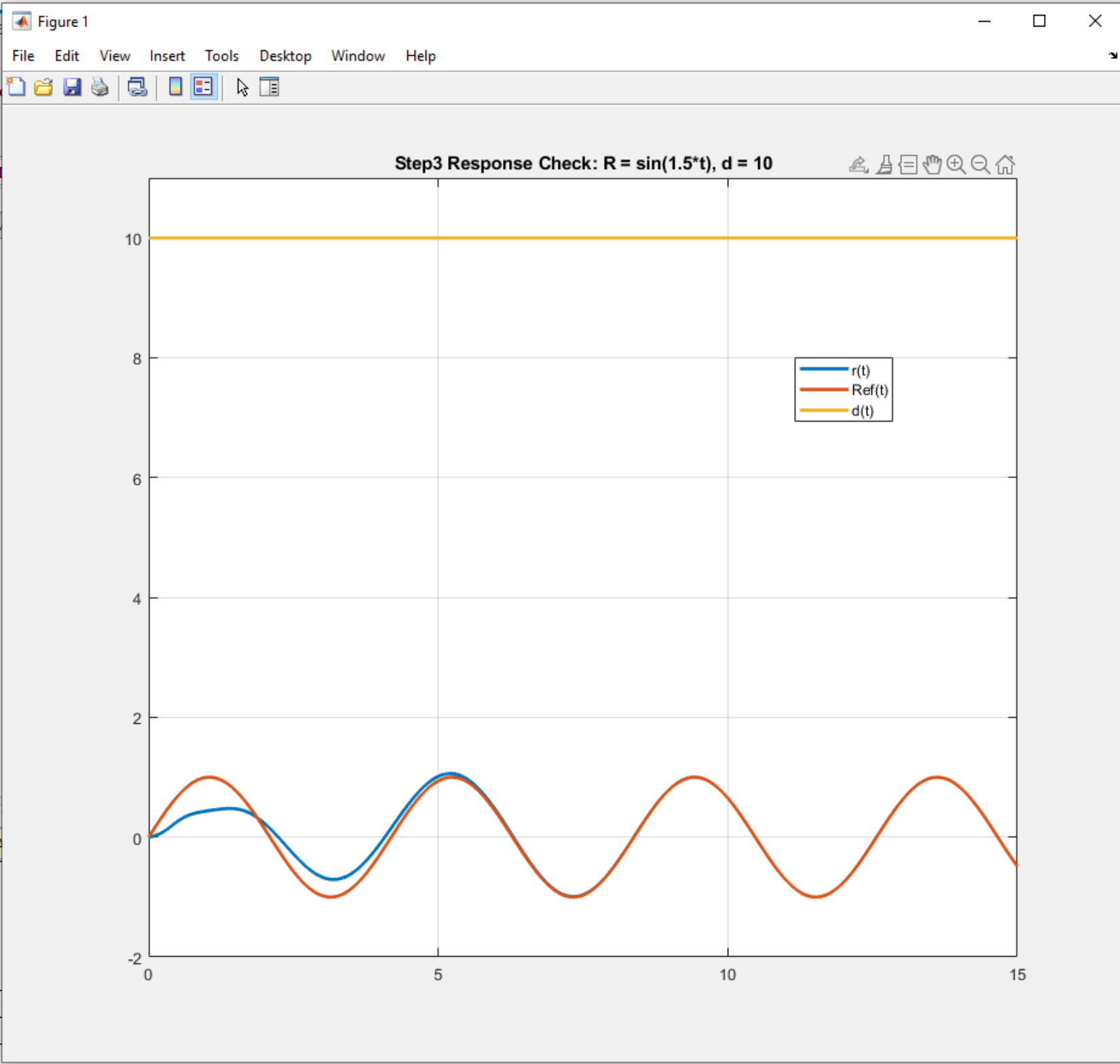
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