22.11.2023

Deadline: 23.11.2023/13:30

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T.C. MARMARA UNIVERSITY INSTITUTE OF PURE AND APPLIED SCIENCE, DEPARTMENT OF MECHANICAL ENGINEERING, ADVANCED MATHEMATIC COURSE MIDTERM EXAM

1-) In a room with a volume of 20000 m^3, 600 m^3 of fresh clean air is given per minute and exposed at the same rate. a) If y(0)=0 initial condition, find the expression that gives the amount of fresh air y(t) at any time. b) Find the time in minutes after which 90 % of the air in the room will be clean air.

2-) A small copper ball whose temperature is Ti = 90 °C is put into ice water (0 °C) at time t = 0. It was observed that after t = 1 minute, the temperature of the ball dropped to 40 °C. Using Newton's law of cooling, find the temperature of the ball at t=5 minutes.

3-) The water temperature in a tank in the room is 200 °C at 1:00 o'clock and the ambient temperature is 80 °C. At 1:30, the temperature of the water is 160 °C. a) What is the temperature of the water at 2:00 and b) when will the temperature be 100 °C?

4-) A person with a mass of 120 kg reaches a constant speed of 200 km/h after jumping from the plane. If he opens his parachute as soon as he reaches a constant speed, how long does it take for him to reach 10% of the speed v = 2.5 m/s in order to land safely on the ground?

5-) Consider the homogenous linear system, and solve it using matrix method.

$$x_1' = 7x_1 - x_2 + 6x_3 + \cos 2t$$
$$x_2' = -10x_1 + 4x_2 - 12x_3$$
$$x_3' = -2x_1 + x_2 - x_3$$

6-) Find series solution of the differential equation

$$6x^2y'' + 7xy' - \left(1 + x^2\right)y = 0 \; about \; x_0 = 0$$