Composite Structures 252 (2020) 112683

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A case for *Tsai's Modulus*, an invariant-based approach to stiffness

Albertino Arteiro [a], Naresh Sharma [b,*], Jose Daniel D. Melo [c], Sung Kyu Ha [d], Antonio Miravete [e], Yasushi Miyano [f], Thierry Massard [g], Pranav D. Shah [h], Surajit Roy [i], Robert Rainsberger [j], Klemens Rother [k], Carlos Cimini Jr. [l], Jocelyn M. Seng [m], Francisco K. Arakaki [n], Tong-Earn Tay [o], Woo Il Lee [p], Sangwook Sihn [q], George S. Springer [r], Ajit Roy [s], Aniello Riccio [t], Francesco Di Caprio [u], Sachin Shrivastava [v], Alan T. Nettles [w], Giuseppe Catalanotti [x], Pedro P. Camanho [a,y], Waruna Seneviratne [z], António T. Marques [a], Henry T. Yang [a], H. Thomas Hahn [a,b]

[a] *Universidade do Porto – Faculdade de Engenharia, Porto, Portugal*
[b] *NASHERO s.r.l., www.nashero.com, Italy*
[c] *Dept. Materials Engineering, Federal University of Rio Grande do Norte (UFRN), Brazil*
[d] *Hanyang University, Republic of Korea*
[e] *Massachusetts Institute of Technology, USA*
[f] *Kanazawa Institute of Technology (KIT), Japan*
[g] *Think Composites, France*
[h] *Aurora Flight Sciences – A Boeing Company, USA*
[i] *California State University, Long Beach, USA*
[j] *XYZ Scientific Applications, Inc., USA*
[k] *Munich University of Applied Sciences, Germany*
[l] *Dept. Structural Engineering, Federal University of Minas Gerais (UFMG), Brazil*
[m] *Institute for Defense Analyses, USA*
[n] *Embraer S.A., São José dos Campos, Brazil*
[o] *National University of Singapore, Singapore*
[p] *Seoul National University, Republic of Korea*
[q] *University of Dayton Research Institute, USA*
[r] *Stanford University, USA*
[s] *USAF AFMC Air Force Research Labs, USA*
[t] *University of Campania, Luigi Vanvitelli, Italy*
[u] *CIRA (Italian Aerospace Research Center), Italy*
[v] *Department of Aerospace Engineering, Indian Institute of Technology Kanpur, India*
[w] *NASA MSFC, USA*
[x] *Advanced Composites Research Group (ACRG), School of Mechanical and Aerospace Engineering, Queen's University Belfast, Ashby Building, Belfast, UK*
[y] *INEGI, Instituto de Ciência e Inovação em Engenharia Mecânica e Engenharia Industrial, Porto, Portugal*
[z] *NIAR, Wichita State University, USA*
[aa] *University of California, Santa Barbara, USA*
[ab] *University of California, Los Angeles, USA*

ARTICLE INFO

Keywords:
Stiffness
Invariants
CFRP
Analysis

ABSTRACT

For the past six years, we have been benefiting from the discovery by Tsai and Melo (2014) that the *trace* of the plane stress stiffness matrix ($\mathrm{tr}(\mathbf{Q})$) of an orthotropic composite is a fundamental and powerful scaling property of laminated composite materials. Algebraically, $\mathrm{tr}(\mathbf{Q})$ turns out to be a measure of the summation of the moduli of the material. It is, therefore, a material property. Additionally, since $\mathrm{tr}(\mathbf{Q})$ is an invariant of the stiffness tensor $\mathbf{Q}$, independently of the coordinate system, the number of layers, layup sequence and loading condition (in-plane or flexural) in a laminate, if the material system remains the same, $\mathrm{tr}(\mathbf{Q}) = \mathrm{tr}(\mathbf{A}^*) = \mathrm{tr}(\mathbf{D}^*)$ is still the same. Therefore, $\mathrm{tr}(\mathbf{Q})$ is the total stiffness that one can work with making it one of the most powerful and fundamental concepts discovered in the theory of composites recently. By reducing the number of variables, this concept shall simplify the design, analysis and optimization of composite laminates, thus enabling lighter, stronger and better parts. The reduced number of variables shall result in reducing the number and type of tests

