ABDULLAH ALMOSALAMI | ECE444 HW#3 | 10/03/2020 Sunday

1) 2.7-6) Find the tf $H(s)$ and plot $|H(\omega)|$ for a BP BWF that satisfies $\alpha_p \le 3dB$, $\alpha_s \ge 17dB$, $\omega_{p1} \le 100$ rad/s, $\omega_{p2} \ge 250$ rad/s, $\omega_{s1} \ge 40$ rad/s, $\omega_{s2} \le 500$ rad/s. and determ LPF prototype

First, translate parameters to a LPF's parameters: $\omega_{p1}, \omega_{p2} \to \omega_p$, $\omega_{s1}, \omega_{s2} \to \omega_s$

This mapping is not unique, so we can start by having $\omega_p = 1$ rad/s and recalling the LP→BP transformation(mapping)..

$$\omega \to \omega_p \frac{\omega^2 - \omega_1\omega_2}{\omega(\omega_2 - \omega_1)},$$

We know $\omega_p \to \omega_{p1}, \omega_{p2}$ under this mapping so...

$$\omega \to \omega_p \frac{\omega^2 - \omega_{p1}\omega_{p2}}{\omega(\omega_{p2} - \omega_{p1})}$$

We know ∃ ω_s that maps to ω_{s1}, ω_{s2}, so

$$\frac{\omega_p(\omega_{s1}^2 - \omega_{p1}\omega_{p2})}{\omega_{s1}(\omega_{p2} - \omega_{p1})} = \frac{(1)(40^2 - 100(250))}{40(250-100)} \approx -3.9$$

$$\frac{\omega_p(\omega_{s2}^2 - \omega_{p1}\omega_{p2})}{\omega_{s2}(\omega_{p2} - \omega_{p1})} = \dots \approx 3$$

Need to ask Dr. Green about this... kind of feel like the mapping should be 1:1; in the sence that $\omega_{Low\text{-}pass}$ should map to a unique $(\omega_1, \omega_2)_{BP}$ pair... so why would we get two ω_s's?

So choosing $\omega_s = 3$ rad/s, with $\alpha_p = 3dB$, $\alpha_s = 17dB$, we can determine the needed order K:

$$K = \left\lceil \frac{\log[(10^{\alpha_s/10} - 1)/(10^{\alpha_p/10} - 1)]}{2\log(\omega_s/\omega_p)} \right\rceil$$

(Remember the BW polynomial is defined w/ respect to K, ω_c)

$$\boxed{K = 2}$$

with this K, a suitable range of values for the cutoff frequency is:

$$\frac{\omega_p}{(10^{\alpha_p/10} - 1)^{1/2K}} \le \omega_c \le \frac{\omega_s}{(10^{\alpha_s/10} - 1)^{1/2K}}$$

$\Rightarrow \quad \omega_c \in [1.0012, 1.1332]$

Choosing ω_c right in the middle $\boxed{\omega_c = 1.0672 \text{ rad/s}}$ (though I should not the amount of clearance at ω_p and amount of clearance at ω_s is not necessarily symmetric with ω_c chosen like this)

So now, with k and ω_c, we can define this LP prototype's polynomial, and so its poles:

Poles: $j\omega_c e^{j\frac{\pi}{2k}(2k-1)} \Rightarrow \boxed{-0.7546 \pm j0.7546}$

corresponds to

$s^2 + 1.5092s + 1.1389$

$$H_p(s) = \frac{1.0672^2}{(s+0.7546+j0.7546)(s+\cdots)}$$

We can apply the LP → BP transform directly now to obtain our BPF but using the book's observation that each prototype pole goes to two BP poles and a zero, we can simplify (skip) some of the tedious algebra:

Zeros: All at $s=0 \Rightarrow (\omega_c^k)(\omega_{p2}-\omega_{p1})^k s^k \doteq 25{,}625 s^2$

Poles: $(p_k)_p \rightarrow s^2 - p_k(\omega_{p2}-\omega_{p1})s + \omega_{p1}\omega_{p2}$

$-365 \pm j102.79, -766.9 \pm j215.98$

So, overall:

$$\boxed{H(s) = \frac{25{,}625 s^2}{(s+365 \pm j102.79)(s \pm 766.9 \pm j215.98)}}$$

