The heat flux at the interface e is determined by:

$$q_e = \frac{k_e\left(T_P - T_E\right)}{\left(\delta x\right)_e} \tag{3.9}$$

Assuming that the control volume surrounding the point P is filled with a material of uniform conductivity k_P and the control volume surrounding the point E with a material of conductivity k_E the heat flux at the interface can be determined from analytical solution (steady and one-dimensional case) as follows:

$$q_e = \frac{T_P - T_E}{\frac{(\delta x)_{e-}}{k_P} + \frac{(\delta x)_{e+}}{k_E}} \tag{3.10}$$

Substituting Eq. (3.8) into Eq. (3.10) gives:

$$k_e = \left(\frac{1-f_e}{k_P} + \frac{f_e}{k_E}\right)^{-1} \tag{3.11}$$

+ Oluştur

3. Heat conduction

The discretization of the heat conduction problem illustrates also the methods suitable to all the physical processes governed by similar mathematical equations

e.g.:

- Potential flows
- Momentum viscous diffusion
- Flows through porous media

3.1. One-dimensional steady heat conduction

The governing differential equation:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+S=0 \tag{3.1}$$

discretization equation:

$$a_P T_P = a_E T_E + a_W T_W + b \tag{3.2}$$

where:

or

$$k_e = \frac{2k_P k_E}{k_P + k_E} \tag{3.13}$$

the thermal conductivity at the interface between two control volumes should be expressed as a harmonic mean of thermal conductivity in both control volumes rather than an arithmetic one

The combination of Eqs (3.13) and (3.3) leads to the following formula for discretization equation coefficient:

$$a_E = \left[\frac{(\delta x)_{e-}}{k_P} + \frac{(\delta x)_{e+}}{k_E}\right]^{-1} \tag{3.14}$$

+ Oluştur

The heat flux at the interface e is determined by:

$$q_e = \frac{k_e(T_P - T_E)}{(\delta x)_e} \tag{3.9}$$

Assuming that the control volume surrounding the point P is filled with a material of uniform conductivity k_P and the control volume surrounding the point E with a material of conductivity k_E the heat flux at the interface can be determined from analytical solution (steady and one-dimensional case) as follows:

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Substituting Eq. (3.8) into Eq. (3.10) gives:

$$k_e = \left(\frac{1 - f_e}{k_P} + \frac{f_e}{k_E}\right)^{-1} \tag{3.11}$$

Moreover, when the interface is placed midway between P and E

$$q_e = \frac{T_P - T_E}{\frac{(\delta x)_{e-}}{k_P} + \frac{(\delta x)_{e+}}{k_E}} \tag{3.10}$$

Substituting Eq. (3.8) into Eq. (3.10) gives:

$$k_e = \left(\frac{1-f_e}{k_P} + \frac{f_e}{k_E}\right)^{-1} \tag{3.11}$$

Moreover, when the interface is placed midway between P and E so:

$$k_e^{-1} = \frac{k_P^{-1} + k_E^{-1}}{2} \tag{3.12}$$

2. $k_P >> k_E \Rightarrow k_e \to \frac{k_E}{f_e}$

Conclusions:

- Thermal conductivity at the interface k_E is independent of the thermal conductivity k_P
- $q_e = \frac{k_E(T_P - T_E)}{(\delta x)_{e+}}$ - because the material surrounding the point P is high-conductivity one, the temperature T_P prevails up to the interface e and the temperature drop takes place actually over the distance $(\delta x)_{e+}$

+ Oluştur

The combination of Eqs (3.13) and (3.3) leads to the following formula for discretization equation coefficient:

$$a_E = \left[\frac{(\delta x)_{e-}}{k_P} + \frac{(\delta x)_{e+}}{k_E}\right]^{-1} \tag{3.14}$$

Limiting cases:

1. $k_E \rightarrow 0$ - a material surrounding the control volume E is a perfect insulator

From Eq. (3.13)

$$k_e \rightarrow 0$$

This implies that the heat flux at the interface of an insulator becomes zero. The arithmetic-mean formulation would have given a nonzero flux in such a situation

Nonlinearity of the differential equation - the discretization equation coefficients are related to the dependent variable

There are two reasons for non-linearity in heat conduction problems:

- thermal conductivity dependence on temperature level
- presence of a source term which is a nonlinear function of the temperature level

To handle non-linear equations –use iterative techniques

1. Start with a guess or estimate for the values of dependent variables at all grid points.

2. From these guessed values calculate tentative values of the coefficients in discretization equation

3. Keeping the values of the discretization equation coefficients constant solve this nominally linear set of algebraic equations to get new values of the dependent variable at all grid points

4. With this better predicted set of values of the dependent

Linear interpolation:

$$k_e = f_e k_P + (1 - f_e) k_E \tag{3.7}$$

where interpolation factor is defined in terms of the distances from Fig.3.1:

$$f_e \equiv \frac{(\delta x)_{e+}}{(\delta x)_e} \tag{3.8}$$

Thermal conductivity at the interface between two control volumes

Thermal conductivity can vary because of the non-homogeneity of the material (e.g. composite slab) or due to temperature variations