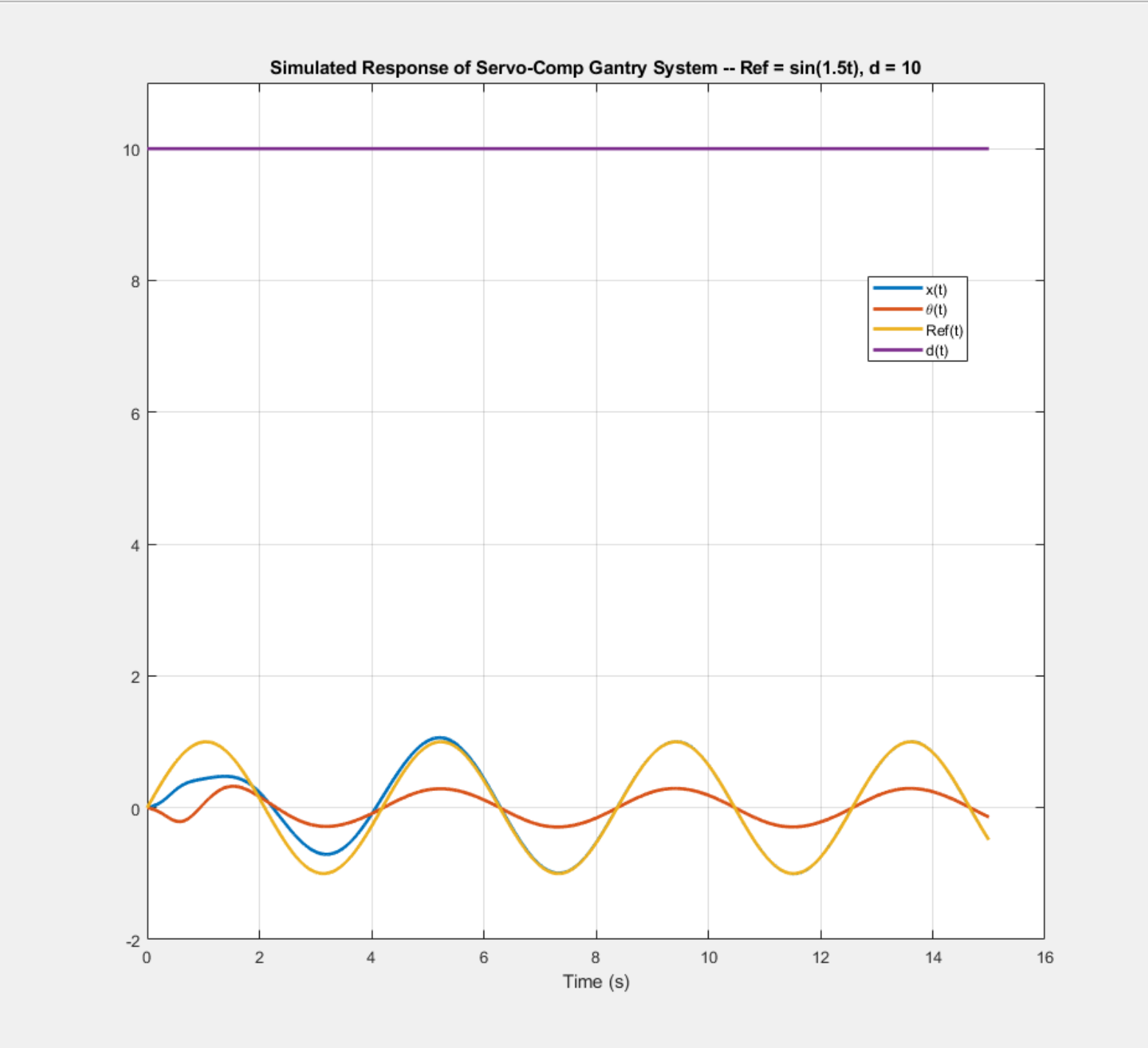