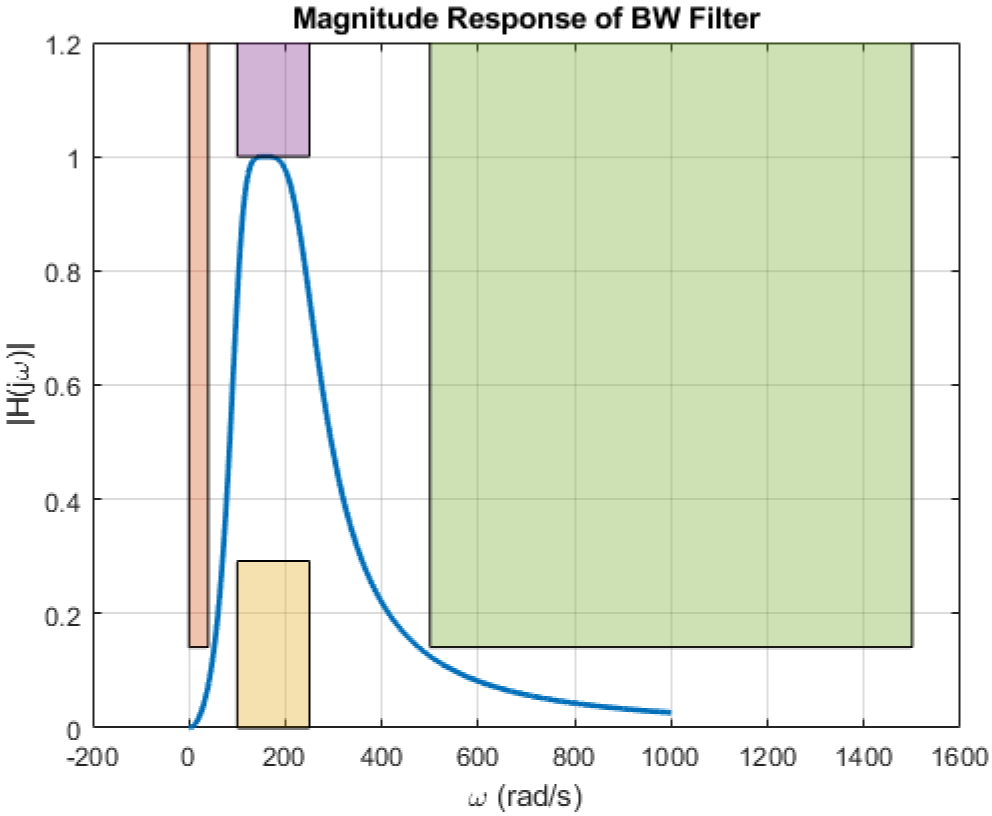
BP Design

BP characteristics in terms of LP

```
wp=1; wp1=100; wp2=250; ws1=40; ws2=500;
ws = abs([ wp*(ws1^2 - wp1*wp2)/(ws1*(wp2-wp1)),...
    wp*(ws2^2 - wp1*wp2)/(ws2*(wp2-wp1)) ]);
ws = min(ws); ap=3; as=17;

% Design LP prototype
p = @(k,K,wc) 1j*wc*exp((1j*pi/(2*K))*((2*k-1)));
K = @(wp, ap, ws, as) ceil( log( (10^(as/10)- 1) / (10^(ap/10)-1) ) /
 (2*log(ws/wp)) );
wc_range = @(wp,ap,ws,as, K) [ (wp/(10^(ap/10)-1)^(1/(2*K))) (ws/
(10^(as/10)-1)^(1/(2*K))) ];
order = K(wp,ap,ws,as)
omegac_range = wc_range(wp,ap,ws,as,order)
wc = (omegac_range(1) + omegac_range(2)) / 2;
k = 1:order;
poles_p = p(k,order,wc)
A = poly(poles_p)

% Now transform
a = 1; b = -poles_p*(wp2-wp1); c = wp1*wp2;
poles = [ (-b + sqrt(b.^2 - 4*a*c))/(2*a),  (-b - sqrt(b.^2 - 4*a*c))/
(2*a) ]
B = (wc^order)*((wp2-wp1)^order)*poly(zeros(order,1)), A = poly(poles)

delta_p = 10^(-ap/20); delta_s = 10^(-as/20);
w = 0:2*ws2; H = polyval(B, 1j*w) ./ polyval(A,1j*w);
plot(w, abs(H), 'LineWidth', 2);
pgon1 = polyshape([0 ws1 ws1 0], [delta_s delta_s 2 2]);
pgon2 = polyshape([wp1 wp1 wp2 wp2], [1-delta_p 0 0 1-delta_p]);
pgon3 = polyshape([wp1 wp1 wp2 wp2], [1 2 2 1]);
pgon4 = polyshape([ws2 ws2 3*ws2 3*ws2], [delta_s 2 2 delta_s]);
hold on;
plot(pgon1);
plot(pgon2);
plot(pgon3);
plot(pgon4);
hold off;
grid on;
xlabel('\omega (rad/s)'), ylabel('|H(j\omega)|'), title('Magnitude
 Response of BW Filter');
ylim([0 1.2]);
```

```
order =

     2


omegac_range =
```

```
    1.0012    1.1332


poles_p =

  -0.7546 + 0.7546i  -0.7546 - 0.7546i


A =

   1.0000 + 0.0000i   1.5092 - 0.0000i   1.1389 - 0.0000i


poles =

   1.0e+02 *

  -0.3650 - 1.0279i  -0.3650 + 1.0279i  -0.7669 + 2.1598i  -0.7669 -
 2.1598i


B =

   1.0e+04 *

    2.5625         0         0


A =

   1.0e+08 *

  Columns 1 through 4

   0.0000 + 0.0000i   0.0000 - 0.0000i   0.0008 - 0.0000i   0.0566 -
 0.0000i

  Column 5

   6.2500 + 0.0000i
```



LP Design

p = @(k,K,wc) 1j*wc*exp((1j*pi/(2*K))*((2*k-1)));

wp=10; ap=2; ws=30; as=20; K = @(wp, ap, ws, as) ceil(log((10^(as/10)- 1) / (10^(ap/10)-1)) / (2*log(ws/wp))); wc_range = @(wp,ap,ws,as, K) [(wp/(10^(ap/10)-1)^(1/(2*K))) (ws/(10^(as/10)-1)^(1/(2*K)))]; order = K(wp,ap,ws,as) wc_range(wp,ap,ws,as,order)

wc = 12.5; k = 1:order; poles = p(k,order,wc) A = poly(poles)

delta_p = 10^(-ap/20); delta_s = 10^(-as/20); w = 0:100; H = wc^order ./ (polyval(A, 1j*w)); plot(w, abs(H), 'LineWidth', 2); pgon1 = polyshape([0 wp wp 0], [0 0 delta_p delta_p]); pgon2 = polyshape([0 wp wp 0], [1 1 2 2]); pgon3 = polyshape([ws ws 3*ws 3*ws], [delta_s 2 2 delta_s]); hold on; plot(pgon1); plot(pgon2); plot(pgon3); hold off; grid on; xlabel('\omega (rad/s)'), ylabel('|H(j\omega)|'), title('Magnitude Response of BW Filter'); ylim([0 1.2]);

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2) 2.7-14) Determine the tf $H(s)$ and plot $|H(\omega)|$ for a 3rd order LP ICH if $\omega_c = 100$ rad/s with 20dB of stopband ripple.

$K = 3$, $\epsilon = \dfrac{1}{\sqrt{10^{\alpha_s/10} - 1}} \approx 0.1005$ Now to relate $\omega_c \to \omega_s$ ... Well ω_s oughtta be decently larger than ω_c ... ~~because ICH easily meet stopband sf~~ If we set $\omega_s = \omega_c$, we'd probably shorten the passband too much...

$$|H(j\omega_c)|^2 \leq \frac{1}{2} \geq \frac{\epsilon^2 C_k^2(\omega_s/\omega_c)}{1 + \epsilon^2 C_k^2(\omega_s/\omega_c)}$$

No time for the algebra. I'll MATLAB this... (using fminsearch)

So I get $\omega_s \lesssim 153.8459$ rad/s. Let's choose $\omega_s = \cancel{150}\ 145$ rad/s.