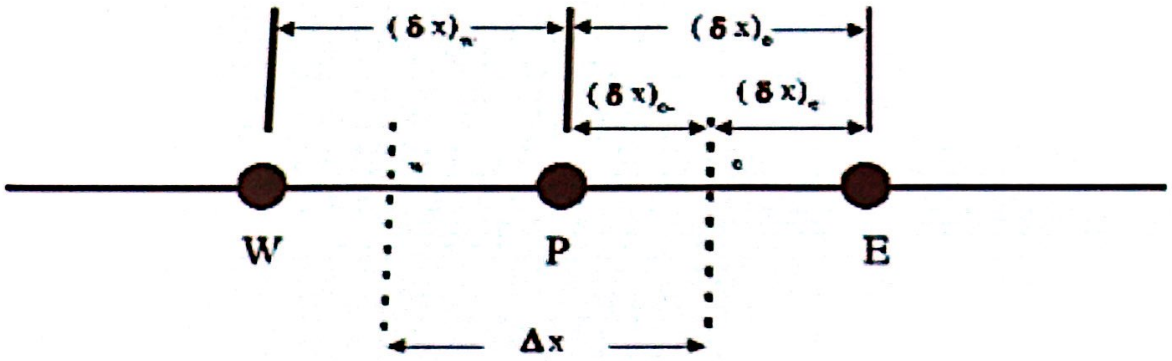


Fig.3.1. Non-uniform grid spacing

Linear interpolation:

$$k_e = f_e k_P + (1 - f_e) k_E \tag{3.7}$$

where interpolation factor is defined in terms of the distances from Fig.3.1:

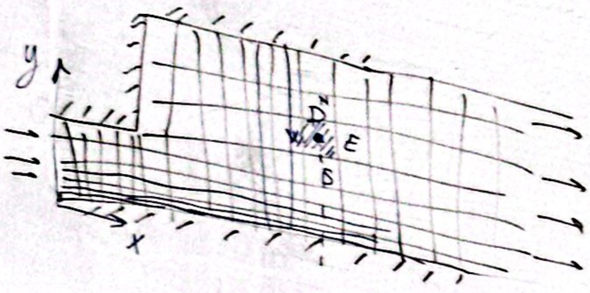
$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=f(x,y,\phi)$$

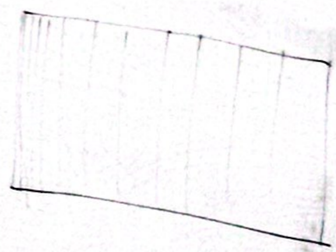
$$\cancel{\frac{\partial T}{\partial t}}+\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+S=0$$

0

$T=T(x,y,t)$

$T=T(x)$





$T(i,j)$

$\phi(i,j)$

$u(i,j)$

The grid spacing

Non-uniform grid spacing in a computational domain corresponding to a dependent variable changes enables to deploy computer power effectively

The grid spacing can be designed by the use of the following rules:

- Some qualitative expectations about the solution imply from analytical or experimental results
- Some preliminary results can be obtained using uniform coarse grid

There are no universal rules allowing to establish the best grid spacing. The proper grid spacing must be established individually to each computational problem

The governing differential equation:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+S=0 \tag{3.1}$$

discretization equation:

$$a_P T_P = a_E T_E + a_W T_W + b \tag{3.2}$$

where:

$$a_E = \frac{k_e}{(\delta x)_e} \tag{3.3}$$

$$a_W = \frac{k_w}{(\delta x)_w} \tag{3.4}$$

$$a_P = a_E + a_W \tag{3.5}$$

$$b = \bar{S}\Delta x \tag{3.6}$$

Thermal conductivity can vary because of the non-homogeneity of the material (e.g. composite slab) or due to temperature variations

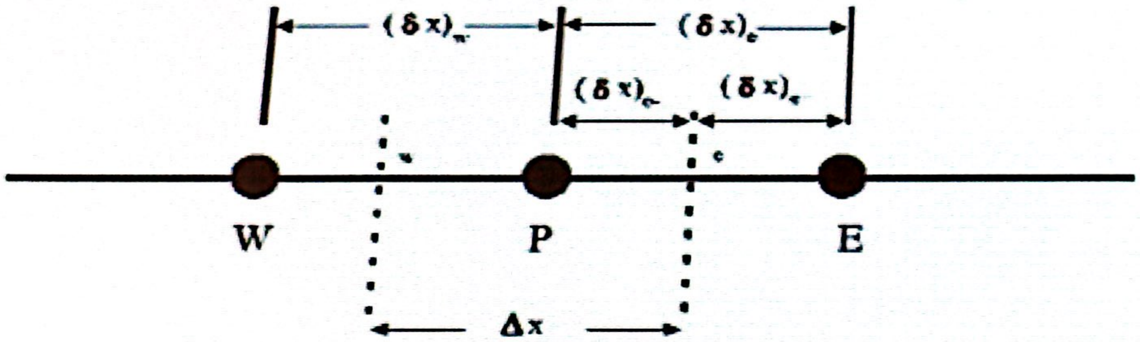


Fig.3.1. Non-uniform grid spacing

Linear interpolation:

$$k_e = f_e k_P + (1 - f_e) k_E \qquad (3.7)$$

where interpolation factor is defined in terms of the distances from Fig.3.1:

$$f_e \equiv \frac{(\delta x)_{e+}}{(\delta x)_e} \qquad (3.8)$$