* Corresponding author.
E-mail address: naresh@nashero.com (N. Sharma).

https://doi.org/10.1016/j.compstruct.2020.112683
Received 29 June 2020; Accepted 30 June 2020
Available online 9 July 2020

required for characterization of composite laminates, thus reducing bureaucratic certification burden. These effects shall enable a new era in the progress of composites in the future. For the above-mentioned reasons, it is proposed here to call this fundamental property, $\mathrm{tr}(\mathbf{Q})$, as *Tsai's Modulus*.

1. The invariant-based approach to stiffness

1.1. Introduction

Tsai and Melo [1] proposed a novel invariant-based approach to describe the stiffness of composite plies and laminates that takes the *trace* of the plane stress stiffness matrix of a unidirectional ply ($\mathrm{tr}(\mathbf{Q})$) as a material property. Since $\mathrm{tr}(\mathbf{Q})$ is an invariant with respect to coordinate transformations, in a laminate, the *trace* of the thickness-normalized in-plane ($\mathbf{A}^*$) and flexural ($\mathbf{D}^*$) stiffness matrices are also the same: $\mathrm{tr}(\mathbf{A}^*) = \mathrm{tr}(\mathbf{D}^*) = \mathrm{tr}(\mathbf{Q})$. Hence, although mathematically *trace* is just a matrix algebraic relation, what it really stands for is different; *trace* is a fundamental property of the material, independent of coordinate system (and material symmetry), number of layers, layup sequence and loading condition (in-plane or flexural). It indicates the total stiffness that one can work with, with implications on how laminated composites are characterized and their elastic behaviour defined.

Tsai and Melo [1] also demonstrated that *trace* can be a powerful scaling property for testing, design and understanding of composites. They observed that the stiffness components from test data normalized by *trace*, $Q^*_{ij} = Q_{ij}/\mathrm{tr}(\mathbf{Q})$, of several carbon fibre-reinforced polymer (CFRP) systems are approximately the same, an observation confirmed by micro-mechanics [2]. In addition, Tsai and Melo [1] demonstrated that the solution of governing equations of plane elasticity using *trace*-normalized stiffness properties for CFRP laminates should be more accurate than can be expected from experimental data, and common examples such as the stress around an open hole, bending, buckling and vibration can be described by two separate parameters: the geometric factors reflected by each laminate, and the material property represented by *trace*. This is a powerful and pivotal discovery that promises to change the way carbon composites are worked with, analysed, manufactured and tested. Many studies have already benefited from the use of *trace* in the mechanics of composite materials, including new laminate concepts [3,4] and structures [5], preliminary design [6–8] and optimization [9,10], fast notched strength prediction tools [11], generation of statistical virtual allowables [12] and fracture mechanics [13,14].

1.2. Trace theory and master ply

Tsai and Melo [1] observed, based on test data, that CFRP systems share common stiffness properties when normalized by their respective *trace* of the stiffness matrix:

$$Q^*_{11} = \frac{Q_{11}}{\mathrm{tr}(\mathbf{Q})},\quad Q^*_{22} = \frac{Q_{22}}{\mathrm{tr}(\mathbf{Q})},\quad Q^*_{12} = \frac{Q_{12}}{\mathrm{tr}(\mathbf{Q})},\quad Q^*_{66} = \frac{Q_{66}}{\mathrm{tr}(\mathbf{Q})} \tag{1}$$

with:

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \tag{2}$$

In particular, the fibre-dominated *trace*-normalized longitudinal stiffness component Q^*_{11} had a coefficient of variation of only 1.5% [1]. The variation of the matrix-dominated transverse and shear components was larger (up to 16.4%); however, their contribution to *trace* in CFRP composites is small in absolute terms, with standard deviations below 0.006 [1].