The poles (roots) of $1 + \epsilon^2 C_k^2(\omega_s/(s/j))$ are tedious to "algebra," so let me just find $\omega_c \to \omega_p$ like I did for ω_s and go $P_k = \omega_p \omega_s / P_k'$.

So let's say $\omega_p = 60$ rad/s

Assumed $\alpha_p = 2$dB...

I get $\omega_p \geq 50$ rad/s. So, now I can get the poles & zeros for the ICH: ...

Ok so I get insanely huge zeros... z_k: $\pm j167.4...$, $j2.368... \times 10^{18}$ rad/s

I effectively get...

P_k: $-40 \pm j91.2$, $-123.75 - j9.96 \times 10^{-15}$

No conjugate?

$$\boxed{H(s) = \frac{1.226 \times 10^6}{s^3 + 203.78s^2 + 19808s + 1.226 \times 10^6}}$$

```
K=3; as=20; ws=145; wp=60; ap=2;
epsilon = 1/sqrt(10^(as/10)-1)

% Now obtain polynomials that define the TF of a prototype Chebyshev
 that
% we will later transform to an inverse Chebyshev's
k = 1:K;
H0 = (mod(K,2)==1) + (mod(K,2)==0)*(1/sqrt(1+epsilon^2));   % For even
 order, H0 always is 1; for odd, it's 1/...
pk = -wp*sinh(asinh(1/epsilon)/K)*sin(pi*(2*k-1)/(2*K)) + ...
    1j*wp*cosh(asinh(1/epsilon)/K)*cos(pi*(2*k-1)/(2*K))

% Now the inverse Chebyshev
pk = wp*ws ./ pk
zk = 1j*ws*sec(pi*(2*k-1)/(2*K))

B = prod(pk./zk)*poly(zk), A = poly(pk)

% Now the TF!
w = 0:ws*3;
H = polyval(B,1j*w) ./ (polyval(A,1j*w));
plot(w, abs(H), 'LineWidth', 2);
delta_p = 10^(-ap/20); delta_s = 10^(-as/20);
pgon1 = polyshape([0 wp wp 0], [0 0 delta_p delta_p]);
pgon2 = polyshape([0 wp wp 0], [1 1 2 2]);
pgon3 = polyshape([ws ws 3*ws 3*ws], [delta_s 2 2 delta_s]);
hold on;
plot(pgon1);
plot(pgon2);
plot(pgon3);
hold off;
grid on;
xlabel('\omega (rad/s)'), ylabel('|H(j\omega)|'), title('Magnitude
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ylim([0 1.2]);
```
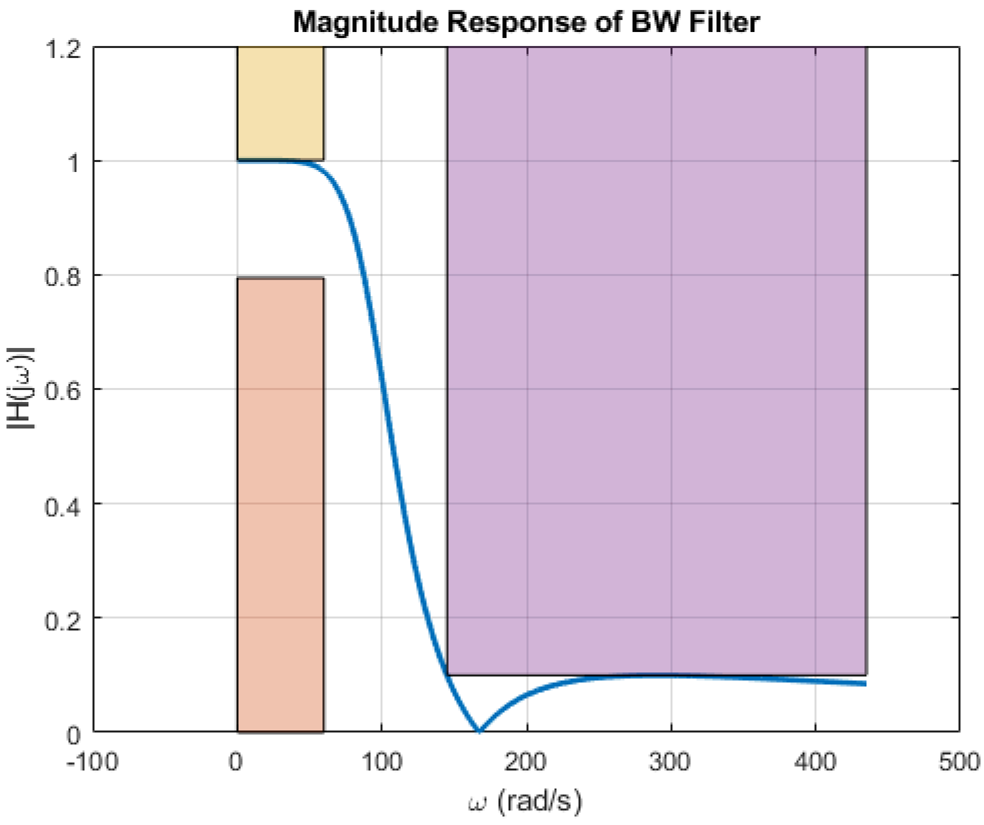

```
epsilon =

    100.503781525921e-003


pk =

  Column 1

   -35.1515487326770e+000 + 80.0430767456955e+000i

  Column 2

   -70.3030974653540e+000 + 5.65944701518947e-015i
```

```
  Column 3

   -35.1515487326770e+000 - 80.0430767456955e+000i


pk =

  Column 1

   -40.0153684073627e+000 - 91.1184092853451e+000i

  Column 2

   -123.749881778501e+000 - 9.96194939215203e-015i

  Column 3

   -40.0153684073627e+000 + 91.1184092853451e+000i


zk =

  Column 1

     0.00000000000000e+000 + 167.431578064991e+000i

  Column 2

     0.00000000000000e+000 + 2.36802970621333e+018i

  Column 3

     0.00000000000000e+000 - 167.431578064991e+000i


B =

  Column 1

   -770.371977754894e-036 + 18.4622451521886e-018i

  Column 2

     43.7191449637757e+000 + 1.82426372815790e-015i

  Column 3

     0.00000000000000e+000 + 0.00000000000000e+000i

  Column 4

     1.22559336381785e+006 + 51.1401931793599e-012i
```

```
A =

  Column 1

    1.00000000000000e+000 + 0.00000000000000e+000i

  Column 2

    203.780618593227e+000 + 14.2108547152020e-015i

  Column 3

    19.8075884389373e+003 + 1.81898940354586e-012i

  Column 4

    1.22559336381785e+006 + 58.2076609134674e-012i
```



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3) 3.1-2(c,d,e) Fig. shows spectra of x_1 and x_2. Nyquist sampling?