Step3 Response Check: $R = \sin(t)$, $d = 0$









Observer Design Assume you can measure both position $x(t)$ and angle $\theta(t)$.

- 4) Design a full-order observer to estimate the states and the disturbance d . Feed back the state estimates rather than the actual states. Give the matrices for the resulting plant, servo compensator, observer, and FSF.

Ok clearly we have an input disturbance. We add the disturbance as a state resulting in the plant + disturbance system:

$$s \begin{bmatrix} X \\ X_i \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ X_i \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} X \\ X_i \end{bmatrix}$$

Just using position

Putting this into MATLAB, we can get our full-order observer:

$$H = \begin{bmatrix} 23, -9.3961, 161, -147.0408, \\ 191.3265 \end{bmatrix}$$

H_d

Since I made the system earlier (w/ dom pole at -1, I'll make the obs. poles $[-3, -5, -5, -5, -5]$)

Now we design the servo-comp separate from the state estimate \Rightarrow we just use the first design: $K_x = [624.6481, \dots, 52.7256]$, $K_z = [414.6627, \dots, 228.5714]$

This leaves us with:

Open-loop:

$$\begin{cases} sX = AX + BU \\ sZ = A_2Z + B_2(CX - R) = (B_2C)X + (A_2)Z - B_2R \\ s\hat{X} = A\hat{X} + BU + H(CX - \hat{X}) = (HC)X + (A - HC)\hat{X} + BR \end{cases}$$

$$\rightarrow s \begin{bmatrix} X \\ \hat{X} \\ Z \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ HC & A - HC & 0 \\ B_2C & 0 & A_2 \end{bmatrix} \begin{bmatrix} X \\ \hat{X} \\ Z \end{bmatrix} + \begin{bmatrix} B \\ B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ -B_2 \end{bmatrix} R$$

Screenshot next page

$$K_{\hat{x}} = [K_x, 0]$$

Closed-Loop:
(separate paper)
for working

$$s \begin{bmatrix} X \\ \hat{X} \\ Z \end{bmatrix} = \begin{bmatrix} A & -BK_{\hat{x}} & -BK_z \\ HC & A - HC - B_s K_{\hat{x}} & -BK_z \\ B_2C & 0 & A_2 \end{bmatrix} \begin{bmatrix} X \\ \hat{X} \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -B_2 \end{bmatrix} R$$

$$K_2 = [K_x, 0]$$

$$u = -K_2 \hat{x} - K_z z$$

↓

$$sX = AX + B(-K_2 \hat{x} - K_z z) = AX + (-BK_2) \hat{x} + (-BK_z) z$$

$$s\hat{x} = (B_z C)X + (0)\hat{x} + (A_z)z + (-B_z)R$$

$$\cancel{s\hat{x}} = (HC)X + (A - HC)\hat{x} + B(-K_2 \hat{x} - K_z z)$$

$$s\hat{x} = (HC)X + (A - HC - BK_2)\hat{x} + (-BK_z)z$$

Aol =

0	0	1.0000	0	0	0	0	0	0	0	0	0	0
0	0	0	1.0000	0	0	0	0	0	0	0	0	0
0	39.2000	0	0	0	0	0	0	0	0	0	0	0
0	-49.0000	0	0	0	0	0	0	0	0	0	0	0
23.0000	0	0	0	-23.0000	0	1.0000	0	0	0	0	0	0
-9.3961	0	0	0	9.3961	0	0	1.0000	0	0	0	0	0
161.0000	0	0	0	-161.0000	39.2000	0	0	1.0000	0	0	0	0
-147.0408	0	0	0	147.0408	-49.0000	0	0	-1.0000	0	0	0	0
191.3265	0	0	0	-191.3265	0	0	0	0	0	0	0	0
1.0000	0	0	0	0	0	0	0	0	0	1.5000	0	0
1.0000	0	0	0	0	0	0	0	0	-1.5000	0	0	0
1.0000	0	0	0	0	0	0	0	0	0	0	0	0

>> Bol

Bol =

0
0
1
-1
0
0
1
-1
0
0
0

```
>> Col
```

```
Col =
```

```
    1    0    0    0    0    0    0    0    0    0    0    0    0
```

```
>> Dol
```

```
Dol =
```

```
    0
```