Using the median values of the *trace*-normalized stiffness components of several CFRPs, Tsai and Melo [1] proposed the definition of a **Master Ply** (Table 1), an universal relation between the components of the orthotropic in-plane stiffness matrix and the *trace*, $\mathrm{tr}(\mathbf{Q})$, of any CFRP:

$$Q_{11} = Q^*_{11}\cdot\mathrm{tr}(\mathbf{Q}),\quad Q_{22} = Q^*_{22}\cdot\mathrm{tr}(\mathbf{Q}),\quad Q_{12} = Q^*_{12}\cdot\mathrm{tr}(\mathbf{Q}),\quad Q_{66} = Q^*_{66}\cdot\mathrm{tr}(\mathbf{Q}) \tag{3}$$

For unidirectional CFRP composites, the longitudinal stiffness component is responsible for about 88% of the *trace* (Table 1), while the matrix-related components are responsible for 12% [1]. Hence, in spite of higher variability, the effect of the matrix-dominated components on the laminate properties is masked by the dominant fibre component, whose variability is small.

Trace-normalized engineering constants (Table 1) can be determined from the components of the *trace*-normalized compliance matrix $\mathbf{S}^* = (\mathbf{Q}^*)^{-1}$:

$$E^*_1 = \frac{1}{S^*_{11}},\ E^*_2 = \frac{1}{S^*_{22}},\ G^*_{12} = \frac{1}{S^*_{66}},\ \nu_{12} = -\frac{S^*_{12}}{S^*_{11}} \tag{4}$$

where:

$$E^*_1 = \frac{E_1}{\mathrm{tr}(\mathbf{Q})},\ E^*_2 = \frac{E_2}{\mathrm{tr}(\mathbf{Q})},\ G^*_{12} = \frac{G_{12}}{\mathrm{tr}(\mathbf{Q})} \tag{5}$$

and E_1, E_2, G_{12} and ν_{12} are the ply engineering constants.

The Classical Laminated Plate Theory (CLPT) can be used to generate *trace*-normalized laminate factors for stiffness components of multi-directional laminates. The *trace*-normalized plane stress stiffness components of the **Master Ply** E^*_1, E^*_2, G^*_{12} and ν_{12} (Table 1) can be used

Table 1
Master ply and examples of laminate factors [3].

Master Ply	Q^*_{11}	Q^*_{22}	Q^*_{12}	Q^*_{66}
[0]	0.885	0.052	0.017	0.031
Master Ply	E^*_1	E^*_2	G^*_{12}	ν_{12}
[0]	0.880	0.052	0.031	0.320
Laminate factors	E^*_x	E^*_y	G^*_{xy}	ν_{xy}
[0/90]	0.468	0.468	0.031	0.036
Conventional 10% rule				
$[0/\pm45/90]$	0.336	0.336	0.129	0.308
$[0_7/\pm45/90]$	0.662	0.175	0.070	0.310
$[0_5/\pm45_2/90]$	0.518	0.208	0.109	0.423
$[0_2/\pm45/90]$	0.445	0.289	0.109	0.308
$[0/\pm45_4/90]$	0.217	0.217	0.187	0.552
Herringbone				
$[0/\pm45]$	0.370	0.155	0.161	0.734
$[0/\pm45/0]$	0.499	0.141	0.129	0.701
$[0/\pm30]$	0.510	0.074	0.129	1.220
$[0/\pm30/0]$	0.611	0.072	0.104	1.079
Angle-ply				
$[\pm12.5]$	0.764	0.053	0.066	0.913
Double angle-ply				
$[\pm15/\pm60]$	0.403	0.266	0.104	0.387
$[0_2/\pm50]$	0.485	0.179	0.126	0.551

to calculate the stiffness matrix of a given laminate $\mathbf{A}^*$. From the compliance matrix $\mathbf{a}^* = (\mathbf{A}^*)^{-1}$, the *trace*-normalized engineering constants for the laminate, or laminate factors, E_x^*, E_y^*, G_{xy}^* and ν_{xy}, can be calculated, where:

$$E_x^* = \frac{E_x}{\mathrm{tr}(\mathbf{A}^*)},\ E_y^* = \frac{E_y}{\mathrm{tr}(\mathbf{A}^*)},\ G_{xy}^* = \frac{G_{xy}}{\mathrm{tr}(\mathbf{A}^*)} \tag{6}$$

Table 1 shows examples of such laminate factors for several layups.