(c) $x_1^2(3t)$

$x_1(3t)$ has 3 times the bandwidth, so $B_{x_1(3t)} = 12\times10^4$ Hz $= 120$kHz

$x_1^2(3t) = x_1(3t)\cdot x_1(3t) \Rightarrow$ Convolve in f-dom.

widths sum → So $B = 240$kHz

So $\boxed{F_{Nyq} = 480\text{kHz}}$

(d) $x_2^3(t)$

$3(6\times10^4) \rightarrow 180$kHz bandwidth → $\boxed{F_s = 360\text{kHz}}$

(e) $x_1 x_2$

40kHz + 60 kHz = 100kHz → $\boxed{F_s = 200\text{kHz}}$

3.1-3 (a,c) Now for:

(a) $x_a(t) = \text{sinc}^2(100t) \Rightarrow$ For $\text{sinc}(100t)$, recall $\frac{B}{\pi}\,\text{sinc}\left(\frac{B}{\pi}t\right)$

$\updownarrow$

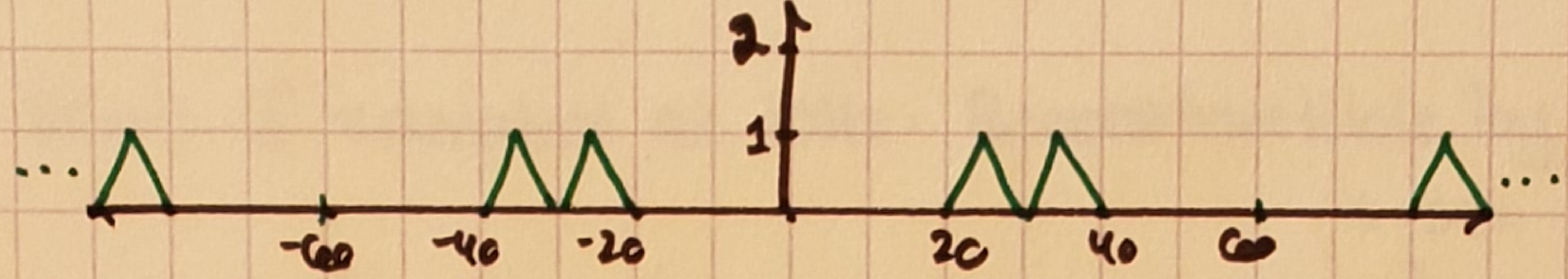
$\Pi\left(\omega/2B\right)$

$\frac{B}{\pi} = 100 \Rightarrow B = 100\pi$ rad/s $= 50$Hz

For $\text{sinc}^2 \rightarrow 100$Hz → $\boxed{F_s = 200\text{Hz}}$

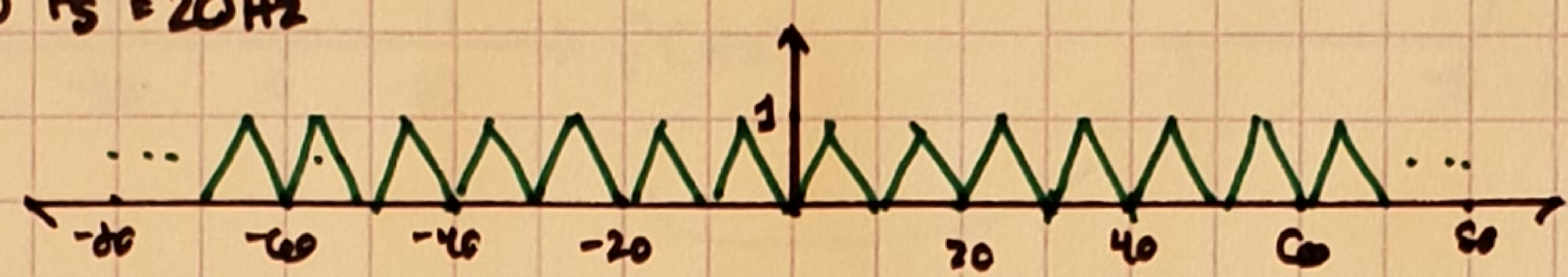
(c) $\underbrace{\text{sinc}(100t)}_{100\text{Hz}} + \underbrace{3\,\text{sinc}^2(60t)}_{120\text{Hz}}$ → $\boxed{F_s = 240\text{Hz}}$