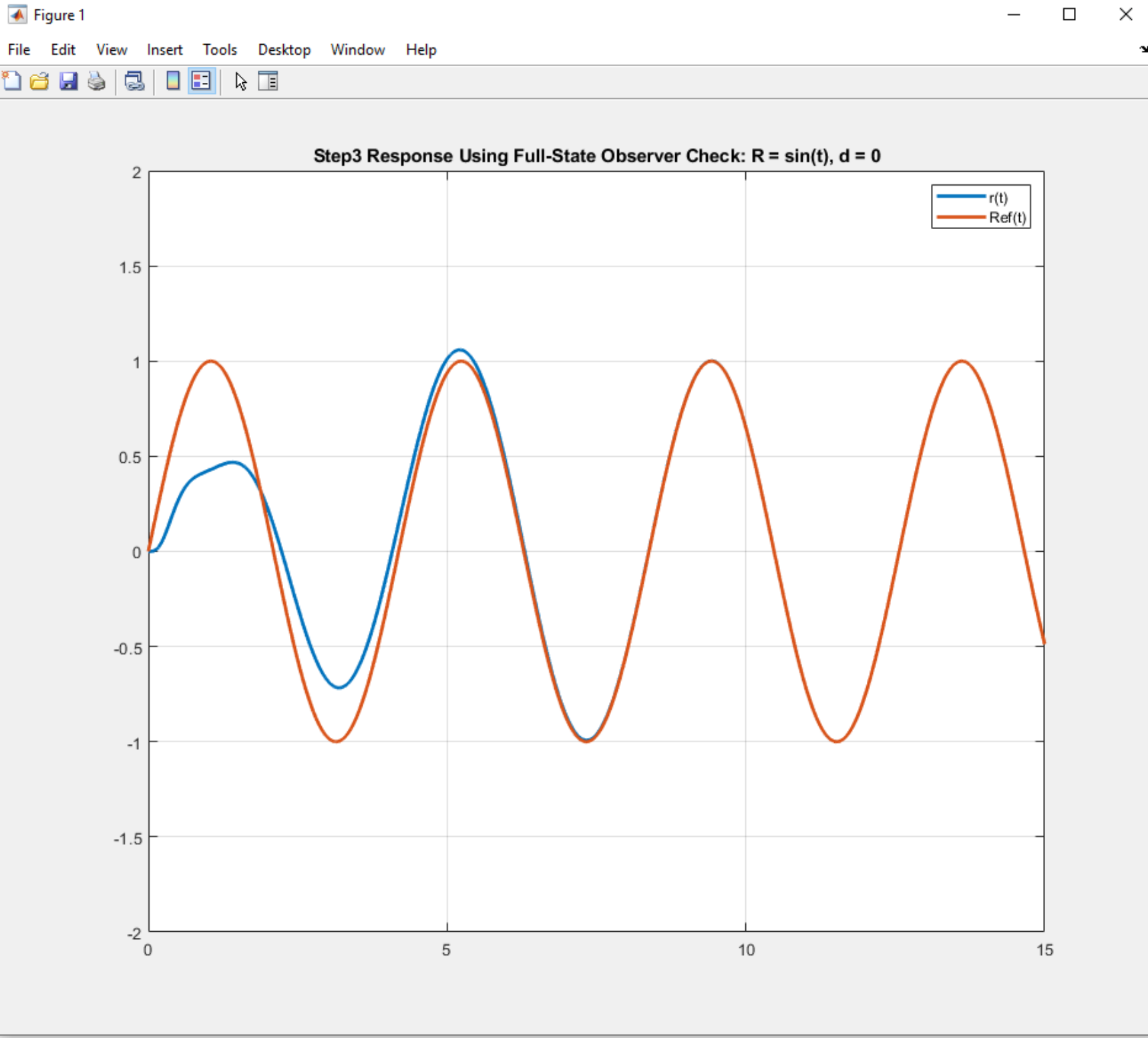
```
fx >> |
```

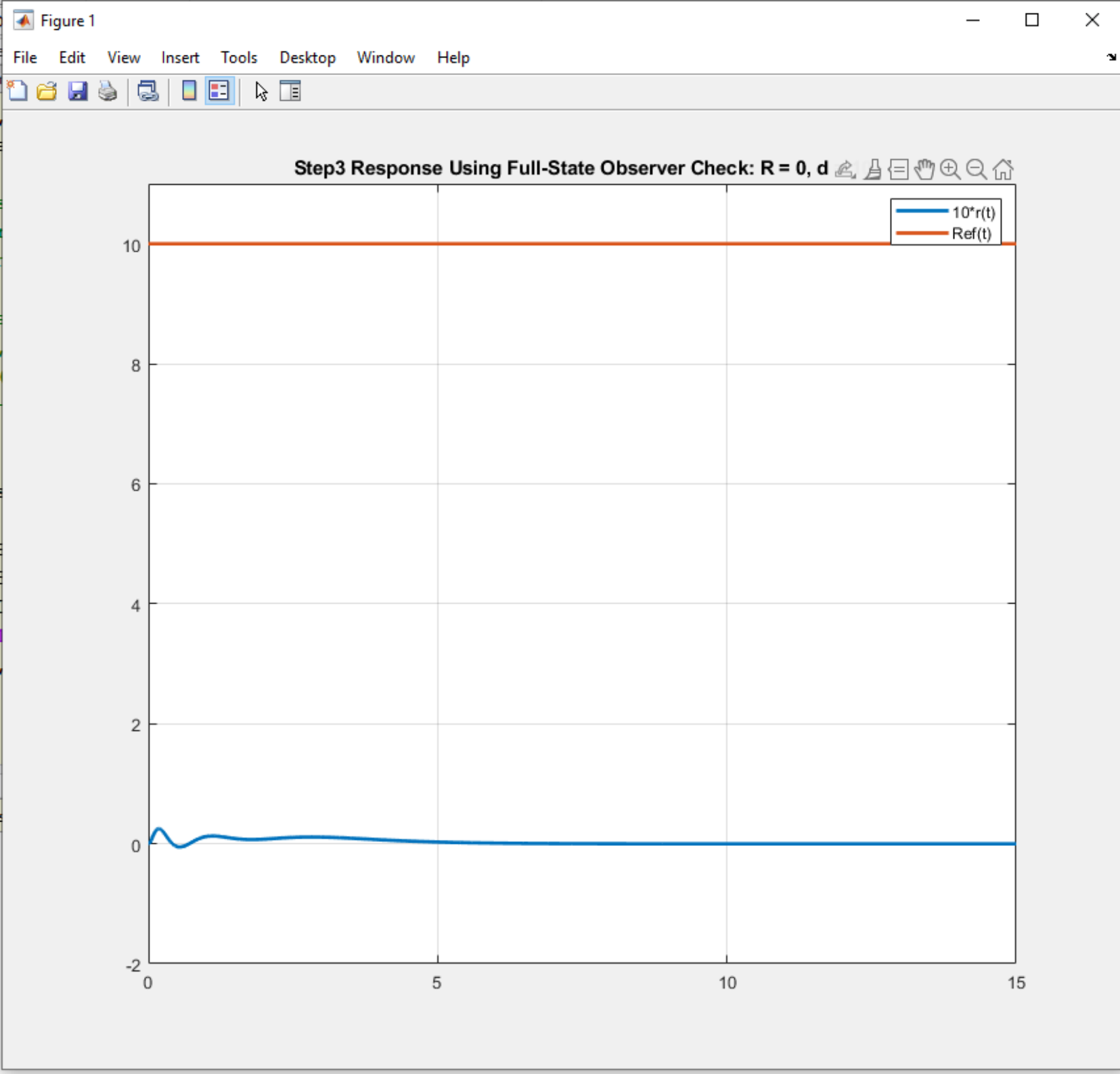
Ac1 =

0	0	1.0000	0	0	0	0	0	0	0	0	0	0
0	0	0	1.0000	0	0	0	0	0	0	0	0	0
0	39.2000	0	0	-624.6481	-353.8981	-80.7256	-52.7256	0	-414.6627	-462.6552	-228.5714	0
0	-49.0000	0	0	624.6481	353.8981	80.7256	52.7256	0	414.6627	462.6552	228.5714	0
23.0000	0	0	0	-23.0000	0	1.0000	0	0	0	0	0	0
-9.3961	0	0	0	9.3961	0	0	1.0000	0	0	0	0	0
161.0000	0	0	0	-785.6481	-314.6981	-80.7256	-52.7256	1.0000	-414.6627	-462.6552	-228.5714	0
-147.0408	0	0	0	771.6889	304.8981	80.7256	52.7256	-1.0000	414.6627	462.6552	228.5714	0
191.3265	0	0	0	-191.3265	0	0	0	0	0	0	0	0
1.0000	0	0	0	0	0	0	0	0	0	1.5000	0	0
1.0000	0	0	0	0	0	0	0	0	-1.5000	0	0	0
1.0000	0	0	0	0	0	0	0	0	0	0	0	0

Bc1 =

0
0
0
0
0
0
0
0
0
-1
-1
-1





s) simulate linearized system /w

• $R = \sin(1.5t)$, $d = 0$

• $R = 0$, $d = 10$

(Attached)

e) Nonlinear sim /w $R = \sin(1.5t)$ and $d = 10$

• Use actual states

• Use estimated states

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\dot{x} = Ax + Bu + Gw$$

this leaves us with:

$$\dot{z} = A_z z + B_z u + G_z w$$

Output:

estimated with both

$$K_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w$$