Given the dominant contribution for *trace* of the longitudinal stiffness, the value of *trace* ($\mathrm{tr}(\mathbf{Q}) = \mathrm{tr}(\mathbf{A}^*)$) can be determined using the longitudinal Young's modulus of a 0° unidirectional laminate, E_1, or the longitudinal Young's modulus of a multi-directional laminate, E_x, by dividing it by the corresponding *trace*-normalized laminate factor, E_1^* or E_x^* (e.g. Table 1), whose coefficients of variation are within 4% for CFRPs [3], reading, respectively:

$$\mathrm{tr}(\mathbf{Q}) = \frac{E_1}{E_1^*} \qquad (\textbf{Master Ply}), \text{or} \tag{7a}$$

$$\mathrm{tr}(\mathbf{Q}) = \mathrm{tr}(\mathbf{A}^*) = \frac{E_x}{E_x^*} \qquad (\text{laminate factor}) \tag{7b}$$

In addition, this concept allows the determination of the elastic properties E_x, E_y and G_{xy} of any laminate by multiplying the *trace*-normalized laminate factors by the value of *trace* of the corresponding CFRP:

$$E_x = E_x^* \cdot \mathrm{tr}(\mathbf{A}^*),\ E_y = E_y^* \cdot \mathrm{tr}(\mathbf{A}^*),\ G_{xy} = G_{xy}^* \cdot \mathrm{tr}(\mathbf{A}^*) \tag{8}$$

Hence, the *trace of the plane stress stiffness matrix*, $\mathrm{tr}(\mathbf{Q}) = \mathrm{tr}(\mathbf{A}^*)$ is the only independent elastic stiffness property of the, otherwise highly complex, CFRP. This observation has overwhelming implications. Characterization of the laminate stiffness can be dramatically simplified, by reducing the number of tests required to characterize the orthotropic in-plane elastic properties of CFRPs from 3 tests to 1 test at the ply level (to obtain E_1 and, from Eq. (7a), $\mathrm{tr}(\mathbf{Q})$); in addition, the highly non-linear in-plane shear test, which adds complexity and cost to the characterization programme, can be avoided. Alternatively, the 3 tests at the ply level can be replaced by 1 test at the laminate level (to obtain E_x and, from Eq. (7b), $\mathrm{tr}(\mathbf{Q}) = \mathrm{tr}(\mathbf{A}^*)$); in this case, lamination effects are intrinsically accounted for, potentially replacing additional tests at the laminate level sometimes required to account for these effects (e.g. aerospace industry) [5].

Although the concept of **Master Ply** originated from empirical observations [1], its fundamental analytical basis is nowadays well understood. Ha and Cimini Jr [15], based on the CLPT, verified analytically that the **Master Ply** concept can be applied for high modulus and standard modulus CFRPs and aramid fibre-reinforced polymers. The theoretical *trace*-normalized longitudinal modulus (E_1^*) and stiffness matrix components ($Q_{11}^*, Q_{22}^*, Q_{12}^*$, and Q_{66}^*), although nonlinear, present an asymptotic behaviour as the material longitudinal modulus (E_1) increases, tending to a plateau for materials of higher longitudinal stiffness. Hence, the *trace*-normalized stiffness properties of the material systems with higher E_1 are less sensitive to small variations of E_1 than material systems with lower E_1, such as glass fibre-reinforced polymers. These observations were validated by empirical data on different grades of CFRPs and on aramid fibre-reinforced polymers.

More recently, Arteiro et al. [2] used micro-mechanical models to study the relationship between the fibre volume fraction, the fibre/matrix stiffness ratios, and the *trace*-normalized engineering constants of unidirectional laminae and multi-directional laminates. For fibre-reinforced composites whose fibres are substantially stiffer than the matrix material (i.e. fibre/matrix stiffness ratios above 50) and fibre volume fractions between 50% and 70%, this study showed that the variation of the *trace*-normalized longitudinal Young's modulus is within 6% for unidirectional laminae (E_1^*) and within 1% for multi-directional laminates (E_x^*), supporting the applicability of the **Master Ply** proposed by Tsai and Melo [3] and the derived laminate factors to virtually any CFRP.