3.1-9) (a) $F_s = 60\text{Hz}$



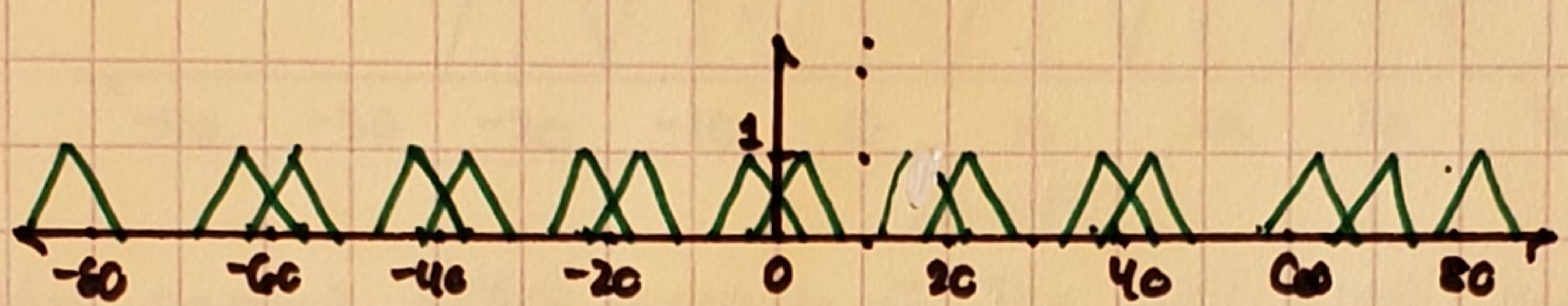
Yes. iLPF /w $\omega_c = 30\text{Hz}$

(b) $F_s = 20\text{Hz}$



Yes! Black magic but BP this signal /w $\omega_1 = 20\text{Hz}$, $\omega_2 = 30\text{Hz}$.

(c) Now Y at 20Hz



Nope. Aliasing is now occuring. f_2 here is 28Hz with $B = 10\text{Hz}$

$$\Rightarrow \frac{f_2}{B} = 2.8 \rightarrow K_{max} = 1$$

Using $K=1$

$$2.8 = \frac{2}{(1+1)}(2.8) \le \frac{F_s}{B} \le \frac{2}{1}(2.8-1) = 3.6$$

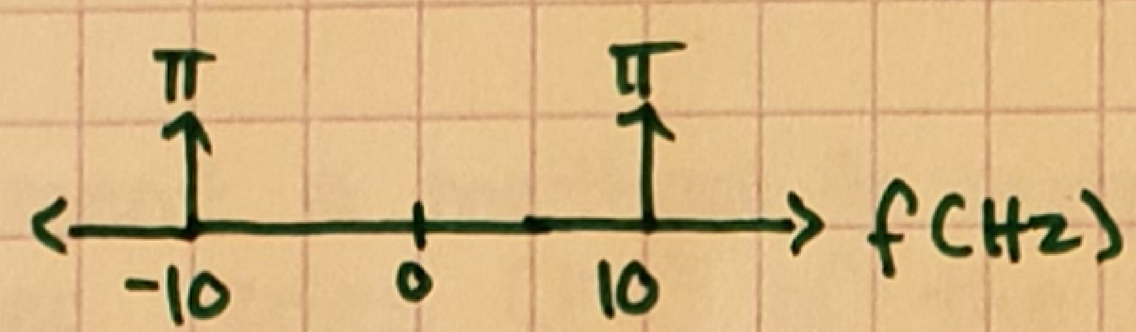
$$\Downarrow$$

$$2.8 \le \frac{F_s}{B} \le 3.6 \Rightarrow \boxed{(28 \le F_s \le 36)\ \text{Hz}}$$

4) 3.1-8) $x(t) = \cos(20\pi t) + 5\,\mathrm{sinc}^2(5t)$

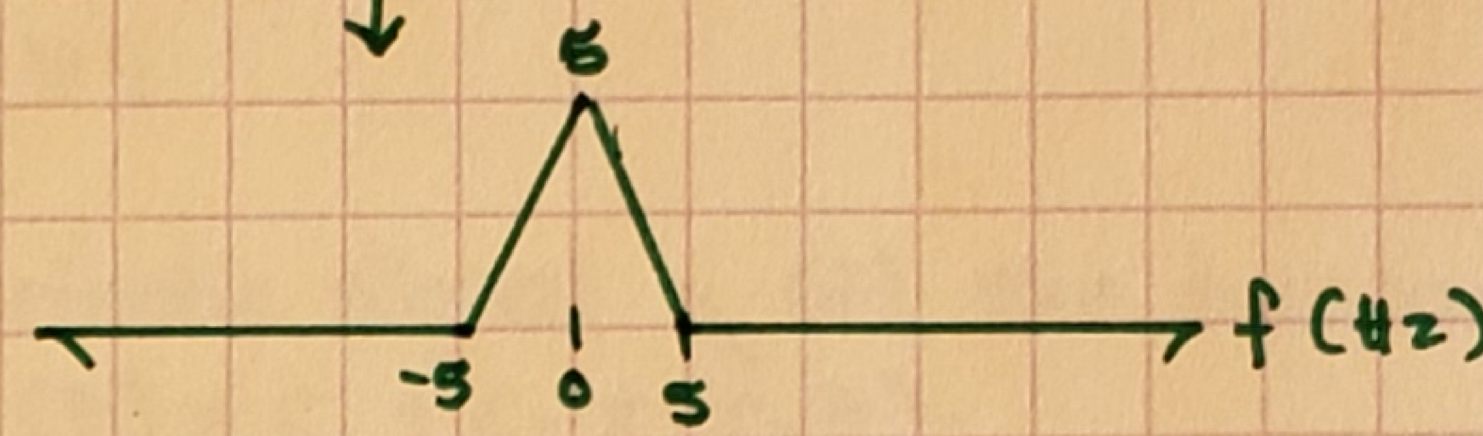
(a) $X(\omega)$ if sampled at 10Hz. Reconstructible by LPF?

