

1) Settling time  $T_s \approx 1.5s$ , DC gain = 11,  $\omega_d \approx \dots T_d = 0.28s \Rightarrow \approx 3.57Hz$

a)

$$\tau = T_s/4 \approx 0.375s \Rightarrow \sigma = -2.667 \quad \omega_d \approx 22.4 \frac{\text{rad}}{s}$$

Dom. Poles @  $-2.667 \pm j22.4$

Hella underdamped

$$G(s) = k \frac{1}{(s + 2.667 \pm j22.4)}$$

$$G(0) = 11 \Rightarrow k = 5597.6$$

$$\Rightarrow G(s) = \frac{5597.6}{(s + 2.667 \pm j22.4)}$$

Checking on MATLAB...  
(see attached)

b) Step Response of  $Y = \frac{2000}{(s+7 \pm j12)(s+40)} X$

$$X(s) = \frac{1}{s} \Rightarrow Y = \frac{2000}{s(s+7 \pm j12)(s+40)} = 2000 \left( \frac{A}{s} + \frac{B}{s+40} + \frac{C}{s+7 \pm j12} + \frac{C^*}{s+7-j12} \right)$$

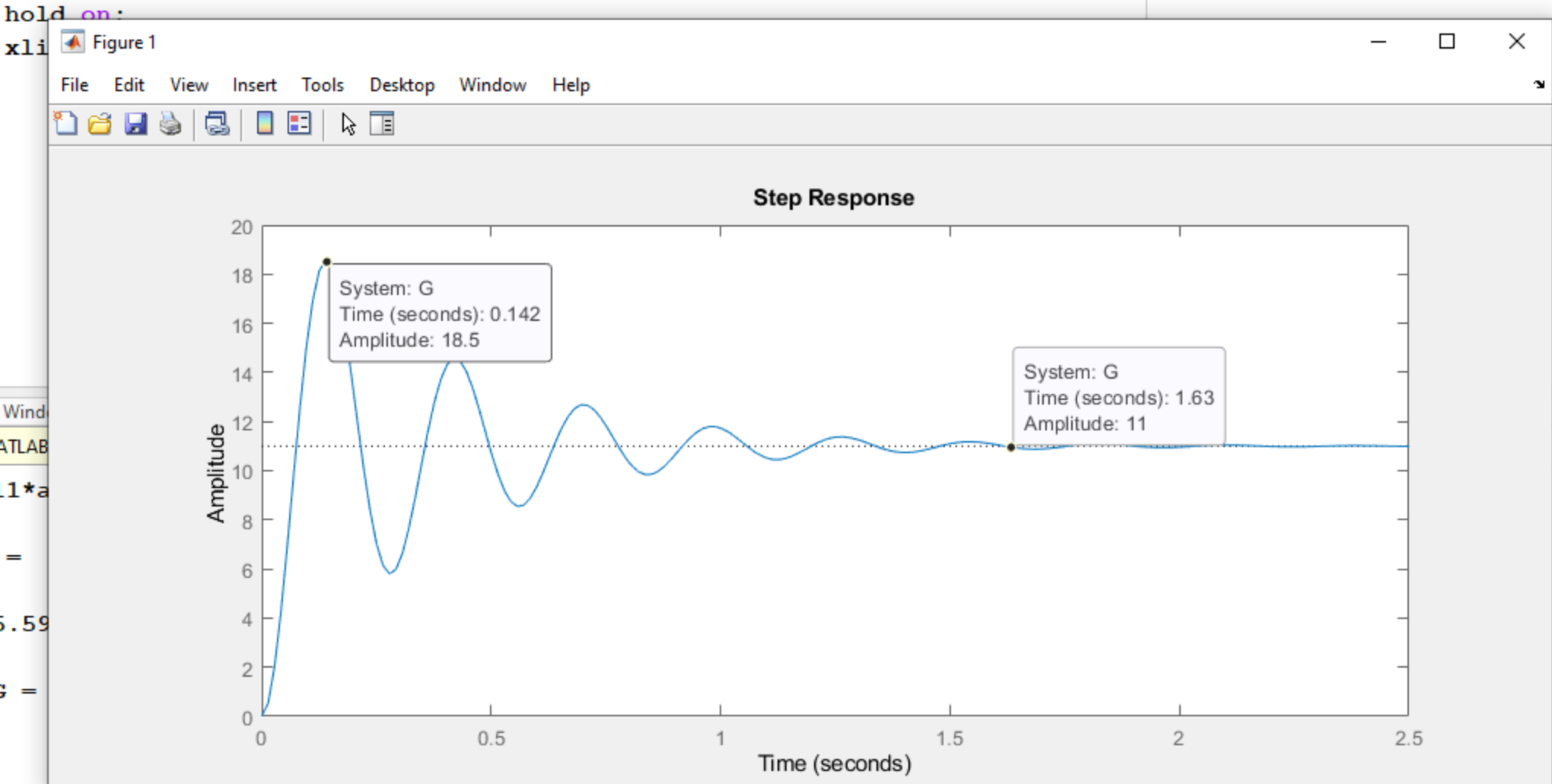
$$A = F(s)(B) \Big|_{s=0} = \frac{1}{117 + j121^2 40} \approx 0.12953 \times 10^{-3}$$

$$B = F(s)(s+40) \Big|_{s=-40} = -2.0276 \times 10^{-3}$$

$$C = F(s)(s+7 \pm j12) \Big|_{s=-7-j12} = (-0.0546 - 0.0656j) \times 10^{-3}$$

$$|C| = 0.1708$$

$$\angle C = -150^\circ$$





$$Y(s) = \frac{0.2590}{s} + -0.04055 \frac{1}{s+40} + (-0.1093 + j0.1313) \frac{1}{s+7-j12} + \dots$$

$$y(t) = 0.2590 u(t) - 0.04055 e^{-40t} u(t) + 2(0.1708) e^{-7t} \cos(12t + 130^\circ)$$

So DC gain: 0.2590

2% Settling: slowest is  $e^{-7t}$  term,  
 $\tau = 1/7 \Rightarrow 2\% T_s = 4/7 s$   
 $= 0.57s$

% OS: Angle of Dam. Pole  $-7 \pm j12$  is ~~45~~  $-60^\circ$  from - real axis  
 This corresponds to  $\zeta = 0.5039$

$$\Rightarrow OS = \exp\left(-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$OS \approx 16\%$$

Checking in MATLAB (w step:

```
>> step(G)
>> stepinfo(G)
```

```
ans =
```

struct with fields:

```
    RiseTime: 0.1277
SettlingTime: 0.5984
SettlingMin: 0.2340
SettlingMax: 0.2976
    Overshoot: 14.8699
Undershoot: 0
      Peak: 0.2976
    PeakTime: 0.2901
```