2. A case for *Tsai's Modulus*

Up to now, this fundamental property — $\mathrm{tr}(\mathbf{Q})$ — has been simply referred to as *trace*. Giving it a name that is just a mathematical definition does not reflect its engineering significance given the implications of its usage and properties in the mechanics of composite materials. We, therefore, propose to call this property as *Tsai's Modulus*, as recognition of Prof. Stephen W. Tsai's pivotal role in this fundamental discovery and his key historical contributions to the mechanics and design of composite materials.

3. Examples of application

The opportunities brought by the invariant-based approach to stiffness [1] are endless. Tsai et al. [6] proposed a direct laminate sizing method using the *trace of the plane stress stiffness matrix*, $\mathrm{tr}(\mathbf{Q}) = \mathrm{tr}(\mathbf{A}^*)$ — hereafter referred to as *Tsai's modulus* — as normalizing factor. Laminate selection could be performed independently of the material system. Material selection would be the last step, according to strength and stiffness requirements. The latter could be matched by direct scaling using the *Tsai's modulus*. The same strategy was followed by Melo et al. [8]. Earlier, Tsai et al. [9] proposed a method for direct scaling of the structural weight of composite laminated beams based on stiffness requirements. Shape optimization of tapered beams could be easily performed, and weight savings of a given material and laminate easily calculated. Direct material and laminate selection could be performed based on the *Tsai's modulus* only, without the need for additional shape iterations. Shrivastava et al. [10] also used stiffness components normalized by *Tsai's modulus* in layup optimization of a wing box, including thermal deformation analysis.

Melo et al. [8] also extended the invariant-based design approach proposing carpet plots normalized by *Tsai's modulus*. Similar normalized carpet plots were proposed by Barbero [7]. These universal carpet plots are valid for any CFRP. They are not sensitive to environmental conditions, such as cold-dry or hot-wet conditions. The actual Young's and shear moduli for any CFRP can be immediately recovered simply by using the corresponding value of the *Tsai's modulus*, at the respective environmental conditions.

Tsai et al. [5] then presented a practical case study towards the simplification of the current design and certification processes without posing risk to the safe design of structures at larger scales. A single test configuration on a double-double (or double angle-ply [16]) $[\pm 15/\pm 60]$ laminate was used to generate *laminate data* — the longitudinal laminate Young's modulus, E_x, from uniaxial tensile tests — and *material data* — the *Tsai's modulus*, Tr, from Eq. (7b). The *Tsai's modulus* was then used in the determination of the moduli of a $[0_2/\pm 50]$ laminate according to Eq. (8). Uniaxial tension tests were performed to validate this analysis. A difference of approximately 3% with respect to the measured value was obtained for the longitudinal laminate Young's modulus, E_x, of the $[0_2/\pm 50]$ laminate when using the *Tsai's modulus* identified from the $[\pm 15/\pm 60]$ laminate. One test at the laminate level replaced the three independent tests at the ply level required by current design and certification processes.

A similar strategy was followed earlier by Furtado et al. [11] to obtain the elastic properties of balanced multi-directional laminates and their sub-laminates, which were required for prediction of the open–hole strength of laminated panels. The elastic properties of quasi-isotropic laminates of different CFRP systems were obtained based on the corresponding *Tsai's moduli* and the **Master ply** concept, with differences below 3% for high-strength CFRP systems and below 7% for high-modulus CFRP systems. Vallmajo et al. [12] employed the strategy proposed by Furtado et al. [11] in the generation of virtual allowables for open–hole tension from just three independent stochastic material properties, with accurate predictions of the mean values and variability of the open–hole tensile strengths of quasi-isotropic laminates.