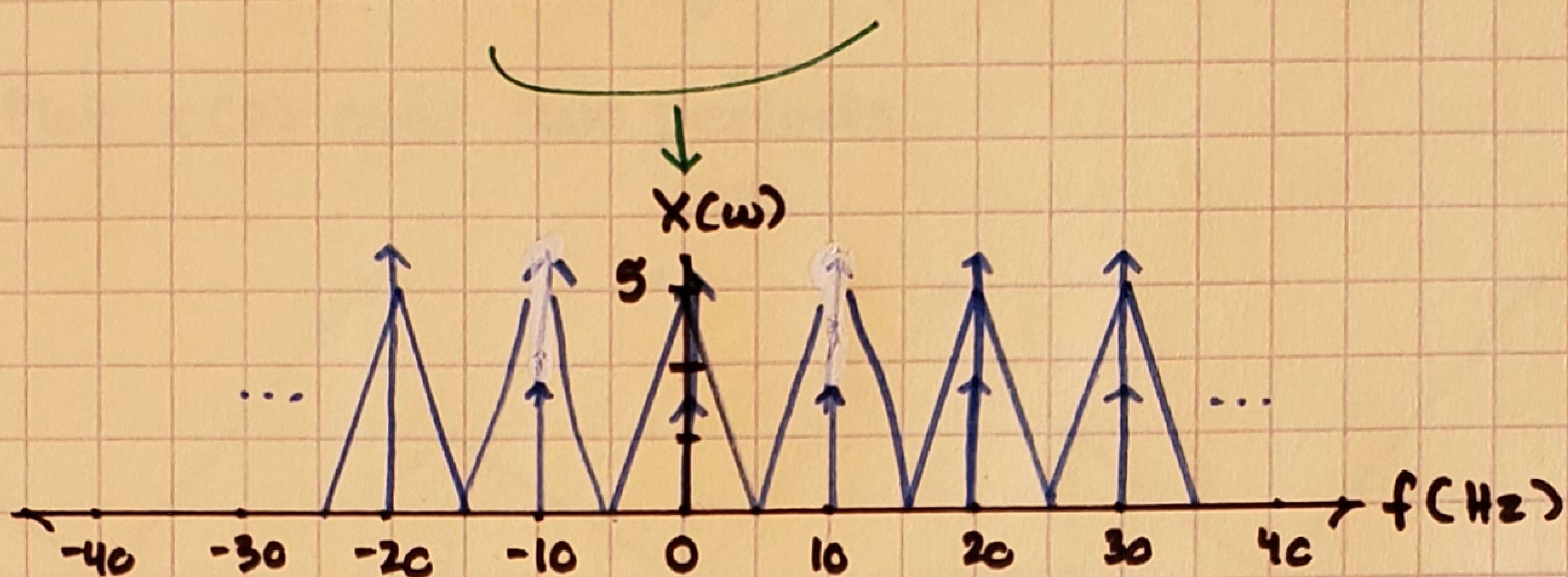
$\cos(2\pi 10 t)$ $\downarrow$



$$B = 5\text{Hz} \quad 5\,\mathrm{sinc}^2(5t) \downarrow$$

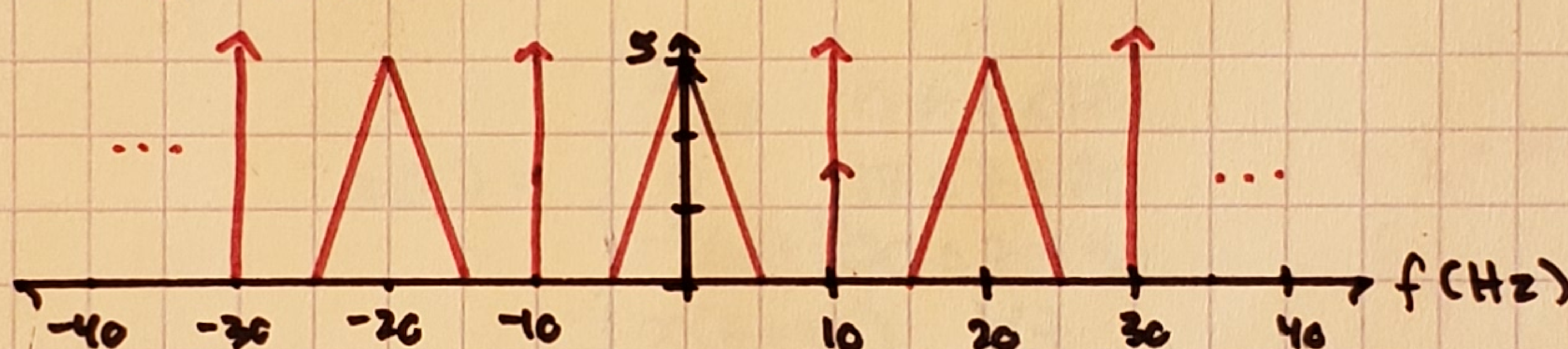
$$\frac{B}{2\pi}\mathrm{sinc}^2\left(\frac{B}{2\pi}t\right) \updownarrow \Lambda\left(\frac{\omega}{2B}\right)$$





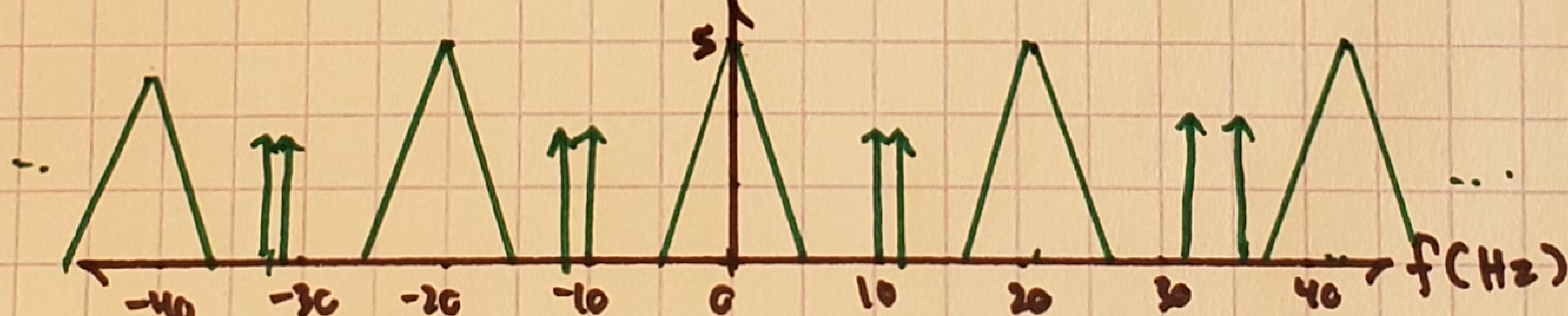
Definitely cannot ~~fully~~ reconstruct at all. The $\cos(2\pi 10t)$ spectrum aliases into any $B = 10$Hz interval.

(b) Now $F_s = 20$Hz.



Still can't fully reconstruct by LPFing. Can recover the $5\,\mathrm{sinc}^2(5t)$ part but not the $\cos(2\pi 10t)$ part due to aliasing.

(c) Now $F_s = 21$Hz.



Now, yes. Let an iLPF with $\omega_c = 2\pi 10$ rad/s do the job.

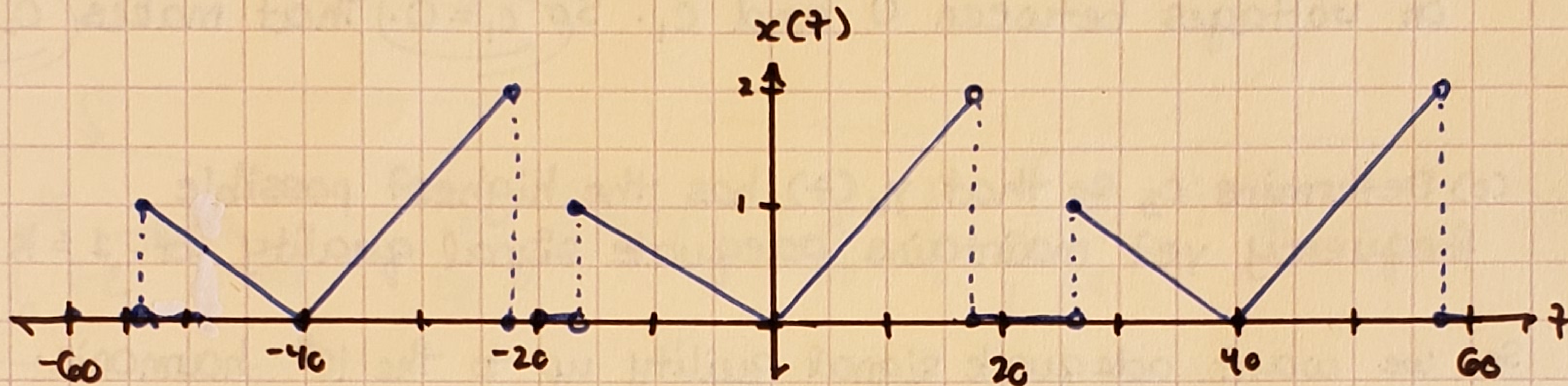
$\rightarrow 7, 4$

5) Let A and B designate the first two digits of your ID. Define a $T = 40s$ periodic signal $x(t)$ as

$$x(t) = \begin{cases} 0 & -20 \le t < \cancel{-11}\ -14 \\ -t/14 & \cancel{-11}\ {-14} \le t < 0 \\ 2t/\cancel{11}\ 17 & 0 \le t < \cancel{11}\ 17 \\ 0 & \cancel{11}\ 17 \le t < 20 \\ x(t+40) & \forall t \end{cases}$$

Implement a method to output a periodic signal $y_k(t)$, the truncated fourier series (FS) of the signal $y(t) = c_1 + c_2 x(c_3 t)$.

(a) Plot $x(t)$ over two periods.



= So on MATLAB, the mod(a,m) function is a little wonky for the negative part of the function definition. So, I'll define the function x based on:

$$x(t) = \begin{cases} 2t/17 & 0 \le t < 17 \\ 0 & 17 \le t < 26 \\ -t/14 + \frac{40}{14} & 26 \le t < 40 \\ x(t+40) & \forall t \end{cases}$$

$y - y_0 = m(x - x_0)$

$\Rightarrow y = -\frac{1}{14}(x - 40)$

$= -\frac{1}{14}x + \frac{40}{14}$

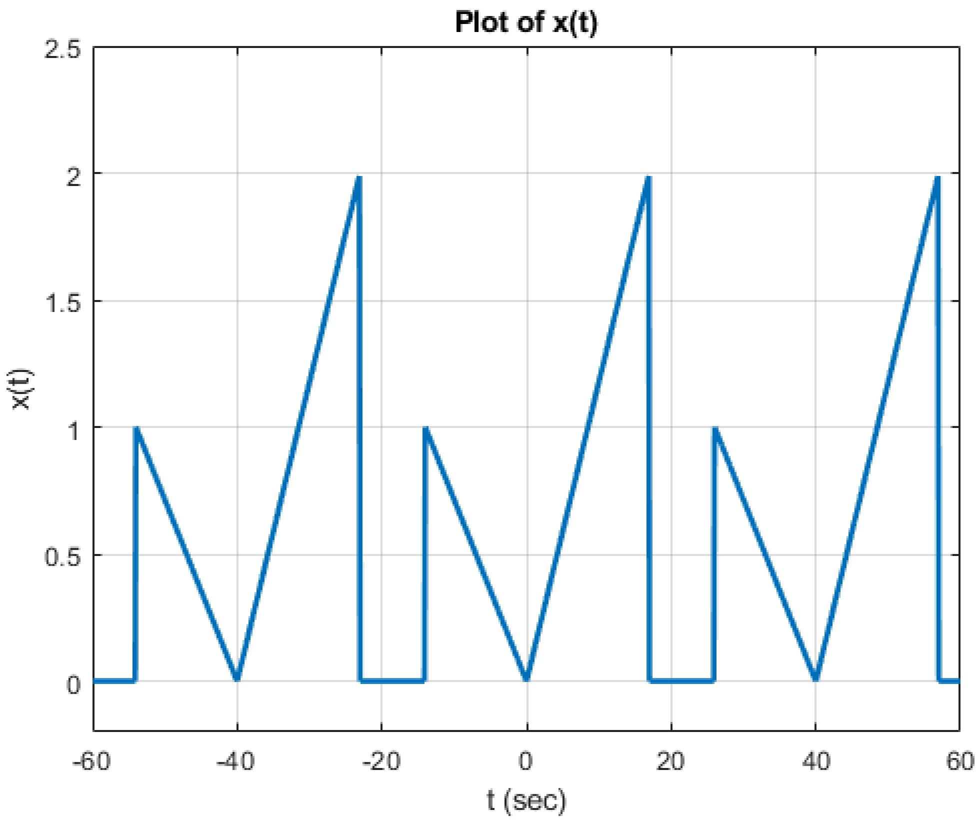
And so, in MATLAB:

* *

I used the back side! Was running out of paper!

```
T = 40;
t = -60:0.1:60;
x = @(t) (2*t/17).*(t>=0 & t<17) + (-t/14 + 40/14).*(t>=26 & t<40);
x_per = @(t) x(mod(t,T));

plot(t, x_per(t), 'LineWidth', 2);
xlabel('t (sec)'), ylabel('x(t)'), title('Plot of x(t)');
ylim([-0.2 2.5]);
grid on;
```



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(b) Determine constants c_1 and c_2 to maximize the output amplitude range of $y_k(t)$ yet provide some buffer against saturation and clipping.