Further simplification of the analysis of notched laminates can be achieved through normalization by *Tsai's modulus*. Melo et al. [8] obtained the stress concentration factors for open–hole laminates using the normalized **Master ply** properties, thus valid for any CFRP. For an infinite open–hole laminate subjected to a remote stress parallel to the *x*-axis, the stress concentration factors at the intersection of the hole edge with the *x*- and *y*-axes yield, respectively:

$$\mathrm{SCF}_x = -\frac{1}{K}, \quad K = \sqrt{\frac{E_x^*}{E_y^*}} \tag{9}$$

$$\mathrm{SCF}_y = N + 1, \quad N = \sqrt{2\left(\sqrt{\frac{E_x^*}{E_y^*}} - \nu_{xy}\right) + \frac{E_x^*}{G_{xy}^*}} \tag{10}$$

where E_x^*, E_y^*, G_{xy}^* and ν_{xy} are the laminate factors normalized by *Tsai's modulus* (Table 1). Very good agreement was obtained with the stress concentration factors for three different composite systems and three different laminates each.

Dalli et al. [13,14] used *trace*-normalized moduli to simplify the expressions of the geometric correction factors used in fracture mechanics of orthotropic bodies. Using *Tsai's modulus* as a normalizing factor, Dalli et al. [13,14] proposed a general formulation for the energy release rate, $\mathscr{G}$, that can be used with the majority of standard balanced woven CFRPs:

$$\mathscr{G} = \frac{E_x^*}{E_x \acute{E}^*} w\sigma^2\kappa^2 \approx \frac{2}{E_x} w\sigma^2\kappa^2 \tag{11}$$

where w is a characteristic dimension of the orthotropic cracked specimen of crack length a, σ is the nominal remote stress, κ is the geometric correction factor and $E_x^*/\acute{E}^* \approx 2$ is the ratio of the *trace*-normalized longitudinal Young's modulus, E_x^*, and *trace*-normalized equivalent modulus, $\acute{E}^* = \acute{E}/\mathrm{Tr}$, of balanced woven CFRPs (equivalent to [0/90] cross-ply laminates, Table 1) loaded in the longitudinal (x) direction, with:

$$\acute{E} = \left(\frac{1+\rho}{2E_x E_y}\right)^{-1/2} \mu^{-1/4} \tag{12}$$

where ρ and μ are two elastic parameters, given, respectively, by [17]:

$$\rho = \frac{\left(E_x E_y\right)^{1/2}}{2G_{xy}} - \left(\nu_{xy}\nu_{yx}\right)^{1/2} \tag{13}$$

$$\mu = \frac{E_y}{E_x} \tag{14}$$

For double edge notch specimens of balanced woven CFRPs [13,14], the geometric correction factor κ depends solely on the shape parameters a/w and l/w, where the characteristic dimension w is half of the total width ($2w$) and $2l$ is the total length, and it does not need to be recalculated for different woven CFRP materials. In addition, the energy release rate, Eq. (11), can be calculated knowing only the longitudinal Young's modulus, E_x, or the *Tsai's modulus*, making $E_x = \mathrm{Tr} \cdot E_x^*$ in Eq. (11), of the woven CFRP material.

4. Final remark

For simple isotropic materials, Young's Modulus defines the stiffness of an elastic material. *Tsai's modulus* expands such simplicity to anisotropic materials and structures and thus offers a much more versatile use for product development as has been explained.

This new material property goes far beyond the benefits of the well-known modulus of elasticity for isotropic bodies. *Tsai's modulus* is a property to simplify and generalize an otherwise very complex material and structural behaviour. Its impact as a basis for new methods of design, analysis and optimization of structures made of CFRP composites, and fibre-reinforced materials, is already proven and such work will continue to find more powerful ways to develop high performing structures.