Well ~~output~~ $x(t)$ ~~goes up to 2 from~~ $\in [0, 2]$, ~~so~~ and DAC clips/saturates after 3.3V and below 0V.
Let's say,

~~$c_1 = 0.65$ and $c_2 =$~~
~~$\Downarrow$~~
~~brings $x \in [0.65, 2.65]$~~
~~with midpoint $1.65 = \frac{3.3}{2}$~~

Nope. $x(t) \geq 0 \ \forall t$, so any DC offset causes us to miss out on voltages between 0 and c_1. So $c_1 = 0$. That makes $c_2 = 3.3/2$
$= 1.65$

(c) Determine c_3 so that $y_k(t)$ has the highest possible frequency yet maintains adequate signal quality for $1 \leq k \leq 10$.

So we want adequate signal quility up to the 10^{th} harmonic.

The DAC will supposedly operate at 10kHz. I know we studied Nyquist in terms of sampling, but if we're reconstructing a 5kHz signal while sampling at 10k, we've got to at least be able to DAC out those two samples per cycle of the signal we sampled ZOH style. So we cap Y_{10}'s frequency at 5kHz $\Rightarrow f_0 = 500$Hz.

$x(t)$'s frequency is 1/40 Hz, so we can scale that by (40)(500) to get a fundamental f of 500Hz. So, $\boxed{c_3 = 20{,}000}$.

(d) Y_k.

We'll go from $X_k \rightarrow$ F. Prop. $\rightarrow Y_k$

$$\boxed{X_0} = \frac{1}{40}\int_0^{40} x(t)\, dt = \frac{1}{40}\left(\frac{1}{2}(17)(2) + \frac{1}{2}(14)(1)\right)$$

$$= \frac{1}{40}(17 + 7) = 24/40 = 12/20 = 6/10 = \boxed{0.6}$$

$\omega_0 = \frac{\pi}{20}$ rad/s

$$X_k = \frac{1}{40}\int_{-20}^{20} x(t)\, e^{-jk\frac{\pi}{20}t}\, dt$$

$$= \frac{1}{40}\left(\int_{-14}^{0}\left(-\frac{1}{14}t\right)e^{-jk\frac{\pi}{20}t}\, dt + \int_0^{17}\left(\frac{2t}{17}\right)e^{-jk\frac{\pi}{20}t}\, dt\right)$$

...

$$X_k = \frac{1}{40}\left(-\frac{1}{11}\underbrace{\int_{-14}^{0} \tau e^{\cdots}\, dt} + \frac{2}{17}\underbrace{\int_{0}^{17} \tau e^{\cdots}\, dt}\right)$$

$\int \tau e^{a\tau} dt$

$u = \tau \quad v = \frac{1}{a}e^{at}$

$du = dt \quad dv = e^{a\tau} dt$

$= \frac{1}{a}\tau e^{a\tau} - \frac{1}{a}\int e^{a\tau} dt$

$= \frac{1}{a}\tau e^{a\tau} - \frac{1}{a^2} e^{a\tau}$

$= \frac{1}{a} e^{a\tau}\left(\tau - \frac{1}{a}\right)$

$$\int_{-14}^{0} \tau e^{-jk\frac{\pi}{20}\tau}\, dt = \frac{1}{jk\frac{\pi}{20}} e^{-jk\frac{\pi}{20}\tau}\left(\tau - \frac{1}{jk\frac{\pi}{20}}\right)\Bigg|_{-14}^{0}$$

$$= \left[\frac{20}{k\pi}\left(\frac{20}{k\pi}\right) - \left(\frac{20}{k\pi} e^{jk\frac{\pi 14}{20}}(-14 - \cdots)\right)\right.$$

$$= \frac{400}{(k\pi)^2} - \frac{20}{k\pi} e^{jk\frac{14\pi}{20}}\left(\frac{20}{k\pi} - 14\right)$$

$$\int_{0}^{17} \tau e^{-jk\frac{\pi}{20}\tau}\, dt = \frac{20}{k\pi} e^{-jk\frac{\pi}{20}\tau}\left(\tau + \frac{20}{k\pi}\right)\Bigg|_{0}^{17} = \frac{20}{k\pi} e^{-jk\frac{17\pi}{20}}\left(17 + \frac{20}{k\pi}\right) - \frac{400}{(k\pi)^2}$$

So,

$$X_k = \frac{1}{40}\left(-\frac{1}{11}\left(\frac{400}{(k\pi)^2} - \frac{20}{k\pi} e^{jk\frac{14\pi}{20}}\left(\frac{20}{k\pi} - 14\right)\right) + \frac{2}{17}\left(\frac{20}{k\pi} e^{-jk\frac{17\pi}{20}}\left(17 + \frac{20}{k\pi}\right) - \frac{400}{(\pi k)^2}\right)\right)$$

Now since $\underbrace{y(\tau) = 1.65\, x(20{,}000\tau)}_{Y_k = 1.65 X_k}$, $(\omega_o)_y = 500\,\text{Hz} = 1000\pi$ rad/s

so replace the $\omega_0 \to (\omega_o)_y$

$Y_0 = (1.65)(0.6) = 0.99$

$$Y_k = \frac{1}{40}\left(\frac{-1}{11}\left(\frac{1}{(1000\pi k)^2} - \frac{1}{1000\pi k} e^{j14{,}000\pi k}\left(\frac{1}{1000k\pi} - 14\right)\right) + \frac{2}{17}\left(\frac{1}{1000k\pi} e^{-j17000\pi k}\left(17 + \frac{1}{1000k\pi} - \left(\frac{1}{1000k\pi}\right)^2\right)\right)\right.$$