Other fields of application of *Tsai's modulus* and the **Master ply** concept, such as bending, buckling and vibration analysis of CFRP laminates, will follow shortly, given the potential of the invariant-based approach to stiffness to simplify the analysis within these domains. The immediate adoption of *Tsai's modulus* and the **Master ply** concept by engineering educators, researchers and industry practitioners will be the key in making the characterization, certification, analysis and design of composite structures easier and, thus, more competitive.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The list of people who support and understand *Tsai's Modulus* is long, we are listing some of the key supporters, who have contributed to and who believe in the groundbreaking potential of *Tsai's Modulus*. Our thanks go to Mr. Michel Cognet, President JEC CPC and Chairman JEC Board of Administration, France; Mr. Manuel Torres, Chairman and founder, MTorres, Spain.

References

[1] Tsai SW, Melo JDD. An invariant-based theory of composites. Compos Sci Technol 2014;100:237–43.
[2] Arteiro A, Pereira LF, Bessa MA, Furtado C, Camanho PP. A micro-mechanics perspective to the invariant-based approach to stiffness. Compos Sci Technol 2019;176:72–80.
[3] Tsai SW, Melo JDD. Composite Materials Design and Testing — Unlocking mystery with invariants. Stanford: Composites Design Group, Department of Aeronautics and Astronautics, Stanford University; 2015.
[4] Tsai SW, Melo JDD, Sihn S, Arteiro A, Rainsberger R. Composite Laminates: Theory and practice of analysis, design and automated layup. Stanford: Composites Design Group, Department of Aeronautics and Astronautics, Stanford University; 2017.
[5] Tsai SW, Sharma N, Arteiro A, Roy S, Rainsberger R. Composite Double-Double and Grid/Skin Structures: Low Weight/Low Cost Design and Manufacturing. Stanford: Composites Design Group, Department of Aeronautics and Astronautics, Stanford University; 2019.
[6] Tsai SW, Arteiro A, Melo JDD. A trace-based approach to design for manufacturing of composite laminates. J Reinf Plast Comp 2016;35(7):589–600.
[7] Barbero EJ. Universal carpet plots for stiffness and strength of carbon/epoxy laminates. In: CAMX — The Composites and Advanced Materials Expo Conference Proceedings. Orlando, FL, USA; 2017, p. 1–11.
[8] Melo JDD, Bi J, Tsai SW. A novel invariant-based design approach to carbon fiber reinforced laminates. Compos Struct 2017;159:44–52.
[9] Tsai SW, Sihn S, Melo JDD. Trace-based stiffness for a universal design of carbon-fiber reinforced composite structures. Compos Sci Technol 2015;118:23–30.
[10] Shrivastava S, Sharma N, Tsai SW, Mohite PM. D and DD-drop layup optimization of aircraft wing panels under multi-load case design environment. Compos Struct 2020;248: 112518.
[11] Furtado C, Arteiro A, Bessa MA, Wardle BL, Camanho PP. Prediction of size effects in open-hole laminates using only the Young's modulus, the strength, and the $\mathscr{R}$-curve of the 0° ply. Compos Part A-Appl S 2017;101:306–17.
[12] Vallmajó O, Cózar IR, Furtado C, Tavares R, Arteiro A, Turon A, et al. Virtual calculation of the B-value allowables of notched composite laminates. Compos Struct 2019;212:11–21.
[13] Dalli D, Catalanotti G, Varandas LF, Falzon BG, Foster S. Mode I intralaminar fracture toughness of 2D woven carbon fibre reinforced composites: A comparison of stable and unstable crack propagation techniques. Eng Fract Mech 2019;214:427–48.
[14] Dalli D, Catalanotti G, Varandas LF, Falzon BG, Foster S. Compressive intralaminar fracture toughness and residual strength of 2D woven carbon fibre reinforced composites: New developments on using the size effect method. Theor Appl Fract Mec 2020;106: 102487.
[15] Ha SK, Cimini Jr CA. Theory and validation of the master ply concept for invariant-based stiffness of composites. J Compos Mater 2018;52(12):1699–708.
[16] York CB New insights into stiffness matching between standard and double angle-ply laminates. In: 11_{th} Asian-Australasian Conference on Composite Materials. Cairns, Australia; 2018, p. 1.
[17] Bao G, Ho S, Suo Z, Fan B. The role of material orthotropy in fracture specimens for composites. Int J Solids Struct 1992;29(9):1105–16.